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Tsukamoto, Yasuo
Ishikawa, Yuichi
Terashima, Masahiro

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Finite Element Analysis for the Stress Distribution in Atherosclerotic Vessels by Using Personal Computer

Yasuo Tsukamoto[1], Yuichi Ishikawa[1], and Masahiro Terashima[2]

One of the hypotheses for the mechanism of plaque rupture in atherosclerotic vessels is the circumferential stress within the plaque. The distribution of circumferential stress in vessels can be obtained by using a finite element method. This method is based on dividing a complex structure into a number of but finite small polygons (i.e. elements) composed of several nodes and reconstructing simultaneous displacement equations for all nodes of elements with boundary and loading conditions. To solve these equations and to obtain the stress and strain distribution, all coefficients of the simultaneous equations for the structure have been stored in the powerful and high-speed computers (e.g. supercomputer or workstation) with a large memory. We proposed a desktop system to solve the finite element model by using a personal computer. The skyline method was utilized in this study to save the memory and the processing time.

Key Words
Finite element method,
Stress,
Plaque,
Atherosclerosis.

Introduction

The circumferential stress within the plaque. has been remarked for the plaque rupture in atherosclerotic vessels[1, 2]. The displacement and stress distribution in the vessel can be analyzed by the finite element method (FEM)[3, 4], which is an engineering technique to estimate the mechanical deformation and the stress–strain distribution of a complex structure under prescribed boundary and loading conditions. This method is based on dividing a complex structure into a number of finite elements composed of several nodes and reconstructing simultaneous displacement equations for all nodes of elements. Within the small deformation,every component of the vessel can be approximated by linear elastic material coefficients[3, 4], and then the finite element model is represented by a set of simultaneous linear equations with all unknown displacements of nodes for the discretized structure.

Large computer memories are necessary to store the coefficients of simultaneous equations in executing the FEM program. A supercomputer and/or workstation may generally be utilized to solve these equations, but such computers are not so convenient to use. We constructed an improved system for solving the FEM on a personal computer, which employed the skyline method[5] to save memories and to cutdown the numerical processing time.

Faculty of Health Sciences[1] and First Department of Internal Medicine[2], Kobe University School of Medicine, Kobe

Methods

The coefficient matrix derived from the simultaneous linear equations is generally symmetric and diagonally dominant but sparse otherwise, and then has diagonal band ; i.e. all coefficients have zero components except for the components within diagonal bandwidth of the matrix. The skyline method is based on the idea that the memory can cut down for zero coefficients out of the band for every column ; the outline of the non–zero columns shows the skyline of the buildings.

We developed the improved two–dimensional orthotropic FEM program and installed on a personal computer VS 30 D (NEC). The orthotropic material has orthogonal symmetry of material properties within the two–dimensional cross section. The bandwidth completely depends on the order of node numbers for the discretized structure. We must assign the order of node number so that adjacent nodes have close numbers to concentrate the non–zero components near the diagonal elements and to eliminate the bandwidth.

Results and discussion

Bearing such attention of assigning the node order in mind, the proposed method was applicable to the model with 3,000 nodes, under 100 boundary nodes and 100 loading nodes. The processing time was within a few seconds.

The simple vessel model for a thick–walled cylinder represented in Fig. 1 was designed to test the accuracy compared with the theoretical result[6] as follows ;

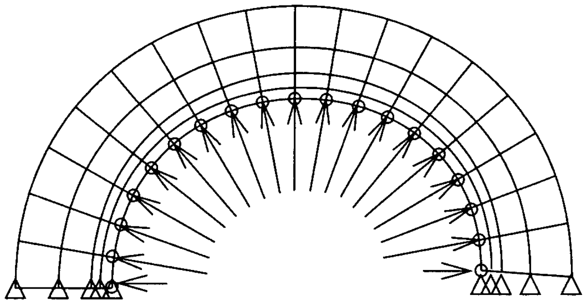

Fig. 1 Finite element mesh of a cylindrical vessel

$$u = \frac{(1+\nu)(1-2\nu)}{E}\left(\frac{a^2 p}{b^2 - a^2}\right) r + \frac{1+\nu}{E}\frac{a^2 b^2 p}{(b^2 - a^2) r} \quad (1)$$

where u is the displacement of cylinder at radius r under the inner blood pressure pa, and a and b are the inner and outer radius of the cylinder, respectively. E is the Young's modulus and ν is the Poisson's ratio. According to the Equation (1), the normal stress component in the radial direction is finally denoted by

$$\sigma_r = \frac{a^2 p}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right) \quad (2)$$

and the normal component in the circumferential direction is

$$\sigma_\theta = \frac{a^2 p}{b^2 - a^2}\left(\frac{b^2}{a^2} - 1\right) \quad (3)$$

The finite element analysis was restricted, as shown in Fig. 1, to the half part of the cylinder because of the symmetrical shape and mechanical conditions. According to the measurment of material properties for normal and atherosclerotic vessels[3, 4], We adopted E=1 $\times 10^6$ Pa, ν= 0.27, p=100 mmHg, a=2.5 mm and b= 3.35 mm, respectively. The result by the proposed FEM was shown in Fig. 2 by stress distribution and also shown in Fig. 3 by the displacement of the vessel together with the theoretical analysis by equation (1). The numerical result by the FEM showed to be markedly consistent with the theoretical one. These results indicated that the proposed tool should be appropriate to obtain the displacement and the stress field for the idealized vessel.

To improve the model, a cylindrical vessel surrounded by soft tissues was designed in Fig. 4 and the vessel with atherosclerosis was proposed in Fig. 5. The material properties used in this study were presented in Table 1, in which 1 means the radial direction and 2 designates the circumferential direction for the vessels. The radial stress in the concentric plaque was dominant in the cylindrical vessel shown in Fig. 4, while, as illustrated in Fig. 5, a part of circumferential stress found to be significant in the eccentric plaque. Employing the mechanical information, clinical discussion and interpretation must be facilitatedto explain the mechanism of plaque rupture.

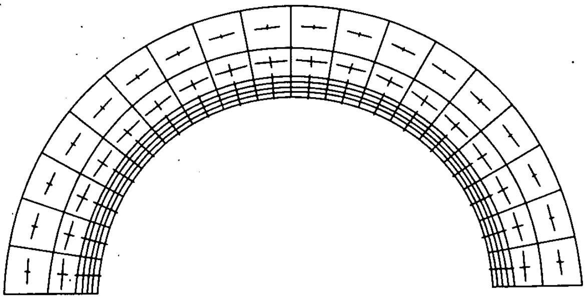

Fig. 2 Stress distribution of the cylindrical vessel

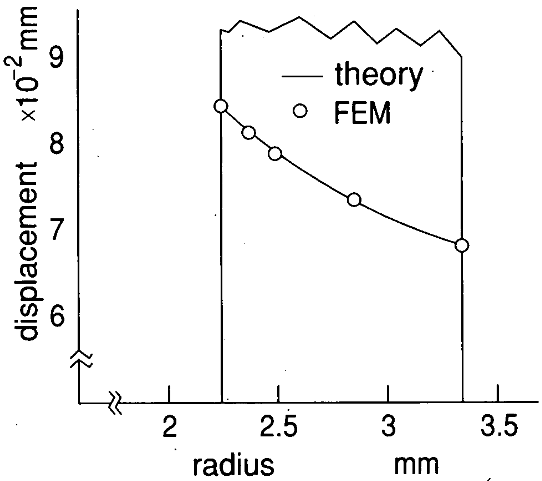

Fig. 3 Radial displacement by the theory and proposed FEM

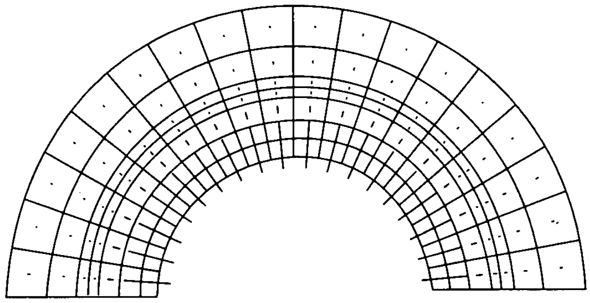

Fig. 4 Stress distribution of the vessel supported by soft tissue

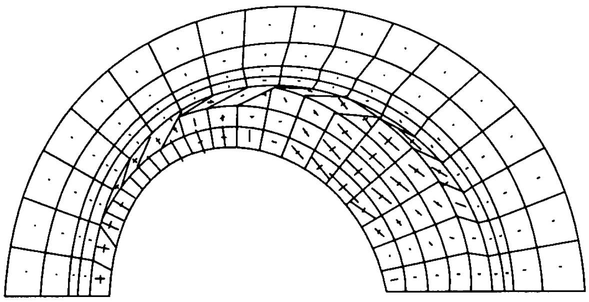

Fig. 5 Stress distribution of the vessel with plaque and soft tissue

1 : circumferential, 2 : radial

	plaque	artery	soft tissue
E_1 (kPa)	1,000	100	10
E_2 (kPa)	50	10	1
G_2 (kPa)	500	50	5
ν_1	0.27	0.27	0.27
ν_2	0.01	0.01	0.01

Table 1. Idealized orthotropic material properties for plaque, artery and soft tissue

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