Kobe University Repository : Kernel

PDF issue: 2025-09-05

Parameter Determination of a Tank Model for Probable Maximum Runoff Prediction

Kawatani, Takeshi
Saito, Masahiko
Asare-Boafo, Emmanuel

(Citation)
神戸大学都市安全研究センター研究報告,4:87-96

(Issue Date)
2000-03

(Resource Type)
departmental bulletin paper

(Version)
Version of Record

(JaLCDOI)
https://doi.org/10.24546/00231309

(URL)
https://hdl.handle.net/20.500.14094/00231309

神戸大学都市安全研究センター
研究報告，第４号，平成１２年３月

PARAMETER DETERMINATION OF A TANK MODEL FOR PROBABLE MAXIMUM RUNOFF PREDICTION

Takeshi Kawatani [1)]
Masahiko Saito [2)]
Emmanuel Asare-Boafo [3)]

Abstract: It is often realized in runoff analysis that model parameters identified from several observed runoff events in a certain watershed vary widely though each event has been well reproduced. Accordingly, when the runoff prediction with that model is required, it is hard to select proper numerical values of the model parameters.

This study aims to propound a procedure of determining the parameters of a tank model that can be used to predict not only the peak discharge but also the hydrograph neighboring the peak due to the heavy rainfall of low occurrence probability even when data for parameter identification was limited to comparatively small runoff.

The variation in the parameters is inferred to be due to variations in the runoff process and the contributing area when rainfall is not heavy enough to make the hydrological condition of runoff fully developed. Hence, multiple regression analysis was performed to examine the correlation of model parameters with rainfall characteristics and runoff characteristics which are expected to reflect the hydrological condition of each runoff event.

It is found that the runoff coefficient and the centroid of rainfall distribution before the occurrence of peak discharge are significant quantities to estimate the proper model parameters. In addition, the regression expressions provide the information of concentration time. Accordingly, it becomes possible to determine the rainfall distribution from the intensity-duration curve for a prescribed return period and to predict the runoff.

Keywords: runoff prediction, tank model, parameter identification, multiple regression

1. INTRODUCTION

A model for runoff prediction is normally desired to be efficient but simple enough for its parameters to be reliably identified through the runoff analysis. In the runoff analysis, once a runoff model is designed, the model parameters would be identified by optimally reproducing observed hydrographs with the expectation of consistent results coming out. However, the numerical values of the parameters identified from several runoff data often tend to vary so widely that the model cannot be used for runoff prediction. The variation in the parameters is inferred to be due to variations in the runoff generating process and the contributing area. When rainfall is not heavy, the hydraulic and hydrologic conditions of runoff are not fully developed but depend on the rainfall characteristics. Therefore, when the observation period is short, the lack of data measured under extreme conditions limits a thorough application of any derived model to runoff prediction. Nevertheless, it is envisaged that a satisfactory model for prediction is attainable using limited data.

The model parameters that have been identified from comparatively small runoff are expected to be closely correlated with the rainfall characteristics as well as the runoff characteristics. In this study, a tank model is employed and its parameters are identified by the reproduction of observed hydrographs. Then the multiple regression analysis is performed to examine the correlation of the model parameters with the factors that reflect the runoff conditions.

2. MODEL DESIGN

The tank model is a simple non-linear rainfall-runoff model composed of one or several tanks with one or more outlets on the side and bottom of each tank [1]. Outputs through the side outlets represent components of the total discharge due to the immediate or delayed response to rainfall. Flow through bottom outlets represents the portion of the water that infiltrates and does not positively contribute to surface flow. The model is such that the exact numbers of tanks and outlets as well as the positions of the side outlets are determined primarily by the desired output.

In this study, for the purpose of generating the direct surface runoff, especially the peak discharge, a single tank with one outlet on the side (the runoff outlet) and one on the bottom (the infiltration outlet) as shown in Figure 1 is considered to be the appropriate arrangement. In addition, a fewer number of parameters increases the uniqueness of any solution in the parameter identification and makes it easy to interpret the effect of each parameter on the generation of hydrograph. The height of the runoff outlet is assumed to be 5*mm* based mainly on observation and other preliminary investigations. Thus, the quantities that are to be identified are α (the runoff parameter) and β (the infiltration parameter) defined by the expressions

$$q_1(t) = \Delta\{S(t) + I(t) - h\} \quad (1)$$
$$q_2(t) = \Delta\{S(t) + I(t)\} \quad (2)$$
$$S(t+\Delta t) = S(t) + I(t) - q_1(t) - q_2(t) \quad (3)$$

where $q_1(t)$ and $q_2(t)$ are the surface runoff and the infiltration at time t, respectively, S(t) is the storage in depth at time t, I(t) is the rainfall intensity in depth at t, h is the height of the side outlet which is assumed to equal 5 *mm* in this study, and Δt denotes the time increment.

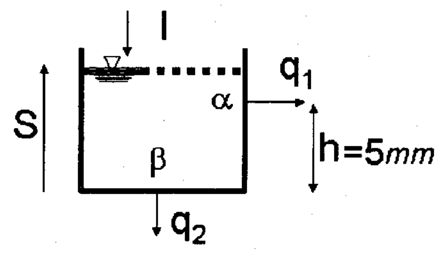


Figure 1 The tank model for the study

3. PARAMETERS IDENTIFICATION

(1) Watershed and available data

The study area is a 1700*ha* watershed of predominantly natural vegetation and hilly terrain located in the northwestern part of the city of Kobe, Japan. Figure 2 is a schematic representation of the area that is drained by two rivers - the Akashi and the Komi. There are three observational points for flow measurements. The first two are referred to as A and B. They are on the Komi with A being upstream of B meanwhile the C point is on the Akashi. Table 1 shows the area of each watershed and characteristics of the two rivers.

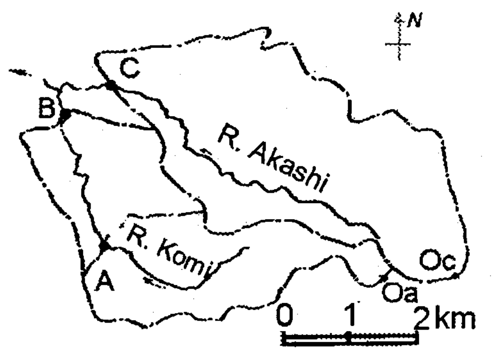


Figure 2 Watershed and observation points (A, B, C)

The points Oa and Oc in the figure are the extreme points of the Komi and the Akashi, respectively.

Table 1 Area and river characteristics

Watershed		A	B	(B-A)	C
Area (*ha*)		365	680	315	995
River		Komi	Komi	(Komi)	Akashi
Elevation (*m*)	Starting pt.	Oa; 280	Oa; 280	A; 155	Oc; 260
	End pt.	A; 155	B; 120	B; 120	C; 130
	Drop	125	160	35	130
Distance (*m*)		6000	8900	2900	8500
Av. Slope		0.021	0.018	0.012	0.015

The entire area has low human settlement which is not in excess of 15% in any of the three composite areas. The vegetation is made up of mainly Japanese pine and conifers. In the portion of watershed B that is not part of the watershed A, a sizeable portion of it has the vegetation cleared for development and has significant areas for rice fields which is feared to affect the actual state. The watershed C is also predominantly forested with some paddy fields.

Measurements of 10*min* rainfall and 5*min* runoff have been going on since 1989. The rain height of 0.5 *mm* is recorded as one pulse by an automated rain gage sited at a location near the point C. For the runoff measurements, water depths in the rivers are recorded continuously using water pressure sensors. Though tens of events have been recorded over the period, some are found to be unacceptable for the analysis because of equipment malfunction, human error or missing data.

(2) Procedure

The parameters of the tank model can be identified by the reproduction of observed hydrographs assuming that the basic watershed characteristics remain unchanged during the observation of events. In the runoff analysis, because all the three watersheds are fairly small in size, the travel time is expected to be relatively short. Thus thirty minutes is considered an appropriate time interval for evaluating and analyzing rainfall intensities and discharges.

The constrained simplex method [(2), (3)] is considered to be appropriate for the parameter identification because this optimization process does not get trapped in the presence of a minor optimum but is able to reach the global optimum [(4)]. The optimum point can be achieved by an iterative procedure in which the values of the objective function at each of the vertices of the simplex are compared and the vertex of the maximum value is reflected with respect to the center of other vertices to form a new simplex. The optimum parameters are the coordinates of the vertex with a minimum value of the objective function. The algorithm involves assigning randomly generated values to the initial coordinates of all the vertices of the simplex.

In this identification process, the coordinates of the k-th vertex are given by (α_k, β_k) where α_k and β_k are the runoff and infiltration parameters, respectively. The objective function F_k that measures the deviation of the computed discharge values from the observed is defined as

$$F_k = \frac{1}{N}\sum_{t=1}^{N}\frac{q_t^o}{q_{mx}^o}\left|q_t^o - q_t^c\right| \qquad (4)$$

where, q^o_t is the observed discharge during the t-th time interval, q^o_{mx} is the observed maximum discharge, q^c_t is the computed discharge during the t-th interval and N is the number of time intervals. To attain good matching especially around the peak, the objective function places much emphasis on the peak discharge employing (q^o_t/q^o_{mx}) as a weighting factor to the absolute error. The objective function is evaluated for discharges that are in excess of 30% of the peak discharge and minimized under the following constraints.

$$0.01 \leq \alpha \leq 0.9 \quad (5)$$
$$0.01 \leq \beta \leq 0.9 \quad (6)$$
$$0.01 \leq \alpha + \beta \leq 0.9 \quad (7)$$

(3) Results

For virtually all the cases analyzed there is satisfactory reproduction of observed hydrographs especially in the neighborhood of the peak. Six out of the eighty-three cases considered (watershed A – 90/05/07, 98/09/22; watershed B – 93/06/30; watershed C – 90/05/19, 90/09/19, 97/09/19) are shown in figure 3 as a confirmation that the selected model is sufficient in reproducing observed hydrographs especially in the region of the peak discharge.

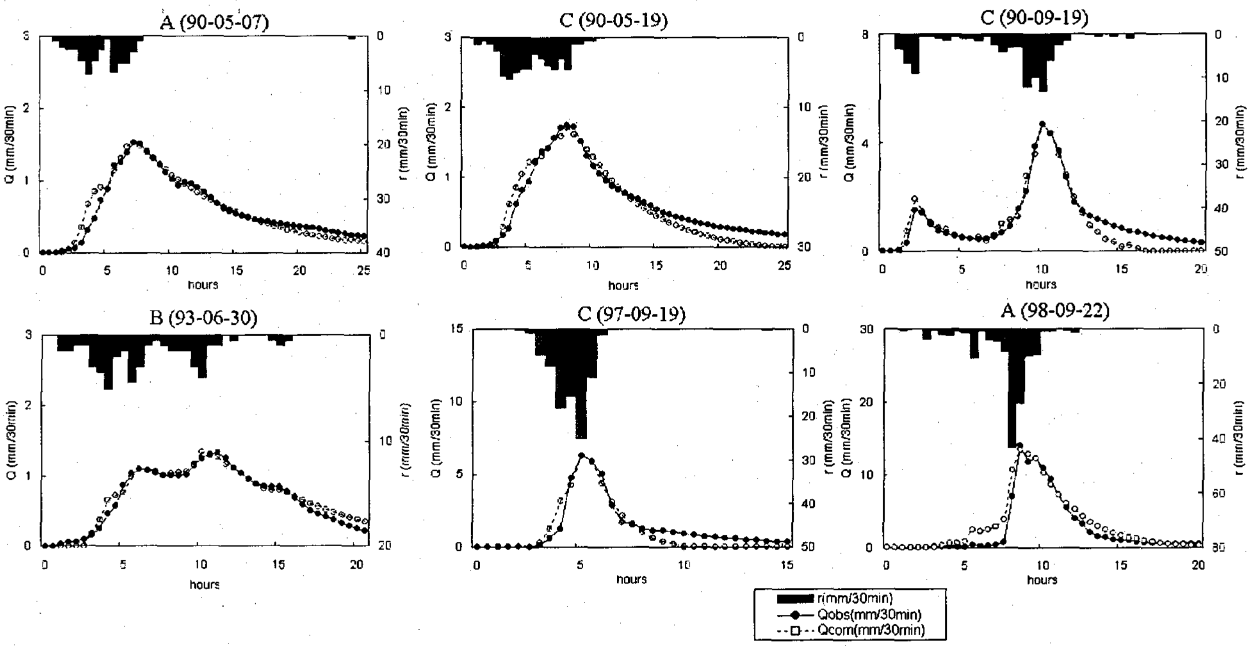


Figure 3 Sample of reproduced hydrographs

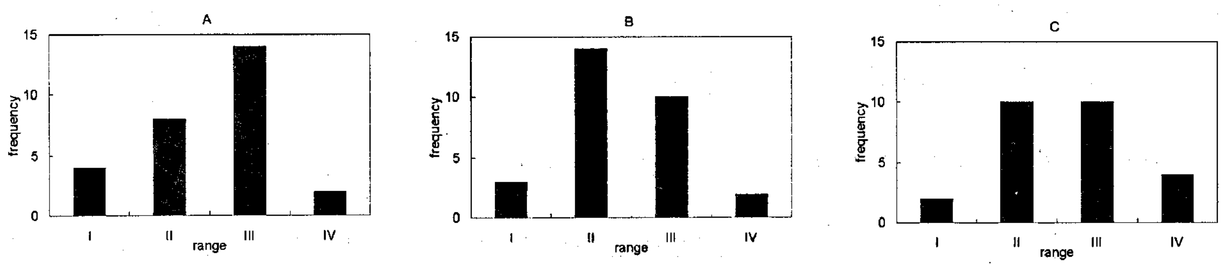
A complete list of the runoff and infiltration parameters (α, β) identified for all the events is in Table 2. It is notable from these results that although each hydrograph is satisfactorily reproduced the values of the model parameters especially α exhibit great dispersion. The values of the runoff parameter α range from 0.024 to 0.269 for the watershed A, from 0.033 to 0.211 for B and from 0.043 to 0.253 for C. Figure 4 shows the distribution of α in each of the three cases. Furthermore, in most of the cases where the value of α is greater than 0.1, α is several times larger than β. Therefore, as far as the tank model employed in this study is concerned, it is expected that the runoff parameter α is more directly related to the amount of discharge than the infiltration parameter β and the larger α is the greater the amount of peak discharge is.

Table 2 Model parameters identified by reproducing observed hydrographs.

Watershed A		
Date	α	β
890622	0.031	0.033
890713	0.034	0.021
890902	0.123	0.022
890914	0.051	0.037
900507	0.049	0.011
900616	0.119	0.001
900630	0.051	0.027
900919	0.107	0.029
901008	0.120	0.000
901104	0.093	0.000
901130	0.094	0.033
910407	0.128	0.013
910508	0.024	0.016
911001	0.072	0.152
930628	0.079	0.038
930710	0.087	0.062
930816	0.132	0.010
930817	0.181	0.218
930818	0.195	0.000
930914	0.160	0.154
930930	0.187	0.013
960626	0.200	0.000
961014	0.148	0.008
970710	0.088	0.119
970726	0.208	0.000
970916	0.161	0.143
980922	0.173	0.000
981018	0.269	0.000

Watershed B		
Date	α	β
890622	0.066	0.046
890623	0.178	0.000
890624	0.102	0.000
890713	0.077	0.038
890914	0.078	0.000
900507	0.033	0.000
900630	0.055	0.102
900919	0.156	0.009
910311	0.236	0.000
910508	0.033	0.011
911001	0.033	0.009
930630	0.062	0.009
930710	0.077	0.014
930816	0.118	0.045
930817	0.084	0.050
930930	0.108	0.037
950511	0.099	0.025
960626	0.092	0.010
960913	0.073	0.018
961014	0.087	0.015
970710	0.052	0.064
970713	0.141	0.001
970726	0.149	0.078
970916	0.140	0.221
980516	0.085	0.016
980711	0.080	0.017
980922	0.135	0.011
981016	0.112	0.000
981018	0.211	0.002

Watershed C		
Date	α	β
890622	0.084	0.079
890623	0.095	0.033
890713	0.124	0.058
890907	0.078	0.000
890914	0.072	0.021
900519	0.061	0.030
900630	0.055	0.064
900919	0.161	0.097
901008	0.221	0.017
910407	0.079	0.000
910508	0.043	0.009
911001	0.253	0.031
920819	0.210	0.078
930630	0.088	0.034
930702	0.198	0.000
930710	0.107	0.036
930816	0.161	0.044
930817	0.163	0.053
930914	0.122	0.000
960913	0.048	0.012
970710	0.074	0.043
970713	0.249	0.146
970726	0.116	0.090
970728	0.111	0.028
970916	0.132	0.140
981017	0.072	0.002



Range I : $\alpha \leq 0.05$
Range II : $0.05 < \alpha \leq 0.10$
Range III : $0.10 < \alpha \leq 0.20$
Range IV : $\alpha > 0.20$

Figure 4 Distribution of runoff parameter α

4. REGRESSION ANALYSIS

The values of α have direct influence on the amount of discharge generated by the tank model studied herein. However, because of the great dispersion in the values of α it is quite difficult to determine a proper value of α for the prediction of probable maximum discharge from the available results of the parameter identification.

It is inferred that when rainfall is not heavy neither hydraulic conditions of runoff process nor the contributing area is fully developed and therefore the identified value of α varies with every rainfall. Accordingly, the value of α is expected to have a certain correlation with some descriptors of the rainfall and runoff characteristics. Multiple regression analyses are performed to examine the correlation.

The descriptor used to characterize the runoff is the runoff coefficient ($F = Q_{tl}/R_{tl}$), where Q_{tl} and R_{tl} are total amount of discharge of the direct runoff and rainfall amount, respectively. On the other hand, the descriptors of the rainfall characteristics employed for the regression analyses are the average thirty-minute rainfall intensity (R_{av}), the maximum rainfall intensity (R_{mx}), the accumulated rainfall amount (R_{sm}), the centroid (R_g) and the kurtosis (R_{kr}) of the hyetograph. The descriptors characterizing the rainfall are evaluated from the portion of rainfall before the occurrence of peak discharge. In this evaluation, the time lag between hyetograph and the hydrograph that has been determined in the reproduction of the observed hydrograph is taken into account. To normalize the centroid R_g, it is defined as

$$R_g = \frac{\sum_{i=1}^{N} R_i \times L_i}{R_T \times L_T} \tag{8}$$

where R_T is the total amount of rainfall up to the time of peak flow, R_i is the rainfall amount during the i-th time interval, L_i is the elapsed time of the i-th interval after start of rain, L_T is the total duration between start of rain and occurrence of peak flow and N the number of time intervals.

(1) Multiple regression expressions obtained using all data

Using all the available data, the multiple regression expression consisting of up to four independent variables (i.e., descriptors) is determined for each watershed. The proportion of each independent variable is judged to ascertain its significance in the overall expression.

All the regression expressions turned out to have the following four descriptors, i.e., the runoff coefficient F, the total rainfall amount R_{sm}, the average rainfall intensity R_{av}, and the centroid R_g. The general expression is of the following form;

$$\alpha = a_1 + a_2 R_g + a_3 R_{av} + a_4 R_{sm} + a_5 F \tag{9}$$

The values for the coefficients a_i ($i = 1, \ldots, 5$) are given in Table 3.

Table 3 Coefficients in regression expression (9)

Watershed	Data used	a_1	a_2	a_3	a_4	a_5
A	28	−0.1624	0.2547	0.0132	−0.0013	0.1892
B	29	−0.0706	0.1228	0.0063	−0.0007	0.1456
C	26	−0.1382	0.2003	0.0128	−0.0009	0.1917

Comparisons between the runoff parameters obtained after reproduction (α) and those computed from the multiple regression expressions (α_{cl}) are shown in Figure 5. It is observed that the runoff parameters α and α_{cl} are well correlated. The correlation coefficient between α and α_{cl} is 0.798 for the watershed A, 0.779 for B and 0.738 for C. Most of the runoff parameters after the reproduction (α) lie in the range given by

$$\alpha < \alpha_{cl} + k \qquad (10)$$

where, k is 0.05 for both watersheds A and B, and 0.08 for C.

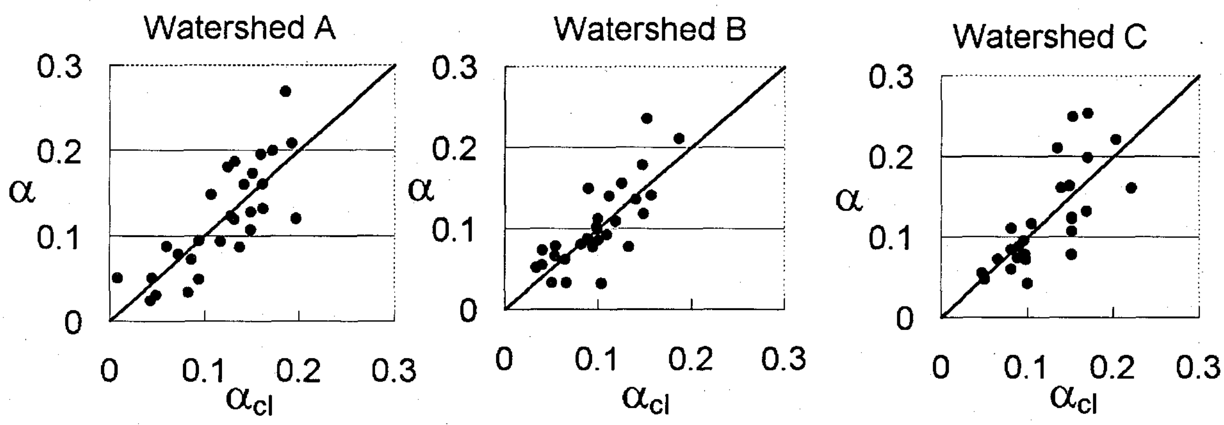


Figure 5 Comparison between runoff parameters obtained after reproduction (α) and those computed from the multiple regression expressions (α_{cl})

(2) Regression expressions obtained from data of weak rainfall

In general, the return period assigned to the rainfall for prediction of probable maximum discharge is long. On the other hand, rainfall events that are likely to occur during a relatively short period of observation are those with shorter return periods. Therefore, it is worth to examine the possibility of establishing a regression expression based only on low intensity and/or amount rainfall that is applicable for all likely events. Events selected for this purpose are those with the total amount of rainfall (R_t) less than 75*mm* and/or the maximum thirty-minute rainfall intensity (R_{mx}) less than 15*mm*.

The general regression expression of α is of the following form;

$$\alpha = a_1 + a_2 F + a_3 X \qquad (11)$$

where, the descriptor X is the total rainfall amount R_{sm} for the watersheds A and B, and the maximum thirty-minute rainfall intensity R_{mx} for C. The values of a_i (i = 1,2,3) are given in Table 4 with the descriptor X.

Table 4 Coefficients and descriptors in expression (11)

Watershed	Data used	a_1	a_2	a_3 X
A	16	0.0316	0.1892	$-0.0011\ R_{sm}$
B	18	0.0802	0.1427	$-0.0021\ R_{sm}$
C	16	-0.1708	0.2361	$0.0173\ R_{mx}$

Fig.6 shows the comparison between α and α_{cl}, the runoff parameter obtained after reproduction of observed hydrographs and that computed from the multiple regression expression (9). The runoff parameter α_{cl} is computed for all the events including those that are not used in the determination of the regression expression. In this figure, the points marked as ● correspond to the runoff events that were used in the regression analysis while those marked as □ are the events not used for the analysis. The correlation coefficients between α and α_{cl} for these two respective cases are shown in Table 5.

The excessive reduction in the correlation coefficients is observed when all the events are included to examine the correlation between α and α_{cl}. This indicates that the regression expression determined only from runoff events due to relatively weak rainfall is inapplicable to the prediction of the runoff with a long return period.

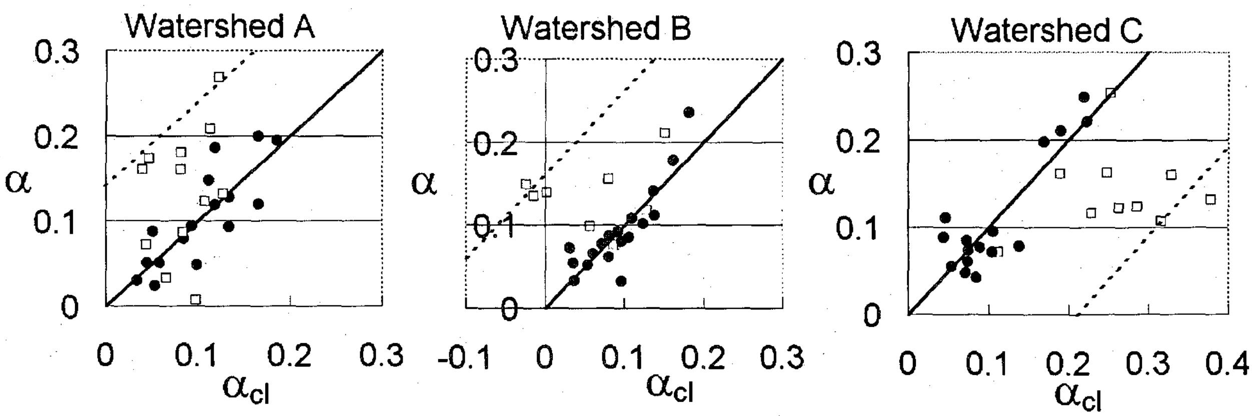


Figure 6 Comparison between the runoff parameters obtained after reproduction (α) and those computed from the multiple regression expressions (α_{cl})

Table 5 Correlation coefficients between α and α_{cl} in figure 6

	Watershed	A	B	C
Regression coefficient	Low rainfall events only	0.823	0.857	0.882
	All events	0.491	0.402	0.580

5. SELECTION OF MODEL PARAMETERS FOR RUNOFF PREDICTION

As mentioned above, the runoff parameter α is considered to be the key parameter that determines the amount of peak discharge and the hydrograph neighboring the peak. Thus the regression expression (9) is examined to select a proper value of α for predicting discharge due to rainfall of a long return period.

The hydraulic mechanism for runoff generation and the area contributing to surface runoff change with rainfall characteristics [5]. The variations in the values of α are considered to be attributable to the changes in these hydrologic conditions for surface runoff generation according to the corresponding rainfall characteristics. Hence it is expected that when the rainfall exceeds a certain threshold the hydrologic conditions are fully developed stage and thus the runoff parameter does not change with amount of rainfall beyond this stage. Considering the regression expression (9) in relation to the fully developed hydrologic conditions, two terms related to the amount of rainfall (i.e., a_3R_{av} and a_4R_{sm}) have to disappear for the value of α to remain virtually unchanged with rainfall. Thus

$$a_3R_{av} + a_4R_{sm} = 0 \qquad (12)$$

and the expression (9) is written as follows;

$$\alpha = a_1 + a_2R_g + a_5F \qquad (13)$$

Firstly, let us consider equation (12). As the idea behind the rational method suggests, if a rainfall of constant intensity continues indefinitely, the rate of runoff will increase until the time of concentration Tc when the entire watershed is contributing to flow. Hence, taking the definitions of R_{av}(*mm*/30*min*) and R_{sm}(*mm*) into account, the concentration time Tc can be estimated as follows;

$$Tc = n\,\Delta T = R_{sm}/R_{av} = -\,a_3/a_4 \qquad (14)$$

where ΔT is the unit time interval of this runoff analysis, i.e., 30 *min*. In the case of the watershed A, $a_3 = 0.0132$ and $a_4 = -\,0.0013$, and therefore Tc = 10 ΔT = 5 hours. Similarly, the concentration time Tc is 4.5 hours for the watershed B and 7 hours for C. The concentration time in the watershed B is shorter than that in the watershed A though the watershed is the upstream half of the watershed B. This reason is believed to be that when the hydraulic mechanism is fully developed the travel time is shorter in the watershed B because of the fact that a sizeable portion of the watershed B downstream of A is cleared of vegetation.

Secondly, let us examine the expression (13) to determine the value of α for predicting the probable maximum discharge. As an extreme situation, the runoff coefficient F is assumed to equal 1 in the expression (13). On the other hand, the centroid R_g reflects the fact that the rainfall distribution has large influence on a hydrograph [6]. It is well known that the rainfall distribution of a delayed type (i.e., backward concentrated type) generates larger peak discharge than the distribution of an intermediate type (i.e., centrally concentrated type) or an advanced type (i.e., forward concentrated type). Therefore, it seems reasonable to employ the value of R_g corresponding to the rainfall distribution of a delayed type that generally lies between 0.65 and 0.68 depending upon the intensity-duration relationship.

The following is an example of the application of the regression expression (13) to the determination of the probable maximum value of α when the intensity-duration relation is prescribed. The intensity-duration relationship for a 30-year return period in the study area is given as

$$i = \frac{536.5}{\sqrt{t} + 0.238} \qquad (15)$$

where, i designates the design rainfall intensity in *mm/hr* and t is the duration in minutes. Using this relationship, it is possible to design the rainfall distribution of a delayed type for the duration that is equal to the concentration time Tc [7]. For the watershed A, the value of R_g of the delayed–type distribution during the concentration time of 300*min* equals 0.66 and substituting this value into the expression (13) the value of α is 0.2. Similar computations yield 0.16 and 0.19 to the values of α for the watersheds B and C, respectively.

Figure 7 shows the hyetographs and the hydrograph with the probable maximum peak discharge due to the rainfall of a 30-year return period. In this generation of the hydrograph, because the value of α is estimated for F = 1, the input rainfall into the model should be the effective rainfall. Accordingly, the value of the infiltration parameter α was assumed to equal 0.001 expecting that the whole of the input rainfall became practically the surface discharge.

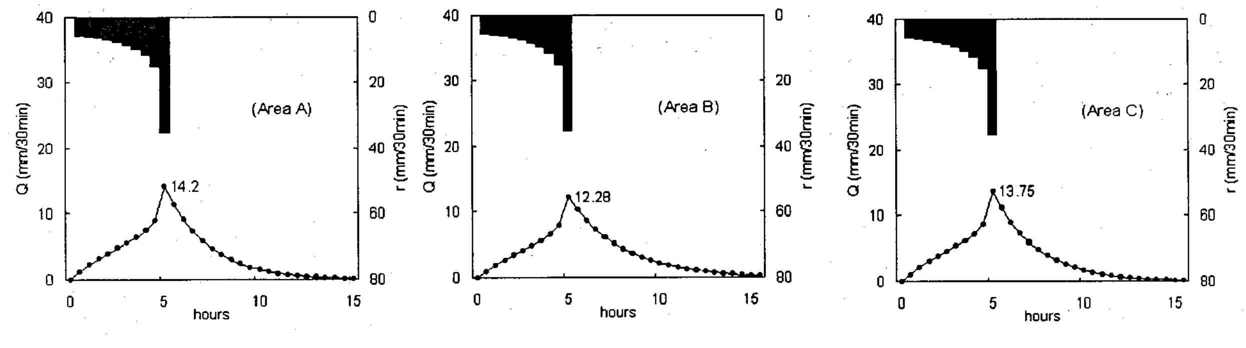


Figure 7 Hydrograph with the probable maximum peak discharge of 30-year return period

6. CONCLUSIONS

The study led to the following conclusions analysis on the prediction of maximum surface runoff in a small watershed using the tank model concept.

i) There is the confirmation of the efficacy of the single tank with one runoff outlet in peak discharge determination.

ii) The runoff parameter can be determined from the multiple regression expression obtained from observed rainfall-runoff events.

iii) Descriptors in the regression expression are identified as the centroid of hyetograph, the total rainfall amount and average rainfall before the occurrence of peak discharge and the runoff coefficient.

iv) The expression provides the estimation of the concentration time that determines the probable maximum rainfall distribution from the intensity-duration curve.

v) It was also confirmed that rainfall of a backward concentrated distribution produces discharge with the highest peak.

vi) It is worth to note that the rainfall events employed in the determination of the runoff parameter were those in excess of a certain amount of precipitation.

vii) The outcome of this analysis provides an approach that is applicable to the runoff prediction for other comparable watersheds where appreciable long period of observation is not feasible.

References

1) Sugawara S., *Runoff Analysis Methods*, Kyoritsu Shuppan Press, pp. 25-69, 1978 (In Japanese).
2) Spendley W., Hext, G. and Himsworth, F., *Sequential Application of Simplex Designs in Optimization and Evolutionary Operation*, Technometrics, Vol. 4, No. 4, pp.441-460, 1962
3) Rao S., *OPTIMIZATION Theory and Application*, Halsted Press, John Wiley & Sons Inc., pp.292-301, 1984
4) Gan, Y. and Biftu, G., *Automatic calibration of conceptual rainfall-runoff models: Optimization algorithms, catchment conditions and model structure*, Water Resources Research, Vol. 32, No.12, pp. 3513-3524, 1996
5) Beven, K. and Wood, E., *Catchment Geomorphology and the dynamics of runoff contributing areas*, J. Hydrology, pp.139 –158, 1993
6) Kawatani, T., Saito, M. and Asare-Boafo, E., *Identification of a Short-Term Storm Drainage Prediction Model in a Small Basin,* Hydrologic Modeling, Proc. of Int'l Conf. WEESHE, pp. 166-177, 1999
7) Japan River Authority, *Design manual for Flood Control Reservoirs*', pp. 65-69, 1987 (In Japanese)

Authors: 1) Takeshi Kawatani, Research Center for Urban Safety and Security, Kobe University, Professor, 2) Masahiko Saito, Research Center for Urban Safety and Security, Kobe University, Research Associate, 3) Emmanuel Asare-Boafo, Graduate School of Science and Technology, Kobe University, Graduate Student