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(Citation)
神戸大学都市安全研究センター研究報告,11:35-42

(Issue Date)
2007-03

(Resource Type)
departmental bulletin paper

(Version)
Version of Record

(JaLCDOI)
https://doi.org/10.24546/00518508

(URL)
https://hdl.handle.net/20.500.14094/00518508

神戸大学都市安全研究センター
研究報告，第11号，平成19年３月

ON THE MECHANICAL PROPERTIES OF A DUAL TUNNEL SUBJECTED TO A VERTICAL DISTURBANCE

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Abstract: This contribution addresses the mechanical characteristics of dual tunnels that are impinged upon by body waves generated due to nearby earthquakes or blasting. A two-dimensional model analysis shows that not only the distance between each tunnel but also the overburden can control the dynamic behavior of this type of underground structures. It is suggested that the conventional assessment criteria for the static stability of dual tunnels are not valid for the dynamic stability related to the incidence of vertical oscillations, if the incident waves are in the high frequency range.

Key words: Wave-induced damage, dynamic stress concentration, dual tunnel, overburden

1. INTRODUCTION

The 1995 Hyogo-ken Nanbu (Kobe) and the 2004 Niigata-ken Chuetsu, Japan earthquakes both caused similar (and previously unrecognized) damage patterns in the tunnels in the mountainous and urban regions, which were possibly induced by the direct interaction of longitudinal (*P*-) body waves with the underground structures[1-2)]: In the mountainous region of Greater Kobe, in the central section of the Bantaki Tunnel where it passes through an originally fractured zone, we observed exfoliation of the lining concrete and buckling of the reinforcing steel on the sidewall, as well as detachment of the subgrade from the invert (about 10cm) (Figure 1)[2-3)]. The facts that (1) the damage occurred in the central section of the 1,766m-long NATM tunnel; and (2) we found no permanent deformation (of the invert, etc.)[2-4)] suggest that the failure was caused not by fault displacement or instability near the portal, but by the dynamic interaction of the seismic waves with the tunnel; The previous investigations into the effect of seismic vibrations on underground structures indicate that only vertical *P*-waves of relatively high frequencies can induce dynamic compression on the sidewall, impart large acceleration to the subgarde, and consequently generate the damage patterns such as observed in the Bantaki Tunnel[2-3)]. Since the number of dual tunnels – typically two tunnels running in parallel – is increasing, especially in urban areas,. it may be important at this moment to investigate the effect of such body waves on the mechanical behavior of dual tunnels[5].

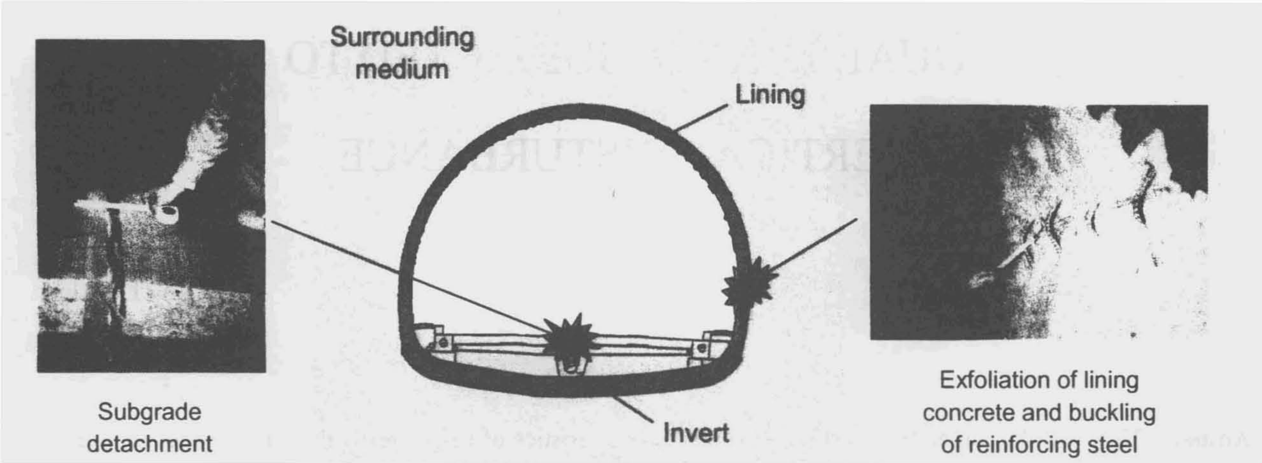


Figure 1 The Damage to the Bantaki Tunnel caused by the 1995 Hyogo-ken Nanbu (Kobe), Japan, earthquake.

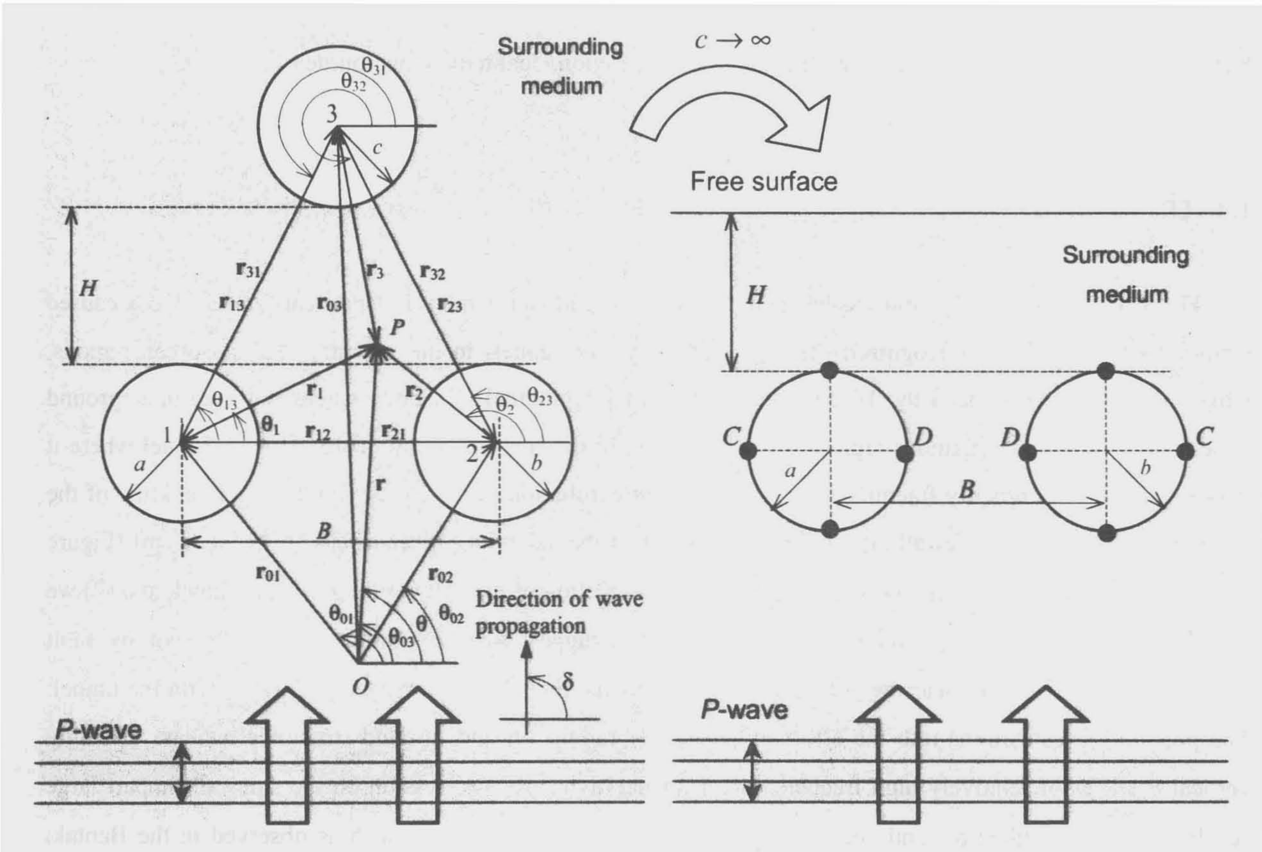


Figure 2 Geometry of the problem.

2. ANALYTICAL MODEL

Consider a two-dimensional model shown in Figure 2. Assume three circular tunnels, under plane strain conditions, are located in an infinitely extending linear elastic body that is free from initial stresses. A plane harmonic P-wave impinges on and interacts with the tunnels (Figure 2 left). The coordinates (r, θ), (r_1, θ_1), (r_2, θ_2) and (r_3, θ_3) are set as shown in Figure 2. The positions of point P and the centers of the tunnels can be expressed respectively as

$$
\begin{aligned}
&\mathbf{r} = \mathbf{r}_{01} + \mathbf{r}_1 = \mathbf{r}_{02} + \mathbf{r}_2 = \mathbf{r}_{03} + \mathbf{r}_3,\\
&\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2 = \mathbf{r}_{02} - \mathbf{r}_{01} = -\mathbf{r}_{21},\\
&\mathbf{r}_{13} = \mathbf{r}_1 - \mathbf{r}_3 = \mathbf{r}_{03} - \mathbf{r}_{01} = -\mathbf{r}_{31},\\
&\mathbf{r}_{23} = \mathbf{r}_2 - \mathbf{r}_3 = \mathbf{r}_{03} - \mathbf{r}_{02} = -\mathbf{r}_{32}.
\end{aligned}
\tag{1}
$$

The stresses and displacements in the medium can be analytically obtained by using the scalar and vector displacement potentials[3,5-11], and the effect of the free (flat) surface can be evaluated quantitatively from the distributions of the stresses and displacements for the limiting case where the radius of Tunnel 3 is infinite ($c \to \infty$; see Figure 2 right). In the following text, the analytical procedure will be described. Then, the results obtained by the analysis will be given and their physical meanings will be discussed. In the following text, ${}^{S}(\ )^{m}$ corresponds to Tunnel S ($S = 1, 2, 3$) and the number of reflections m.

(1) Incident wave

Using the scalar and vector displacement potentials, ϕ_i and $\boldsymbol{\psi}_i$, the incident plane harmonic P-wave propagating in the surrounding medium is given by

$$
\phi_i = \phi_0 e^{-i\omega t} e^{i\alpha r\cos(\theta-\delta)}, \quad \boldsymbol{\psi}_i = \mathbf{0}, \tag{2}
$$

where ϕ_0 is a constant related to the amplitude of the wave, t is time, ω is the angular frequency of the harmonic wave, and the dilatational wave number $\alpha = \omega/c_P$ with c_P being the P-wave velocity in the surrounding medium. By applying Jacobi's expansion theorem, one can express $\phi_i = {}^{1}\phi_i$ (incident wave for Tunnel 1) in polar coordinates (r_1, θ_1) as

$$
{}^{1}\phi_i = \phi_0 e^{-i\omega t} e^{i\alpha r_{01}\cos(\theta_{01}-\delta)} \sum_{n=-\infty}^{\infty} J_n(\alpha r_1) e^{in(\theta_1-\delta+\pi/2)}, \tag{3}
$$

where $J_n(\ )$ denotes the Bessel function of the first kind of order n. Similarly, ${}^{2}\phi_i$ and ${}^{3}\phi_i$ are respectively given by

$$
\begin{aligned}
&{}^{2}\phi_i = \phi_0 e^{-i\omega t} e^{i\alpha r_{02}\cos(\theta_{02}-\delta)} \sum_{n=-\infty}^{\infty} J_n(\alpha r_2) e^{in(\theta_2-\delta+\pi/2)},\\
&{}^{3}\phi_i = \phi_0 e^{-i\omega t} e^{i\alpha r_{03}\cos(\theta_{03}-\delta)} \sum_{n=-\infty}^{\infty} J_n(\alpha r_3) e^{in(\theta_3-\delta+\pi/2)}.
\end{aligned}
\tag{4}
$$

(2) Reflected waves

When the incident P-wave ${}^1\phi_i$ (${}^2\phi_i$, ${}^3\phi_i$) impinges upon Tunnel 1 (2, 3), it is reflected, refracted and diffracted, and both P- and shear (S-) waves are generated. In the surrounding medium, reflected waves ${}^1\phi_r^1$, ${}^1\psi_r^1$ (${}^2\phi_r^1$, ${}^2\psi_r^1$; ${}^3\phi_r^1$, ${}^3\psi_r^1$) are generated around Tunnel 1 (2, 3), respectively. The reflected waves ${}^1\phi_r^1$, ${}^1\psi_r^1$ (${}^2\phi_r^1$, ${}^2\psi_r^1$; ${}^3\phi_r^1$, ${}^3\psi_r^1$) from Tunnel 1 (2, 3) impinges on and interacts with Tunnels 2 and 3 (1 and 3; 1 and 2) as the incident waves ${}^2\phi_i^1$, ${}^2\psi_i^1$ and ${}^3\phi_i^1$, ${}^3\psi_i^1$ (${}^1\phi_i^1$, ${}^1\psi_i^1$ and ${}^3\phi_i^1$, ${}^3\psi_i^1$; ${}^1\phi_i^1$, ${}^1\psi_i^1$ and ${}^2\phi_i^1$, ${}^2\psi_i^1$), respectively. As a result, reflected waves ${}^1\phi_r^2$, ${}^1\psi_r^2$ (${}^2\phi_r^2$, ${}^2\psi_r^2$; ${}^3\phi_r^2$, ${}^3\psi_r^2$) are generated around Tunnel 1 (2, 3). This reflection process is repeated infinitely. The reflected waves around Tunnel S (S = 1, 2, 3) generated by the m-th reflection are expressed as

$$ {}^S\phi_r^m = e^{-i\omega t}\sum_{n=-\infty}^{\infty} {}^SA_n^m H_n^{(1)}(\alpha r_S) e^{in(\theta_S-\delta+\pi/2)}, \qquad {}^S\psi_r^m = e^{-i\omega t}\sum_{n=-\infty}^{\infty} {}^SB_n^m H_n^{(1)}(\beta r_S) e^{in(\theta_S-\delta+\varepsilon_n)}, \tag{5} $$

where ${}^SA_n^m$ and ${}^SB_n^m$ are constants to be determined for each m from the boundary conditions, $H_n^{(1)}(\ )$ is the Hankel function of the first kind of order n, the distortional wave number $\beta = \omega/c_S$ with c_S being the S-wave velocity in the surrounding medium, and $\varepsilon_n = \pi/(2n)$ if n is a positive even number and $\varepsilon_n = 0$ for the other integers.

The potentials associated with the total reflected waves are obtained by adding each reflected wave and they are given by

$$ \phi_r = e^{-i\omega t}\sum_{S=1}^{3}\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty} {}^SA_n^m H_n^{(1)}(\alpha r_S) e^{in(\theta_S-\delta+\pi/2)}, $$

$$ \psi_r = e^{-i\omega t}\sum_{S=1}^{3}\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty} {}^SB_n^m H_n^{(1)}(\beta r_S) e^{in(\theta_S-\delta+\varepsilon_n)}, \tag{6} $$

and finally, the full wave field can be expressed using the following two potentials:

$$ \phi = \phi_i + \phi_r = \phi_0 e^{-i\omega t} e^{i\alpha r_{0S}\cos(\theta_{0S}-\delta)} \sum_{n=-\infty}^{\infty} J_n(\alpha r_S) e^{in(\theta_S-\delta+\pi/2)} + e^{-i\omega t}\sum_{S=1}^{3}\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty} {}^SA_n^m H_n^{(1)}(\alpha r_S) e^{in(\theta_S-\delta+\pi/2)}, $$

$$ \psi = \psi_r = e^{-i\omega t}\sum_{S=1}^{3}\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty} {}^SB_n^m H_n^{(1)}(\beta r_S) e^{in(\theta_S-\delta+\varepsilon_n)}. \tag{7} $$

The displacements and stresses are given by

$$ u_r = \frac{\partial\phi}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}, $$

$$ u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} - \frac{\partial\psi}{\partial r}, $$

$$ \sigma_r = (\lambda+2\mu)\frac{\partial u_r}{\partial r} + \lambda\left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right), $$

$$\sigma_\theta = (\lambda + 2\mu)\left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right) + \lambda \frac{\partial u_r}{\partial r},$$

$$\tau_{r\theta} = \mu\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right). \tag{8}$$

Here, λ and μ are Lamé's constants.

(3) Boundary conditions

At the traction-free surface of each Tunnel S, the radial and shearing stresses vanish:

$$(\sigma_r)_{r_S = a_S} = 0, \quad (\tau_{r\theta})_{r_S = a_S} = 0, \quad (S = 1, 2, 3;\, m = 1, 2, \ldots, \infty) \tag{9}$$

where $a_1 = a$, $a_2 = b$ and $a_3 = c$. From equations (7)-(9), one obtains the coefficients for $m = 1$ first:

$$\begin{aligned} {}^S A_n^1 &= \phi_0 e^{i\alpha r_{os} \cos(\theta_{os} - \delta)}\, {}^S A_n, \\ {}^S B_n^1 &= \phi_0 e^{i\alpha r_{os} \cos(\theta_{os} - \delta)}\, {}^S B_n. \end{aligned} \tag{10}$$

Here,

$$\begin{aligned} {}^S A_n &= (-D_{na}\overline{K}_{na} + E_{na}\overline{H}_{na})/\Delta, \\ {}^S B_n &= e^{in(\pi/2 - \varepsilon_n)}(D_{na}\varepsilon_{na} - E_{na}\overline{D}_{na})/\Delta, \\ \Delta &= \overline{D}_{na}\overline{K}_{na} - \varepsilon_{na}\overline{H}_{na}. \end{aligned} \tag{11}$$

Then, by applying Neumann's addition theorem on Bessel functions[12)]

$$H_{n'}^{(1)}(\alpha r_{S'}) e^{in'\theta_{S'}} = \sum_n H_{n-n'}^{(1)}(\alpha r_{SS'}) J_n(\alpha r_S) e^{-i(n-n')\theta_{SS'}} e^{in\theta_S}, \tag{12}$$

the coefficients for $m \geq 2$ can be obtained and they are given by

$$\begin{aligned} {}^S A_n^m = {} & [-\{\sum_{S \neq S'}\sum_{n'}({}^{S'}A_{n'}^{m-1} e^{in'(-\delta+\pi/2)} H_{n-n'}^{(1)}(\alpha r_{SS'}) e^{-i(n-n')\theta_{SS'}} D_{na} + {}^{S'}B_{n'}^{m-1} e^{in'(-\delta+\varepsilon_{n'})} H_{n-n'}^{(1)}(\beta r_{SS'}) e^{-i(n-n')\theta_{SS'}} iH_{na})\}\overline{K}_{na} \\ & + \{\sum_{S \neq S'}\sum_{n'}({}^{S'}A_{n'}^{m-1} e^{in'(-\delta+\pi/2)} H_{n-n'}^{(1)}(\alpha r_{SS'}) e^{-i(n-n')\theta_{SS'}} E_{na} + {}^{S'}B_{n'}^{m-1} e^{in'(-\delta+\varepsilon_{n'})} H_{n-n'}^{(1)}(\beta r_{SS'}) e^{-i(n-n')\theta_{SS'}} iK_{na})\}\overline{H}_{na}] \\ & \cdot e^{-in(-\delta+\pi/2)}/\Delta, \\ {}^S B_n^m = {} & [\{\sum_{S \neq S'}\sum_{n'}({}^{S'}A_{n'}^{m-1} e^{in'(-\delta+\pi/2)} H_{n-n'}^{(1)}(\alpha r_{SS'}) e^{-i(n-n')\theta_{SS'}} D_{na} + {}^{S'}B_{n'}^{m-1} e^{in'(-\delta+\varepsilon_{n'})} H_{n-n'}^{(1)}(\beta r_{SS'}) e^{-i(n-n')\theta_{SS'}} iH_{na})\}\varepsilon_{na} \\ & - \{\sum_{S \neq S'}\sum_{n'}({}^{S'}A_{n'}^{m-1} e^{in'(-\delta+\pi/2)} H_{n-n'}^{(1)}(\alpha r_{SS'}) e^{-i(n-n')\theta_{SS'}} E_{na} - {}^{S'}B_{n'}^{m-1} e^{in'(-\delta+\varepsilon_{n'})} H_{n-n'}^{(1)}(\beta r_{SS'}) e^{-i(n-n')\theta_{SS'}} iK_{na})\}\overline{D}_{na}] \\ & \cdot e^{-in(-\delta+\varepsilon_n)}/\Delta, \end{aligned} \tag{13}$$

where

$$D_{nr} = (n^2 + n - 0.5\beta^2 r_S^2) J_n(\alpha r_S) - \alpha r_S J_{n-1}(\alpha r_S),$$

$$
\begin{aligned}
\overline{D}_{nr} &= (n^2 + n - 0.5\beta^2 r_S^2) H_n^{(1)}(\alpha r_S) - \alpha r_S H_{n-1}^{(1)}(\alpha r_S), \\
E_{nr} &= n(n+1) J_n(\alpha r_S) - n\alpha r_S J_{n-1}(\alpha r_S), \\
\varepsilon_{nr} &= n(n+1) H_n^{(1)}(\alpha r_S) - n\alpha r_S H_{n-1}^{(1)}(\alpha r_S), \\
H_{nr} &= -n(n+1) J_n(\beta r_S) + n\beta r_S J_{n-1}(\beta r_S), \\
\overline{H}_{nr} &= -n(n+1) H_n^{(1)}(\beta r_S) + n\beta r_S H_{n-1}^{(1)}(\beta r_S), \\
K_{nr} &= -(n^2 + n - 0.5\beta^2 r_S^2) J_n(\beta r_S) + \beta r_S J_{n-1}(\beta r_S), \\
\overline{K}_{nr} &= -(n^2 + n - 0.5\beta^2 r_S^2) H_n^{(1)}(\beta r_S) + \beta r_S H_{n-1}^{(1)}(\beta r_S),
\end{aligned} \tag{14}
$$

and D_{na}, $\overline{D}_{na}$, E_{na}, ε_{na}, H_{na}, $\overline{H}_{na}$, K_{na}, $\overline{K}_{na}$ are obtained using equation (14) with $r_S = a_S$. The circumferential stress $\sigma_{\theta\theta}$ at an arbitrary point in the surrounding medium is expressed as

$$
\begin{aligned}
\sigma_\theta = 2\mu r_S^{-2}\Big[&\phi_0 e^{i\alpha r_{0S}\cos(\theta_{0S}-\delta)} \sum_{n=-\infty}^{\infty} F_{nr} e^{in(\theta_S-\delta+\pi/2)} + \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty} {}^{S}A_n^m \overline{F}_{nr} e^{in(\theta_S-\delta+\pi/2)} + \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty} {}^{S}B_n^m (-i\overline{H}_{nr}) e^{in(\theta_S-\delta+\varepsilon_n)} \\
&+ \sum_{m=2}^{\infty}\sum_{S\neq S'}\sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty} {}^{S'}A_{n'}^{m-1} e^{in'(-\delta+\pi/2)} H_{n-n'}^{(1)}(\alpha r_{SS'}) e^{-i(n-n')\theta_{SS'}} F_{nr} e^{in\theta_S} \\
&+ \sum_{m=2}^{\infty}\sum_{S\neq S'}\sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty} {}^{S'}B_{n'}^{m-1} e^{in'(-\delta+\varepsilon_{n'})} H_{n-n'}^{(1)}(\beta r_{SS'}) e^{-i(n-n')\theta_{SS'}} (-iH_{nr}) e^{in\theta_S}\Big] e^{-i\omega t},
\end{aligned} \tag{15}
$$

where

$$
\begin{aligned}
F_{nr} &= -(n^2 + n - \alpha^2 r_S^2 + 0.5\beta^2 r_S^2) J_n(\alpha r_S) + \alpha r_S J_{n-1}(\alpha r_S), \\
\overline{F}_{nr} &= -(n^2 + n - \alpha^2 r_S^2 + 0.5\beta^2 r_S^2) H_n^{(1)}(\alpha r_S) + \alpha r_S H_{n-1}^{(1)}(\alpha r_S).
\end{aligned} \tag{16}
$$

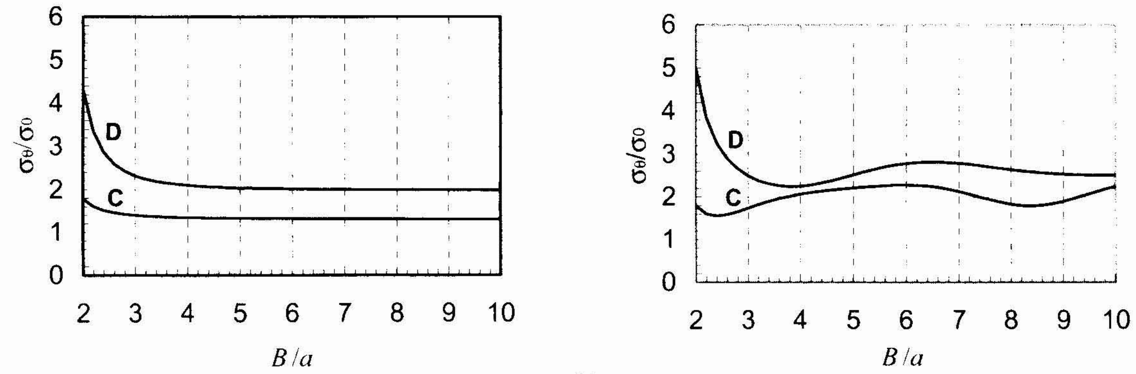
The stress induced by the incident wave is

$$
\sigma_0 = -\beta^2 \mu \phi_0, \tag{17}
$$

and finally, the dynamic stress concentration factor σ_θ/σ_0 can be obtained.

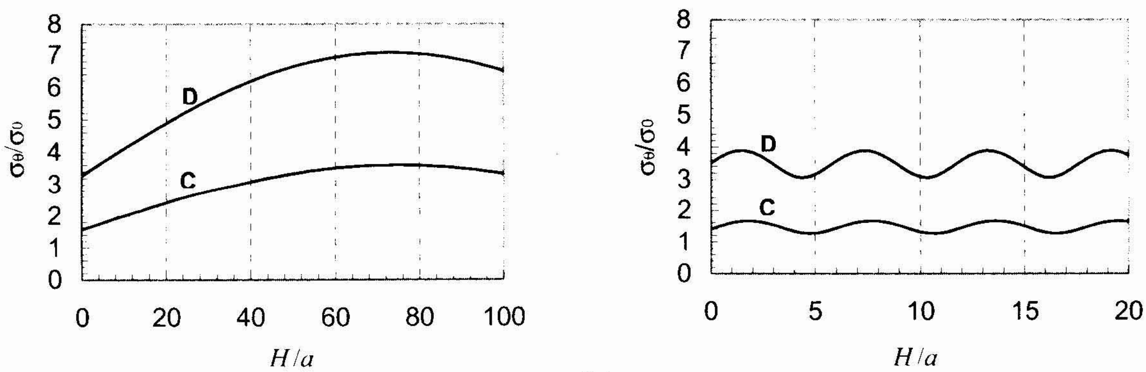
3. RESULTS AND DISCUSSION

Following the mathematical procedure described above, one can evaluate quantitatively the amplification and reduction of the stresses and displacements (particle velocities, accelerations) during the dynamic *P*-wave-tunnel interaction process. Referring to the actual dimensions of the Bantaki Tunnel, we set the mass density 2200kg/m^3, Poisson's ratio 0.25, shear modulus 16GPa, and the radius of Tunnel 1 and 2 as $a = b = 4$m. In practice, the surface of Tunnel 3 can be regarded as a flat plane when its radius c is sufficiently large. Therefore, in the analysis, c/a is set to be $c/a = 1000$, and the effect of the overburden H and the distance B is evaluated computationally from those stress and displacement distributions. For example, Figure 3 indicates that, at shallow depth ($H/a = 1$), the normal stresses induced on the sidewall at points C and D are sensitive to the change of distance B when B/a is smaller. In the case of low frequency [Figure 3(a)], when $B/a > 4$, the values of the stresses become nearly constant (up to a certain value of B/a), clearly showing the effect of distance B. In the case of a high frequency wave [Figure 3(b)], the induced dynamic stress drops at points D with the increase of B/a (< 4), but it increases when $4 < B/a < 6.5$. The circumferential stress at points C increases even in the range of $B/a < 4$. Conventional assessment criteria generally recommend that the distance should be at least $B/a > 4$ for the static stability of a dual tunnel[5], but the present study indicates if the incident waves are in the high frequency range, damage may be caused even when the distance between the tunnels is sufficient for the static stability. Given the same B/a (= 2.2), Figure 4 shows there is a certain depth (H/a ~ 75 for the low frequency incident wave; 1.6 for high frequencies) where the stress amplification on the sidewall becomes maximum, implying a shallow tunnel is not always more stressed than a tunnel at depth.



(a) (b)

Figure 3 The relation between the maximum circumferential stresses induced on the sidewall (at points C, D in Figure 2) and the distance B/a between the centers of the tunnels (overburden $H/a = 1$). The incident wave frequency is f = 3Hz (a) and 100Hz (b).



(a) (b)

Figure 4 The relationship between the maximum circumferential stresses induced on the sidewall (at points C, D) and the overburden H/a (distance between the centers of the tunnels $B/a = 2.2$). As before, the incident wave frequency is f = 3Hz (a) and 100Hz (b).

4. CONCLUSIONS

We have considered the dynamic stability of underground structures, more specifically, dual tunnels, which are subjected to body waves. The simple elastodynamic analysis has shown that vertical (longitudinal) disturbances may induce large compressive stress on the sidewalls of the dual tunnels and hence generate the unique damage patterns such as observed on the occasion of the Kobe earthquake, depending on the overburden and the distance between tunnels. These damage patterns have not previously been predicted using the conventional analytical and numerical methods that primarily take account of the shear (horizontal) vibrations. This contribution also suggests that underground structures subjected to seismic waves vibrate with their surroundings and they are expected to function as sensors that respond only to waves of specific characters.

REFERENCES

1. Uenishi, K., and S. Sakurai. Characteristic of the Vertical Seismic Waves Associated with the 1995 Hyogo-Ken Nanbu (Kobe), Japan Earthquake Estimated from the Failure of the Daikai Underground Station. Earthquake Engineering and Structural Dynamics, Vol. 29, No. 6, pp. 813-821, 2000.
2. Sakurai, S., and K. Uenishi. On the 2004 Niigata-Ken Chuetsu, Japan Earthquake. Journal of the Japan Society of Civil Engineers, Vol. 90, No. 6, pp. 48-51, 2005 (in Japanese).
3. Uenishi, K., and S. Sakurai. Dynamic Failure of an Underground Structure: Could It be Induced by a Horizontal Disturbance? In: Rock Mechanics in the National Interest (D. Elsworth, J.P. Tinucci, and K.A. Heasley [editors]), pp. 1107-1114, Swets & Zeitlinger B.V., Lisse, Netherlands, 2001.
4. Koyama, Y., T. Asakura and Y. Sato. Hyogo-ken Nanbu earthquake-induced damage and reconstruction of the tunnels in the mountains. Tunnels and Underground, vol.27, pp.51-61, 1996 (in Japanese).
5. Uenishi, K., and K. Tsuji. Theoretical Study on the Mechanical Behavior of Dual Tunnels Subjected to Vertical Oscillations. In: Proceedings of the 2006 JSCE (The Japan Society of Civil Engineers) Annual Meeting, 3-017 (pp.III-33 - III-34), The Japan Society of Civil Engineers, Tokyo, Japan, 2006.
6. Uenishi, K. Dynamic Behavior of an Underground Structure with Small Overburden Subjected to Vertical Oscillations. In: Soil and Rock America 2003 (P.J. Culligan, H.H. Einstein, and A.J. Whittle [editors]), pp. 2259-2264, Verlag Glückauf, Essen, Germany, 2003.
7. Pao, Y.H. Dynamical stress concentration in an elastic plate. J. Appl. Mech., vol.29, pp.299-305, 1962.
8. Cheng, S.L. Dynamic stress in a plate with circular holes. J. Appl. Mech., vol.39, pp.129-132, 1972.
9. Kasai, S. Theoretical Study on the Mechanical Behavior of Tunnels Subjected to Wave Loading. Master's Thesis, Kobe University, 1974 (in Japanese).
10. Balendra, T., D.P. Thambiratnam, C.G. Koh and S.L. Lee. Dynamic response of twin circular tunnels due to incident SH-waves. Earthquake Engineering and Structural Dynamics, vol.12, pp.181-201, 1984.
11. Moore, I.D., and F. Guan. Three-dimensional dynamic response of lined tunnels due to incident seismic waves. Earthquake Engineering and Structural Dynamics, vol.25, pp.357-369, 1996.
12. Watson, G.N. A Treatise on the Theory of Bessel Functions. Cambridge University Press, Cambridge, 1922.

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