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DOWNSTREAM CROSS-HOLDINGS AND UPSTREAM R&D*

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Cross-holdings affect firms' behavior in other vertically related markets. We consider a vertical market with two downstream firms and an upstream firm engaging in cost-reducing R&D. Since downstream cross-holdings weaken downstream competition, the upstream firm also decreases its investment. Hence, we find that as the degree of downstream cross-holdings increases, input price increases and investment level decreases. Although cross-holdings have this negative effect on downstream firms' profits, they increase the downstream firms' profits if the investment technology is inefficient. Finally, we show that with inefficient upstream investment, total surplus increases with cross-holdings, while consumer surplus always decreases with cross-holdings.

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I. INTRODUCTION

Cross-holdings are widely observed in many industries.[1] For example, shareholding in the automobile, airlines, financial, broadcasting, steel, insurance, and telecommunication industries have cross-ownership patterns (Alley [1997], Dietzenbacher et al. [2000], Gilo [2000], Jürgens et al. [2000], Clayton and Jorgensen [2005], Brito et al. [2014], López and Vives [2019]).

Since a firm acquiring a rival's stock obtains a share of the rival's profit, cross-holdings make the acquiring firm less aggressive and weaken competition. Hence, competition authorities prohibit firms from acquiring the shares of rivals if their holdings would substantially restrain competition.[2] In addition, academics have analyzed cases in which cross-holdings have a detrimental effect on competition, among them Clayton and Jorgensen [2005] and Reynolds and Snapp [1986].

We also look at the undesirable effects of cross-holdings. Cross-holdings change the market structure, and thus, affect the behavior of firms in other vertically related markets. We incorporate an upstream firm engaging in cost-reducing R&D and show that cross-holdings in a downstream market hinder innovation in an upstream market. In addition, we find that cross-holdings are always bad for consumer surplus.

More formally, we consider a two-stage game with an upstream firm and two downstream firms. In the first stage, the upstream firm decides the input price and investment level for marginal cost reduction. In the second stage, the downstream firms compete in quantity. We present the results of comparative statics for the degree of cross-holdings.

An analysis of our model shows that an increase in the degree of cross-holdings boosts the input price and lowers the cost-reducing investment of upstream firms. This is because the cross-holdings shrink the downstream market, which reduces the profitability of cost-reducing investment.[3] In addition, a high rate of cross-holdings increases downstream producer surplus

[1] While there are many types of cross-holdings, we focus only on direct cross-holdings. For details on the types of cross-holdings, see Adams [1999].

[2] For instance, Article 10 of the Japanese Antimonopoly Act does not allow cross-holdings to weaken competition.

[3] Our result is indirectly related to empirical evidence. Using data for the U.S., Nain and Wang [2018]

if the upstream firm uses an inefficient investment technology. On the contrary, consumer surplus and the upstream firm's profit always decrease with the degree of cross-holdings. Finally, we show that the positive effect of cross-holdings on downstream producer surplus dominates the negative effects on consumers and the upstream firm if upstream investment is not efficient.

Our results are useful for competition policy. Since most competition authorities use a consumer surplus standard (Vergé [2010]), any increase in the degree of cross-holdings is detrimental from the viewpoint of consumer surplus standard. Meanwhile, from the viewpoint of total surplus standard, some increase in the degree of cross-holdings is permissible if investment technology in an upstream sector is inefficient.

The effects of cross-holdings have received considerable attention in the literature on industrial organization. While some studies find anti-competitive effects of cross-holdings (Bresnahan and Salop [1986], O'Brien and Salop [2000], Reynolds and Snapp [1986]), other studies show welfare-increasing effects of cross-holdings (Farrell and Shapiro [1990], López and Vives [2019]).[4] In this body of literature, Fanti [2013, 2015, 2016] and Shuai et al. [2018] consider a vertical market with exogenous cross-holdings; and Bárcena-Ruiz and Olaizola [2007], Bayona and López [2018], and López and Vives [2019] consider cost-reducing R&D and cross-holdings in non-vertical markets. Since we focus on the profitability of cross-holdings, our study is also related to previous works on endogenous cross-holdings (Clayton and Jorgensen [2005], Flath [1991], Ghosh and Morita [2017], Li et al. [2015], Serbera [2011]). However, all the abovementioned studies consider models without upstream R&D.

The rest of this paper is organized as follows. In the next section, we present the model. In section 3, we analyze the model under linear inverse demand and quadratic investment cost. In section 4, we extend the analysis to a model with a larger class of demand and cost

show that cross-holdings weaken competition and raise prices. Fontana and Guerzoni [2008], using data for EU countries, show that process R&D becomes less important as market size decreases.

[4]For empirical studies on the anti-competitive effects of cross-holdings, see Allen and Phillips [2000] and Parker and Röller [1997]. Some studies develop theory without vertical structure: for environmental policy, see Bárcena-Ruiz and Campo [2012, 2017]; for cartels, see Gilo et al. [2006] and Malueg [1992]; for licensing, see Chen et al. [2010]; and, for asymmetric information, see Liu et al. [2018].

functions. The final section concludes.

II. MODEL

We assume a market with an upstream firm and two downstream firms i and j ($i, j = 1, 2$ and $i \neq j$). The upstream firm produces input and sells it to the downstream firms at input price w. To produce one unit of final product, the downstream firms use one unit of input. Each downstream firm produces homogeneous product and incurs no cost except input price. The inverse demand is $p(Q)$, where $Q \equiv q_i + q_j$ is the aggregate output.

The upstream firm can make investments to reduce own marginal cost. To reduce the marginal cost by x, the upstream firm incurs investment cost $\Gamma(x)$. We denote the marginal cost without investment as c. Hence, the marginal cost of upstream firm is $c - x$. Then, the profit of the upstream firm is $\pi_U \equiv [w - (c - x)]Q - \Gamma(x)$.

We denote the operating profit of downstream firm i as $\pi_i \equiv [p(Q) - w]q_i$. We assume that the downstream firms are symmetric and each firm has $100s$ percent of the equity of its rival in the form of passive investments with no control rights (e.g., nonvoting shares; Gilo et al. [2006]). We call s the cross-holding rate. The total value of firm i, V_i, is represented as $V_i = \pi_i + sV_j$ where $0 \leq s < 1$. Note that there exists a chain effect (Gilo et al. [2006]). Solving $V_i = \pi_i + sV_j$ for V_i, the total value of firm i can be rewritten as $V_i \equiv (\pi_i + s\pi_j)/(1 - s^2)$.

While we assume the linear pricing, it is crucial for our results. Consider a case with a two-part tariff. Since for any degree of cross-holdings, the upstream firm adjusts input price to always achieve monopoly output, the upstream investment level is constant for the degree of cross-holdings. However, the linear pricing is one of standard assumptions in the literature on vertical markets (Gaudin [2019], Tyagi [1999]). In addition, linear contracts are typically used in some industries: semiconductor and agriculture industries (Hwang et al. [2018]).

We assume that each downstream firm competes in quantity to maximize the total value of firm, V_i. We define the downstream producer surplus as $PS^D \equiv V_i + V_j = [p(Q) - w]Q/(1 - s)$

and the total surplus as $TS \equiv \int_0^Q p(y)dy - p(Q)Q + PS^D + \pi_U$.

The timing of the game is as follows. In the first stage, the upstream firm chooses the investment level x and the input price w; in the second stage, each downstream firm compete in quantity. Using backward induction, we solve this game.

III. CASE WITH LINEAR INVERSE DEMAND AND QUADRATIC INVESTMENT COST

To present our intuition behind the main results, we consider a case with linear inverse demand and quadratic investment cost; the inverse demand is $p(Q) = a - bQ$, where $a, b > 0$; the investment cost is $\Gamma(x) = \gamma x^2/2$, where γ relates to investment efficiency and we assume $b\gamma \geq a/[c(s+3)] \equiv \underline{b\gamma}$.[5]

III(i). *Calculating Equilibrium*

In the second stage, the first-order condition $\partial V_i/\partial q_i = 0$ yields aggregate output as

$$Q_L(w, s) = \frac{2(a-w)}{b(3+s)}, \tag{1}$$

where subscript L represents the case with linear inverse demand and quadratic investment cost.

In the first stage, taking (1) into account, the upstream firm maximizes π_U with respect to w and x. Then, the input price, investment level and downstream producer surplus are

$$w_L(s) = \frac{(a+c)(3+s)b\gamma - 2a}{2(3+s)b\gamma - 2}, \; x_L(s) = \frac{a-c}{(3+s)b\gamma - 1}, \; PS_L^D(s) = \frac{(a-c)^2(1+s)b\gamma^2}{2(1-s)[(3+s)b\gamma - 1]^2}.$$

[5] This assumption is needed for the second-order condition and non-negative marginal cost for the upstream firm in equilibrium, $c - x \geq 0$.

III(ii). *Comparative Statics*

Next, we provide the results of comparative statics in the upstream market. Differentiating the outcomes $w_L(s)$ and $x_L(s)$ with respect to s yields

$$\frac{\partial w_L(s)}{\partial s} = \frac{(a-c)b\gamma}{2[(3+s)b\gamma - 1]^2} > 0, \quad \frac{\partial x_L(s)}{\partial s} = -\frac{(a-c)b\gamma}{[(3+s)b\gamma - 1]^2} < 0.$$

Hence, we obtain the following result.

Result 1. With linear inverse demand and quadratic investment cost, an increase in the degree of cross-holdings boosts the input price and reduces the investment level.

This result can be interpreted as follows. An increase in cross-holdings softens competition in the downstream market, which reduces input demand for the upstream market. From the perspective of the upstream firm, a decrease in downstream output weakens the incentive to invest, $\partial x_L(s)/\partial s < 0$. Meanwhile, since the upstream firm has a higher marginal cost, it sets a higher input price, $\partial w_L(s)/\partial s > 0$.

We now study the effects of cross-holdings on the downstream producer surplus. Differentiating $PS_L^D(s)$ with respect to s yields

$$\frac{\partial PS_L^D(s)}{\partial s} = \frac{(a-c)^2 b\gamma^2[(2+s+s^2)b\gamma - 1]}{(1-s)^2[(3+s)b\gamma - 1]^3}.$$

Note that from the second-order condition for the upstream firm, the denominator is positive. Solving $\partial PS_L^D(s)/\partial s < 0$ for $b\gamma$, we obtain the following result.

Result 2. With a linear inverse demand and quadratic investment cost, the downstream producer surplus decreases with the degree of cross-holdings if $\underline{b\gamma} < b\gamma < 1/(2+s+s^2)$.

As explained above, the degree of cross-holdings s has a positive effect on the downstream producer surplus $PS_L^D(s)$, because the market becomes less competitive with s. However, the downstream producer surplus is harmed by an increase in the input price w, since an increase in s makes the upstream firm less efficient. Therefore, s also has an indirect negative effect on $PS_L^D(s)$. The impact of s on the equilibrium $PS_L^D(s)$ depends on which of the two effects dominates. In particular, when investment technology is inefficient (large γ), the indirect negative effect of a higher input price is small, since the upstream firm spends too little on R&D; by contrast, when the investment technology is sufficiently efficient (small γ), then the negative effect of the cross-holdings, s, on input price and investment level becomes strong, and may dominate the positive direct effect of weaker competition, in which case the downstream producer surplus decreases with s.

IV. CASE WITH A LARGER CLASS OF INVERSE DEMAND AND INVESTMENT COST FUNCTIONS

In this section, we analyze the model by generalizing to a larger class of inverse demand and investment cost functions and show the main results, which correspond to those in the previous section.

First, we assume that inverse demand is continuously differentiable and has a constant curvature. That is, the curvature of inverse demand $z \equiv -Qp''(Q)/p'(Q)$ does not depend on Q where $p'(Q)$, $p''(Q)$, and $p'''(Q)$ are the first, second, and third derivatives, respectively. Specifically, we assume that the inverse demand is $p(Q) = a - bQ^{1-z}/(1-z)$, where $a, b \geq 0$ and $z < 1$. The last inequality guarantees second-order conditions for the downstream firms and positive outcome in the downstream market.[6]

Next, to reduce the marginal cost by x, the upstream firm incurs investment cost $\Gamma(x)$ where $\Gamma(0) = 0$ and for any $x > 0$, $\Gamma(x) > 0$, $\Gamma'(x) > 0$, and $\Gamma''(x) > 0$. Moreover, we assume the investment cost function has a constant curvature: $t \equiv x\Gamma''(x)/\Gamma'(x)$, where $t > 0$. Therefore, we identify the investment cost function as $\Gamma(x) = \gamma x^{t+1}/(t+1)$.[7]

[6]This inverse demand includes familiar inverse demand. For example, the inverse demand becomes linear at $z = 0$ and log-linear with $z \to 1$ (Ritz [2008]).

[7]Note that, at $t = 1$, $\Gamma(x)$ takes a quadratic form.

IV(i). *Downstream Decision*

We consider the competition in the downstream market. The first-order condition, $\partial V_i/\partial q_i = 0$, leads to the downstream output.

$$q_i(w,s) = -\frac{p(Q)-w}{(1+s)p'(Q)}, \quad Q(w,s) = -\frac{2[p(Q)-w]}{(1+s)p'(Q)}. \tag{2}$$

Moreover, substituting $p(Q) = a - bQ^{1-z}/(1-z)$ into the aggregate outcomes in the above equation and solving it for Q^{1-z}, we obtain the following closed form.

$$Q(w,s)^{1-z} = \frac{2(a-w)(1-z)}{b\xi}, \tag{3}$$

where $\xi \equiv (1-z)(1+s)+2$. Note that, $\xi > 2$ because of $z < 1$.

Here, we provide the results of comparative statics in the downstream market. Differentiating $Q(w,s)$ with respect to w and/or s, we obtain Lemma 1.

Lemma 1. The results of comparative statics are as follows.

$$\frac{\partial Q(w,s)}{\partial w} = \frac{2}{\xi p'(Q)} < 0,\ \frac{\partial Q(w,s)}{\partial s} = \frac{2[p(Q)-w]}{(1+s)\xi p'(Q)} < 0,\ \frac{\partial^2 Q(w,s)}{\partial w \partial s} = -\frac{2}{\xi^2 p'(Q)} > 0,$$
$$\frac{\partial^2 Q(w,s)}{\partial w^2} = -\frac{2(1+s)z}{\xi^2 [p(Q)-w]p'(Q)} > 0,\ \frac{\partial^2 Q(w,s)}{\partial s^2} = -\frac{2(2-z)[p(Q)-w]}{(1+s)\xi^2 p'(Q)} > 0.$$

Proof. Differentiating $z = -p''(Q)Q/p'(Q)$ with respect to Q, substituting $p''(Q) = -zp'(Q)/Q$, and solving it for $p'''(Q)$ yields

$$p'''(Q) = \frac{z(1+z)p'(Q)}{Q^2}. \tag{4}$$

Substituting $p''(Q) = -zp'(Q)/Q$, (4), and (2) into the first and second derivatives of $Q(w,s)$ and solving them for $\partial Q(w,s)/\partial w$, $\partial Q(w,s)/\partial s$, $\partial^2 Q(w,s)/\partial w^2$, $\partial^2 Q(w,s)/\partial s^2$, and $\partial^2 Q(w,s)/\partial w \partial s$, we obtain this lemma.■

Lemma 1 can be explained as follows. It is well known that the input price w has a negative effect on total output: $\partial Q(w,s)/\partial w < 0$. As the degree of cross-holdings s increases, the market becomes less competitive, which reduces total output $\partial Q(w,s)/\partial s < 0$. Since with large w and s, the total output become small, the downstream firms have a weak reaction to a parameter change as w and s increase: $\partial^2 Q(w,s)/\partial w \partial s > 0$, $\partial^2 Q(w,s)/\partial w^2 > 0$, and $\partial^2 Q(w,s)/\partial s^2 > 0$.

We discuss the effects of the degree of cross-holdings s and input price w on the downstream producer surplus. Substituting $Q = Q(w,s)$ into PS^D and differentiating it with respect to either s or w yields

$$\frac{\partial PS^D(w,s)}{\partial s} = \frac{p(Q) - w + Qp'(Q)}{1-s}\frac{\partial Q(w,s)}{\partial s} - \frac{[p(Q)-w]Q}{(1-s)^2}, \tag{5}$$

$$\frac{\partial PS^D(w,s)}{\partial w} = \frac{p(Q) - w + Qp'(Q)}{1-s}\frac{\partial Q(w,s)}{\partial w} - \frac{Q}{1-s}. \tag{6}$$

Then, substituting (2) into (5) and (6), and using Lemma 1, we obtain Lemma 2.

Lemma 2. For a given input price, the downstream producer surplus increases with the degree of cross-holdings, and for a given degree of cross-holdings, the downstream producer surplus decreases with the input price. That is,

$$\frac{\partial PS^D(w,s)}{\partial s} = -\frac{2\left[\xi + 1 + s(\xi - 2 + s)\right]\left[p(Q) - w\right]^2}{(1-s^2)^2 \xi p'(Q)} > 0,$$

$$\frac{\partial PS^D(w,s)}{\partial w} = \frac{2(\xi - 1 + s)[p(Q) - w]}{(1-s^2)\xi p'(Q)} < 0.$$

Since the downstream producer surplus decreases with downstream competition and/or its marginal cost, this result is very intuitive.

IV(ii). *Upstream Decision*

In the first stage, the profit of upstream firm is $\pi_U(w,x) \equiv [w-(c-x)]Q(w,s)-\Gamma(x)$. From the first-order conditions $\partial\pi_U(w,x)/\partial w = 0$ and $\partial\pi_U(w,x)/\partial x = 0$ and using the results of comparative statics (Lemma 1), we obtain

$$\text{(7)} \qquad \frac{\partial\pi_U(w,x)}{\partial w} = Q + \frac{2(w-c+x)}{\xi p'(Q)} = 0, \quad \frac{\partial\pi_U(w,x)}{\partial x} = Q - \Gamma'(x) = 0.$$

Solving these first-order conditions and $Q = Q(w,s)$ in (2) for w, x, and $\Gamma'(x)$, we obtain outcomes in the first stage.

$$\text{(8)} \quad w(s) = \frac{1}{2}Q(s+1)p'(Q) + p(Q), \quad x(s) = c - \frac{1}{2}Q(\xi+1+s)p'(Q) - p(Q), \quad \Gamma'(x) = Q.$$

Case without upstream R&D. Here, we show that the equilibrium input price is independent from the degree of cross-holdings, if the upstream firm does not engage in cost-reducing R&D. Using (3) and $\partial\pi_U(w,x)/\partial w = 0$ in (7), we obtain equilibrium input price.

$$w = \frac{a(1-z)+c-x}{2-z}.$$

Since the input price does not depend on the degree of cross-holdings, the effects of degree of cross-holdings on equilibrium outcomes are exactly the same as those in Lemma 1 and 2.

Lemma 3. If the upstream firm cannot reduce own marginal cost, increasing the degree of cross-holdings reduces the total output and increases the downstream producer surplus.

Second-order necessary condition. We derive the second-order necessary condition in the first stage.[8] Since we have $\partial^2\pi_U(w,x)/\partial x^2 = -\Gamma''(x)$ and assume $\Gamma''(x) > 0$, we consider

[8] The second-order condition for global optima is satisfied if t is large. However, we derive only the second-order necessary condition, since we focus on the effects of cross-holdings on equilibrium outcomes. Moreover, in Section 2, we provide an example in which with a linear inverse demand and quadratic investment cost, the profit function of the upstream firm is concave for w and x.

only the condition in which $\partial^2\pi_U(w,x)/\partial w^2 \cdot \partial^2\pi_U(w,x)/\partial x^2 - [\partial^2\pi_U(w,x)/\partial w\partial x]^2 > 0$.

Substituting the result of Lemma 1, (8), and $\Gamma''(x) = t\Gamma'(x)/x$ into the second-order condition, we obtain the following inequality.

$$\frac{2[-2x - t(2-z)\xi p'(Q)\Gamma'(x)]}{x\xi^2 p'(Q)^2} = \frac{2\psi^{SOC}}{x\xi^2 p'(Q)^2} > 0,$$

where $\psi^{SOC} \equiv -2x - t(2-z)\xi p'(Q)\Gamma'(x)$. Note that, ψ^{SOC} is a linear function of t and the coefficient of t is positive. Therefore, solving $\psi^{SOC} > 0$ for t, we obtain the following lemma.

Lemma 4. The second-order necessary condition for the upstream firm is satisfied if

$$t > -\frac{2x}{(2-z)\xi p'(Q)\Gamma'(x)} \equiv t^{SOC}.$$

IV(iii). *Comparative Statics for the Degree of Cross-holdings*

Input price and investment. Next, we provide the results of comparative statics in the upstream market. Differentiating the first-order conditions for the upstream firm with respect to s, and then substituting the result of Lemma 1, (8), $p''(Q) = -zp'(Q)/Q$, $\Gamma''(x) = t\Gamma'(x)/x$ and $d\xi/ds = 1 - z$ into them, we derive the following equations.

$$\frac{(2-z)w'(s) + x'(s)}{\xi p'(Q)} = 0, \quad \frac{Qp'(Q) - 2w'(s)}{\xi p'(Q)} + \frac{tx'(s)\Gamma'(x)}{x} = 0.$$

Solving these equations for $w'(s)$ and $x'(s)$ and using the definition of ψ^{SOC}, we obtain the results of comparative statics for input price and investment level.

Proposition 1. An increase in the degree of cross-holdings boosts input prices and reduces the investment level in the upstream firm. That is,

$$w'(s) = -\frac{Qxp'(Q)}{\psi^{SOC}} > 0, \quad x'(s) = \frac{Qx(2-z)p'(Q)}{\psi^{SOC}} < 0.$$

Since Proposition 1 is parallel to Result 1, the intuition behind it is exactly the same. Moreover, from Lemma 3, upstream R&D is needed for this result.

Downstream producer surplus. Now, we show the effect of cross-holdings on the downstream producer surplus, PS^D. We define equilibrium downstream producer surplus as $PS^D(w(s), s) \equiv [p(Q(w(s), s)) - w(s)]Q(w(s), s)$. Differentiating it with respect to s yields

$$\frac{dPS^D(w(s), s)}{ds} = \underbrace{\frac{\partial PS^D(w, s)}{\partial w}}_{(-)} \underbrace{\frac{\partial w(s)}{\partial s}}_{(+)} + \underbrace{\frac{\partial PS^D(w, s)}{\partial s}}_{(+)}. \tag{9}$$

where the signs in the equation are as follows from Lemma 2 and Proposition 1. Since each term has a different sign in the above equation, the net effect of s on $PS^D(w(s), s)$ can be either positive or negative. After some calculations, we obtain the following proposition.

Proposition 2. The downstream producer surplus decreases with the degree of cross-holdings, $dPS^D(w(s), s)/ds < 0$ if $t^{SOC} < t < t^{PSD}$; the downstream producer surplus increases with the degree of cross-holdings if $t > t^{PSD}$, where

$$t^{PSD} \equiv -\frac{4x}{(2-z)[2\xi - (1-s^2)(2-z)]p'(Q)\Gamma'(x)}.$$

Proof. See Appendix.

Proposition 2 is parallel to Result 2. Hence, the intuition behind Proposition 2 is the same as that of Result 2.

Consumer surplus and upstream firm's profit. Since consumer surplus increases with total output, we consider the effect of cross-holdings on $Q(w(s), s)$.

$$\frac{dQ(w(s), s)}{ds} = \underbrace{\frac{\partial Q(w, s)}{\partial w}}_{(-)} \underbrace{w'(s)}_{(+)} + \underbrace{\frac{\partial Q(w, s)}{\partial s}}_{(-)} < 0,$$

where Lemma 1 and Proposition 1 guarantee the sign of the above derivatives. Hence, an increase in the degree of cross-holdings reduces total output, which decreases consumer surplus.

Next, we consider the effect of cross-holdings on the upstream firm's profit. From Lemma 1, the total output decreases with the degree of cross-holdings: $\partial Q(w,s)/\partial s < 0$. Then, for any w, s_1 and s_2 ($s_1, s_2 \in [0,1)$ and $s_1 < s_2$), $Q(w,s_1) > Q(w,s_2)$. Hence, the upstream firm's profit decreases with the degree of cross-holdings, as shown by the following inequality.[9]

$$\begin{aligned}\pi_U(w(s_1), x(s_1); s_1) &= [w(s_1) - c + x(s_1)]Q(w(s_1), s_1) - \Gamma(x(s_1)) \\ &\geq [w(s_2) - c + x(s_2)]Q(w(s_2), s_1) - \Gamma(x(s_2)) \\ &> [w(s_2) - c + x(s_2)]Q(w(s_2), s_2) - \Gamma(x(s_2)) = \pi_U(w(s_2), x(s_2); s_2),\end{aligned}$$

where $\pi_U(w_0, x_0; s_0)$ means that at $s = s_0$, the upstream firm chooses $w = w_0$ and $x = x_0$. Summarizing these results, we obtain Proposition 3.

Proposition 3. Consumer surplus and the upstream firm's profit decrease with the degree of cross-holdings.

An intuition behind this proposition is as follows. An increase in the degree of cross-holdings softens competition in the downstream market. In addition, from Proposition 1, the increase in s raises the input price, which decreases total output. Hence, consumer surplus decreases with s. Moreover, with large s, the upstream firm faces little derived demand, which reduces its profit.

Total surplus. Since there is a chain effect, the downstream producer surplus $PS^D = [p(Q) - w]Q/(1-s)$ diverges to infinity as $s \to 1$. Hence, for some parameter range, the positive

[9]Since, at $s = s_1$, the profit of the upstream firm is maximized with $w = w(s_1)$ and $x = x(s_1)$, for any w and x, we have $\pi_U(w(s_1), x(s_1); s_1) \geq \pi_U(w, x; s_1)$. Hence, the first inequality in the second line of following equation is satisfied.

effect of s on downstream produce surplus dominates the negative effects of s on consumer surplus and the upstream firm's profit. Here, we show a condition for it.

Substituting $Q(w(s), s)$, $w(s)$, and $x(s)$ into TS, we obtain equilibrium total surplus as $TS(s)$. After differentiating $TS(s)$ with respect to s and doing some calculations, we obtain the following proposition.

Proposition 4. Total surplus increase with s if $t > t^{TS}$ and $z < -(1 - 11s + s^2 + s^3)/[s(3 + 2s - s^2)]$, where

$$t^{TS} \equiv \frac{2(1 + 2s - s^2)x}{(2 - z)[1 - 11s + s^2 + s^3 + s(3 + 2s - s^2)z]p'(Q)\Gamma'(x)}.$$

Proof. See Appendix.

An intuition behind this proposition is as follows. Since our model contains a chain effect (Gilo et al. [2006]), the marginal effect of s on PS^D diverges to infinity as $s \to 1$. This marginal effect on PS^D dominates those on the other surpluses if s is sufficiently large. This condition is represented by $z < -(1 - 11s + s^2 + s^3)/[s(3 + 2s - s^2)]$.

Moreover, when t is sufficiently large, the upstream firm rarely invests for marginal cost reduction. Then, an increase of s causes little inefficiency for upstream production, which means that the negative effect of s on $TS(s)$ becomes small. Hence, with large t, the marginal effect of s on PS^D tends to dominate those on the other surpluses.

When competition authorities challenge cross-holdings with no control rights, our results can be applied. When an upstream firm uses an efficient investment technology, unilateral effects of cross-holdings become more harmful. In this case, the authorities should tend to challenge the cross-holdings.

V. Conclusions

We consider a relationship between downstream cross-holdings and upstream R&D. We construct a vertically related model with an upstream firm and two downstream firms, and show that the amount of upstream R&D decreases with the degree of downstream cross-holdings, which increases input price. Although the downstream cross-holdings have negative effects on downstream profit, the cross-holdings are beneficial for downstream firms if the R&D technology for the upstream firm is inefficient. Because of this positive effect, the total surplus may increase with the degree of cross-holdings, while the consumer surplus always decreases with cross-holdings.

Our analysis has some limitations. First, we consider the case in which all competitors participate in cross-holdings. If some outsiders exist, the profitability of cross-holdings decreases. Second, we assume a homogeneous product and Cournot competition in the downstream market. It would be worthwhile to consider product differentiation and/or price competition in the downstream market. We leave these topics for future research.

APPENDIX

A1. Proof of Proposition 2

Substituting the results of Lemma 1, Proposition 1, and $w(s)$ in (8) into (9) leads to

$$\frac{dPD^D(w(s),s)}{ds} = -\frac{Q^2p'(Q)[-2x(1-s)(\xi-1+s)+(1+\xi+s(\xi-2+s))\psi^{SOC}]}{2(1-s)^2\xi\psi^{SOC}}.$$

Then, using the definitions of ψ^{SOC} and ξ, we obtain the above first derivative as follows.

$$\frac{dPD^D(w(s),s)}{ds} = -\frac{Q^2p'(Q)[-4x-t(2-z)(2\xi-(1-s^2)(2-z))p'(Q)\Gamma'(x)]}{2(1-s)^2\psi^{SOC}}.$$

From $\psi^{SOC} > 0$, the sign of the above derivative is the same as that of the terms in square brackets. It is a linear function of t and the coefficient of t is positive. Hence, solving $dPD^D(w(s),s)/ds < 0$ for t, we obtain $t < t^{PSD}$. Finally, comparing t^{PSD} with t^{SOC}, we can easily show $t^{SOC} < t^{PSD}$ and complete the proof.■

A2. Proof of Proposition 4

Differentiating $TS(s)$ with respect to s, and substituting the results of Lemma 1, Proposition 1, and (8) into it, we obtain

$$\frac{dTS(s)}{ds} = \frac{Q^2p'(Q)}{2(1-s)^2\xi\psi^{SOC}}\left[\begin{array}{l}((1-s)^2-(3-s)s\xi)\psi^{SOC} \\ \quad +(1-s)(1-s+\xi)(2c-2p(Q)-Q(1+s+\xi)p'(Q))\end{array}\right].$$

From (8), $2x = 2c-2p(Q)-Q(1+s+\xi)p'(Q)$. Substituting it into the derivative and using the definition of ψ^{SOC} and ξ, we obtain the following expression.

$$\frac{dTS(s)}{ds} = -\frac{Q^2p'(Q)}{2(1-s)^2\psi^{SOC}}\left[\begin{array}{l}-2(1+2s-s^2)x \\ +t(2-z)(1-11s+s^2+s^3+s(3+2s-s^2)z)p'(Q)\Gamma'(x)\end{array}\right].$$

The sign of the above derivative depends only on the terms in the square brackets, which is a linear function of t. Since the coefficient of t is positive if $z < -(1-11s+s^2+s^3)/[s(3+2s-s^2)]$, the sign of $dTS(s)/ds$ is as follows.

$$\begin{cases} \dfrac{dTS(s)}{ds} > 0 \ \text{ if } \ t > t^{TS} \text{ and } z < -\dfrac{1-11s+s^2+s^3}{s(3+2s-s^2)}, \\ \qquad\qquad\quad \text{or } \ t < t^{TS} \text{ and } z > -\dfrac{1-11s+s^2+s^3}{s(3+2s-s^2)}, \\ \dfrac{dTS(s)}{ds} \leq 0 \ \text{ otherwise,} \end{cases}$$

where

$$t^{TS} \equiv \frac{2(1+2s-s^2)x}{(2-z)[1-11s+s^2+s^3+s(3+2s-s^2)z]p'(Q)\Gamma'(x)}.$$

Next, we compare t^{TS} with t^{SOC}. Using the definition of ξ, $t^{TS} - t^{SOC}$ yields

$$t^{TS} - t^{SOC} = \frac{2(1-s)x(4-z-sz)}{(2-z)[1-11s+s^2+s^3+s(3+2s-s^2)z]p'(Q)\Gamma'(x)}.$$

Since the numerator is positive, solving $t^{TS} - t^{SOC} > 0$ for z, we obtain $z < -(1-11s+s^2+s^3)/[s(3+2s-s^2)]$.

Summarizing the abovementioned results, for $t > t^{SOC}$, we identify the sign of $dTS(s)/ds$ as follows.

$$\begin{cases} \dfrac{dTS(s)}{ds} > 0 \ \text{ if } \ t > t^{TS} \text{ and } z < -\dfrac{1-11s+s^2+s^3}{s(3+2s-s^2)}, \\ \dfrac{dTS(s)}{ds} \leq 0 \ \text{ otherwise.} \end{cases}$$

Therefore, we complete the proof.■

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