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Offshoring and the Distribution of Skills

Hiroshi Goto Yan Ma Nobuyuki Takeuchi

Discussion Paper Series

Offshoring and the Distribution of Skills*

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Abstract

How does the distribution of human capital affect offshoring? To study this question, we extend Grossman and Maggi (2000) to allow offshoring, which means that workers in different countries can collaborate in teams. To investigate how the distribution of skills affects the possibility of offshoring, we first analyze how changes in skill diversity and the average skill level affect the relative price of the supermodular good, matching rules, and wage schedules under autarky. Next, we investigate the effects of offshoring and compare the equilibria under offshoring with its corresponding equilibria under free trade (without offshoring). We show that there is a possibility that the relative price of the supermodular good under offshoring is higher than both countries' autarky relative prices. In addition, we demonstrate that if two countries differ in skill diversity but share the same average skill level, the wages of workers with the same skill level are equalized under free trade; thus, there is no incentive for offshoring to occur. However, if two countries differ in only the average skill, the wages of workers with the same skill level under free trade (without offshoring) are not equalized across countries, thus opening a door for offshoring. Furthermore, we demonstrate that offshoring and free trade (without offshoring) have different effects on the welfare of workers with skills at the upper and lower ends of the distribution.

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1 Introduction

Production processes have increasingly involved multiple countries, with each country specializing in certain tasks, and this phenomenon of offshoring has attracted considerable attention in the literature from both policy makers and economists.[1] The phenomenon of offshoring has been followed by a considerable portion of the economic literature. Several studies, most notably those from Feenstra and Hanson (1996), Yi (2003), and Grossman and Rossi-Hansberg (2008), have investigated the effects of offshoring on trade volumes, trade patterns, and income distribution. Costinot, Vogel, and Wang (2013) highlight how global or local technological changes affect countries participating in the same global supply chain. Antras and Chor (2013) emphasize the optimal allocation of ownership rights along the global value chain. Baldwin and Venables (2013) reveal the implications of production processes for offshoring. However, how the distribution of human capital affects offshoring is more or less ignored in the literature. In this paper, we aim to fill this gap by extending the seminal paper of Grossman and Maggi (2000) (henceforth, GM (2000)).

GM (2000) examine how the distribution of human capital affects the pattern of trade in a two-sector model where technology in each sector involves tasks performed by a pair of workers. They show that when one sector involves a matching process where workers' skills are complementary (supermodular technology) and the other involves a matching process where skills are substitutable (submodular technology), workers in the middle of the skill distribution will be employed in the sector where skills are complementary and those in the extremes of the skill distribution will be employed in the sector where skills are substitutable. They then establish that if two countries have the same average skill level, the country with a more diverse distribution of skills will have a comparative advantage in the sector where skills are substitutable. Intuitively, the country with a more diverse distribution of skills has an abundance of the extremes in skills used in the sector where skills are substitutable.

While GM (2000) allows for trade in goods, they do not allow for the possibility of trade in tasks between countries. As noted by the existing studies, offshoring is determined by international cost differences and frictions related to the costs of separating production stages or tasks spatially.[2] In this paper, by extending GM (2000), we focus on differences across countries in the mean level and diversity of skills as determinants of the pattern of offshoring. This approach is of interest because countries differ in the distribution of skills, as found

[1] This phenomenon is referred to as fragmentation in Jones and Kierzkowski (1990) and Bond (2001), trade in tasks in Grossman and Rossi-Hansberg (2008), and vertical specialization in Hummels, Ishii, and Yi (2001).

[2] For example, Baldwin and Venables (2013) and Grossman and Rossi-Hansberg (2008, 2012).

by Bombardini, Gallipoli, and Pupato (2012).[3] In light of the revolutionary advances in transportation and communications technology having reduced the frictions to trade in tasks, we believe this is an important extension of GM (2000).

Offshoring refers to international teams collaborated by workers in different countries.[4] To investigate how the distribution of skills affects the possibility of offshoring, we first analyze how changes in skill diversity and the average skill level affect the relative price, matching rules, and wage schedules under autarky. An increase in skill diversity increases the relative price of the supermodular good because an increase in skill diversity raises the relative productivity of the submodular sector; thus, the relative supply of supermodular good declines. Moreover, an increase in skill diversity leads to a decrease in nominal wages (in terms of the submodular good) of workers with skills at the lower end of the distribution, while the rest of the workers experience an increase in nominal wages. There are two effects of an increase in skill diversity on nominal wages: one is matching effects, which means workers change their team partners and sectors to work; and the other is the price effect, which means that in the numeraire sector (submodular sector), the nominal wages of workers with extreme skills go in opposite directions. In addition, we show that an increase in the average skill level decreases the relative price of the supermodular sector and increases the nominal wages of workers with skills at the lower end of the distribution because they become relatively scarce.

We demonstrate that if two countries have the same average skill level but differ in skill diversity, as in GM (2000), there are no incentives for offshoring to occur because the wages of workers with the same skill levels are equalized under free trade (without offshoring). In other words, the free-trade equilibrium is the same as the equilibrium under offshoring. However, if two countries have the same level of skill diversity but differ in the average skill level, the wages of workers with the same skill level are not equalized under free trade (without offshoring), thereby opening a door for offshoring. Even in the case where the free-trade equilibrium has no trade in final goods, the wages of some workers with the same skill level are not equalized in the equilibrium.[5] In addition, in the case of offshoring between two countries with different average skill levels but the same level of skill diversity, we demonstrate that there is a possibility that

[3] See Figure 1 in Bombardini, Gallipoli, and Pupato (2012). They use scores on the International Adult Literacy Survey (IALS), an internationally comparable measure of work-related skills, to document the differences in the mean and standard deviation of skills among 19 countries during the 1994–1998 period. Their results support the assertion that skill dispersion affects the pattern of trade, as predicted by Grossman and Maggi (2000).

[4] Offshoring refers to international teams in Antras, Garricano and Rossi-Hansberg (2006), in which hierarchies of teams are considered.

[5] The case where the free-trade equilibrium has no trade in final goods refers to the case where the autarky relative prices in both countries are equal.

the relative price of the supermodular good under offshoring is higher than both countries' autarky relative prices due to a high skill diversity effect.

We show that offshoring leads to an increase in the nominal wages of workers at the upper end of the distribution of skills while decreasing the nominal wages of workers at the lower end of the distribution in a country with a higher average skill level. Hummels, Jorgensen, Munch, and Xiang (2014) estimate how offshoring and exporting affect wages by skill type using matched worker-firm data from Denmark. They find that within job spells, offshoring tends to increase high-skilled wages and decrease low-skilled wages.[6] As shown in Figure 1 in Bombardini, Gallipoli, and Pupato (2012), Denmark has higher average skill levels than do most high-income countries.

We also separately examine the effects of trade and offshoring on income distribution. We demonstrate that free trade (without offshoring) and offshoring have the qualitatively same effect on the welfare of workers with skills at the lower end of the distribution in the submodular sector, in which workers with skills at the upper and lower ends of distribution collaborate. However, free trade (without offshoring) and offshoring may have different effects on the welfare of workers at the upper end of the skill distribution. Our numerical examples show that trade affects the welfare of workers in the submodular sector in the same direction: all workers in the export sector gain from trade, or all workers in the import sector lose from trade. The intuition is that workers with skills at the lower and upper ends of the distribution form teams in the submodular sector and thus share the same effect from trade shock. However, offshoring affects the welfare of workers in the submodular sector in opposite directions. Intuitively, in the country with a higher average skill level, workers with skills at the upper end of the distribution have the opportunity to form teams with better partners in the foreign country; thus, they gain from offshoring. It follows that lower-skilled workers who are replaced by foreign workers lose. These results are consistent with the findings in Artuc and McLaren (2015, p.1), who argue that "a worker's industry of employment is much more important than either the worker's occupation or skill class in determining whether she is harmed by a trade shock, but occupation is crucial in determining who is harmed by an offshoring shock".[7]

Our paper is related to two strands of the trade literature. One is the literature on off-

[6] As noted by Hummels, Jorgensen, Munch, and Xiang (2014), most Danish trade is with other high-income countries, and when the sample was restricted to include only Danish trade with high-income partners, a similar sign pattern for offshoring was still found. How the distribution of skills affects the pattern of offshoring is not the focus in Hummels, Jorgensen, Munch, and Xiang (2014).

[7] Artuc and McLaren (2015) develop a model of offshoring involving task-by-task comparative advantage and conduct dynamic structural estimation based on a simplified version of the model. How the distribution of skills affect offshoring is not their focus.

shoring, which is large and diverse.[8] Our paper is closely related to, among others, Antras, Garricano, and Rossi-Hansberg (2006) and Costinot and Vogel (2010). Antras, Garricano, and Rossi-Hansberg (2006) analyze the impact of the formation of cross-country teams with one manager and several workers on the organization of production and wages in a one-sector model. In contrast, our paper investigates how the average skill level and skill diversity affect offshoring and income distribution in a two-sector model. Costinot and Vogel (2010) develop a one-good model with heterogeneous workers who differ in skill levels. The production of the final good requires many tasks differing in skill intensity. Worker productivity is assumed to be log supermodular in task intensity and skill; thus, their matching represents positive assortative matching between workers and tasks. The key feature of their model is that they assume away spillovers between workers; thus, factor prices are always equalized across countries. Our paper considers two types of production technologies that lead to positive assortative matching in the supermodular sector and negative assortative matching in the submodular sector. In addition, our matching features matching workers with workers to sectors.

Moreover, our paper is related to a growing body of literature that uses matching and assignment models in an international context, (e.g., Nocke and Yeaple 2008 and Costinot 2009). Our article is closely connected with, among others, studies on talent (human capital) and trade pioneered by Grossman and Maggi (2000), who analyze how skill distribution affects a country's comparative advantage and the pattern of trade. Research in this area includes Grossman (2004), Bougheas and Riezman (2007), Ohnsorge and Trefler (2007), Bombardini, Giovanni, and Germán (2012, 2014), and Chang and Huang (2014). None of these articles, however, considers offshoring, whereas our paper studies how the distribution of human capital affects offshoring.

Our paper is organized as follows. Section 2 describes the basic set-up and equilibrium under autarky. In Section 3, we investigate how changes in skill diversity and the average skill level affect the relative price of the supermodular good, matching rules and wage schedules under autarky. We examine the effects of offshoring and compare the equilibria under offshoring with its corresponding equilibria under free trade in Section 4. In Section 5, we consider how offshoring and free trade affect income distribution. Section 6 provides concluding remarks. All proofs are provided in Appendices.

[8] See Antras and Rossi-Hansberg (2009) for a review.

2 The Closed Economy

In this section, we introduce the setup and some results in GM (2000). There is a continuum of workers who differ in their skill levels, t. The skill distribution is represented by a cumulative distribution function $\Phi(t)$, $t \in [t_{\min}, t_{\max}]$, where $t_{\min}$ and $t_{\max}$ denote the minimum and maximum skill levels. Let L and $\bar{t}$ represent the measure of the labor force and the average skill level, respectively. Following GM (2000), we assume that the density function $\phi \equiv d\Phi/dt$ is symmetric about its mean, $\bar{t}$.

There are two sectors: sector C and sector S. In each sector, a team of two workers is required in the production process, with each worker performing a different task. In sector i ($i = C, S$), output by a pair of workers is $F^i(t_1, t_2)$, where t_j ($j = 1, 2$) represents the skill level of the worker conducting task j. F^i is assumed to be monotonically increasing in t_j, is symmetric, and exhibits constant returns to skills. In sector C, two workers perform complementary tasks and the production function $F^C(t_1, t_2)$ exhibits supermodularity. As argued by Milgrom and Roberts (1990, p.517), supermodularity can be equivalent to the cross derivative $F^C_{12} > 0$; the more able a worker's team partner is, the greater the marginal product of the worker's skill. The examples of sector C in GM (2000) are automobiles and high-end consumer electronics. In sector S, the production process exhibits submodularity, which implies that the marginal product of a worker's skill decreases in the skill level of his co-worker, i.e., $F^S_{12} < 0$. One example of S sector is the software industry in GM (2000). Given constant returns to skills, the supermodular production process in sector C implies decreasing returns to a single worker's skill ($F^C_{jj} < 0$, $j = 1, 2$), while the submodular production process implies increasing returns to a single worker's skill ($F^S_{jj} > 0$).

All markets are perfectly competitive. Due to the nature of the production process in each sector, the output of sector C is maximized through self-matching of workers' skill levels, while the output of sector S is maximized through cross-matching of workers' skill levels. The optimal allocation of labor across sectors is summarized as follows[9]:

Lemma 1 *(GM (2000), Lemmas 1 and 3) For any given output Y_S of good S, the output Y_C of good C is maximized by (1) allocating all workers with skill levels $t < \hat{t}$ and all workers with skill levels $t > m(\hat{t})$ to sector S, where the matching function $m(t)$ is defined implicitly by $\Phi[m(t)] = 1 - \Phi(t)$, and $\hat{t}$ solves $Y_S = L \int_{t_{min}}^{\hat{t}} F^S(t, m(t))\, d\Phi(t)$; (2) allocating the remaining workers with $t \in [\hat{t}, m(\hat{t})]$ to sector C such that $t_1 = t_2$ in all teams, i.e., $Y_C = \frac{\lambda_C L}{2} \int_{\hat{t}}^{m(\hat{t})} t d\Phi(t)$, where $\lambda_C = F^C(1, 1)$.*

[9] We follow Chang and Huang (2014, Lemma 1) to summarize Lemmas 1 and 3 in GM (2000).

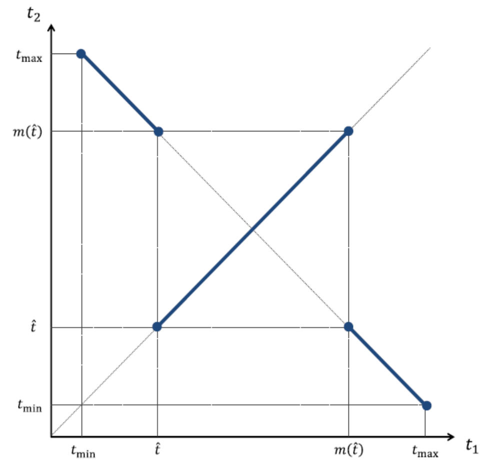


Figure 1: Matching Rules

Since the density function ϕ is symmetric about its mean, $\bar{t}$, it is clear that $m(t) = 2\bar{t} - t$. Figure 1 shows the matching rules described by Lemma 1: the 45-degree line for $t \in [\hat{t}, m(\hat{t})]$ reflects that workers with skill level $t \in [\hat{t}, m(\hat{t})]$ will match workers with the same skill levels in sector C, while the negatively sloped line $m(t) = 2\bar{t} - t$ for $t \in [t_{\min}, \hat{t})$ and $t \in (m(\hat{t}), m(t_{\min})]$ reflects that workers with skill $t \in [t_{\min}, \hat{t})$ will cross-match workers with skill $t \in (m(\hat{t}), m(t_{\min})]$ in sector S.[10] Thus, matching in sector C represents positive assortative matching while matching in sector S represents negative assortative matching.

In a competitive equilibrium, all firms maximize their profits, and all markets clear. Since workers with skill levels between $\hat{t}$ and $m(\hat{t})$ are allocated to sector C and other workers are cross-matched into sector S, the outputs of two goods are given as follows:

$$Y_C = \frac{L}{2}\lambda_C \bar{t} \int_{\hat{t}}^{2\bar{t}-\hat{t}} \phi(t)dt, \tag{1}$$

$$Y_S = L \int_{t_{\min}}^{\hat{t}} F^S \left(t, 2\bar{t} - t\right) \phi(t)dt. \tag{2}$$

The preference is assumed to be homothetic. Thus, the ratio of quantities consumed of good S (X_S) and good C (X_C) depends only on the relative price of good C as in equation (3)

$$\frac{X_S}{X_C} = f(p), \tag{3}$$

[10]Note that $m(t_{\min}) = t_{\max}$.

where $p = p_C/p_S$ denotes the relative price of good C and we use good S as the numeraire. Let $MRT \equiv -\frac{dY_S/d\hat{t}}{dY_C/d\hat{t}}$ denote the marginal rate of transformation (MRT) of the production possibility frontier (PPF), and we have

$$MRT = \frac{F^S\left(\hat{t}, 2\bar{t} - \hat{t}\right)}{\lambda_C \bar{t}}. \tag{4}$$

Clearly, MRT depends on the average skill level $\bar{t}$ and the least-skilled level in sector C, i.e., $\hat{t}$. In the competitive equilibrium, the relative price p is equal to MRT. Thus, the relative price p and $\hat{t}$ are determined as follows:

$$p = \frac{F^S\left(\hat{t}, 2\bar{t} - \hat{t}\right)}{\lambda_C \bar{t}}, \tag{5}$$

$$\frac{Y_S}{Y_C} = f(p). \tag{6}$$

In sector C, two workers with the same skill levels form a team, so they contribute to the output equally and split the total revenue equally. Thus, we have

$$w(t) = \frac{p\lambda_C t}{2}, \text{ for } t \in [\hat{t}, m(\hat{t})]. \tag{7}$$

In sector S, since a worker with skill level $t \in [t_{\min}, \hat{t})$ teams with a worker with skill level $m(t) = 2\bar{t} - t$, the zero profit condition leads to

$$w(t) + w[m(t)] = F^S[t, m(t)], \text{ for } t \in [t_{\min}, \hat{t}). \tag{8}$$

As shown in GM (2000, pp.1266–1267), profit maximization implies that $F_1^S[s, m(s)] = w'(s)$ for all $s < \hat{t}$. Since the wages of workers with skill level $\hat{t}$ satisfy (7), the wages of workers with skills at the lower end of the distribution are yielded as

$$w(t) = \frac{p\lambda_C \hat{t}}{2} - \int_t^{\hat{t}} F_1^S\left(\tau, 2\bar{t} - \tau\right) d\tau, \text{ for } t \in [t_{\min}, \hat{t}). \tag{9}$$

Combining (8) and (9), we obtain the wages of workers with skills at the upper end of the distribution.

$$w(t) = F^S\left(t, 2\bar{t} - t\right) - \frac{p\lambda_C \hat{t}}{2} + \int_{2\bar{t}-t}^{\hat{t}} F_1^S\left(\tau, 2\bar{t} - \tau\right) d\tau, \text{ for } t \in (2\bar{t} - \hat{t}, t_{\max}]. \tag{10}$$

Thus, a competitive equilibrium is a set of equations of (1), (2), (5), (6), (7), (9), and (10).

We prove in Appendix A that a unique equilibrium exists.

3 Comparative Statics in a Closed Economy

In this section, we investigate how exogenous changes in skill diversity and average skill level affect the relative price of good C, the matching rules that are captured by $\hat{t}$ and $m(\hat{t})$, and the wage schedule. Such changes in the distribution of skills may capture the effects of educational reforms.

3.1 A Change in Skill Diversity

We begin by examining how an increase in skill diversity with the average skill level fixed affects the relative price of good C, p, and marginal skill levels in sector C, $\hat{t}$ and $m(\hat{t})$. We define skill diversity as follows:

Definition 1 *For distributions Φ and Φ^*, which have the same average skill level, the distribution of skill Φ is more diverse than that of Φ^* if and only if there exist $t', t'', t''' \in (t_{\min}, t_{\max})$ such that $t'' \leqq t' \leqq t'''$, and they satisfy the following two conditions:*

(a): $\Phi(t) \geqq \Phi^*(t)$ for $t \leqq t'$ and $\Phi(t) \leqq \Phi^*(t)$ for $t \geqq t'$.

(b): $\Phi(t) > \Phi^*(t)$ for $t_{\min} < t < t''$ and $\Phi(t) < \Phi^*(t)$ for $t_{\max} > t > t'''$.

Note that if two distributions of skills have different average skill levels, we compare their skill diversity by shifting one distribution parallel to share the same mean level with the other. If a parallel shift of one distribution of skills coincides completely with the other distribution of skills, we say that these two skill distributions share the same skill diversity.

Intuitively, the definition of skill diversity reflects that there are relatively more workers with extreme skill levels (either high or low) under Φ than under Φ^*. GM (2000) allows two cumulative distribution functions to cross only once. If two cumulative distribution functions cross more than twice, the proof for Proposition 4 in GM (2000) may fail.[11] Our definition slightly relaxes the definition of skill diversity in GM (2000) in the sense that we do not allow the two cumulative distribution functions to cross more than twice, but we allow them to coincide for some range of skill levels. The definition of skill diversity in GM (2000) is

[11] Proposition 4 in GM (2000, p.1266) is as follows: "Suppose $\bar{t} = \bar{t}^*$, and Φ is more diverse than Φ^*. Then the home country exports good S and imports good C in a free-trade equilibrium".

relaxed for the purpose of analyzing the effects of offshoring in Section 4.[12] Under the relaxed definition of skill diversity, we also obtain the results (Lemma 2), the same as Proposition 4 in GM (2000).[13]

Lemma 2 *An increase in skill diversity leads to an increase in the relative price of the good produced in the supermodular sector, p. In addition, $\hat{t}$ decreases while $m(\hat{t})$ increases.*

The intuition is that an increase in skill diversity relatively increases the supply of workers with extreme skills, who choose to work in sector S, and therefore the productivity of good S is relatively improved. It follows that the relative supply of good S increases with $\hat{t}$ fixed, which leads to a rise in the relative price of good C. This, in turn, enlarges sector C and shrinks sector S. As a result, $\hat{t}$ decreases while $m(\hat{t})$ increases due to the symmetry of the density function. Since we know that the relative price of good C in equilibrium increases, the equilibrium relative supply of good S, $\frac{Y_S}{Y_C}$, increases, which implies that the effect of an increase in skill diversity on $\frac{Y_S}{Y_C}$ dominates the effect of a decrease in $\hat{t}$ on $\frac{Y_S}{Y_C}$.[14]

Next, we turn to the effects of a change in skill diversity on the wage schedule and welfare of workers. We focus on changes in welfare for the workers with same skill levels under different skill distributions. We use the indirect utility function, $V(p, 1, w(t))$, to measure workers' welfare. Note that how a change in skill diversity affects the wage schedule is not the focus of GM (2000), which only investigates the effects of opening up to trade on two countries' wage schedules with skill diversity fixed in each country. Figure 2 shows the effects of an increase in skill diversity on the wage schedule. For workers in sector C before and after a change in skill diversity, their nominal wages (in terms of good S) increase in proportion to the increase in p due to (7). For the remaining workers, an increase in skill diversity increases the nominal wages of workers at the upper end of the distribution, while decreasing the nominal wages of workers at the lower end of the distribution.

[12] If the distribution of skills in each country follows a uniform distribution function, the cumulative distribution function of each country and the cumulative distribution worldwide under offshoring coincide for some range of skill levels. Since the definition of diversity in GM (2000) cannot be satisfied for this case, we relax their definition to allow this case.

[13] Following GM (2000), we can prove that if $\Phi(t)$ and $\Phi^*(t)$ are symmetric, and share the same average skill level, i.e., $\bar{t} = \bar{t}^*$, and $\Phi(t)$ is more diverse than $\Phi^*(t)$, $\Phi(t)$ can be generated from $\Phi^*(t)$ by a sequence of single, symmetric mean-preserving spreads (SSMPS). At the equilibrium autarky relative price of good C under $\Phi^*(t)$, p^*, each SSMPS either increases or does not change the relative supply of good S. Therefore, a homothetic preference implies that the autarky relative price under $\Phi(t)$, p, is higher than p^*.

[14] The result that an increase in skill diversity leads to an increase in the relative supply of good S, $\frac{Y_S}{Y_C}$ is not a standard Rybczinski result because the middle-skilled workers are used in the supermodular sector. Workers with low skills and workers with high skills are used in the submodular sector. The intuition of the Rybczinski theorem is useful for understanding the effects of an increase in skill diversity under autarky because it increases high-skilled and low-skilled workers who are used in the submodular sector.

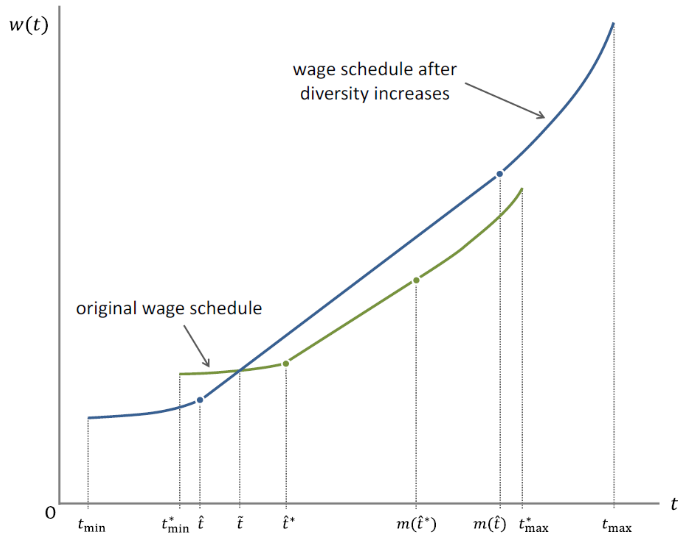


Figure 2: Skill Diversity and Wage Schedules

Proposition 1 *An increase in skill diversity leads to a decrease in the nominal wages of workers with* $t \in [t^*_{\min}, \tilde{t})$ *but leads to an increase in the nominal wages of workers with skill* $t \in (\tilde{t}, t^*_{\max}]$. *Moreover, an increase in skill diversity leads to a decline in the welfare of workers with* $t \in [t^*_{\min}, \tilde{\tilde{t}})$, *where* $\tilde{\tilde{t}} \in (\tilde{t}, \hat{t}^*)$, *and leads to an increase in the welfare of workers with* $t \in (\tilde{\tilde{t}}, m(\hat{t}^*)]$.

The result that an increase in skill diversity affects the nominal wages of workers at the upper and lower ends of the distribution in opposite directions is interesting, as the supply of workers with extreme skills (either high or low) increases simultaneously. An increase in skill diversity affects the nominal wages of workers in sector S through two effects: the price effect and the matching effect. The price effect means that the nominal wages of workers at the lower end and workers at the upper end of the distribution go in opposite directions because there is no change in the price of the numeraire good S, as implied in (8). The matching effect means that the increase in p due to an increase in skill diversity leads to an expansion of sector C, i.e., a fall in $\hat{t}$ and an increase in $m(\hat{t})$, which in turn leads to workers changing their team partners and sectors. Intuitively, the increase in p gives workers in sector S two choices: switching to sector C or remaining in sector S. Some workers in sector S change their partners and switch to sector C for higher nominal wages, for example, workers with $t \in [m(\hat{t}^*), m(\hat{t})]$ and workers with $t \in [\tilde{t}, \hat{t}^*)$ in Figure 2. The increase in p raises the nominal wages of workers with the original marginal skill levels of sector C, i.e., $\hat{t}^*$ and $m(\hat{t}^*)$. Thus, workers with skill level higher than $m(\hat{t}^*)$, such as workers with $t \in [m(\hat{t}^*), m(\hat{t})]$, will receive higher nominal wages by switching to sector C. Workers with skill levels higher than $m(\hat{t})$ who remain in sector S must obtain higher nominal wages than what they can receive if switching to sector C because the nominal wages of workers are linear with the skill levels in sector C. Hence, the nominal wages of workers with skill levels higher than $m(\hat{t})$ increase and their partners, i.e., workers with $t \in [t^*_{\min}, \hat{t}]$ experience a decrease in nominal wages due to the price effect. Note that workers with $t > m(\hat{t})$ pair with workers with $t < \hat{t}$ before and after the change in skill diversity; therefore, the movements in their nominal wages reflect changes in their shares of the revenues from $F^S[t, m(t)]$, where $t < \hat{t}$. Workers with $t \in [\hat{t}, \tilde{t})$ have to switch to sector C even if their nominal wages decrease because they lose their original partners who switch to sector C for better nominal wages.

Since the relative price increases due to an increase in skill diversity, the workers at the lower end of the distribution are worse off because their nominal wages decline. Workers with middle skill levels are better off because their nominal wages increase in proportion to the increase in the relative price. The net effect of an increase in skill diversity on the welfare of workers at the upper end of distribution is ambiguous because both their nominal wages and the relative price increase, and we do not know which effect dominates.

3.2 A Change in the Average Skill Level

We investigate how an increase in the average skill level with constant skill diversity affects the relative price, p, and matching rules reflected by $\hat{t}$ and $m(\hat{t})$. We assume that the average skill level $\bar{t}$ increases from $\bar{t}^*$, i.e., $\bar{t} = \beta\bar{t}^*$, where $\beta > 1$. Thus, β reflects a rise in the average skill level. Keeping the skill diversity constant means that the new density function $\phi(t)$ and the original density function $\phi^*(t)$ satisfy $\phi(t) = \phi^*[t - (\beta - 1)\bar{t}^*]$ for $t \in [t_{\min}, t_{\max}]$ and $t_{\max} - t_{\min} = t^*_{\max} - t^*_{\min}$.[15] $\Phi^*(t)$ represents the original cumulative distribution function, and $\Phi(t)$ represents that with the average skill level $\bar{t}$.

We begin with the effect of an increase in the average skill level on the relative price of good C, p. Intuitively, workers with middle skill levels are allocated to sector C, so an increase in the average skill level relatively raises the productivity of sector C, which in turn leads to a

[15] Note that our definition that two countries differ in the average skill level but share a common skill diversity is different from scaling down the cumulative distribution function of $\Phi(t)$ to obtain $\Phi^{ma}(t/\beta)$ in GM (2000). Scaling up or down the cumulative distribution function in GM (2000, p.1266) is defined as follows: "Let $\Phi(t)$ and $\Phi^*(t)$ be symmetric distributions. Define $\Phi^{ma}(\cdot)$ such that $\Phi^{ma}(rt) = \Phi(t)$ for all t, where $r = \bar{t}^*/\bar{t}$." Such a scaling down of the cumulative distribution function requires that all $t \in [t_{\min}, t_{\max}]$ be increased $1/\beta$ times; thus, both the average skill level and the skill diversity change at the same time.

decrease in the relative price of good C (see Appendix A for the proof). It follows that sector C will shrink while sector S will expand. Therefore, there are two effects of an increase in the average skill level on both $\hat{t}$ and $m(\hat{t})$. One effect is the direct effect due to the density function shifting to the right, which increases both $\hat{t}$ and $m(\hat{t})$. The other effect is the indirect effect through the decrease in the relative price p, which increases $\hat{t}$ but decreases $m(\hat{t})$. We demonstrate in Appendix A that the equilibrium $\hat{t}$ increases, and under a sufficient condition, i.e., $\min\{\phi(t)\} \geqq \frac{1}{2t}$ for $t \in [t_{\min}, t_{\max}]$, the equilibrium $m(\hat{t})$ increases, which implies that the direct effect dominates the indirect effect.

Lemma 3 *An increase in the average skill level leads to a decrease in the relative price of the good that is produced in the supermodular sector, p. Moreover, $\hat{t}$ increases, and if* $\min\{\phi(t)\} \geqq \frac{1}{2t}$ *for $t \in [t_{\min}, t_{\max}]$, $m(\hat{t})$ increases.*

Note that the condition of $\min\{\phi(t)\} \geqq \frac{1}{2t}$ is a sufficient condition. For example, uniform distribution functions satisfy this condition. Intuitively, this sufficient condition requires that the density of any skill not be too low. If the density of $m(\hat{t})$ is too low, a decrease in p due to an increase in the average skill level can lead to a large decline in $m(\hat{t})$, and thus, this indirect effect dominates its direct effect. The effects of an increase in the average skill level on the nominal wages of workers are shown in Figure 3. Clearly, an increase in the average skill level and an increase in skill diversity have opposite effects on the wage schedule (see Appendix A for the proof of Proposition 2).

Proposition 2 *If* $\min\{\phi(t)\} \geqq \frac{1}{2t}$ *is satisfied everywhere, an increase in the average skill level leads to an increase in the nominal wages of workers with $t \in [t_{\min}, \tilde{t}_m)$, but leads to a decline in the nominal wages of workers with $t \in (\tilde{t}_m, t^*_{\max}]$. In addition, an increase in the average skill level improves the welfare of workers with $t \in [t_{\min}, \tilde{\tilde{t}}_m)$, where $\tilde{\tilde{t}}_m \in (\tilde{t}_m, \hat{t})$, but decreases the welfare of workers with $t \in (\tilde{\tilde{t}}_m, m^*(\hat{t}^*)]$.*

Intuitively, an increase in the average skill level provides relatively more high-skilled workers and less low-skilled workers. As a result, the nominal wages of workers with relatively high skill levels decrease while the nominal wages of workers with relatively low skill levels increase. Since the relative price declines due to an increase in the average skill level, the welfare of workers at the lower end of the distribution increase. However, the welfare of workers with middle skill levels decline because their nominal wages decrease in proportion to the decline in the relative price. The changes in the welfare of the workers at the high end of the distribution

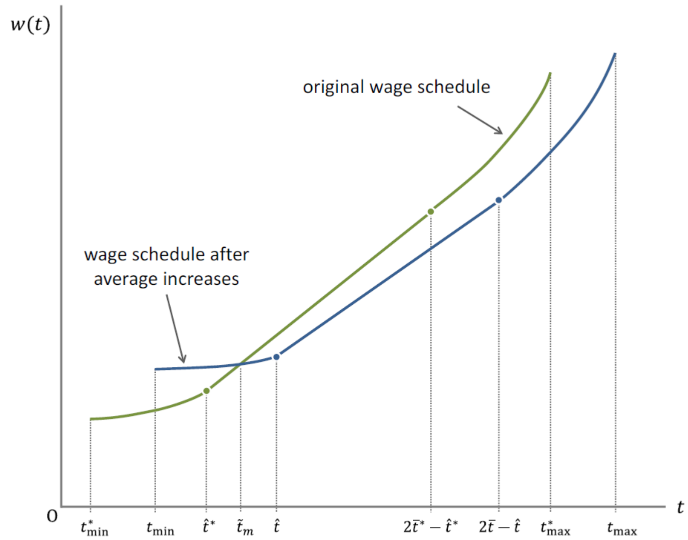


Figure 3: The Average Skill Level and Wage Schedules

is ambiguous because both their nominal wages and the relative price decline, and it is not clear which change will dominate.

4 The World Economy

We consider a world economy consisting of two countries, Home (H) and Foreign (F), in the remainder of this paper. The two countries share the same preferences, and workers are internationally immobile. In each country, we assume that production is as described in Section 2. Let $\phi^i(t)$ and $\Phi^i(t)$, $i = H, F$, $t \in [t^i_{\min}, t^i_{\max}]$, represent the density function and cumulative distribution function of skills with the minimum skill level $t^i_{\min}$ and the maximum skill level $t^i_{\max}$ in country i. Let $\bar{t}^i$ and L_i denote the average skill level and the labor size in country i.

Offshoring means that workers in different countries can collaborate in teams. As the theory of trade generally ignores trade costs in explaining the trade pattern, we investigate the offshoring pattern by assuming that there are no offshoring costs.[16] If two countries share the same labor size, when the density function of skills in each country is symmetric and the two countries differ only in the average skill level or skill diversity, the world density function under

[16] Grossman and Rossi-Hansberg (2008,2012) assume that the cost of offshoring varies by task. In their studies, tasks with a low cost of offshoring are offshored, while tasks with a high cost of offshoring are not.

offshoring is symmetric.[17] Since the world economy under offshoring is equivalent to a closed economy with a larger labor size, we can use the results obtained in the closed economy to generate new insights about the consequences of offshoring. The equilibrium under offshoring in our study refers to the integrated equilibrium with both trade and offshoring, because trade in goods and offshoring (trade in tasks) coexist. Since the density function under offshoring mixes each country's density function, it is clear that the average skill level of the world is the weighted average of two countries' average skill levels. However, the skill diversity of the world's distribution of skills is not necessary between the skill diversity of each country's distribution of skills. We first examine the effects of offshoring between two countries that differ either in skill diversity or in the average skill level. In either case of offshoring, we focus on the equilibrium with the least amount of offshoring, which means that there is no offshoring if the wages of workers with the same skill levels are equalized across countries.

Next, we compare each equilibrium under offshoring with its corresponding equilibrium under free trade without offshoring. Hereafter, free trade means free trade in final goods and no offshoring.

4.1 The World Economy with Skill Diversity

We begin by exploring the effects of offshoring between two countries that differ in skill diversity. Next, we turn to the effects of free trade between these two countries and compare the equilibrium under offshoring with the equilibrium under free trade.

4.1.1 Offshoring

We assume that the two countries share the same average skill level, but Home has a more diverse distribution of skills than Foreign. Since workers can collaborate in international teams, we need to consider the global distribution of skills. Let $\phi^W(t)$ represent the density function of skills under offshoring, which is given by

$$\phi^W(t) = \frac{L_H}{L_H + L_F}\phi^H(t) + \frac{L_F}{L_H + L_F}\phi^F(t), \tag{11}$$

where $t \in [t^W_{\min}, t^W_{\max}]$, with $t^W_{\min} = \min\{t^H_{\min}, t^F_{\min}\}$ and $t^W_{\max} = \max\{t^H_{\max}, t^F_{\max}\}$. Since the average skill level in Home, $\bar{t}^H$, is equal to that in Foreign, $\bar{t}^F$, and the density function of skills in each country is symmetric about its mean level of skill, $\phi^W(t)$ is symmetric about the

[17] If two countries differ in not only the average skill level but also skill diversity, the density function under offshoring will be asymmetric.

average skill level worldwide, $\bar{t}^W \equiv (\bar{t}^H + \bar{t}^F)/2 = \bar{t}^H = \bar{t}^F$. It follows that Lemma 1 holds under offshoring with $\hat{t}^W$ and $m^W(\hat{t}^W)$ representing the marginal skill levels in sector C. Thus, we have

$$Y_C^W = \frac{\lambda_C}{2}(L_H + L_F)\bar{t}^W \int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^W(t)dt, \tag{12}$$

$$Y_S^W = (L_H + L_F) \int_{t_{\min}^W}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right) \phi^W(t)dt, \tag{13}$$

where Y_C^W and Y_S^W are denoted by the outputs of good C and good S in the world under offshoring, respectively.

The relative price of good C under offshoring, p^W, and $\hat{t}^W$ are determined by the following two equations:

$$p^W = \frac{F^S\left(\hat{t}^W, 2\bar{t}^W - \hat{t}^W\right)}{\lambda_C \bar{t}^W}, \tag{14}$$

$$\frac{Y_S^W}{Y_C^W} = f(p^W). \tag{15}$$

The first equation represents that the marginal rate of transformation is equal to the price under offshoring, and the second equation reflects that the relative supply is equal to the relative demand in the world.

Since the world economy under offshoring can be considered as a closed economy, the wage schedule under offshoring is derived as follows:

$$w(t) = \frac{p^W \lambda_C t}{2}, \text{ for } t \in [\hat{t}^W, 2\bar{t}^W - \hat{t}^W], \tag{16}$$

$$w(t) = \frac{p^W \lambda_C \hat{t}^W}{2} - \int_t^{\hat{t}^W} F_1^S\left(\tau, 2\bar{t}^W - \tau\right) d\tau, \text{ for } t \in [t_{\min}^W, \hat{t}^W), \tag{17}$$

$$w(t) = F^S\left(t, 2\bar{t}^W - t\right) - \frac{p^W \lambda_C \hat{t}^W}{2} + \int_{2\bar{t}^W - t}^{\hat{t}^W} F_1^S\left(\tau, 2\bar{t}^W - \tau\right) d\tau, \text{ for } t \in (2\bar{t}^W - \hat{t}^W, t_{\max}^W]. \tag{18}$$

Thus, a competitive equilibrium under offshoring is a set of equations of (12), (13), (14), (15), (16), (17), and (18). As proved in Appendix A, there exists a unique equilibrium under offshoring.

Let p^H and p^F represent the autarky relative prices of good C in Home and Foreign, respectively. If the distribution of skills in Home is more diverse than that in Foreign, it is clear from (11) that Home has a more diverse distribution of skills relative to the world and the world has a more diverse distribution of skills relative to Foreign. Following Lemma 2, we have the following.

Lemma 4 *Suppose that the distribution of skills in Home is more diverse than that in Foreign, p^W satisfies $p^H > p^W > p^F$. In addition, we have $\hat{t}^H < \hat{t}^W < \hat{t}^F$ and $m^F(\hat{t}^F) < m^W(\hat{t}^W) < m^H(\hat{t}^H)$.*

Since offshoring decreases the relative price of good C in Home, which leads to $\hat{t}^W > \hat{t}^H$ and $m^W(\hat{t}^W) < m^H(\hat{t}^H)$, then workers move out of sector C in Home under offshoring. In Foreign, since the relative price of good C increases, workers exit sector S.

4.1.2 Free Trade

We consider a situation in which workers cannot collaborate in international teams and goods are traded freely between the two countries as described above. In the free-trade equilibrium, the relative prices of good C in both countries are equalized. Using (5), we obtain

$$\frac{F^S\left(\hat{t}_T^H, 2\bar{t}^H - \hat{t}_T^H\right)}{\lambda_C \bar{t}^H} = \frac{F^S\left(\hat{t}_T^F, 2\bar{t}^F - \hat{t}_T^F\right)}{\lambda_C \bar{t}^F},$$

where $\hat{t}_T^H$ and $\hat{t}_T^F$ represent the marginal skill levels of sector C under free trade in Home and in Foreign, respectively. Since tasks are symmetric, $\bar{t}^H$ is equal to $\bar{t}^F$, and $F^S(\cdot)$ is homogeneous of degree one, $\hat{t}_T^H$ is equal to $\hat{t}_T^F$. In other words, the marginal skill levels of sector C are equalized across countries under free trade. In addition, the equations of (12), (13), (14), (15), (16), (17) and (18) also hold in the free-trade equilibrium with $\hat{t}^W = \hat{t}_T^H = \hat{t}_T^F$. It follows that the nominal wages of workers with the same skill level are equalized across countries under free trade. In other words, the wage schedule under offshoring is the same as that under free trade. Thus, we have the following.

Proposition 3 *If Home and Foreign share the same average skill level but differ in skill diversity, the nominal wages of workers with the same skill levels are equalized across countries under free trade; thus, there is no incentive for offshoring to occur. In addition, the offshoring equilibrium is the same as the free-trade equilibrium.*

When the two countries share the same average skill level but differ in skill diversity, workers in each country have matched with their best partners under free trade. Therefore, there are no incentives for workers to change their partner; thus, offshoring is not possible. If two countries have the same labor sizes, this free-trade equilibrium is the same as that in GM (2000); thus, the effects of offshoring on welfare are the same as the effects of free trade on welfare in GM (2000).

Interestingly, the offshoring equilibrium is the same as the free-trade equilibrium in this one-continuum-factor model. The classic paper of Mundell (1957) argues that free trade produces the same outcome as full integration with perfect factor mobility in a Heckscher-Ohlin context.

4.2 Average Skill Level and the World Economy

First, we investigate the effects of offshoring between two countries that differ in the average skill level. Next, we examine how free trade affects these two countries when offshoring is not allowed; then we compare the offshoring equilibrium with the free-trade equilibrium.

4.2.1 Average Skill Level and Offshoring

We assume that two countries share the same skill diversity but differ in the average skill level, i.e., Home has a higher average skill level than Foreign, i.e., $\bar{t}^H = \beta \bar{t}^F$, where $\beta > 1$, while the density functions of Home and Foreign satisfy $\phi^H(t) = \phi^F[t - (\beta - 1)\bar{t}^F]$ for $t \in [t^H_{\min}, t^H_{\max}]$ and $t^H_{\max} - t^H_{\min} = t^F_{\max} - t^F_{\min}$. For simplicity, we assume that both countries have the same labor size, i.e., $L_H = L_F$.[18] When two countries differ in the average skill level, the density function of skills under offshoring $\phi^W(t)$ is given by

$$\phi^W(t) = \begin{cases} \dfrac{\phi^F(t)}{2} & t \in \left[t^F_{\min}, t^H_{\min}\right) \\ \dfrac{\phi^H(t)}{2} + \dfrac{\phi^F(t)}{2} & t \in \left[t^H_{\min}, t^F_{\max}\right) \\ \dfrac{\phi^H(t)}{2} & t \in \left[t^F_{\max}, t^H_{\max}\right] \end{cases} . \tag{19}$$

Clearly, $\phi^W(t)$ is symmetric about the mean level of skills worldwide, $\bar{t}^W \equiv (\bar{t}^H + \bar{t}^F)/2$. Thus, Lemma 1 holds under offshoring with $\hat{t}^W$ and $m^W(\hat{t}^W)$ being the marginal skill levels in sector C, and the equations of (12), (13), (14), (15), (16), (17), and (18) also hold. Thus, there exists a unique equilibrium in this case of offshoring.

We begin with the relationship between the relative price of good C under offshoring and that of each country under autarky. Since the density function under offshoring mixes two countries' density functions, offshoring has two effects on the relative price of good C: the mean

[18] If $L_H \neq L_F$, $\phi^W(t)$ will not be symmetric about $\bar{t}^W$.

effect and the skill diversity effect, which will be investigated separately. We first examine the mean effect using Lemma 3, i.e., we consider only how offshoring affects the relative price of good C through the differences among the mean skill levels of the world and of each country. The density function under offshoring has a lower (higher) average skill level than Home's (Foreign's). The mean effect leads to the relative price under offshoring, p^W, being higher than Home's autarky relative price, p^H, while lower than Foreign's autarky relative price, p^F, i.e., $p^H < p^W < p^F$.

Next, we turn to the skill diversity effect. Similarly, we consider only how offshoring affects the relative price of good C through comparing skill diversity between the distribution of skills under offshoring and each country's distribution of skills. Since a parallel shift $(\beta - 1)\bar{t}^F$ of Foreign's density function of skills coincides completely with Home's density function, the distributions of skills in Home and in Foreign share the same skill diversity. If each country's density function is an increasing function at $[t^i_{\min}, \bar{t}^i]$, i.e., $d\phi^i(t)/dt \geqq 0$, where $i = H, F$, we show in Appendix B that the distribution of skills under offshoring is more diverse than each country's distribution of skills. For example, truncated normal distributions, uniform distributions, and beta distributions with two parameters equal and their values larger than 1 satisfy this assumption.

The skill diversity effect is obtained by using Lemma 2: the relative price under offshoring p^W is higher than both p^H and p^F. Clearly, both the mean effect and the skill diversity effect lead to $p^W > p^H$ and $\hat{t}^H > \hat{t}^W$. However, we obtain $m^W(\hat{t}^W) > m^H(\hat{t}^H)$ due to the skill diversity effect (Lemma 2), while $m^W(\hat{t}^W) < m^H(\hat{t}^H)$ due to the mean effect (Lemma 3). We prove in Appendix B that the mean effect dominates the skill diversity effect such that we have $m^W(\hat{t}^W) < m^H(\hat{t}^H)$ under the sufficient condition that $\min\{\phi^F(t)\} \geqq \frac{1}{2\bar{t}^F}$, for $t \in [t^F_{\min}, t^F_{\max}]$.

We turn to the relationship between p^W and p^F. As discussed in the above, the mean effect contradicts the skill diversity effect. We prove in Appendix B that there exists a unique threshold gap between the average skill levels in both countries, β^*, such that if $\beta < \beta^*$, we have $t^H_{\min} < \hat{t}^W < 2\bar{t}^W - \hat{t}^W < t^F_{\max}$, and if $\beta \geqq \beta^*$, $\hat{t}^W \leqq t^H_{\min} < t^F_{\max} \leqq 2\bar{t}^W - \hat{t}^W$ holds.[19] When $\beta < \beta^*$, we demonstrate that if β sufficiently closes to 1, we have $p^F > p^W$, which implies that the mean effect dominates the skill diversity effect. It follows that $\hat{t}^W > \hat{t}^F$. When $\beta \geqq \beta^*$, we show that there is a possibility that $p^F < p^W$. That is, if the gap between the average skill levels in both countries is sufficiently large, the mean effect is overwhelmed by the skill diversity effect; thus, there is a possibility that $p^F < p^W$. Regardless of the value of β, both the mean effect and the skill diversity effect lead to $m^F(\hat{t}^F) < m^W(\hat{t}^W)$ under the sufficient condition that $\min\{\phi^F(t)\} \geqq \frac{1}{2\bar{t}^F}$, for $t \in [t^F_{\min}, t^F_{\max}]$.

[19] We focus on the case in which $t^H_{\min} < t^F_{\max}$ in this paper.

Proposition 4 *Suppose that Home has a higher average skill level than Foreign. If the density function of skills in each country is single-peaked, i.e., $d\phi^i(t)/dt \geqq 0$ for $t \in [t^i_{\min}, \bar{t}^i]$, where $i = H, F$, we have $p^W > p^H$ and $\hat{t}^H > \hat{t}^W$. In addition, we have $m^H(\hat{t}^H) > m^W(\hat{t}^W) > m^F(\hat{t}^F)$ under the sufficient condition that $\min\{\phi^F(t)\} \geqq \frac{1}{2\bar{t}^F}$, for $t \in [t^F_{\min}, t^F_{\max}]$. Moreover, there exists a unique β^*, if $\beta(< \beta^*)$ is sufficiently close to 1, we have $p^F > p^W > p^H$ and $\hat{t}^H > \hat{t}^W > \hat{t}^F$; if $\beta \geqq \beta^*$, there is a possibility of $p^W > p^F > p^H$.*

If the distribution of skills in each country follows a uniform distribution function, we prove that if $\beta < \beta^*$, the mean effect dominates the skill diversity effect, and we have $p^F > p^W$. If $\beta > \beta^*$, the skill diversity effect dominates the mean effect so that there is a possibility of $p^F < p^W$. Moreover, the threshold gap between the average skill levels in both countries, β^* is also the threshold value at which the mean effect is offset by the skill diversity effect.[20]

Corollary 1 *Suppose that Home has a higher average skill level than Foreign. If the distribution of skills in each country follows a uniform distribution function, there exists a unique β^*such that if $\beta < \beta^*$, we have $p^F > p^W > p^H$, $\hat{t}^H > \hat{t}^W > \hat{t}^F$ and $m^H(\hat{t}^H) > m^W(\hat{t}^W) > m^F(\hat{t}^F)$, and if $\beta > \beta^*$, there is a possibility of $p^W > p^F > p^H$.*

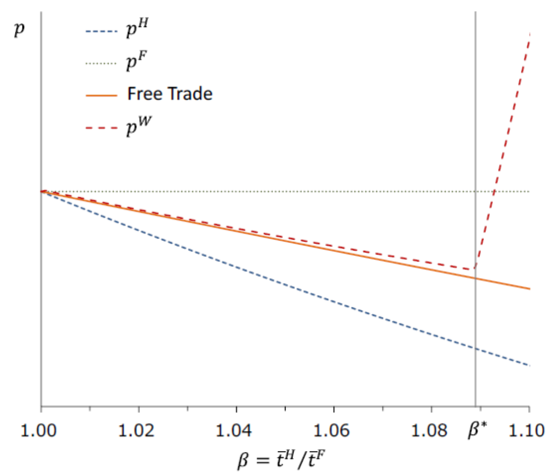
Intuitively, the mean effect (a higher mean) implies that the productivity of the supermodular good is improved more than that of submodular good, while the skill diversity effect (a higher skill diversity) implies the opposite. Therefore, when we compare the relative price of supermodular good under offshoring p^W with the Home autarky relative price p^H, both effects lead to a relative productivity gain in the submodular good under offshoring; consequently, we have $p^W > p^H$. When we compare p^W and p^F, if $\beta < \beta^*$, the mean effect dominates the skill diversity effect, and the net effect leads to a relative increase in the productivity of the supermodular good under offshoring; thus, p^W is lower than p^F. If $\beta > \beta^*$, the skill diversity effect overwhelms the mean effect so that the net effect brings about a relative improvement in the productivity of the submodular good under offshoring; thus, there is possibility of $p^W > p^F$.

Figure 4 shows that we can obtain $p^W > p^F > p^H$ when β is sufficiently large.[21] Stated differently, if there is a sufficiently large difference in the average skill levels between two countries,

[20] If each country's density function is a strictly increasing function at $[t^i_{\min}, \bar{t}^i]$, i.e., $d\phi^i(t)/dt > 0$, the threshold gap between the average skill levels in both countries, β^*, is different from the threshold value at which the mean effect is offset by the skill diversity effect.

[21] Following Bombardini, Gallipoli and Pupato (2012), we set the mean $\bar{t}^F = 5.59$ and the range of skills $t_{\max} - t_{\min} = 1$, which are consistent with that of the 1994–1998 IALS Log scores in United States. In the numerical example, we use a CES production function $F(t_1, t_2) = (t_1^\theta + t_2^\theta)^{1/\theta}$ with $\theta = 4$ for sector S and $\theta = 0.5$ for sector C. We assume the distribution of skills is uniform. The parameter α represents the consumption share of good C of a Cobb-Douglas utility function. In Figure 4, we change β in the range of 1 to 1.10 with $\alpha = 0.5$.

the relative price of C under offshoring could be higher than either autarky relative price. The possibility of $p^W > p^F > p^H$ is interesting. When two countries trade with each other without offshoring, the world relative price is always between the two autarky prices under incomplete specialization. Intuitively, offshoring combines the labor markets of two countries with widely different level of skills, which creates a labor force with greater skill diversity than that of either country under autarky; thus, the relative price of C under offshoring is higher than either autarky relative price. Moreover, if the preference is represented by a Cobb-Douglas utility function, β^* decreases in the consumption share of good C (see Appendix B). Thus, we have $p^W > p^F > p^H$ when the consumption share of C is sufficiently high, as shown in Figure 5.[22] As the consumption share of C increases, only the most productive teams remain in sector S, i.e., the pairs of workers with the highest and lowest skill levels. It follows that a lower threshold difference in the mean skill levels between two countries β^* is required to create a greater skill diversity to dominate the mean skill effect.



The Average Skill Level and Relative Price Comparison

Next, we turn to the effects of offshoring on nominal wages. We focus on the case of a small gap between the average skill levels in two countries, i.e., $\beta < \beta^*$. Clearly, the nominal wages of workers with the same skill levels are equalized between Home and Foreign, as shown in Figure 6. We can see that offshoring has opposite effects on the nominal wages of workers in the two countries. Offshoring affects the wage schedule through two effects: the skill diversity

[22] As in footnote 21, we set the mean $\bar{t}^H = 5.59$ and the range of skills $t_{\max} - t_{\min} = 1$. We use a CES production function $F(t_1, t_2) = (t_1^\theta + t_2^\theta)^{1/\theta}$ with $\theta = 4$ for sector S and $\theta = 0.5$ for sector C. In Figure 5, we change α in a range from 0.004 to 0.968 with $\beta = 1.07$, consistent with the ratio of the highest average IALS score to the lowest one in Table 2 in Bombardini, Gallipoli and Pupato (2012, page 2338).

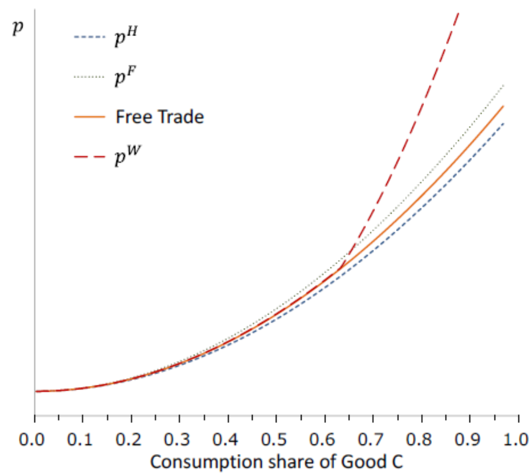


Figure 4: Consumption Share of Good C and Relative Price Comparison

effect (Proposition 1) and the mean effect (Proposition 2). Compared with Home's density function of skills, the density function under offshoring is more diverse and has a lower average skill level. Both effects lead to an increase in the nominal wages of workers who remain in the supermodular sector after offshoring, because both effects lead to an increase in the relative price of good C. For remainders in the submodular sector, the nominal wages of workers with skills at the upper end of the distribution increase, while the nominal wages of workers at the lower end of the distribution decline due to both effects. In addition, among workers with skill $t \in [\hat{t}^W, \hat{t}^H)$ who switch from sector S to sector C, some with $t \in (t_{HO}, \hat{t}^H)$, where $\hat{t}^W < t_{HO} < \hat{t}^H$, experience an increase in nominal wages because the increase in the relative price of good C dominates the effects of switching their partners from a higher skill level to a skill level equal to theirs, and others with $t \in [\hat{t}^W, t_{HO})$ suffer from switching sectors because the increase in the relative price of good C cannot offset the effects of losing their previous partners. Workers with $t \in (m^W(\hat{t}^W), m^H(\hat{t}^H)]$ obtain higher nominal wages by switching from sector C to sector S because they have sufficient skills to earn better nominal wages in sector S (see proof in Appendix B).

We know that Foreign's density function of skills has a lower average skill level and is less diverse than that under offshoring. The two effects affect the nominal wages of workers in opposite directions. We prove in Appendix B that the mean effect dominates the skill diversity effect so that the nominal wages of remainders in the supermodular sector decline due to a decrease in the relative price of good C when β is sufficiently close to 1, or each country's

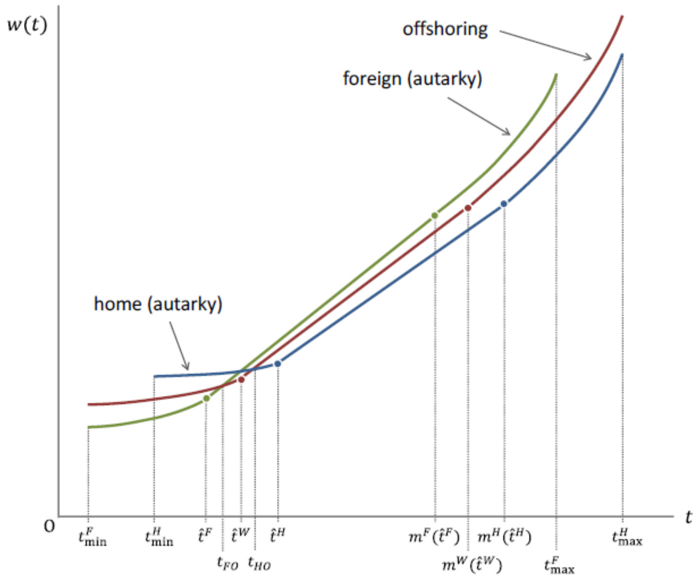


Figure 5: The Average Skill Level and Wage Schedules under Offshoring

distribution of skills follows a uniform distribution function. Since the mean effect dominates the skill diversity effect, workers with the lowest skill levels benefit from offshoring. Hence, in the submodular sector, the nominal wages of remainders with skills at the upper end of the distribution decrease, while the nominal wages of remainders with skills at the lower end of the distribution rise. Moreover, switchers with $t \in [\hat{t}^F, t_{FO})$, i.e., relatively low-skilled levels, enjoy an increase in nominal wages due to switching from sector C to sector S, while workers with $t \in (t_{FO}, \hat{t}^W)$ and workers with $t \in (m^F(\hat{t}^F), m^W(\hat{t}^W)]$, i.e., relatively high-skilled levels, experience a decrease in nominal wages due to switching sectors.

Proposition 5 *Suppose that the gap between the average skill levels in two countries is sufficiently small. The nominal wages of workers with the same skill levels are equalized between Home and Foreign under offshoring. In the country with a higher average skill level, offshoring decreases the nominal wages of workers with skills* $t \in [t_{\min}^H, t_{HO})$ *while increasing the nominal wages of workers with skills* $t \in (t_{HO}, t_{\max}^H]$*. In the country with a lower average skill level, the nominal wages of workers with skills* $t \in [t_{\min}^F, t_{FO})$ *increase while the nominal wages of the rest of the workers decrease.*

When two countries differ in only the average skill levels, offshoring allows the most skilled workers in the home country, i.e., the highest skilled workers in the world, to match with their best partners in the world, i.e., workers with lowest skills in the foreign country and also in the world, which leads to an increase in the nominal wages of both parties due to the nature of submodularity ($F^S_{ij} < 0$). Thus, offshoring gives workers opportunities to find better partners in the world, which can be supported by the study of Bernard, Moxnes, and Saito (2019). They show that the 2004 opening of the southern portion of the high-speed rail lines in Japan (Kyushu Shinkansen) leads to the reductions in search costs and buyer-seller inefficiencies, which allow firms to match with more and better suppliers. However, the lowest-skilled workers in the home country lose because they are displaced by foreign counterparts and thus have to team with workers having lower skill levels than their original partners.

4.2.2 Average Skill Level and Free Trade

We investigate the effects of trade between Home and Foreign when offshoring is not allowed. As in Section 4.2.1, the two countries only differ in the average skill level, and Home has a higher average skill level than Foreign, i.e., $\bar{t}^H = \beta\bar{t}^F$, where $\beta > 1$. Following Lemma 3, it is clear that Home has a comparative advantage in the supermodular good (good C). Under free trade, the two countries share the same marginal rate of substitution of the production possibility frontier, i.e., MRT^H is equal to MRT^F. Substituting $\bar{t}^H = \beta\bar{t}^F$ into (4), we obtain

$$\frac{F^S\left(\hat{t}^H_T, 2\beta\bar{t}^F - \hat{t}^H_T\right)}{\lambda_C\beta\bar{t}^F} = \frac{F^S\left(\hat{t}^F_T, 2\bar{t}^F - \hat{t}^F_T\right)}{\lambda_C\bar{t}^F},$$

where $\hat{t}^H_T$ and $\hat{t}^F_T$ represent the marginal skill levels of sector C in Home and Foreign, respectively. Since tasks are symmetric, and $F^S(\cdot)$ is homogeneous of degree one, we have $\hat{t}^H_T = \beta\hat{t}^F_T$.

Next, we examine the wage schedules of the two countries under free trade. Since $\hat{t}^H_T$ is equal to $\beta\hat{t}^F_T$, and the two countries share the same relative price of good C in the free-trade equilibrium, wage schedules are not equalized between the countries as illustrated in Figure 7 (see the proof in Appendix B). Thus, we have the following proposition.

Proposition 6 *If two countries differ in only the average skill level, the nominal wages of some workers with the same skill levels are not equalized under free trade; thus, offshoring is possible.*

Clearly, whether nominal wages of workers are equalized across countries depends on the relationship between $\hat{t}^H_T$ and $\hat{t}^F_T$ in the free-trade equilibrium. In GM (2000), the two countries share the same average skill level, which leads to $\hat{t}^H_T$ equal to $\hat{t}^F_T$; thus, wage schedules are equalized across countries in the free-trade equilibrium. When countries differ in average skill

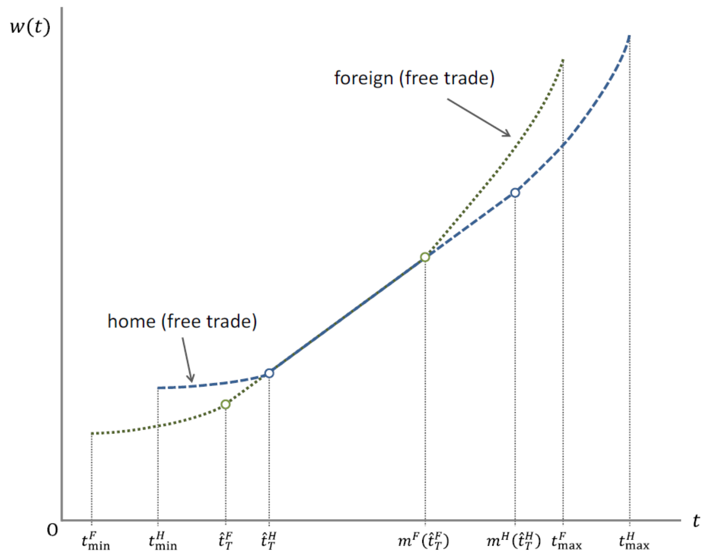


Figure 6: The Average Skill Levels and Wages Under Free Trade

levels, $\hat{t}^H_T$ and $\hat{t}^F_T$ are not equal. Thus, the difference in the average skill levels across countries leads to wage unequalization and opens the door for offshoring.

Interestingly, even in the case in which the free-trade equilibrium has no trade in final goods, as shown by Proposition 3 in GM (2000), the nominal wages of some workers with the same skill levels are not equalized if the two countries differ in the average skill level. GM (2000, Proposition 3) demonstrates that if $\Phi^H(\beta t) = \Phi^F(t)$, $\beta \neq 0$, for all $t \in [t^F_{\min}, t^F_{\max}]$, then the free-trade equilibrium has no trade in final goods.[23] If $\beta \neq 1$, $\hat{t}^H_T$ and $\hat{t}^F_T$ are not equal, the nominal wages of some workers with the same skill level are not equalized in the free-trade equilibrium. Thus, we obtain

Corollary 2 *If* $\Phi^H(\beta t) = \Phi^F(t)$, $\beta \neq 0$, *then the free-trade equilibrium has no trade in final goods. However, the nominal wages of workers with the same level of skills are not equalized across countries; thus, offshoring is possible.*

Whether wage schedules across countries are equalized depends solely on the average skill levels under free trade. If two countries share the same average skill level, all workers have paired with their best partners under free trade; thus, there is no incentive for offshoring. However, if two countries differ in average skill levels, there exist incentives for workers to find better partners in the foreign country; thus offshoring is possible.

[23] In this case, the two countries' autarky relative prices are equal.

5 Income Distribution

In this section, we examine how offshoring and free trade affect the income distribution. We assume that the density function of skills in each country is single-peaked, i.e., $d\phi^i(t)/dt \gtreqq 0$ for $t \in [t^i_{\min}, \bar{t}^i]$, where $i = H, F$, and the two countries differ in only the average skill levels.[24] We use the indirect utility function, $V(p, 1, w(t))$, to measure workers' welfare. We first investigate the effects of offshoring on workers' welfare. Next, we examine the effects of free trade on the welfare of workers.

5.1 Income Distribution under Offshoring

We consider a small gap between the average skill levels in the two countries, i.e., $\beta < \beta^*$. In particular, we focus on the case where $p^F > p^W > p^H$ holds. We begin with the effects of offshoring on the welfare of workers in Home (see proof in Appendix B). Compared with offshoring, Home has a lower autarky relative price of the supermodular good (good C). Since offshoring leads to a decline in the nominal wages (in terms of good S) of workers with skills $t \in [t^H_{\min}, t_{HO})$; thus, they are worse off. Workers with skills $t \in [t_{HO}, \tilde{t}_{HO})$, where $\tilde{t}_{HO} \in [t_{HO}, \hat{t}^H]$, are worse off because the increase in their nominal wages is dominated by the increase in the relative price. For workers with $t \in (\tilde{t}_{HO}, m^W(\hat{t}^W)]$, the increase in the nominal wages dominates the increase in the relative price, and thus, they benefit from offshoring. For the remaining workers, the effects of offshoring on welfare are ambiguous because both their nominal wages and the relative price of good C increase. Figure 8 shows how offshoring affects the income distribution in Home, the country with the higher average skill level, with a numerical example.[25] Interestingly, offshoring affects workers in the submodular sector in opposite directions: workers at the upper end of the skill distribution benefit from offshoring, while workers at the lower end of the skill distribution lose.

We turn to the effects of offshoring on the welfare of workers in Foreign (see proof in Appendix B). We know that the relative price of good C under offshoring is lower than Foreign's autarky relative price if β is sufficiently close to 1 or each country's distribution of skills follows a uniform distribution function. Since offshoring affects the wage schedule in Home and that in Foreign in opposite directions, workers with skills $t \in [t^F_{\min}, t_{FO})$ experience an increase in their wages, and thus, they are better off. Workers with skills $t \in [t_{FO}, \tilde{t}_{FO})$, where $\tilde{t}_{FO} \in [t_{FO}, \hat{t}^W]$,

[24] The effects of offshoring on welfare when two countries differ only in skill diversity are qualitatively similar to the effects of trade on welfare in GM (2000).

[25] As in footnote 21, we set the mean $\bar{t}^H = 5.59$ and $t^H_{\max} - t^H_{\min} = 1$. We use a CES production function $F(t_1, t_2) = (t_1^\theta + t_2^\theta)^{1/\theta}$ with $\theta = 4$ for sector S and $\theta = 0.5$ for sector C. In Figures 8 and 9, we change α in the range from 0.004 to 0.652 with $\beta = 1.07$.

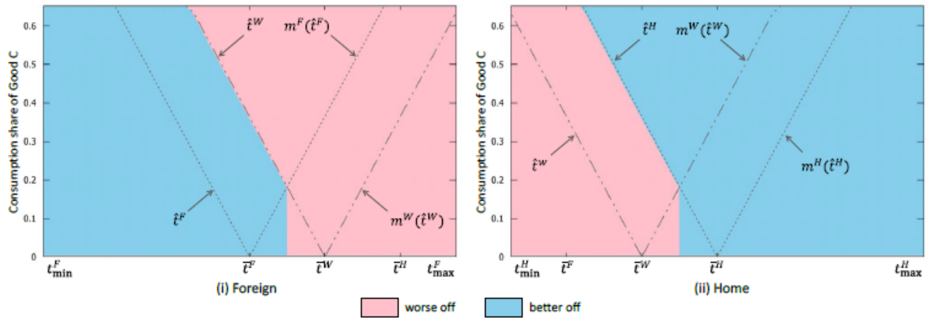


Figure 7: The Income Distribution under Offshoring

also gain from offshoring because the decrease in their nominal wages overwhelms the decline in the relative price. For workers with skills $t \in (\tilde{t}_{FO}, m^F(\hat{t}^F)]$, they lose because the decrease in their nominal wages dominates the decline in the relative price. The changes in welfare of the remaining workers are ambiguous because offshoring leads to a decrease in both their nominal wages and the relative price. Our numerical example for a foreign country illustrates that the effects of offshoring on the welfare of Foreign workers are the opposite of those on Home workers as in Figure 8. That is, workers at the upper end of the skill distribution lose from offshoring, while workers at the lower end of the skill distribution are better off.

5.2 Income Distribution under Free Trade

We turn to the effects of opening to trade on income distribution (see proof in Appendix B). We begin with the effects of trade on income distribution in Home. Next, we examine how trade affects the welfare of workers in Foreign.

Since Home has a higher average skill level than Foreign, Home will export good C in the free-trade equilibrium. It follows that the relative world price of supermodular good (good C) is higher than its autarky relative price. Hence, trade benefits workers who remain in the supermodular sector (export sector) after trade because their nominal wages increase proportion to the increase in the relative price. For workers who remain in the submodular sector (import competing sector) after trade, workers with skills at the lower end of the distribution lose because their nominal wages decrease. The effects of trade on workers at the upper end of the skill distribution are ambiguous because both their wages and the relative price of good C increase. Figure 9 shows the effects of trade on income distribution in Home based on a numerical example. We can see that workers at the upper and lower ends of the skill distribution lose from trade.

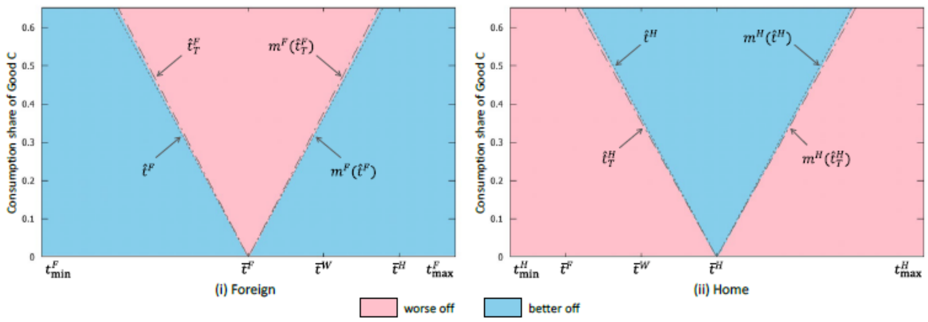


Figure 8: The Income Distribution under Free Trade

We turn to how trade affects the welfare of workers in Foreign (see proof in Appendix B). Since Foreign will import good C in the free-trade equilibrium, its autarky relative price of good C is higher than its relative world price. It follows that trade will affect the welfare of workers in Foreign and in Home in opposite directions. Comparing the effects of offshoring and trade on income distribution in both countries, we obtain the following proposition.

Proposition 7 *Offshoring and free trade have the qualitatively same effects on the welfare of workers in the supermodular sector. In the submodular sector, offshoring and free trade have the qualitatively same effects on workers with skills at the lower end of the distribution but may have different effects on the welfare of workers at the upper end of the skill distribution.*

Our numerical example shows that trade affects the welfare of workers in the same direction: all workers in the export sector gain from trade and all workers in the import sector lose from trade. However, offshoring affects the welfare of workers at the higher end and lower end of the distribution of skills in opposite directions. Offshoring offers an incentive for workers in the submodular sector to form international teams, which leads to labor reallocation. Workers with skill levels at the upper end of distribution in the country with the higher average skill level have the opportunity to form teams with better partners in the country with the lower average skill level. Therefore, those workers gain from offshoring, and low-skilled workers who are replaced by foreign workers lose. Offshoring affects workers in the country with with the lower average skill level in opposite directions since workers with skill levels at the lower end of distribution find better partners, and workers with skill levels at the upper end of the distribution lose their best partners.

6 Concluding Remarks

In this paper, we investigate how skill diversity and average skill level affect the possibility of offshoring. We demonstrate that if two countries have the same average skill level but differ in skill diversity, as in GM (2000), the wages of workers with the same level of skills are equalized under free trade; thus, there are no incentives for offshoring to occur. If two countries differ in only the average skill level, the wages of some workers with the same skill levels will not be equalized across countries; thus, offshoring is possible. In addition, we demonstrate that there is a possibility for the relative price under offshoring to be higher than both autarky relative prices due to a high skill diversity effect.

We also show that offshoring leads to a wage decline for workers with skills at the lower end of distribution in the country with a higher average skill level because some are displaced by their counterparts in the foreign countries. These displaced workers have to switch sectors and change their tasks (occupations), while there are also some workers with high skills who switch sectors to generate higher wages. Moreover, we demonstrate that free trade and offshoring have different effects on the welfare of workers with skills at the upper and lower ends of the distribution of skills. Our results are consistent with Artuc and McLaren (2015), who highlight that a worker's industry of employment determines whether she is harmed by a trade shock, while occupation determines who loses due to an offshoring shock.

Our paper is the first two-sector model of offshoring in which the production technologies of the two sectors are fundamentally different. Workers' skills are complementary (supermodular technology) in one sector, and the other sector features substitutable skills of workers (submodular technology). Thus, our model provides a variety of predictions that are empirically testable. First, controlling for skill diversity, the difference in the average skill levels between two countries is the determinant of offshoring and provides the predictions regarding wage distributions. Next, our model predicts that offshoring easily occurs in the sector in which workers' skills are substitutable. Finally, if we interpret tasks conducted by workers with high skills in the submodular sector as managerial tasks, our model provides a prediction that labor share (at least in managerial occupations) in the submodular sector increases with offshoring in a country with a higher average skill level.

In our paper, for simplicity we only consider one dimension of skills. In general, skills are multidimensional and some skills are sector-specific. For instance, one dimension is the language of English, which is essential for communication in most multinational teams. Thus, we interpret skills, which are owned by the lowest skill workers who work in multinational teams in a country with a lower average skill level, as sector-specific skills in our paper. The extension of multi-dimensional skills is left for future work.

Our paper abstracts from offshoring costs and imperfect observations of workers' skills in the current setup. In addition, to compare the effects of trade in goods and those of trade in tasks, we keep a key assumption of GM (2000) that the density functions of skills in each country are symmetry about the mean level of the skills. When this assumption is relaxed, offshoring may be possible when two countries share the same average skill level. These issues should be focused on in our future work.

Appendix A: Proofs for the Closed Economy

Proof for the Existence of a Unique Equilibrium

Using (3) and (5), we obtain the relative demand for good S

$$\frac{X_S}{X_C} \equiv D(\hat{t}) = f\left(\frac{F^S\left(\hat{t}, 2\bar{t}-\hat{t}\right)}{\lambda_C \bar{t}}\right), \quad \hat{t} \in [t_{\min}, \bar{t}].$$

Differentiating the above equation with respect to $\hat{t}$ yields

$$D'(\hat{t}) \equiv \left(\frac{F_1^S\left(\hat{t}, 2\bar{t}-\hat{t}\right) - F_2^S\left(\hat{t}, 2\bar{t}-\hat{t}\right)}{\lambda_C \bar{t}}\right) \cdot f'\left(\frac{F^S\left(\hat{t}, 2\bar{t}-\hat{t}\right)}{\lambda_C \bar{t}}\right).$$

Since $f'(\cdot) > 0$, the sign of $D'(\hat{t})$ depends on the sign of $F_1^S\left(\hat{t}, 2\bar{t}-\hat{t}\right) - F_2^S\left(\hat{t}, 2\bar{t}-\hat{t}\right)$.

We prove $F_1^S(t_1, t_2) - F_2^S(t_1, t_2) < 0$, if $t_1 < t_2$. Since the tasks are symmetric, we have $F^S(t_1, t_2) = F^S(t_2, t_1)$ and $F_2^S(t_1, t_2) = F_1^S(t_2, t_1)$. It follows that

$$F_1^S(t_1, t_2) - F_2^S(t_1, t_2) = F_1^S(t_1, t_2) - F_1^S(t_2, t_1) = F_1^S\left(\frac{t_1}{t_2}, 1\right) - F_1^S\left(\frac{t_2}{t_1}, 1\right).$$

The last equality is obtained because $F^S(t_1, t_2)$ is homogeneous of degree one; thus, $F_1^S(t_1, t_2)$ is homogeneous of degree zero. Since $F^S(t_1, t_2)$ exhibits submodularity, we have $F_{11}^S > 0$, yielding that $F_1^S(t_1/t_2, 1) - F_1^S(t_2/t_1, 1) < 0$, if $t_1 < t_2$. Therefore, we have $F_1^S\left(\hat{t}, 2\bar{t}-\hat{t}\right) - F_2^S\left(\hat{t}, 2\bar{t}-\hat{t}\right) < 0$ if $\hat{t} \in [t_{\min}, \bar{t})$. It follows that $D(\hat{t})$ is decreasing in $\hat{t}$, leading to $D(t_{\min}) > D(\bar{t}) > 0$.

Next, we turn to the supply side. Using (1) and (2), the relative supply of good S is shown by

$$\frac{Y_S}{Y_C} \equiv \mathcal{K}(\hat{t}) = \frac{2}{\lambda_C \bar{t}} \frac{\displaystyle\int_{t_{\min}}^{\hat{t}} F^S\left(t, 2\bar{t}-t\right)\phi(t)dt}{\displaystyle\int_{\hat{t}}^{2\bar{t}-\hat{t}} \phi(t)ds}, \quad \hat{t} \in [t_{\min}, \bar{t}]. \tag{A1}$$

Differentiating (A1) with respect to $\hat{t}$ yields

$$\frac{\partial \mathcal{K}}{\partial \hat{t}} = \frac{2\phi(\hat{t})}{\displaystyle\int_{\hat{t}}^{2\bar{t}-\hat{t}} \phi(t)dt} \left(p + \frac{Y_S}{Y_C}\right) > 0.$$

Therefore, $\mathcal{K}(\hat{t})$ is increasing in $\hat{t}$. In addition, we have $\mathcal{K}(t_{\min}) = 0$ and $\lim_{\hat{t}\to\bar{t}} \mathcal{K}(t) = +\infty$.

Since $D(\hat{t})$ is decreasing in $\hat{t}$ and $K(\hat{t})$ is increasing in $\hat{t}$, $D(t_{\min}) > K(t_{\min})$, and $D(\bar{t}) < \lim_{t\to\bar{t}} K(t)$, there exists a unique $\hat{t}$ satisfying $D(\hat{t}) = K(\hat{t})$. It follows that there exists a unique equilibrium relative price p due to $p = \frac{F^S\left(\hat{t}, 2\bar{t}-\hat{t}\right)}{\lambda_C \bar{t}}$ and $F_1^S\left(t, 2\bar{t}-t\right) - F_2^S\left(t, 2\bar{t}-t\right) < 0$.

Proof for Proposition 1

Let $w^*(t)$ represent the original wage schedule relative to the price of good S and $w(t)$ represent the wage schedule relative to the price of good S after a change in skill diversity. We first investigate how an increase in skill diversity affects the nominal wages of workers. Next, we examine the effects of an increase in skill diversity on the real wages of workers.

We start with the effects of an increase in skill diversity on the nominal wages of workers who remain in the same sectors as before. Note that a change in skill diversity leads to $\hat{t} < \hat{t}^* < m(\hat{t}^*) = 2\bar{t} - \hat{t}^* < m(\hat{t}) = 2\bar{t} - \hat{t}$. For workers with $t \in [\hat{t}^*, 2\bar{t} - \hat{t}^*]$, we have $w^*(t) < w(t)$ from (7) because an increase in skill diversity leads to an increase in p. For workers with $t \leqq \hat{t}$, the effects of a change in relative price on their nominal wages are obtained from (9):

$$\frac{dw(t)}{dp} = \left[\frac{\lambda_C}{2} p - F_1^S\left(\hat{t}, 2\bar{t}-\hat{t}\right)\right] \frac{\partial \hat{t}}{\partial p} + \frac{\lambda_C}{2} \hat{t}.$$

From (5), we obtain

$$\frac{p\lambda_C}{2} - F_1^S(\hat{t}, 2\bar{t}-\hat{t}) = \frac{2\bar{t}-\hat{t}}{2\bar{t}} \Big[F_2^S(\hat{t}, 2\bar{t}-\hat{t}) - F_1^S(\hat{t}, 2\bar{t}-\hat{t})\Big],$$

and

$$\frac{\partial \hat{t}}{\partial p} = \frac{\lambda_C \bar{t}}{F_1^S(\hat{t}, 2\bar{t} - \hat{t}) - F_2^S(\hat{t}, 2\bar{t} - \hat{t})}.$$

Therefore, we have

$$\frac{dw(t)}{dp} = -\frac{\lambda_C(2\bar{t} - \hat{t})}{2} + \frac{\lambda_C}{2}\hat{t} = -\lambda_C(\bar{t} - \hat{t}) < 0.$$

From Lemma 2, we know $p > p^*$. Therefore, we have $w(t) < w^*(t)$ for $t \leqq \hat{t}$. From (10), we obtain that the nominal wages for workers with skills at the upper end of the distribution increase, i.e.,$w^*(t) < w(t)$, for $t \geqq m(\hat{t})$.

Next, we turn to the nominal wages of workers who change sectors. We define $w_D(t) \equiv w(t) - w^*(t)$. For workers with $t \in [\hat{t}, \hat{t}^*]$, they shift from sector S to sector C; thus, we have

$$w_D(t) = \frac{p\lambda_C t}{2} - \frac{p^* \lambda_C \hat{t}^*}{2} + \int_t^{\hat{t}^*} F_1^S(\tau, 2\bar{t} - \tau) d\tau, \tag{A2}$$

where p^* represents the original relative price. From the above analysis, we have $w_D(\hat{t}) < 0$ and $w_D(\hat{t}^*) > 0$. Differentiating (A2) with respect to t and using (5), we obtain

$$\begin{aligned}
\frac{\partial w_D(t)}{\partial t} &= \frac{F^S(\hat{t}, 2\bar{t} - \hat{t})}{2\bar{t}} - F_1^S(t, 2\bar{t} - t) \\
&> \frac{F^S(\hat{t}^*, 2\bar{t} - \hat{t}^*)}{2\bar{t}} - F_1^S(\hat{t}^*, 2\bar{t} - \hat{t}^*) \\
&= \frac{2\bar{t} - \hat{t}^*}{2\bar{t}} \Big[F_2^S(\hat{t}^*, 2\bar{t} - \hat{t}^*) - F_1^S(\hat{t}^*, 2\bar{t} - \hat{t}^*) \Big] > 0.
\end{aligned}$$

where the second inequality is obtained from $F^S(\hat{t}, 2\bar{t} - \hat{t}) > F^S(\hat{t}^*, 2\bar{t} - \hat{t}^*)$ due to $F_1^S(\hat{t}, 2\bar{t} - \hat{t}) - F_2^S(\hat{t}, 2\bar{t} - \hat{t}) < 0$ and $F_1^S(t, 2\bar{t} - t) \leqq F_1^S(\hat{t}^*, 2\bar{t} - \hat{t}^*)$ due to $F_{11}^S(t, 2\bar{t} - t) - F_{12}^S(t, 2\bar{t} - t) > 0$. Since $w_D(\hat{t}) < 0 < w_D(\hat{t}^*)$ and $\partial w_D / \partial t > 0$ for $t \in [\hat{t}, \hat{t}^*]$, there exists a unique $\tilde{t} \in [\hat{t}, \hat{t}^*]$, at which $w_D(\tilde{t}) = 0$. Clearly, we have $w(t) < w^*(t)$ for $t \in [\hat{t}, \tilde{t})$ and $w^*(t) < w(t)$ for $t \in (\tilde{t}, \hat{t}^*]$.

Workers with $t \in [m(\hat{t}^*), m(\hat{t})]$ also shift from sector S to sector C. Thus, from (7) and (10), we have

$$\begin{aligned}
w_D(t) &= \frac{p\lambda_C t}{2} - F^S(t, 2\bar{t} - t) + \frac{p^* \lambda_C \hat{t}^*}{2} - \int_{2\bar{t}-t}^{\hat{t}^*} F_1^S(\tau, 2\bar{t} - \tau) d\tau \\
&= \frac{tF^S(\hat{t}, 2\bar{t} - \hat{t})}{2\bar{t}} - F^S(t, 2\bar{t} - t) + \frac{\hat{t}^* F^S(\hat{t}^*, 2\bar{t} - \hat{t}^*)}{2\bar{t}} - \int_{2\bar{t}-t}^{\hat{t}^*} F_1^S(\tau, 2\bar{t} - \tau) d\tau.
\end{aligned} \tag{A3}$$

Differentiating (A3) with respect to t and using $F_1(t,s) = F_2(s,t)$, we have

$$\begin{aligned}\frac{\partial w_D(t)}{\partial t} &= \frac{F^S(\hat{t}, 2\bar{t}-\hat{t})}{2\bar{t}} + F_2^S(t, 2\bar{t}-t) - F_1^S(t, 2\bar{t}-t) - F_1^S(2\bar{t}-t, t) \\ &= \frac{F^S(\hat{t}, 2\bar{t}-\hat{t})}{2\bar{t}} - F_1^S(t, 2\bar{t}-t).\end{aligned}$$

Thus, the second derivative of $w_D(t)$ is obtained as

$$\frac{\partial^2 w_D(t)}{\partial t^2} = F_{12}^S(t, 2\bar{t}-t) - F_{11}^S(t, 2\bar{t}-t) < 0.$$

Therefore, $w_D(t)$ is a concave function for $t \in [m(\hat{t}^*),\, m(\hat{t})]$. Due to $w_D(m(\hat{t}^*)) > 0$ and $w_D(m(\hat{t})) > 0$ from the above analysis, we have $w(t) > w^*(t)$, for $t \in [m(\hat{t}^*),\, m(\hat{t})]$.

Next, we turn to the welfare of workers. Since the preference is homothetic, the indirect utility function satisfies $V(p_C, p_S, W(t)) = v(p_C, p_S)W(t)$, where $W(t)$ is the wage for skill level t. Because $V(p_C, p_S, W(t))$ is homogeneous of degree zero with respect to prices and income, we have

$$V(p_C, p_S, W(t)) = V(p, 1, w(t)) = \nu(p)w(t),$$

where $\nu(p) \equiv v(p, 1)$, $p \equiv p_C/p_S$, and $w(t) \equiv W(t)/p_S$. Using Euler's homogeneous function theorem, we have

$$\frac{d\nu}{dp} = -\frac{1}{p}\left[v_2(p,1) + \nu(p)\right] < 0.$$

For workers with skill level $t \leqq \tilde{t}$, we have $w(t) \leqq w^*(t)$. Since $p > p^*$, we have $V(t) < V^*(t)$ for $t \leqq \tilde{t}$. For workers who remain in sector C when the skill diversity increases, we have

$$\begin{aligned}\frac{\partial V}{\partial p} &= w(t)\frac{d\nu(p)}{dp} + \nu(p)\frac{\partial w(t)}{\partial p} \\ &= \left[p\frac{d\nu(p)}{dp} + \nu(p)\right]\frac{\lambda_C t}{2} \\ &= -v_2(p,1)\frac{\lambda_C t}{2} > 0,\end{aligned}$$

Therefore, we obtain $V^*(t) < V(t)$ for $t \in [\hat{t}^*,\, m(\hat{t}^*)]$.

For workers with skill level $t \in (\tilde{t},\, \hat{t}^*]$, we have

$$V(t) - V^*(t) = \nu(p)\frac{p\lambda_C t}{2} - \nu(p^*)\left[\frac{p^*\lambda_C \hat{t}^*}{2} - \int_t^{\hat{t}^*} F_1^S(\tau, 2\bar{t}-\tau)d\tau\right].$$

Differentiating $V(t) - V^*(t)$ with respect to t yields

$$\frac{d}{dt}\left[V(t) - V^*(t)\right] = \nu(p)\frac{F^S\left(\hat{t}, 2\bar{t} - \hat{t}\right)}{2\bar{t}} - \nu(p^*)F_1^S\left(t, 2\bar{t} - t\right).$$

The second derivative of $V(t) - V^*(t)$ with respect to t is obtained as

$$\frac{d^2}{dt^2}\left[V(t) - V^*(t)\right] = \nu(p^*)\left[F_{12}^S(t, 2\bar{t} - t) - F_{11}^S(t, 2\bar{t} - t)\right] < 0,$$

implying that $V(t) - V^*(t)$ is a concave function. Since we already know that

$$\lim_{t \to \tilde{t}}\left[V(t) - V^*(t)\right] < 0 < \lim_{t \to \hat{t}^*}\left[V(t) - V^*(t)\right],$$

there uniquely exists $\tilde{\tilde{t}} \in (\tilde{t}, \hat{t}^*)$ such that $V(t) < V^*(t)$ for $t \in [\tilde{t}, \tilde{\tilde{t}})$ and $V^*(t) < V(t)$ for $t \in (\tilde{\tilde{t}}, \hat{t}^*]$.

Summarizing the above results, we have

$$\begin{cases} V(t) < V^*(t), & t^*_{\min} \leqq t < \tilde{\tilde{t}} \\ V^*(t) < V(t), & \tilde{\tilde{t}} < t \leqq m(\hat{t}^*) \end{cases}.$$

Proof for Lemma 3

Since the average skill level $\bar{t}$ increases from $\bar{t}^*$, i.e., $\bar{t} = \beta\bar{t}^*$, the equations of (1), (2), and (5) are rewritten as follows:

$$Y_C = \frac{L}{2}\lambda_C\beta\bar{t}^*\int_{\hat{t}}^{2\beta\bar{t}^* - \hat{t}}\phi(t)dt, \tag{A4}$$

$$Y_S = L\int_{t^*_{\min} + (\beta-1)\bar{t}^*}^{\hat{t}} F^S\left(t, 2\beta\bar{t}^* - t\right)\phi(t)dt, \tag{A5}$$

$$p = \frac{F^S\left(\hat{t}, 2\beta\bar{t}^* - \hat{t}\right)}{\lambda_C\beta\bar{t}^*}, \tag{A6}$$

where $\phi(t) = \phi^*[t - (\beta - 1)\bar{t}^*]$, $t_{\min} = t^*_{\min} + (\beta - 1)\bar{t}^*$, and $t_{\max} = t^*_{\max} + (\beta - 1)\bar{t}^*$.

From (A4), (A5), and (6), we have

$$\frac{Y_S}{Y_C}(\hat{t}, \beta) = f(p). \tag{A7}$$

From (A6), we obtain

$$\hat{t} = \hat{t}(p, \beta). \tag{A8}$$

From (A8), we have

$$\begin{aligned}
\frac{\partial \hat{t}}{\partial p}(p, \beta) &= \frac{\lambda_C \beta \bar{t}^*}{F_1^S\left(\hat{t}, 2\beta\bar{t}^* - \hat{t}\right) - F_2^S\left(\hat{t}, 2\beta\bar{t}^* - \hat{t}\right)} < 0, \\
\frac{\partial \hat{t}}{\partial \beta}(p, \beta) &= \frac{\hat{t}}{\beta} > 0.
\end{aligned}$$

due to $F_1^S\left(\hat{t}, 2\beta\bar{t}^* - \hat{t}\right) - F_2^S\left(\hat{t}, 2\beta\bar{t}^* - \hat{t}\right) < 0$.

Substituting (A8) into (A7) yields

$$\frac{Y_S}{Y_C}(\hat{t}(p, \beta), \beta) = f(p).$$

Differentiating this equation we obtain

$$\frac{dp}{d\beta} = -\frac{\dfrac{\partial Y_S/Y_C}{\partial \hat{t}}\dfrac{\partial \hat{t}}{\partial \beta} + \dfrac{\partial Y_S/Y_C}{\partial \beta}}{\dfrac{\partial Y_S/Y_C}{\partial \hat{t}}\dfrac{\partial \hat{t}}{\partial p} - f'(p)}. \tag{A9}$$

From (A4) and (A5), we obtain

$$\begin{aligned}
\frac{\partial (Y_S/Y_C)}{\partial \hat{t}} &= \frac{2\phi(\hat{t})}{\displaystyle\int_{\hat{t}}^{2\bar{t}-\hat{t}} \phi(t)dt}\left(p + \frac{Y_S}{Y_C}\right) > 0 \\
\frac{\partial (Y_S/Y_C)}{\partial \beta} &= \frac{2\bar{t}^*}{\lambda_C \bar{t}\displaystyle\int_{\hat{t}}^{2\bar{t}-\hat{t}} \phi(t)dt} \times \left\{\int_{t_{min}}^{\hat{t}} \left[F_1^S + F_2^S\right]\phi(t)dt - F^S(\hat{t}, 2\bar{t} - \hat{t})\phi(\hat{t})\right. \\
&\qquad \left. - \frac{\lambda_C}{2}\frac{Y_S}{Y_C}\left[2\bar{t}\phi(\hat{t}) + \int_{\hat{t}}^{2\bar{t}-\hat{t}} \phi(t)dt\right]\right\}.
\end{aligned}$$

Using the above equations, the numerator of (A9) is obtained as follows:

$$\frac{\partial Y_S/Y_C}{\partial \hat{t}}\frac{\partial \hat{t}}{\partial \beta}+\frac{\partial Y_S/Y_C}{\partial \beta}$$
$$=-\frac{2}{\beta\displaystyle\int_{\hat{t}}^{2\bar{t}-\hat{t}}\phi(t)dt}\times\left\{(\bar{t}-\hat{t})\phi(\hat{t})\left(p+\frac{Y_S}{Y_C}\right)-\frac{1}{\lambda_C}\int_{t_{min}}^{\hat{t}}(F_1^S+F_2^S)\phi(t)dt+\frac{1}{\lambda_c\bar{t}}\int_{t_{min}}^{\hat{t}}F^S\phi(t)dt\right\}$$
$$=-\frac{2}{\beta\displaystyle\int_{\hat{t}}^{2\bar{t}-\hat{t}}\phi(t)dt}\times\left\{(\bar{t}-\hat{t})\phi(\hat{t})\left(p+\frac{Y_S}{Y_C}\right)+\frac{1}{\lambda_C\bar{t}}\int_{t_{min}}^{\hat{t}}(\bar{t}-t)(F_2^S-F_1^S)\phi(t)dt\right\}<0.$$

Since the denominator of (A9) is negative, we have $\frac{dp}{d\beta}<0$.

From (A8), we have

$$d\hat{t}=\frac{\partial \hat{t}}{\partial p}dp+\frac{\partial \hat{t}}{\partial \beta}d\beta.$$

Since $\partial\hat{t}/\partial p<0$, $dp/d\beta<0$, and $\partial\hat{t}/\partial\beta>0$, an increase in β leads to an increase in $\hat{t}$.

Differentiating $m(\hat{t})=2\beta\bar{t}^*-\hat{t}$ with respect to β, we obtain

$$\frac{dm(\hat{t})}{d\beta}=\frac{d(2\beta\bar{t}^*-\hat{t})}{d\beta}=2\bar{t}^*-\frac{d\hat{t}}{d\beta}=\frac{2\bar{t}-\hat{t}}{\beta}-\frac{\partial\hat{t}}{\partial p}\frac{dp}{d\beta}.$$

From (A9), we obtain

$$\frac{dp}{d\beta}=-\frac{\frac{\partial Y_S/Y_C}{\partial\hat{t}}\frac{\partial\hat{t}}{\partial\beta}+\frac{\partial Y_S/Y_C}{\partial\beta}}{\frac{\partial Y_S/Y_C}{\partial\hat{t}}\frac{\partial\hat{t}}{\partial p}-f'(p)}>-\frac{\frac{\partial Y_S/Y_C}{\partial\hat{t}}\frac{\partial\hat{t}}{\partial\beta}+\frac{\partial Y_S/Y_C}{\partial\beta}}{\frac{\partial Y_S/Y_C}{\partial\hat{t}}\frac{\partial\hat{t}}{\partial p}},$$

where the inequality is due to $f'(p)>0$. Thus, we have

$$\begin{aligned}\frac{dm(\hat{t})}{d\beta}&=\frac{2\bar{t}-\hat{t}}{\beta}-\frac{\partial\hat{t}}{\partial p}\frac{dp}{d\beta}\\&>\frac{2\bar{t}-\hat{t}}{\beta}+\frac{\partial\hat{t}}{\partial\beta}+\frac{\partial Y_S/Y_C}{\partial\beta}\Big/\frac{\partial Y_S/Y_C}{\partial\hat{t}}=\frac{1}{\frac{\partial Y_S/Y_C}{\partial\hat{t}}}\left[\frac{2\bar{t}}{\beta}\frac{\partial Y_S/Y_C}{\partial\hat{t}}+\frac{\partial Y_S/Y_C}{\partial\beta}\right],\end{aligned}$$

where we have

$$\frac{2\bar{t}}{\beta}\frac{\partial Y_S/Y_C}{\partial \hat{t}}+\frac{\partial Y_S/Y_C}{\partial \beta}$$
$$=\frac{2}{\beta\int_{\hat{t}}^{2\bar{t}-\hat{t}}\phi(t)dt}\left[\bar{t}\phi(\hat{t})\left(p+\frac{Y_S}{Y_C}\right)-\frac{1}{\lambda_C\bar{t}}\int_{t_{min}}^{\hat{t}}(\bar{t}-t)(F_2^S-F_1^S)\phi(t)dt\right],$$
$$=\frac{2}{\beta\int_{\hat{t}}^{2\bar{t}-\hat{t}}\phi(t)dt}[\bar{t}\phi(\hat{t})p+\frac{1}{\lambda_C}\int_{t_{\min}}^{\hat{t}}(F_2^S+F_1^S)\phi(t)dt+\bar{t}\phi(\hat{t})\frac{Y_S}{Y_C}-\frac{1}{\lambda_C\bar{t}}\int_{t_{\min}}^{\hat{t}}F^S(t,2\bar{t}-t)\phi(t)dt].$$

Note that

$$\begin{aligned}
&\bar{t}\phi(\hat{t})\frac{Y_S}{Y_C}-\frac{1}{\lambda_C\bar{t}}\int_{t_{\min}}^{\hat{t}}F^S(t,2\bar{t}-t)\phi(t)dt\\
&=\frac{2\int_{t_{\min}}^{\hat{t}}F^S(t,2\bar{t}-t)\phi(t)dt}{\lambda_C\bar{t}\int_{\hat{t}}^{2\bar{t}-\hat{t}}\phi(t)dt}[2\bar{t}\phi(\hat{t})-\int_{\hat{t}}^{2\bar{t}-\hat{t}}\phi(t)dt]\\
&=\frac{2\int_{t_{\min}}^{\hat{t}}F^S(t,2\bar{t}-t)\phi(t)dt}{\lambda_C\bar{t}\int_{\hat{t}}^{2\bar{t}-\hat{t}}\phi(t)dt}[2\bar{t}\phi(\hat{t})-2\int_{\hat{t}}^{\bar{t}}\phi(t)dt]\\
&\geqq\frac{2\int_{t_{\min}}^{\hat{t}}F^S(t,2\bar{t}-t)\phi(t)dt}{\lambda_C\bar{t}\int_{\hat{t}}^{2\bar{t}-\hat{t}}\phi(t)dt}(2\bar{t}\phi(\hat{t})-1)
\end{aligned}$$

where the second inequality is derived because $\phi(t)$ is symmetric. Thus, if $\min\{\phi(t)\}\geqq\frac{1}{2\bar{t}}$, we have $\frac{dm(\hat{t})}{d\beta}>0$.

Proof for Proposition 2

We first prove the effects of an increase in the average skill level on the nominal wages. Next, we prove the effects of an increase in the average skill level on the real wages. Let $w^*(t)$, $t\in[t^*_{\min},t^*_{\max}]$, represent the original wage schedule and $w(t)$, $t\in[t_{\min},t_{\max}]$, represent the wage schedule after an increase in the average skill level. Since the average skill level increases from $\bar{t}^*$ to $\bar{t}=\beta\bar{t}^*$, $\beta>1$, we have $\hat{t}=\hat{t}(\beta)$ and $p=p(\beta)$ from (A8) and (A9). Thus, the wage schedule $w(t)$ is rewritten as follows:

$$w(t,\beta)=\begin{cases}\dfrac{\lambda_C p(\beta)\hat{t}(\beta)}{2}-\displaystyle\int_t^{\hat{t}(\beta)} F_1^S\left(\tau,2\beta\bar{t}^*-\tau\right)d\tau, & t<\hat{t}(\beta)\\ \dfrac{p(\beta)\lambda_C t}{2}, & \hat{t}(\beta)\leqq t\leqq 2\beta\bar{t}^*-\hat{t}(\beta)\\ F^S(t,2\beta\bar{t}^*-t)-\dfrac{p(\beta)\lambda_C\hat{t}(\beta)}{2}+\displaystyle\int_{2\beta\bar{t}^*-t}^{\hat{t}(\beta)} F_1^S\left(\tau,2\beta\bar{t}^*-\tau\right)d\tau, & t>2\beta\bar{t}^*-\hat{t}(\beta)\end{cases}. \tag{A10}$$

Note that $w^*(t)=w(t,1)$. Following Lemma 3, if $\min\{\phi(t)\}\geqq\frac{1}{2\bar{t}}$ is satisfied everywhere, we have $\hat{t}^*<\hat{t}$ and $m^*(\hat{t}^*)=2\bar{t}^*-\hat{t}^*<2\bar{t}-\hat{t}=m(\hat{t})$. Differentiating $w(t,\beta)$ with respect to β yields

$$\frac{\partial w(t,\beta)}{\partial\beta}=\begin{cases}-\lambda_C\left(\bar{t}-\hat{t}\right)\dfrac{dp}{d\beta}+\dfrac{\left(2\bar{t}-\hat{t}\right)\left[F_2^S\left(\hat{t},2\bar{t}-\hat{t}\right)-F_1^S\left(\hat{t},2\bar{t}-\hat{t}\right)\right]}{2\bar{t}}\dfrac{\partial\hat{t}}{\partial\beta}-2\bar{t}^*\displaystyle\int_t^{\hat{t}} F_{12}^S\left(\tau,2\bar{t}-\tau\right)d\tau>0\\ \dfrac{\lambda_C t}{2}\dfrac{dp}{d\beta}<0\\ \lambda_C\left(\bar{t}-\hat{t}\right)\dfrac{dp}{d\beta}-\dfrac{\left(2\bar{t}-\hat{t}\right)\left[F_2^S-F_1^S\right]}{2\bar{t}}\dfrac{\partial\hat{t}}{\partial\beta}+2\bar{t}^*\displaystyle\int_{2\bar{t}-t}^{\hat{t}} F_{12}^S(\tau,2\bar{t}-\tau)d\tau<0\end{cases}. \tag{A11}$$

We begin by comparing the nominal wages of workers who stay in the same sector before and after the increase in the average skill level. Using (A10) and (A11), we obtain

$$\begin{aligned} w^*(t)<w(t), &\quad t\in[t_{\min},\,\hat{t}^*],\\ w(t)<w^*(t), &\quad t\in[\hat{t},\,2\bar{t}^*-\hat{t}^*],\\ w(t)<w^*(t), &\quad t\in[2\bar{t}-\hat{t},\,t^*_{\max}]. \end{aligned}$$

Next, we turn to comparing the nominal wages of workers who change their sectors after an increase in the average skill level, i.e., workers with $t\in[\hat{t}^*,\,\hat{t}]$ and $t\in[2\bar{t}^*-\hat{t}^*,\,2\bar{t}-\hat{t}]$. Let $w_D(t)\equiv w(t)-w^*(t)$. For $t\in[\hat{t}^*,\,\hat{t}]$, we have

$$w_D(t)=\frac{p\lambda_C\hat{t}}{2}-\int_t^{\hat{t}} F_1^S\left(\tau,2\bar{t}-\tau\right)d\tau-\frac{p^*\lambda_C t}{2}. \tag{A12}$$

Note that we have already proved that $w_D(\hat{t}^*)>0$ and $w_D(\hat{t})<0$. Differentiating (A12) with respect to t, we obtain

$$\begin{aligned}
\frac{\partial w_D(t)}{\partial t} &= F_1^S\left(t, 2\bar{t}-t\right) - \frac{p^*\lambda_C}{2} \\
&< F_1^S\left(t, 2\bar{t}-t\right) - \frac{p\lambda_C}{2} \\
&= F_1^S\left(t, 2\bar{t}-t\right) - \frac{F^S\left(\hat{t}, 2\bar{t}-\hat{t}\right)}{2\bar{t}} \\
&\leqq F_1^S\left(\hat{t}, 2\bar{t}-\hat{t}\right) - \frac{F^S\left(\hat{t}, 2\bar{t}-\hat{t}\right)}{2\bar{t}} \\
&= \frac{2\bar{t}-\hat{t}}{2\bar{t}}\left[F_1^S(\hat{t}, 2\bar{t}-\hat{t}) - F_2^S(\hat{t}, 2\bar{t}-\hat{t})\right] < 0.
\end{aligned}$$

Therefore, there exists a unique $\tilde{t}_m \in [\hat{t}^*, \hat{t}]$ such that $w_D(\tilde{t}_m)$ equals zero. It follows that $w^*(t) < w(t)$, for $t \in [\hat{t}^*, \tilde{t}_m)$, and $w(t) < w^*(t)$, for $t \in (\tilde{t}_m, \hat{t}]$.

For $t \in [2\bar{t}^* - \hat{t}^*,\ 2\bar{t} - \hat{t}]$, we have

$$w_D(t) = \frac{p\lambda_C t}{2} - \left[F^S(t, 2\bar{t}^*-t) - \frac{p^*\lambda_C\hat{t}^*}{2} + \int_{2\bar{t}^*-t}^{\hat{t}^*} F_1^S\left(\tau, 2\bar{t}^*-\tau\right) d\tau\right]. \tag{A13}$$

Note that we have proved that $w_D(2\bar{t}^* - \hat{t}^*) < 0$ and $w_D(2\bar{t} - \hat{t}) < 0$. Differentiating (A13) with respect to t yields

$$\begin{aligned}
\frac{\partial w_D}{\partial t} &= \frac{p\lambda_C}{2} - F_1^S(t, 2\bar{t}^*-t) \\
&< \frac{p^*\lambda_C}{2} - F_1^S(t, 2\bar{t}^*-t) \\
&= \frac{F^S\left(\hat{t}^*, 2\bar{t}^*-\hat{t}^*\right)}{2\bar{t}^*} - F_1^S(t, 2\bar{t}^*-t) \\
&\leqq \frac{F^S\left(\hat{t}^*, 2\bar{t}^*-\hat{t}^*\right)}{2\bar{t}^*} - F_1^S(2\bar{t}^*-\hat{t}^*, \hat{t}^*) \\
&= \frac{F^S\left(\hat{t}^*, 2\bar{t}^*-\hat{t}^*\right)}{2\bar{t}^*} - F_2^S(\hat{t}^*, 2\bar{t}^*-\hat{t}^*) \\
&= \frac{\hat{t}^*}{2\bar{t}^*}\left[F_1^S(\hat{t}^*, 2\bar{t}^*-\hat{t}^*) - F_2^S(\hat{t}^*, 2\bar{t}^*-\hat{t}^*)\right] < 0.
\end{aligned}$$

Therefore, we have $w(t) < w^*(t)$ for $t \in [2\bar{t}^* - \hat{t}^*,\ 2\bar{t} - \hat{t}]$. Summarizing the above results, we obtain the effects of an increase in the average skill level on the nominal wages of workers as follows:

$$\begin{cases} w^*(t) < w(t), & t_{\min} \leqq t < \tilde{t}_m \\ w(t) < w^*(t), & \tilde{t}_m < t \leqq t^*_{\max} \end{cases}.$$

Next, we turn to the real wages of workers. For workers with skill level $t \leqq \tilde{t}_m$, we have $w^*(t) \leqq w(t)$. The result of $dp/d\beta < 0$ leads to $\partial\nu/\partial\beta > 0$, which follows $V^*(t) < V(t)$ for $t \leqq \tilde{t}$. For workers who remain in C sector when the average skill level increases, we have

$$\begin{aligned} \frac{\partial V}{\partial \beta} &= \left[w(t)\frac{d\nu(p)}{dp} + \nu(p)\frac{\partial w(t)}{\partial p}\right]\frac{dp}{d\beta} \\ &= \left[p\frac{d\nu(p)}{dp} + \nu(p)\right]\frac{\lambda_C t}{2}\frac{dp}{d\beta} \\ &= -v_2(p,1)\frac{\lambda_C t}{2}\frac{dp}{d\beta} < 0. \end{aligned}$$

Thus, we obtain that $V(t) < V^*(t)$ for $t \in [\hat{t},\, m^*(\hat{t}^*)]$. For workers with skill level $t \in (\tilde{t}_m,\, \hat{t}]$, $V(t) - V^*(t)$ is obtained as follows:

$$V(t) - V^*(t) = \nu(p)\left[\frac{p\lambda_C \hat{t}}{2} - \int_t^{\hat{t}} F_1^S\left(\tau, 2\bar{t} - \tau\right) d\tau\right] - \nu(p^*)\frac{p^*\lambda_C t}{2}.$$

Differentiating $V(t) - V^*(t)$ with respect to t yields

$$\frac{d}{dt}\left[V(t) - V^*(t)\right] = \nu(p)F_1^S\left(t, 2\bar{t} - t\right) - \nu(p^*)\frac{F^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)}{2\bar{t}^*}.$$

The second derivative of $V(t) - V^*(t)$ with respect to t is obtained as

$$\frac{d^2}{dt^2}\left[V(t) - V^*(t)\right] = \nu(p)\left[F_{11}^S(t, 2\bar{t} - t) - F_{12}^S(t, 2\bar{t} - t)\right] > 0,$$

implying that $V(t) - V^*(t)$ is a convex function. Since we already know that

$$\lim_{t \to \hat{t}}\left[V(t) - V^*(t)\right] < 0 < \lim_{t \to \tilde{t}_m}\left[V(t) - V^*(t)\right],$$

there uniquely exists $\tilde{\tilde{t}}_m \in (\tilde{t}_m,\, \hat{t})$ such that $V^*(t) < V(t)$ for $t \in [\tilde{t}_m,\, \tilde{\tilde{t}}_m)$ and $V(t) < V^*(t)$ for $t \in (\tilde{\tilde{t}}_m,\, \hat{t}]$.

Summarizing the above results regarding real wages, we have

$$\begin{cases} V^*(t) < V(t), & t_{\min} \leqq t < \tilde{\tilde{t}}_m \\ V(t) < V^*(t), & \tilde{\tilde{t}}_m < t \leqq m^*(\hat{t}^*) \end{cases}.$$

Appendix B: Proofs for the World Economy

Comparison of Diversity

The accumulative distribution of skills under offshoring, Φ^W, is shown as follows:

$$\Phi^W(t) = \begin{cases} \dfrac{1}{2}\displaystyle\int_{t_{\min}^F}^{t} \phi^F(\tau)d\tau, & t \in [t_{\min}^F, t_{\min}^H) \\ \dfrac{1}{2}\displaystyle\int_{t_{\min}^F}^{t} \phi^F(\tau)d\tau + \dfrac{1}{2}\displaystyle\int_{t_{\min}^H}^{t} \phi^H(\tau)d\tau, & t \in [t_{\min}^H, t_{\max}^F] \\ \dfrac{1}{2} + \dfrac{1}{2}\displaystyle\int_{t_{\min}^H}^{t} \phi^H(\tau)d\tau, & t \in (t_{\max}^F, t_{\max}^H] \end{cases}.$$

We focus on the case in which $t_{\min}^H < t_{\max}^F$, which requires that

$$\beta < \bar{\beta} \equiv 1 + \frac{t_{\max}^F - t_{\min}^F}{\bar{t}^F},$$

due to $t_{\min}^H = t_{\min}^F + (\beta - 1)\bar{t}^F$. We construct an accumulative distribution function $\Phi^X(t)$ as

$$\Phi^X(t) = \Phi^H\left(t + \frac{\beta - 1}{2}\bar{t}^F\right), \text{ for } t \in \left[t_{\min}^F, t_{\max}^H\right],$$

leading to

$$\phi^X(t) = \phi^H\left(t + \frac{\beta - 1}{2}\bar{t}^F\right), \text{ for } t \in \left[t_{\min}^F, t_{\max}^H\right].$$

Note that $\bar{t}^W = (1+\beta)\bar{t}^H/2\beta$ and $\Phi^W(\bar{t}^W) = \Phi^X(\bar{t}^W) = 1/2$. Thus, we obtain

$$\Phi^W(t) - \Phi^X(t)$$
$$= \begin{cases} \dfrac{1}{2}\displaystyle\int_{t^F_{\min}}^{t} \phi^F(\tau)d\tau, & t \in \left[t^F_{\min},\, t^H_{\min} - \dfrac{\beta-1}{2\beta}\bar{t}^H\right) \\ \dfrac{1}{2}\displaystyle\int_{t^H_{\min}}^{t+(\beta-1)\bar{t}^F} \phi^H(\tau)d\tau - \int_{t^H_{\min}}^{t+\frac{\beta-1}{2}\bar{t}^F} \phi^H(\tau)d\tau, & t \in \left[t^H_{\min} - \dfrac{\beta-1}{2\beta}\bar{t}^H,\, t^H_{\min}\right) \\ \dfrac{1}{2}\left[\displaystyle\int_{t^H_{\min}}^{t+(\beta-1)\bar{t}^F} \phi^H(\tau)d\tau + \int_{t^H_{\min}}^{t} \phi^H(\tau)d\tau\right] - \int_{t^H_{\min}}^{t+\frac{\beta-1}{2}\bar{t}^F} \phi^H(\tau)d\tau, & t \in [t^H_{\min},\, t^F_{\max}] \\ \displaystyle\int_{t+\frac{\beta-1}{2}\bar{t}^F}^{t^H_{\max}} \phi^H(\tau)d\tau - \dfrac{1}{2}\int_{t}^{t^H_{\max}} \phi^H(\tau)d\tau, & t \in \left(t^F_{\max},\, t^H_{\max} - \dfrac{\beta-1}{2\beta}\bar{t}^H\right] \\ -\dfrac{1}{2}\displaystyle\int_{t}^{t^H_{\max}} \phi^H(\tau)d\tau, & t \in \left(t^H_{\max} - \dfrac{\beta-1}{2\beta}\bar{t}^H,\, t^H_{\max}\right] \end{cases}$$

We start with the assumption that the symmetric density function $\phi^H(t)$ satisfies that $\frac{d\phi^H(t)}{dt} > 0$, for $t \in (t^H_{\min},\, \bar{t}^H)$, implying that $\frac{d\phi^H(t)}{dt} < 0$, for $t \in (\bar{t}^H,\, t^H_{\max})$. It follows that the cumulative distribution function $\Phi^H(t)$ is strictly convex at $[t^H_{\min},\, \bar{t}^H]$, while strictly concave at $[\bar{t}^H,\, t^H_{\max}]$. Before investigating the sign of $\Phi^W(t) - \Phi^X(t)$, we will prove that

$$\frac{\Phi^H(t)}{t - t^H_{\min}} > \frac{\Phi^H\left(\bar{t}^H\right)}{\bar{t}^H - t^H_{\min}},\ \text{for}\ t \in (\bar{t}^H,\, t^H_{\max}). \tag{B1}$$

Due to the convexity of $\Phi^H(t)$ at $[t^H_{\min},\, \bar{t}^H]$, for any $s, t \in (t^H_{\min},\, \bar{t}^H)$ and $t < s$, we obtain

$$\frac{d}{ds}\left(\frac{\Phi^H(s) - \Phi^H(t)}{s-t}\right) = \frac{\Phi^H(t) - \left[\Phi^H(s) + \phi^H(s)\,(t-s)\right]}{(s-t)^2} > \frac{\Phi^H(t) - \Phi^H(t)}{(s-t)^2} = 0. \tag{B2}$$

The concavity of $\Phi^H(t)$ at $[\bar{t}^H,\, t^H_{\max}]$ yields

$$\Phi^H(t) > \alpha_1\Phi^H(\bar{t}^H) + (1-\alpha_1)\Phi^H(t^H_{\max}),$$

where $t = \alpha_1\bar{t}^H + (1-\alpha_1)t^H_{\max} \in (\bar{t}^H,\, t^H_{\max})$ and $0 < \alpha_1 < 1$. It follows that

$$\begin{aligned}\Phi^H(t) &> \frac{t - t^H_{\max}}{\bar{t}^H - t^H_{\max}}\Phi^H\left(\bar{t}^H\right) + \frac{\bar{t}^H - t}{\bar{t}^H - t^H_{\max}}\Phi^H\left(t^H_{\max}\right) \\ &= \frac{t - t^H_{\min}}{\bar{t}^H - t^H_{\min}}\Phi\left(\bar{t}^H\right),\end{aligned}$$

leading to (B1).

We turn to the sign of $\Phi^W(t) - \Phi^X(t)$ for $t \in \left[t^H_{\min} - \frac{\beta-1}{2\beta}\bar{t}^H,\, t^H_{\min}\right)$. We have

$$\begin{aligned}
\Phi^W(t) - \Phi^X(t) &= \frac{1}{2}\left[\int_{t^H_{\min}}^{t+(\beta-1)\bar{t}^F} \phi^H(\tau)d\tau - 2\int_{t^H_{\min}}^{t+\frac{\beta-1}{2}\bar{t}^F} \phi^H(\tau)d\tau\right] \\
&= \frac{1}{2}\left[\Phi^H\left(t+(\beta-1)\bar{t}^F\right) - 2\Phi^H\left(t+\frac{\beta-1}{2}\bar{t}^F\right)\right] \\
&\geqq \frac{1}{2}\left[\Phi^H\left(2t+(\beta-1)\bar{t}^F - t^H_{\min}\right) - 2\Phi^H\left(t+\frac{\beta-1}{2}\bar{t}^F\right)\right],
\end{aligned}$$

where the inequality is due to $2t+(\beta-1)\bar{t}^F - t^H_{\min} \leqq t+(\beta-1)\bar{t}^F$ for $t \leqq t^H_{\min}$. If $2t+(\beta-1)\bar{t}^F - t^H_{\min} \leqq \bar{t}^H$, using (B2), we obtain

$$\frac{\Phi^H\left(t+\frac{\beta-1}{2}\bar{t}^F\right) - \Phi^H(t^H_{\min})}{t+\frac{\beta-1}{2}\bar{t}^F - t^H_{\min}} < \frac{\Phi^H\left(2t+(\beta-1)\bar{t}^F - t^H_{\min}\right) - \Phi^H(t^H_{\min})}{[2t+(\beta-1)\bar{t}^F - t^H_{\min}] - t^H_{\min}}.$$

Since $\Phi^H(t^H_{\min}) = 0$, we have

$$\Phi^H\left(2t+(\beta-1)\bar{t}^F - t^H_{\min}\right) > 2\Phi^H\left(t+\frac{\beta-1}{2}\bar{t}^F\right).$$

If $2t+(\beta-1)\bar{t}^F - t^H_{\min} > \bar{t}^H$, by using (B1) and (B2), we have

$$\frac{\Phi^H\left(2t+(\beta-1)\bar{t}^F - t^H_{\min}\right)}{[2t+(\beta-1)\bar{t}^F - t^H_{\min}] - t^H_{\min}} > \frac{\Phi^H\left(\bar{t}^H\right)}{\bar{t}^H - t^H_{\min}} > \frac{\Phi^H\left(t+\frac{\beta-1}{2}\bar{t}^F\right) - \Phi^H(t^H_{\min})}{t+\frac{\beta-1}{2}\bar{t}^F - t^H_{\min}},$$

leading to

$$\Phi^H\left(2t+(\beta-1)\bar{t}^F - t^H_{\min}\right) > 2\Phi^H\left(t+\frac{\beta-1}{2}\bar{t}^F\right).$$

Thus, we obtain

$$\Phi^W(t) - \Phi^X(t) > 0, \text{ for } t \in \left[t^H_{\min} - \frac{\beta-1}{2\beta}\bar{t}^H,\, t^H_{\min}\right). \tag{B3}$$

For $t \in [t^H_{\min},\, \bar{t}^W)$, $\Phi^W(t) - \Phi^X(t)$ is derived as follows:

$$\begin{aligned}
\Phi^W(t) - \Phi^X(t) &= \frac{1}{2}\left[\int_{t^H_{\min}}^{t+(\beta-1)\bar{t}^F} \phi^H(\tau)d\tau + \int_{t^H_{\min}}^{t} \phi^H(\tau)d\tau - 2\int_{t^H_{\min}}^{t+\frac{\beta-1}{2}\bar{t}^F} \phi^H(\tau)d\tau\right] \\
&= \frac{1}{2}\left[\int_{t^H_{\min}}^{t+(\beta-1)\bar{t}^F} \phi^H(\tau)d\tau - \int_{t^H_{\min}}^{t} \phi^H(\tau)d\tau - 2\int_{t}^{t+\frac{\beta-1}{2}\bar{t}^F} \phi^H(\tau)d\tau\right] \\
&= \frac{1}{2}\left[\int_{t}^{t+(\beta-1)\bar{t}^F} \phi^H(\tau)d\tau - 2\int_{t}^{t+\frac{\beta-1}{2}\bar{t}^F} \phi^H(\tau)d\tau\right] \\
&= \frac{1}{2}\left\{\Phi^H\left(t+(\beta-1)\bar{t}^F\right) - \Phi^H(t) - 2\left[\Phi^H\left(t+\frac{\beta-1}{2}\bar{t}^F\right) - \Phi^H(t)\right]\right\}
\end{aligned}$$

If $t + (\beta-1)\bar{t}^F \leqq \bar{t}^H$, using (B2), we have

$$\frac{\Phi^H\left(t+\frac{\beta-1}{2}\bar{t}^F\right) - \Phi^H(t)}{t+\frac{\beta-1}{2}\bar{t}^F - t} < \frac{\Phi^H\left(t+(\beta-1)\bar{t}^F\right) - \Phi^H(t)}{t+(\beta-1)\bar{t}^F - t},$$

leading to

$$\Phi^H\left(t+(\beta-1)\bar{t}^F\right) - \Phi^H(t) > 2\left[\Phi^H\left(t+\frac{\beta-1}{2}\bar{t}^F\right) - \Phi^H(t)\right].$$

If $t+(\beta-1)\bar{t}^F > \bar{t}^H$, there exists a $\alpha_2 \in (0,\, 1)$ such that $t+(\beta-1)\bar{t}^F = \alpha_2\bar{t}^H + (1-\alpha_2)(2\bar{t}^H - t)$ because $\phi^H(\cdot)$ is symmetric with respect to $\bar{t}^H$, and $\Phi^H(t)$ is convex at $[t^H_{\min}, \bar{t}^H]$ and concave at $[\bar{t}^H,\, t^H_{\max}]$. It follows that

where the second inequality is derived from (B1). Thus, we obtain

$$\Phi^W(t) - \Phi^X(t) > 0, \text{ for } t \in [t^H_{\min},\, \bar{t}^W). \tag{B4}$$

From (B3) and (B4), we have $\Phi^W(t) - \Phi^X(t) > 0$ for $[t^F_{\min},\, \bar{t}^W)$.

Since $\phi^W(t)$ and $\phi^X(t)$ are symmetric with respect to $\bar{t}^W$, for $t > \bar{t}^W$, we have

$$\Phi^W(t) - \Phi^X(t) = -\left[\Phi^W\left(2\bar{t}^W - t\right) - \Phi^X\left(2\bar{t}^W - t\right)\right].$$

If $t > \bar{t}^W$, then $2\bar{t}^W - t < \bar{t}^W$. Therefore, we obtain $\Phi^W(t) - \Phi^X(t) < 0$ for $(\bar{t}^W,\, t^H_{\max}]$.

Next, we consider the case in which the symmetric density function $\phi^H(t)$ satisfies that $\frac{d\phi^H(t)}{dt} = 0$ for $t \in (t^H_{\min},\, \bar{t}^H)$, implying that both $\phi^H(t)$ and $\phi^F(t)$ follow uniform distribution function. Thus, we have

$$
\begin{aligned}
&\Phi^W(t)-\Phi^X(t)\\
&=\begin{cases}
\dfrac{1}{2}\dfrac{t-t^F_{\min}}{t^F_{\max}-t^F_{\min}}>0, & t\in\left(t^F_{\min},\,t^H_{\min}-\dfrac{\beta-1}{2\beta}\bar{t}^H\right)\\
\dfrac{1}{2}\dfrac{t^H_{\min}-t}{t^H_{\max}-t^H_{\min}}>0, & t\in\left[t^H_{\min}-\dfrac{\beta-1}{2\beta}\bar{t}^H,\,t^H_{\min}\right)\\
0, & t\in[t^H_{\min},\,t^F_{\max}]\\
\dfrac{1}{2}\dfrac{t^H_{\max}-(\beta-1)\bar{t}^F-t}{t^H_{\max}-t^H_{\min}}<0, & t\in\left(t^F_{\max},\,t^H_{\max}-\dfrac{\beta-1}{2\beta}\bar{t}^H\right]\\
-\dfrac{1}{2}\dfrac{t^H_{\max}-t}{t^H_{\max}-t^H_{\min}}<0, & t\in\left(t^H_{\max}-\dfrac{\beta-1}{2\beta}\bar{t}^H,\,t^H_{\max}\right)
\end{cases}.
\end{aligned}
$$

Following the definition of diversity, $\Phi^W(t)$ is more diverse than $\Phi^X(t)$. Since Home's and Foreign's cumulative distribution functions share the same diversity, the cumulative distribution function under offshoring is more diverse than both countries' autarky cumulative distribution functions.

Proof for Proposition 4

1. The relationship between p^W and p^F.

The relative supply of good S is rewritten as follows:

$$
\frac{Y^W_S}{Y^W_C}=\mathcal{K}(\hat{t}^W,\beta)\equiv\frac{2}{\lambda_C}\frac{\displaystyle\int_{t^F_{\min}}^{\hat{t}^W}F^S\left(t,2\bar{t}^W-t\right)\phi^W(t)dt}{\displaystyle\bar{t}^W\int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W}\phi^W(t)dt},\tag{B5}
$$

where $\bar{t}^W\equiv(\bar{t}^F+\bar{t}^H)/2=(1+\beta)\bar{t}^F/2$ and $\phi^W(t)$ is shown by (19). Note that $\phi^H(t)=\phi^F(t-(\beta-1)\bar{t}^F)$, $t^H_{\min}=t^F_{\min}+(\beta-1)\bar{t}^F$, and $t^H_{\max}=t^F_{\max}+(\beta-1)\bar{t}^F$. If $\beta=1$, we obtain the Foreign equilibrium under autarky, and if $\beta>1$, we obtain the equilibrium under offshoring.

From (14), we obtain

$$
\hat{t}^W=\hat{t}^W(p^W,\beta).\tag{B6}
$$

Substituting (B6) and (15) into (B5) yields

$$\mathcal{K}\left(\hat{t}^W\left(p^W,\beta\right),\beta\right) = f(p^W).$$

Totally differentiating the above equation, we obtain

$$\frac{dp^W}{d\beta} = -\frac{\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial \beta} + \frac{\partial \mathcal{K}}{\partial \beta}}{\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial p^W} - f'}.$$

From (B6), we obtain

$$\frac{d\hat{t}^W}{d\beta} = \frac{\partial \hat{t}^W}{\partial p^W}\frac{dp^W}{d\beta} + \frac{\partial \hat{t}^W}{\partial \beta}.$$

From (B5) and (14), we have

$$\begin{aligned}
\frac{\partial \hat{t}^W}{\partial p^W} &= \frac{\lambda_C \bar{t}^W}{F_1^S\left(\hat{t}^W, 2\bar{t}^W - \hat{t}^W\right) - F_2^S\left(\hat{t}^W, 2\bar{t}^W - \hat{t}^W\right)} < 0,\\
\frac{\partial \hat{t}^W}{\partial \beta} &= \frac{\hat{t}^W}{1+\beta} > 0,\\
\frac{\partial \mathcal{K}}{\partial \hat{t}^W} &= \frac{2\phi^W(\hat{t}^W)}{\displaystyle\int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^W(t)dt}\left\{p^W + \frac{Y_S^W}{Y_C^W}\right\} > 0.
\end{aligned}$$

Since $f' > 0$, the sign of $\frac{dp^W}{d\beta}$ depends on $\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial \beta} + \frac{\partial \mathcal{K}}{\partial \beta}$.

To investigate the sign of $\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial \beta} + \frac{\partial \mathcal{K}}{\partial \beta}$, we consider two cases: one in which the gap between $\bar{t}^H$ and $\bar{t}^F$, β, is small such that $t_{\min}^H < \hat{t}^W < 2\bar{t}^W - \hat{t}^W < t_{\max}^F$; and the other in which β is large such that $\hat{t}^W \leqq t_{\min}^H < t_{\max}^F \leqq 2\bar{t}^W - \hat{t}^W$. Note that $t_{\min}^H < t_{\max}^F$ implies that $\beta < \bar{\beta} \equiv 1 + \frac{t_{\max}^F - t_{\min}^F}{\bar{t}^F}$. First, we derive the threshold value of β, β^*, which separates the above two cases. Next, we investigate the sign of $\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial \beta} + \frac{\partial \mathcal{K}}{\partial \beta}$ with two cases.

The condition of $t_{\min}^H < \hat{t}^W < 2\bar{t}^W - \hat{t}^W < t_{\max}^F$ requires that there exists an excess demand for good S at $\hat{t}^W = t_{\min}^H$, i.e.,

$$f\left(\frac{F^S\left(t_{\min}^H, t_{\max}^F\right)}{\lambda_C \bar{t}^W}\right) > \frac{2}{\lambda_C \bar{t}^W}\frac{\displaystyle\int_{t_{\min}^F}^{t_{\min}^H} F^S\left(t, 2\bar{t}^W - t\right)\phi^F(t)dt}{\displaystyle\int_{t_{\min}^H}^{t_{\max}^F}(\phi^F(t) + \phi^H(t))dt}. \tag{B7}$$

The condition of $\hat{t}^W \leqq t^H_{\min} < t^F_{\max} \leqq 2\bar{t}^W - \hat{t}^W$ requires that there exist an excess supply of good S at $\hat{t}^W = t^H_{\min}$, i.e., (B7) fails.

Clearly, when β converges to 1, the right-hand side of (B7) converges to 0. When β converges to $\bar{\beta}$, $t^H_{\min}$ converges to $t^F_{\max}$; thus, the right-hand side of (B7) goes to infinity. Differentiating the left-hand side of (B7) with respect to β, we obtain

$$\frac{d}{d\beta} f\left(\frac{F^S\left(t^H_{\min}, t^F_{\max}\right)}{\lambda_C \bar{t}^W}\right) = -f'(\cdot)\frac{t^F_{\max}[F^S_2\left(t^H_{\min}, t^F_{\max}\right) - F^S_1\left(t^H_{\min}, t^F_{\max}\right)]}{(1+\beta)\lambda_C \bar{t}^W} \leqq 0.$$

Differentiating the right-hand side of (B7) yields

$$\begin{aligned}
&\frac{d}{d\beta}\left[\frac{2}{\lambda_C \bar{t}^W}\frac{\displaystyle\int_{t^F_{\min}}^{t^H_{\min}} F^S\left(t, 2\bar{t}^W - t\right)\phi^F(t)dt}{\displaystyle\int_{t^H_{\min}}^{t^F_{\max}}(\phi^F(t) + \phi^H(t))dt}\right] \\
&= \frac{2\bar{t}^F}{\lambda_C \bar{t}^W \int_{t^H_{\min}}^{t^F_{\max}}(\phi^F(t)+\phi^H(t))dt}\left\{\frac{1}{2\bar{t}^W}\int_{t^F_{\min}}^{t^H_{\min}} t(F^S_2 - F^S_1)dt + F^S(t^H_{\min}, t^F_{\max})\phi^F(t^H_{\min})\right. \\
&\left. + \left[\frac{\int_{t^F_{\min}}^{t^H_{\min}} F^S\left(t, 2\bar{t}^W - t\right)\phi^F(t)dt}{\int_{t^H_{\min}}^{t^F_{\max}}(\phi^F(t)+\phi^H(t))dt}\right]\left[\phi^F(t^H_{\min}) + \phi^H(t^F_{\max})\right]\right\} \geqq 0.
\end{aligned}$$

Therefore, there exists a unique $\beta^* < \bar{\beta}$ such that

$$f\left(\frac{F^S\left(t^H_{\min}, t^F_{\max}\right)}{\lambda_C \bar{t}^W}\right) = \frac{2}{\lambda_C \bar{t}^W}\frac{\displaystyle\int_{t^F_{\min}}^{t^H_{\min}} F^S\left(t, 2\bar{t}^W - t\right)\phi^F(t)dt}{\displaystyle\int_{t^H_{\min}}^{t^F_{\max}}(\phi^F(t)+\phi^H(t))dt}. \tag{B8}$$

Clearly, if $\beta < \beta^*$, $t^H_{\min} < \hat{t}^W < 2\bar{t}^W - \hat{t}^W < t^F_{\max}$ holds and if $\beta^* \leqq \beta < \bar{\beta}$, $\hat{t}^W \leqq t^H_{\min} < t^F_{\max} \leqq 2\bar{t}^W - \hat{t}^W$ holds.

Note that, under the Cobb-Douglas preference, (B8) is equivalent to

$$\frac{1-\alpha}{\alpha} = \frac{2}{\lambda_C \bar{t}^W} \frac{\displaystyle\int_{t_{\min}^F}^{t_{\min}^F+(\beta^*-1)\bar{t}^F} F^S\left(t,(1+\beta^*)\bar{t}^F - t\right)\phi^F(t)dt}{\displaystyle\int_{t_{\min}^F+(\beta^*-1)\bar{t}^F}^{t_{\max}^F} (\phi^F(t)+\phi^F(t-(\beta^*-1)\bar{t}^F))dt},$$

where α represents the consumption share of good C. It is clear that the left-hand side of this equation decreases in α. Since the right-hand side of this equation increases in β^*, we obtain that β^* decreases in α.

Next, we examine the sign of $\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial \beta}+\frac{\partial \mathcal{K}}{\partial \beta}$.

(1) If $\beta<\beta^*$, i.e., $t_{\min}^H < \hat{t}^W < 2\bar{t}^W - \hat{t}^W < t_{\max}^F$, we investigate the sign of $\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial \beta}+\frac{\partial \mathcal{K}}{\partial \beta}$.

We rewrite (B5) as

$$\mathcal{K}(\hat{t}^W,\beta) = \frac{2}{\lambda_C \bar{t}^W} \frac{\displaystyle\int_{t_{\min}^F}^{\hat{t}^W} F^S\left(t,2\bar{t}^W - t\right)\phi^F(t)dt + \int_{t_{\min}^H}^{\hat{t}^W} F^S\left(t,2\bar{t}^W - t\right)\phi^H(t)dt}{\displaystyle\int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W} \phi^F(t)dt + \int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W} \phi^H(t)dt}.$$

Differentiating $\mathcal{K}$ with respect to β yields

$$\frac{\partial \mathcal{K}}{\partial \beta} = \frac{2}{\lambda_C} \frac{1}{(1+\beta)\displaystyle\int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W} \phi^W(t)dt} \times \mathcal{A}_F,$$

where

$$\begin{aligned}\mathcal{A}_F = &\int_{t_{\min}^F}^{t_{\min}^H} \frac{t}{2\bar{t}^W}(F_2^S - F_1^S)\phi^F(t)dt + \int_{t_{\min}^H}^{\hat{t}^W} (F_2^S - F_1^S)\left[\frac{t}{2\bar{t}^W}\phi^F(t) + \frac{t-2\bar{t}^W}{2\bar{t}^W}\phi^H(t)\right]dt \\ &- \frac{\lambda_C}{2}\frac{Y_S^W}{Y_C^W}\bar{t}^W[\phi^F(2\bar{t}^W - \hat{t}^W)+\phi^H(\hat{t}^W)] - F^S\left(\hat{t}^W, 2\bar{t}^W - \hat{t}^W\right)\phi^H(\hat{t}^W).\end{aligned}$$

It follows that

$$\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial \beta}+\frac{\partial \mathcal{K}}{\partial \beta} = \frac{2}{\lambda_C(1+\beta)\displaystyle\int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W} \phi^W(t)dt} \times \mathcal{B}_F,$$

where $\mathcal{B}_F$ is given by

$$\begin{aligned}\mathcal{B}_F = &\frac{\hat{t}^W \phi^W(\hat{t}^W) - \bar{t}^W \phi^H(\hat{t}^W)}{\bar{t}^W} F^S\left(\hat{t}^W, 2\bar{t}^W - \hat{t}^W\right) \\ &+ \frac{\lambda_C}{2}\frac{Y_S^W}{Y_C^W}[2\hat{t}^W \phi^W(\hat{t}^W) - \bar{t}^W \phi^H(\hat{t}^W) - \bar{t}^W \phi^F(2\bar{t}^W - \hat{t}^W)] \\ &+ \frac{1}{\bar{t}^W}\int_{t_{\min}^F}^{\hat{t}^W} t(F_2^S - F_1^S)\phi^W(t)dt - \int_{t_{\min}^H}^{\hat{t}^W}(F_2^S - F_1^S)\phi^H(t)]dt.\end{aligned}$$

If β approaches 1, $\mathcal{B}_F$ becomes negative as follows:

$$\lim_{\beta\to 1}\mathcal{B}_F = -(\bar{t}^F - \hat{t}^F)\phi^F(\hat{t}^F)[F^S\left(\hat{t}^F, 2\bar{t}^F - \hat{t}^F\right) + \frac{\lambda_C}{2}\frac{Y_S^F}{Y_C^F}] - \frac{1}{\bar{t}^F}\int_{t_{\min}^F}^{\hat{t}^W}(\bar{t}^F - t)(F_2^S - F_1^S)\phi^F(t)dt.$$

Thus, the sign of $\mathcal{B}_F$ is negative when β is sufficiently close to 1. Therefore, for a general density function, we have that $p^W < p^F$ and $\hat{t}^W > \hat{t}^F$ with β sufficiently close to 1.

If both ϕ^H and ϕ^F follow a uniform distribution function, we have $\phi^H(t) = \frac{1}{I}$ for $t \in [t_{\min}^H, t_{\max}^H]$, and $\phi^F(t) = \frac{1}{I}$ for $t \in [t_{\min}^F, t_{\max}^F]$ where $I = t_{\max}^H - t_{\min}^H = t_{\max}^F - t_{\min}^F$. Thus, we have

$$\begin{aligned}B_F = &\frac{1}{I\bar{t}^W}[\hat{t}^W F^S\left(\hat{t}^W, 2\bar{t}^W - \hat{t}^W\right) - \bar{t}^W F^S(t_{\min}^H, t_{\max}^F)] \\ &+ \frac{1}{I\bar{t}^W}[\int_{t_{\min}^F}^{\hat{t}^W}[(t - \bar{t}^W)F_2^S - tF_1^S]dt + \int_{t_{\min}^H}^{\hat{t}^W}[(t - \bar{t}^W)F_2^S - tF_1^S]dt].\end{aligned}$$

Since $F^S(t, 2\bar{t}^W - t)$ decreases in t, we have $\hat{t}^W F^S\left(\hat{t}^W, 2\bar{t}^W - \hat{t}^W\right) - \bar{t}^W F^S(t_{\min}^H, t_{\max}^F) < 0$, yielding $\mathcal{B}_F < 0$. It follows that $\frac{\partial\mathcal{K}}{\partial\hat{t}^W}\frac{\partial\hat{t}^W}{\partial\beta} + \frac{\partial\mathcal{K}}{\partial\beta} < 0$, leading to $dp^W/d\beta < 0$ and $d\hat{t}^W/d\beta > 0$. Therefore, we obtain that $p^W < p^F$ and $\hat{t}^W > \hat{t}^F$ when both ϕ^H and ϕ^F follow a uniform distribution function.

(2) We prove that $\frac{\partial\mathcal{K}}{\partial\hat{t}^W}\frac{\partial\hat{t}^W}{\partial\beta} + \frac{\partial\mathcal{K}}{\partial\beta} > 0$, if $\hat{t}^W < t_{\min}^H < t_{\max}^F < 2\bar{t}^W - \hat{t}^W$.

For this case, (B5) is rewritten as

$$\mathcal{K}(\hat{t}^W, \beta) = \frac{2}{\lambda_C\bar{t}^W}\frac{\int_{t_{\min}^F}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right)\phi^F(t)dt}{\int_{\hat{t}^W}^{t_{\min}^H}\phi^F(t)dt + \int_{t_{\min}^H}^{t_{\max}^F}[\phi^H(t) + \phi^F(t)]dt + \int_{t_{\max}^F}^{2\bar{t}^W - \hat{t}^W}\phi^H(t)dt}.$$

Differentiating this equation with respect to β, we obtain

$$\frac{\partial \mathcal{K}}{\partial \beta} = \frac{2}{\lambda_C(1+\beta)\displaystyle\int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W} \phi^W(t)dt} \frac{\displaystyle\int_{t^F_{\min}}^{\hat{t}^W} t[F_2^S\left(t, 2\bar{t}^W - t\right) - F_1^S\left(t, 2\bar{t}^W - t\right)]\phi^F(t)dt}{2\bar{t}^W}.$$

Thus, we have

$$\begin{aligned}
&\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial \beta} + \frac{\partial \mathcal{K}}{\partial \beta} \\
&= \frac{\displaystyle\int_{t^F_{\min}}^{\hat{t}^W} t(F_2^S - F_1^S)\phi^F(t)dt + 2\bar{t}^W\lambda_C\hat{t}^W\phi^W(\hat{t}^W)\left(p^W + \frac{Y_S^W}{Y_C^W}\right)}{(1+\beta)\lambda_C\bar{t}^W\displaystyle\int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W} \phi^W(t)dt} > 0
\end{aligned}$$

It follows that $dp^W/d\beta > 0$. Therefore, there is a possibility that $p^W > p^F$.

To compare $m^W(\hat{t}^W)$ with $m^F(\hat{t}^F)$, we differentiate $m^W(\hat{t}^W) = 2\bar{t}^W - \hat{t}^W$ with respect to β to obtain

$$\begin{aligned}
\frac{d(2\bar{t}^W - \hat{t}^W)}{d\beta} &= \bar{t}^F - \frac{d\hat{t}^W}{d\beta} \\
&= \bar{t}^F - \frac{\partial \hat{t}^W}{\partial \beta} + \frac{\dfrac{\partial \hat{t}^W}{\partial p^W}(\dfrac{\partial \mathcal{K}}{\partial \hat{t}^W}\dfrac{\partial \hat{t}^W}{\partial \beta} + \dfrac{\partial \mathcal{K}}{\partial \beta})}{\dfrac{\partial \mathcal{K}}{\partial \hat{t}^W}\dfrac{\partial \hat{t}^W}{\partial p^W} - f'} \\
&> \frac{2\bar{t}^W - \hat{t}^W}{1+\beta} > 0.
\end{aligned}$$

It follows that $m^W(\hat{t}^W) > m^F(\hat{t}^F)$.

2. The relationship between p^W and p^H.

Similarly, to compare p^W with p^H, (B5) is rewritten as

$$\frac{Y_S^W}{Y_C^W} = \mathcal{K}(\hat{t}^W, \delta) \equiv \frac{2}{\lambda_C} \frac{\displaystyle\int_{t_{\min}^H-(1-\delta)\bar{t}^H}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right)\phi^W(t)dt}{\displaystyle\bar{t}^W \int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W} \phi^W(t)dt}, \tag{B9}$$

where $\delta = 1/\beta$, and thus $0 < \delta < 1$, $\bar{t}^W \equiv (\bar{t}^F + \bar{t}^H)/2 = (1+\delta)\bar{t}^H/2$, and $\phi^W(t)$ is shown by (19). Note that, in this case, $\phi^F(t) = \phi^H(t + (1-\delta)\bar{t}^H)$, $t_{\min}^F = t_{\min}^H - (1-\delta)\bar{t}^H$, and $t_{\max}^F = t_{\max}^H - (1-\delta)\bar{t}^H$. If $\delta = 1$, we obtain the Home equilibrium under autarky, and if $\delta < 1$, we have the equilibrium under offshoring.

From (14), we obtain

$$\hat{t}^W = \hat{t}^W(p^W, \delta). \tag{B10}$$

Substituting (B10) and (15) into (B9) yields

$$\mathcal{K}\left(\hat{t}^W\left(p^W, \delta\right), \delta\right) = f(p^W).$$

Totally differentiating the above equation and (B10), we obtain

$$\begin{aligned}
\frac{dp^W}{d\delta} &= -\frac{\dfrac{\partial \mathcal{K}}{\partial \hat{t}^W}\dfrac{\partial \hat{t}^W}{\partial \delta} + \dfrac{\partial \mathcal{K}}{\partial \delta}}{\dfrac{\partial \mathcal{K}}{\partial \hat{t}^W}\dfrac{\partial \hat{t}^W}{\partial p^W} - f'}, \\
\frac{d\hat{t}^W}{d\delta} &= \frac{\partial \hat{t}^W}{\partial p^W}\frac{dp^W}{d\delta} + \frac{\partial \hat{t}^W}{\partial \delta}.
\end{aligned}$$

From (B9) and (14), we have

$$\begin{aligned}
\frac{\partial \hat{t}^W}{\partial p^W} &= \frac{\lambda_C \bar{t}^W}{F_1^S\left(\hat{t}^W, 2\bar{t}^W - \hat{t}^W\right) - F_2^S\left(\hat{t}^W, 2\bar{t}^W - \hat{t}^W\right)} < 0, \\
\frac{\partial \hat{t}^W}{\partial \delta} &= \frac{\hat{t}^W}{1+\delta} > 0, \\
\frac{\partial \mathcal{K}}{\partial \hat{t}^W} &= \frac{2\phi^W(\hat{t}^W)}{\displaystyle\int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W}\phi^W(t)dt}\left\{p^W + \frac{Y_S^W}{Y_C^W}\right\} > 0.
\end{aligned}$$

In the following, we investigate the sign of $\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial \delta} + \frac{\partial \mathcal{K}}{\partial \delta}$ by considering two cases.

(1) $\delta > \delta^* \equiv 1/\beta^*$, i.e., the case of $t^H_{\min} < \hat{t}^W < 2\bar{t}^W - \hat{t}^W < t^F_{\max}$

In this case, $K(\hat{t}^W, \delta)$ is rewritten as

$$\mathcal{K}(\hat{t}^W, \delta) = \frac{2}{\lambda_C \bar{t}^W} \frac{\displaystyle\int_{t^F_{\min}}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right)\phi^F(t)dt + \int_{t^H_{\min}}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right)\phi^H(t)dt}{\displaystyle\int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^F(t)dt + \int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^H(t)dt}.$$

Differentiating $\mathcal{K}$ with respect to δ yields

$$\frac{\partial \mathcal{K}}{\partial \delta} = \frac{2}{(1+\delta)\lambda_C \displaystyle\int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^W(t)dt} \times \mathcal{A}_H, \tag{B11}$$

where

$$\begin{aligned}
\mathcal{A}_H &= \int_{t^F_{\min}}^{\hat{t}^W} F^S_2 \phi^F(t)dt + \int_{t^H_{\min}}^{\hat{t}^W} F^S_2 \phi^H(t)dt - \int_{t^F_{\min}}^{\hat{t}^W} F^S (\phi^F)'(t)dt - F^S(t^F_{\min}, t^H_{\max})\phi^F(t^F_{\min}) \\
&\quad - \frac{\displaystyle\int_{t^F_{\min}}^{\hat{t}^W} F^S \phi^W(t)dt}{\bar{t}^W \displaystyle\int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^W(t)dt} \left[\int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^W(t)dt + \bar{t}^W\left(\phi^F(\hat{t}^W) + \phi^H(2\bar{t}^W - \hat{t}^W)\right)\right] \\
&= \frac{1}{\bar{t}^W} \int_{t^F_{\min}}^{\hat{t}^W} t(F^S_2 - F^S_1)\phi^W(t)dt - \int_{t^F_{\min}}^{\hat{t}^W} F^S (\phi^F)'(t)dt - F^S(t^F_{\min}, t^H_{\max})\phi^F(t^F_{\min}) \\
&\quad - \frac{\lambda_C \bar{t}^W}{2} \frac{Y^W_S}{Y^W_C} \left[\phi^F(\hat{t}^W) + \phi^H(2\bar{t}^W - \hat{t}^W)\right].
\end{aligned} \tag{B12}$$

It follows that

$$\frac{\partial \mathcal{K}}{\partial \hat{t}^W} \frac{\partial \hat{t}^W}{\partial \delta} + \frac{\partial \mathcal{K}}{\partial \delta} = \frac{2}{(1+\delta)\lambda_C \displaystyle\int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^W(t)dt} \times \mathcal{B}_H,$$

where

$$\mathcal{B}_H = \frac{\hat{t}^W}{\bar{t}^W} F^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W)\phi^W(\hat{t}^W) + \frac{1}{\bar{t}^W}\int_{t^F_{\min}}^{\hat{t}^W} t(F^S_2 - F^S_1)\phi^W(t)dt - \int_{t^F_{\min}}^{\hat{t}^W} F^S(\phi^F)'(t)dt$$
$$- F^S(t^F_{\min}, t^H_{\max})\phi^F(t^F_{\min}) + \frac{\lambda_C}{2}\frac{Y^W_S}{Y^W_C}\left[2\hat{t}^W\phi^W(\hat{t}^W) - \bar{t}^W\left(\phi^F(\hat{t}^W) + \phi^H(2\bar{t}^W - \hat{t}^W)\right)\right].$$

Note that $\phi^H(2\bar{t}^W - \hat{t}^W) = \phi^F(\hat{t}^W)$ and

$$\int_{t^F_{\min}}^{\hat{t}^W} F^S(\phi^F)'dt = \int_{t^F_{\min}}^{\hat{t}^W} (F^S_2 - F^S_1)\phi^F dt + \left[F^S\phi^F\right]^{\hat{t}^W}_{t^F_{\min}}.$$

Thus,

$$\mathcal{B}_H = \frac{F^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W)}{\bar{t}^W}\left[\hat{t}^W\phi^W(\hat{t}^W) - \bar{t}^W\phi^F(\hat{t}^W)\right] + \frac{1}{\bar{t}^W}\int_{t^F_{\min}}^{\hat{t}^W}(F^S_2 - F^S_1)\left[t\phi^W(t) - \bar{t}^W\phi^F(t)\right]dt$$
$$+ \lambda_C\frac{Y^W_S}{Y^W_C}\left[\hat{t}^W\phi^W(\hat{t}^W) - \bar{t}^W\phi^F(\hat{t}^W)\right].$$

Note that $\phi^W(\hat{t}^W) - \phi^F(\hat{t}^W) = \frac{1}{2}(\phi^H(\hat{t}^W) - \phi^F(\hat{t}^W))$ and $\phi^F(t) = \phi^H(t + (1-\delta)\bar{t}^H)$. Since $\phi^H(t)$ is nondecreasing at $[t^H_{\min}, \bar{t}^H)$, we have $\phi^H(\hat{t}^W) - \phi^F(\hat{t}^W) \leqq 0$ for $\hat{t}^W \in (t^H_{\min}, \bar{t}^W)$. Therefore, we have $\mathcal{B}_H < 0$, leading to $p^H < p^W$ and $\hat{t}^W < \hat{t}^H$.

We now turn to the effects of offshoring on $2\bar{t}^W - \hat{t}^W$.

$$\begin{aligned}
\frac{d(2\bar{t}^W - \hat{t}^W)}{d\delta} &= \bar{t}^H - \frac{d\hat{t}^W}{d\delta} \\
&= \bar{t}^H - \frac{\partial \hat{t}^W}{\partial \delta} + \frac{\frac{\partial \hat{t}^W}{\partial p^W}\left[\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial \delta} + \frac{\partial \mathcal{K}}{\partial \delta}\right]}{\frac{\partial \mathcal{K}}{\partial \hat{t}^W}\frac{\partial \hat{t}^W}{\partial p^W} - f'} \\
&> \bar{t}^H + \frac{\partial \mathcal{K}}{\partial \delta}\Big/\frac{\partial \mathcal{K}}{\partial \hat{t}^W} \\
&= \frac{\bar{t}^H(1+\delta)\lambda_C \phi^W(\hat{t}^W)\left[p^W + \frac{Y_S^W}{Y_C^W}\right] + \mathcal{A}_H}{(1+\delta)\lambda_C \phi^W(\hat{t}^W)\left[p^W + \frac{Y_S^W}{Y_C^W}\right]},
\end{aligned}$$

where

$$\begin{aligned}
&\bar{t}^H(1+\delta)\lambda_C \phi^W(\hat{t}^W)\left[p^W + \frac{Y_S^W}{Y_C^W}\right] + \mathcal{A}_H \\
&= \bar{t}^H(1+\delta)\lambda_C \phi^W(\hat{t}^W)\left[p^W + \frac{Y_S^W}{Y_C^W}\right] - \int_{t_{\min}^F}^{\hat{t}^W} F^S(\phi^F)'(t)dt - F^S(t_{\min}^F, t_{\max}^H)\phi^F(t_{\min}^F) \\
&+ \frac{1}{\bar{t}^W}\int_{t_{\min}^F}^{\hat{t}^W} t(F_2^S - F_1^S)\phi^W(t)dt - \frac{\lambda_C \bar{t}^W}{2}\frac{Y_S^W}{Y_C^W}\left[\phi^F(\hat{t}^W) + \phi^H(2\bar{t}^W - \hat{t}^W)\right] \\
&= F^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W)\phi^H(\hat{t}^W) + \lambda_C \bar{t}^W \phi^H(\hat{t}^W)\frac{Y_S^W}{Y_C^W} + \frac{1}{\bar{t}^W}\int_{t_{\min}^F}^{\hat{t}^W}(F_2^S - F_1^S)[t\phi^W(t) - \bar{t}^W\phi^F(t)]dt \\
&= F^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W)\phi^H(\hat{t}^W) + \frac{\phi^H(\hat{t}^W)}{\int_{\hat{t}^W}^{\bar{t}^W}\phi^W(t)dt}\int_{t_{\min}^F}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right)\phi^W(t)dt \\
&+ \frac{1}{\bar{t}^W}\int_{t_{\min}^F}^{\hat{t}^W}(F_2^S - F_1^S)[t\phi^W(t) - \bar{t}^W\phi^F(t)]dt.
\end{aligned}$$

Under the condition that $\min\{\phi^F(t)\} \geqq \frac{1}{2\bar{t}^F}$, since $\phi^F(t) = \phi^H(t+(1-\delta)\bar{t}^H)$, we have $\phi^H(\hat{t}^W) \geqq$

$\frac{1}{2\bar{t}^F} > \frac{1}{2\bar{t}^W}$. Since $\int_{\hat{t}^W}^{\bar{t}^W} \phi^W(t)dt < \frac{1}{2}$, we have

$$
\begin{aligned}
&\frac{1}{\bar{t}^W}\int_{t^F_{\min}}^{\hat{t}^W}(F_2^S - F_1^S)[t\phi^W(t) - \bar{t}^W\phi^F(t)]dt + \frac{\phi^H(\hat{t}^W)}{\int_{\hat{t}^W}^{\bar{t}^W}\phi^W(t)dt}\int_{t^F_{\min}}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right)\phi^W(t)dt \\
&> \frac{1}{\bar{t}^W}\int_{t^F_{\min}}^{\hat{t}^W}\{(F_2^S - F_1^S)[t\phi^W(t) - \bar{t}^W\phi^F(t)] + F^S\left(t, 2\bar{t}^W - t\right)\phi^W(t)\}dt \\
&= \int_{t^F_{\min}}^{\hat{t}^W}[F_2^S\phi^H(t) + F_1^S\phi^F(t)]dt > 0.
\end{aligned}
$$

Thus, we have $\bar{t}^H(1+\delta)\lambda_C\phi^W(\hat{t}^W)\left[p^W + \frac{Y_S^W}{Y_C^W}\right] + \mathcal{A}_H > 0$, leading to $\frac{d(2\bar{t}^W - \hat{t}^W)}{d\delta} > 0$. It follows that $2\bar{t}^W - \hat{t}^W < 2\bar{t}^H - \hat{t}^H$.

(2) $\delta < \delta^* \equiv 1/\beta^*$, i.e.,the case of $\hat{t}^W \leqq t^H_{\min} < t^F_{\max} \leqq 2\bar{t}^W - \hat{t}^W$

We rewrite $\mathcal{K}(\hat{t}^W, \delta)$ as follows:

$$
\mathcal{K}(\hat{t}^W, \delta) = \frac{2}{\lambda_C\bar{t}^W}\frac{\displaystyle\int_{t^H_{\min}-(1-\delta)\bar{t}^H}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right)\phi^F(t)dt}{\displaystyle\int_{\hat{t}^W}^{t^H_{\max}-(1-\delta)\bar{t}^H}\phi^F(t)dt + \int_{t^H_{\min}}^{2\bar{t}^W-\hat{t}^W}\phi^H(t)dt}.
$$

Differentiating $\mathcal{K}(\hat{t}^W, \delta)$ with respect to δ yields

$$
\frac{\partial\mathcal{K}}{\partial\delta} = \frac{2}{(1+\delta)\lambda_C\displaystyle\int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W}\phi^W(t)dt} \times \mathcal{A}'_H, \tag{B13}
$$

where

$$\begin{aligned}
\mathcal{A}'_H &= \int_{t^F_{\min}}^{\hat{t}^W} F^S_2 \phi^F(t)dt + \int_{t^H_{\min}}^{\hat{t}^W} F^S_2 \phi^H(t)dt - \int_{t^F_{\min}}^{\hat{t}^W} F^S (\phi^F)'(t)dt - F^S(t^F_{\min}, t^H_{\max})\phi^F(t^F_{\min}) \\
&\quad - \frac{\displaystyle\int_{t^F_{\min}}^{\hat{t}^W} F^S \phi^W(t)dt}{\displaystyle\bar{t}^W \int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^W(t)dt} \left[\int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^W(t)dt + \bar{t}^W \left(\phi^F(\hat{t}^W) + \phi^H(2\bar{t}^W - \hat{t}^W) \right) \right] \\
&= \frac{1}{\bar{t}^W} \int_{t^F_{\min}}^{\hat{t}^W} t(F^S_2 - F^S_1)\phi^W(t)dt - \int_{t^F_{\min}}^{\hat{t}^W} F^S (\phi^F)'(t)dt - F^S(t^F_{\min}, t^H_{\max})\phi^F(t^F_{\min}) \\
&\quad - \frac{\lambda_C \bar{t}^W}{2} \frac{Y^W_S}{Y^W_C} \left[\phi^F(\hat{t}^W) + \phi^H(2\bar{t}^W - \hat{t}^W) \right].
\end{aligned}$$

Note that $\mathcal{A}'_H$ is equal to $\mathcal{A}_H$ in (B12); thus, (B13) is the same as (B11). It follows that $p^H < p^W$.

Proof for Proposition 5

We focus on the case that $p^H < p^W < p^F$. We rewrite the wage schedules in Home, in Foreign and under offshoring as follows:

$$w^H(t) = \begin{cases} \dfrac{p^H \lambda_C \hat{t}^H}{2} - \displaystyle\int_t^{\hat{t}^H} F_1^S\left(\tau, 2\bar{t}^H - \tau\right) d\tau, & t < \hat{t}^H \\ \dfrac{p^H \lambda_C t}{2}, & \hat{t}^H \leqq t \leqq 2\bar{t}^H - \hat{t}^H \\ F^S(t, 2\bar{t}^H - t) - \dfrac{p^H \lambda_C \hat{t}^H}{2} + \displaystyle\int_{2\bar{t}^H - t}^{\hat{t}^H} F_1^S\left(\tau, 2\bar{t}^H - \tau\right) d\tau, & t > 2\bar{t}^H - \hat{t}^H \end{cases},$$

$$w^F(t) = \begin{cases} \dfrac{p^F \lambda_C \hat{t}^F}{2} - \displaystyle\int_t^{\hat{t}^F} F_1^S\left(\tau, 2\bar{t}^F - \tau\right) d\tau, & t < \hat{t}^F \\ \dfrac{p^F \lambda_C t}{2}, & \hat{t}^F \leqq t \leqq 2\bar{t}^F - \hat{t}^F \\ F^S(t, 2\bar{t}^F - t) - \dfrac{p^F \lambda_C \hat{t}^F}{2} + \displaystyle\int_{2\bar{t}^F - t}^{\hat{t}^F} F_1^S\left(\tau, 2\bar{t}^F - \tau\right) d\tau, & t > 2\bar{t}^F - \hat{t}^F \end{cases},$$

$$w^W(t) = \begin{cases} \dfrac{p^W \lambda_C \hat{t}^W}{2} - \displaystyle\int_t^{\hat{t}^W} F_1^S\left(\tau, 2\bar{t}^W - \tau\right) d\tau, & t < \hat{t}^W \\ \dfrac{p^W \lambda_C t}{2}, & \hat{t}^W \leqq t \leqq 2\bar{t}^W - \hat{t}^W \\ F^S(t, 2\bar{t}^W - t) - \dfrac{p^W \lambda_C \hat{t}^W}{2} + \displaystyle\int_{2\bar{t}^W - t}^{\hat{t}^W} F_1^S\left(\tau, 2\bar{t}^W - \tau\right) d\tau, & t > 2\bar{t}^W - \hat{t}^W \end{cases}.$$

(1) Home and Offshoring

Differentiating w^W with respect to δ yields

$$
\frac{\partial w^W(t)}{\partial \delta} =
\begin{cases}
-\lambda_C(\bar{t}^W - \hat{t}^W)\dfrac{dp^W}{d\delta} + \dfrac{\hat{t}^W}{1+\delta}\dfrac{2\bar{t}^W - \hat{t}^W}{2\bar{t}^W}\left[F_2^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W) - F_1^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W)\right] \\
-\bar{t}^H \displaystyle\int_t^{\hat{t}^W} F_{12}^S(\tau, 2\bar{t}^W - \tau)d\tau > 0, \\
\\
\dfrac{\lambda_C t}{2}\dfrac{dp^W}{d\delta} < 0, \\
\\
\lambda_C(\bar{t}^W - \hat{t}^W)\dfrac{dp^W}{d\delta} - \dfrac{\hat{t}^W}{1+\delta}\dfrac{2\bar{t}^W - \hat{t}^W}{2\bar{t}^W}\left[F_2^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W) - F_1^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W)\right] \\
+\bar{t}^H \displaystyle\int_{2\bar{t}^W - t}^{\hat{t}^W} F_{12}^S(\tau, 2\bar{t}^W - \tau)d\tau < 0.
\end{cases}
.
$$

This outcome leads to

$$
\begin{cases}
w^H(t) > w^W(t), & t \in [t_{\min}^H,\, \hat{t}^W] \\
w^H(t) < w^W(t), & t \in [\hat{t}^H,\, 2\bar{t}^W - \hat{t}^W] \\
w^H(t) < w^W(t), & t \in [2\bar{t}^H - \hat{t}^H,\, t_{\max}^H]
\end{cases}
.
$$

If $t \in (\hat{t}^W,\, \hat{t}^H)$, we have

$$
w^H(t) - w^W(t) = \frac{p^H \lambda_C \hat{t}^H}{2} - \frac{p^W \lambda_C t}{2} - \int_t^{\hat{t}^H} F_1^S\left(\tau, 2\bar{t}^H - \tau\right) d\tau.
$$

Note that

$$
\lim_{t \to \hat{t}^H}\left[w^H(t) - w^W(t)\right] \leqq 0 \leqq \lim_{t \to \hat{t}^W}\left[w^H(t) - w^W(t)\right].
$$

Differentiating $w^H(t) - w^W(t)$ yields

$$
\begin{aligned}
\frac{d\left[w^H(t) - w^W(t)\right]}{dt} &= F_1^S(t, 2\bar{t}^H - t) - \frac{p^W \lambda_C}{2} \\
&\leqq F_1^S(\hat{t}^H, 2\bar{t}^H - \hat{t}^H) - \frac{p^H \lambda_C}{2} \\
&= \frac{2\bar{t}^H - \hat{t}^H}{2\bar{t}^H}\left[F_1^S(\hat{t}^H, 2\bar{t}^H - \hat{t}^H) - F_2^S(\hat{t}^H, 2\bar{t}^H - \hat{t}^H)\right] < 0.
\end{aligned}
$$

Therefore, there exists a $t_{HO} \in (\hat{t}^W, \hat{t}^H)$ such that

$$\begin{cases} w^H(t) > w^W(t), & \hat{t}^W < t < t_{HO} \\ w^H(t) < w^W(t), & t_{HO} < t < \hat{t}^H \end{cases}.$$

Similarly, if $t \in (2\bar{t}^W - \hat{t}^W,\, 2\bar{t}^H - \hat{t}^H)$, we have

$$w^H(t) - w^W(t) = \frac{p^H \lambda_C t}{2} + \frac{p^W \lambda_C \hat{t}^W}{2} - F^S(t, 2\bar{t}^W - t) - \int_{2\bar{t}^W - t}^{\hat{t}^W} F_1^S(\tau, 2\bar{t}^W - \tau) d\tau.$$

Note that

$$\lim_{t \to 2\bar{t}^W - \hat{t}^W} \left[w^H(t) - w^W(t) \right] < 0.$$

In addition, we have

$$\begin{aligned} \frac{d\left[w^H(t) - w^W(t)\right]}{dt} &= \frac{p^H \lambda_C}{2} - F_1^S(t, 2\bar{t}^W - t) \\ &\leq \frac{p^W \lambda_C}{2} - F_1^S(2\bar{t}^W - \hat{t}^W, \hat{t}^W) \\ &= \frac{p^W \lambda_C}{2} - F_2^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W) \\ &= \frac{\hat{t}^W}{2\bar{t}^W} \left[F_1^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W) - F_2^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W) \right] < 0. \end{aligned}$$

Therefore, we obtain that $w^H(t) < w^W(t)$ for any $t \in [2\bar{t}^W - \hat{t}^W,\, 2\bar{t}^H - \hat{t}^H]$.

Summarizing these results, we have

$$\begin{cases} w^H(t) > w^W(t), & t \in [t^H_{\min},\, t_{HO}) \\ w^H(t) < w^W(t), & t \in (t_{HO},\, t^H_{\max}] \end{cases}.$$

(2) Foreign and Offshoring

Differentiating w^W with respect β yields

$$\frac{\partial w^W(t)}{\partial \beta} = \begin{cases} -\lambda_C(\bar{t}^W - \hat{t}^W)\dfrac{dp^W}{d\beta} + \dfrac{\hat{t}^W}{1+\beta}\dfrac{2\bar{t}^W - \hat{t}^W}{2\bar{t}^W}\left[F_2^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W) - F_1^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W)\right] \\ -\bar{t}^F \int_t^{\hat{t}^W} F_{12}^S(\tau, 2\bar{t}^W - \tau)d\tau > 0 \\ \dfrac{\lambda_C t}{2}\dfrac{dp^W}{d\beta} < 0 \\ \lambda_C(\bar{t}^W - \hat{t}^W)\dfrac{dp^W}{d\beta} - \dfrac{\hat{t}^W}{1+\beta}\dfrac{2\bar{t}^W - \hat{t}^W}{2\bar{t}^W}\left[F_2^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W) - F_1^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W)\right] \\ +\bar{t}^F \int_{2\bar{t}^W - t}^{\hat{t}^W} F_{12}^S(\tau, 2\bar{t}^W - \tau)d\tau < 0 \end{cases}.$$

This outcome leads to

$$\begin{cases} w^F(t) < w^W(t), & t \in [t_{\min}^F,\, \hat{t}^F] \\ w^F(t) > w^W(t), & t \in [\hat{t}^W,\, 2\bar{t}^F - \hat{t}^F] \\ w^F(t) > w^W(t), & t \in [2\bar{t}^W - \hat{t}^W,\, t_{\max}^F] \end{cases}.$$

If $t \in (\hat{t}^F,\, \hat{t}^W)$, we have

$$w^F(t) - w^W(t) = \frac{p^F \lambda_C t}{2} - \frac{p^W \lambda_C \hat{t}^W}{2} + \int_t^{\hat{t}^W} F_1^S\left(\tau, 2\bar{t}^W - \tau\right) d\tau.$$

Since

$$\lim_{t \to \hat{t}^F}\left[w^F(t) - w^W(t)\right] \leqq 0 \leqq \lim_{t \to \hat{t}^W}\left[w^F(t) - w^W(t)\right],$$

and

$$\begin{aligned} \frac{d\left[w^F(t) - w^W(t)\right]}{dt} &= \frac{p^F \lambda_C}{2} - F_1^S(t, 2\bar{t}^W - t) \\ &> \frac{p^W \lambda_C}{2} - F_1^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W) \\ &= \frac{2\bar{t}^W - \hat{t}^W}{2\bar{t}^W}\left[F_2^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W) - F_1^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W)\right] > 0, \end{aligned}$$

there exists a $t_{FO} \in (\hat{t}^F,\, \hat{t}^W)$ such that

$$\begin{cases} w^F(t) < w^W(t), & \hat{t}^F \leqq t < t_{FO} \\ w^F(t) > w^W(t), & t_{FO} < t \leqq \hat{t}^W \end{cases}.$$

Similarly, if $t \in (2\bar{t}^F - \hat{t}^F,\, 2\bar{t}^W - \hat{t}^W)$, then we obtain

$$w^F(t) - w^W(t) = F^S(t, 2\bar{t}^F - t) + \int_{2\bar{t}^F - t}^{\hat{t}^F} F_1^S\left(\tau, 2\bar{t}^F - \tau\right) d\tau - \frac{p^F \lambda_C \hat{t}^F}{2} - \frac{p^W \lambda_C t}{2}.$$

Since

$$\lim_{t \to 2\bar{t}^F - \hat{t}^F} \left[w^F(t) - w^W(t)\right] \geqq 0,$$

and

$$\begin{aligned} \frac{d\left[w^F(t) - w^W(t)\right]}{dt} &= F_1^S(t, 2\bar{t}^F - t) - \frac{p^W \lambda_C}{2} \\ &\geqq F_1^S(2\bar{t}^F - \hat{t}^F, \hat{t}^F) - \frac{p^F \lambda_C}{2} \\ &= F_2^S(\hat{t}^F, 2\bar{t}^F - \hat{t}^F) - \frac{p^F \lambda_C}{2} \\ &= \frac{\hat{t}^F}{2\bar{t}^F}\left[F_2^S(\hat{t}^F, 2\bar{t}^F - \hat{t}^F) - F_1^S(\hat{t}^F, 2\bar{t}^F - \hat{t}^F)\right] > 0, \end{aligned}$$

we obtain that $w^F(t) > w^W(t)$ for any $t \in [2\bar{t}^F - \hat{t}^F,\, 2\bar{t}^W - \hat{t}^W]$.

Summarizing these results, we have

$$\begin{cases} w^F(t) < w^W(t), & t \in [t^F_{\min},\, t_{FO}) \\ w^F(t) > w^W(t), & t \in (t_{FO},\, t^F_{\max}] \end{cases}.$$

Proof for Proposition 6

Under free trade, the comparison of wage schedules between Home and Foreign is as follows:

$$
\begin{aligned}
&w^F(t) \lesseqqgtr w^H(t), \quad \text{if } t \in [t^H_{\min},\, \hat{t}^F_T) \text{ and } t \in [\hat{t}^F,\, \hat{t}^H_T),\\
&w^F(t) = w^H(t), \quad \text{if } t \in [\hat{t}^H_T,\, 2\bar{t}^F - \hat{t}^F_T],\\
&w^F(t) \gtreqqless w^H(t), \quad \text{if } t \in (2\bar{t}^F - \hat{t}^F_T,\, 2\bar{t}^H - \hat{t}^H_T) \text{ and } t \in [2\bar{t}^H - \hat{t}^H_T,\, t^F_{\max}].
\end{aligned}
$$

p^T represents the relative price under free trade without offshoring. Using (7), (9), and (10), the wage schedule of Home, $w^H(t)$, $t \in [t^H_{\min},\, t^H_{\max}]$, and that of Foreign, $w^F(t)$, $t \in [t^F_{\min},\, t^F_{\max}]$, are obtained as follows:

$$
w^H(t) = \begin{cases}
\dfrac{p^T\lambda_C\hat{t}^H_T}{2} - \displaystyle\int_t^{\hat{t}^H_T} F_1^S\left(\tau, 2\bar{t}^H - \tau\right) d\tau, & t \in [t^H_{\min},\, \hat{t}^H_T)\\
\dfrac{p^T\lambda_C t}{2}, & t \in [\hat{t}^H_T,\, 2\bar{t}^H - \hat{t}^H_T]\\
F^S\left(2\bar{t}^H - t, t\right) - \dfrac{p^T\lambda_C\hat{t}^H_T}{2} + \displaystyle\int_{2\bar{t}^H - t}^{\hat{t}^H_T} F_1^S\left(\tau, 2\bar{t}^H - \tau\right) d\tau, & t \in (2\bar{t}^H - \hat{t}^H_T,\, t_{\max}]
\end{cases},
$$

$$
w^F(t) = \begin{cases}
\dfrac{p^T\lambda_C\hat{t}^F_T}{2} - \displaystyle\int_t^{\hat{t}^F_T} F_1^S\left(\tau, 2\bar{t}^F - \tau\right) d\tau, & t \in [t^F_{\min},\, \hat{t}^F_T)\\
\dfrac{p^T\lambda_C t}{2}, & t \in [\hat{t}^F,\, 2\bar{t}^F - \hat{t}^F_T]\\
F^S\left(2\bar{t}^F - t, t\right) - \dfrac{p^T\lambda_C\hat{t}^F_T}{2} + \displaystyle\int_{2\bar{t}^F - t}^{\hat{t}^F} F_1^S\left(\tau, 2\bar{t}^F - \tau\right) d\tau, & t \in (2\bar{t}^F - \hat{t}^F_T,\, t^F_{\max}]
\end{cases}.
$$

(1) If $t \in [t^H_{\min},\, \hat{t}^F_T)$, we have

$$
\begin{aligned}
&w^F(t) - w^H(t)\\
&= \frac{p^T\lambda_C\hat{t}^F_T}{2} - \int_t^{\hat{t}^F_T} F_1^S\left(\tau, 2\bar{t}^F - \tau\right) d\tau - \left[\frac{p^T\lambda_C\hat{t}^H_T}{2} - \int_t^{\hat{t}^H_T} F_1^S\left(\tau, 2\bar{t}^H - \tau\right) d\tau\right]\\
&= \frac{(1-\beta)p^T\lambda_C\hat{t}^F_T}{2} + \int_{\hat{t}^F_T}^{\beta\hat{t}^F_T} F_1^S\left(\tau, 2\beta\bar{t}^F - \tau\right) d\tau + \int_t^{\hat{t}^F_T} \left[F_1^S\left(\tau, 2\beta\bar{t}^F - \tau\right) - F_1^S\left(\tau, 2\bar{t}^F - \tau\right)\right] d\tau
\end{aligned}
$$

Because of $F^S_{12} < 0$, for $\tau \in [t,\, \hat{t}^F_T)$, we obtain

$$
F_1^S\left(\tau, 2\beta\bar{t}^F - \tau\right) - F_1^S\left(\tau, 2\bar{t}^F - \tau\right) < 0.
$$

In addition, we have

$$\begin{aligned}
&\frac{(1-\beta)p^T\lambda_C\hat{t}_T^F}{2}+\int_{\hat{t}_T^F}^{\beta\hat{t}_T^F}F_1^S\left(\tau,2\beta\bar{t}^F-\tau\right)d\tau\\
&\leqq\frac{(1-\beta)p^T\lambda_C\hat{t}_T^F}{2}+\left(\beta\hat{t}_T^F-\hat{t}_T^F\right)F_1^S\left(\beta\hat{t}_T^F,2\beta\bar{t}^F-\beta\hat{t}_T^F\right)\\
&=\frac{(1-\beta)\hat{t}_T^F}{2}\left[p^T\lambda_C-2F_1^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)\right]\\
&=\frac{(1-\beta)\hat{t}_T^F\left(2\bar{t}^F-\hat{t}_T^F\right)}{2\bar{t}^F}\left[F_2^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)-F_1^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)\right]<0.
\end{aligned}$$

Therefore, $w^F(t)<w^H(t)$.

(2) If $t\in[\hat{t}_T^F,\,\hat{t}_T^H)$, we have

$$\begin{aligned}
w^F(t)-w^H(t)&=\frac{p^T\lambda_C t}{2}-\left[\frac{p^T\lambda_C\hat{t}_T^H}{2}-\int_t^{\hat{t}_T^H}F_1^S\left(\tau,2\bar{t}^H-\tau\right)d\tau\right]\\
&=\frac{p^T\lambda_C\left(t-\hat{t}_T^H\right)}{2}+\int_t^{\hat{t}_T^H}F_1^S\left(\tau,2\bar{t}^H-\tau\right)d\tau\\
&\leqq\frac{p^T\lambda_C\left(t-\hat{t}_T^H\right)}{2}+\left(\hat{t}_T^H-t\right)F_1^S\left(\hat{t}_T^H,2\bar{t}^H-\hat{t}_T^H\right)\\
&=\left(t-\hat{t}_T^H\right)\left[\frac{p^T\lambda_C}{2}-F_1^S\left(\hat{t}_T^H,2\bar{t}^H-\hat{t}_T^H\right)\right]\\
&=\frac{t-\hat{t}_T^H}{2\bar{t}^H}\left[F^S\left(\hat{t}_T^H,2\bar{t}^H-\hat{t}_T^H\right)-2\bar{t}^HF_1^S\left(\hat{t}_T^H,2\bar{t}^H-\hat{t}_T^H\right)\right]\\
&=\frac{\left(t-\hat{t}_T^H\right)\left(2\bar{t}^H-\hat{t}_T^H\right)}{2\bar{t}^H}\left[F_2^S\left(\hat{t}_T^H,2\bar{t}^H-\hat{t}_T^H\right)-F_1^S\left(\hat{t}_T^H,2\bar{t}^H-\hat{t}_T^H\right)\right]<0.
\end{aligned}$$

(3) If $t\in(2\bar{t}^F-\hat{t}_T^F,\,2\bar{t}^H-\hat{t}_T^H)$, we have

$$\begin{aligned}
w^F(t)-w^H(t)&=F^S\left(2\bar{t}^F-t,t\right)-\frac{p^T\lambda_C\hat{t}_T^F}{2}+\int_{2\bar{t}^F-t}^{\hat{t}_T^F}F_1^S\left(\tau,2\bar{t}^F-\tau\right)d\tau-\frac{p^T\lambda_C t}{2}\\
&=F^S\left(2\bar{t}^F-t,t\right)+\int_{2\bar{t}^F-t}^{\hat{t}_T^F}F_1^S\left(\tau,2\bar{t}^F-\tau\right)d\tau-\frac{p^T\lambda_C\left(t+\hat{t}_T^F\right)}{2}.
\end{aligned}$$

When t converges to $2\bar{t}^F-\hat{t}_T^F$, we obtain $\lim_{t\to2\bar{t}^F-\hat{t}^F}\left(w^F(t)-w^H(t)\right)=0$. In addition, we have

$$
\begin{aligned}
&\frac{\partial\left(w^F(t)-w^H(t)\right)}{\partial t}\\
&=F_2^S\left(2\bar{t}^F-t,t\right)-F_1^S\left(2\bar{t}^F-t,t\right)+F_1^S\left(2\bar{t}^F-t,t\right)-\frac{p\lambda_C}{2}\\
&=F_2^S\left(2\bar{t}^F-t,t\right)-\frac{1}{2\bar{t}^F}F^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)\\
&=\left[F_2^S\left(2\bar{t}^F-t,t\right)-F_2^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)\right]+\frac{\hat{t}_T^F}{2\bar{t}^F}\left[F_2^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)-F_1^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)\right].
\end{aligned}
$$

Since $F_{22}^S>0>F_{21}^S$, we obtain $F_2^S\left(2\bar{t}^F-t,t\right)>F_2^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)$ for $t>2\bar{t}^F-\hat{t}_T^F$. It follows that $\partial\left(w^F(t)-w^H(t)\right)/\partial t>0$. Therefore, we have $w^F(t)>w^H(t)$ for $t\in\left(2\bar{t}^F-\hat{t}_T^F,\,2\bar{t}^H-\hat{t}_T^H\right)$.

(4) If $t\in[2\bar{t}^H-\hat{t}_T^H,\,t_{\max}^F]$, we have

$$
\begin{aligned}
&w^F(t)-w^H(t)\\
&=F^S\left(2\bar{t}^F-t,t\right)+\frac{p^T\lambda_C(\hat{t}_T^H-\hat{t}_T^F)}{2}+\int_{2\bar{t}^F-t}^{\hat{t}_T^F}F_1^S\left(\tau,2\bar{t}^F-\tau\right)d\tau-F^S\left(2\beta\bar{t}^F-t,t\right)\\
&\qquad-\int_{2\bar{t}^H-t}^{\hat{t}_T^H}F_1^S\left(\tau,2\bar{t}^H-\tau\right)d\tau\\
&=-\frac{(1-\beta)p^T\lambda_C\hat{t}_T^F}{2}+\left[F^S\left(2\bar{t}^F-t,t\right)-F^S\left(2\beta\bar{t}^F-t,t\right)\right]+\int_{2\bar{t}^F-t}^{\hat{t}_T^F}F_1^S\left(\tau,2\bar{t}^F-\tau\right)d\tau\\
&\qquad-\int_{2\beta\bar{t}^F-t}^{\beta\hat{t}_T^F}F_1^S\left(\tau,2\beta\bar{t}^F-\tau\right)d\tau.
\end{aligned}
$$

Clearly, $\lim_{\beta\to1}\left(w^F(t)-w^H(t)\right)=0$. In addition, we have

$$
\begin{aligned}
&\frac{\partial\left(w^F(t)-w^H(t)\right)}{\partial\beta}\\
&=\frac{p^T\lambda_C\hat{t}_T^F}{2}-\hat{t}_T^F F_1^S\left(\beta\hat{t}_T^F,2\beta\bar{t}^F-\beta\hat{t}_T^F\right)-2\bar{t}^F\int_{2\beta\bar{t}^F-t}^{\beta\hat{t}_T^F}F_{12}^S\left(\tau,2\beta\bar{t}^F-\tau\right)d\tau\\
&=\frac{\hat{t}_T^F}{2\bar{t}^F}\left[F^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)-2\bar{t}^F F_1^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)\right]-2\bar{t}^F\int_{2\beta\bar{t}^F-t}^{\beta\hat{t}_T^F}F_{12}^S\left(\tau,2\beta\bar{t}^F-\tau\right)d\tau\\
&=\frac{\hat{t}_T^F\left(2\bar{t}^F-\hat{t}_T^F\right)}{2\bar{t}^F}\left[F_2^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)-F_1^S\left(\hat{t}_T^F,2\bar{t}^F-\hat{t}_T^F\right)\right]-2\bar{t}^F\int_{2\beta\bar{t}^F-t}^{\beta\hat{t}_T^F}F_{12}^S\left(\tau,2\beta\bar{t}^F-\tau\right)d\tau.
\end{aligned}
$$

Since $F_{12}^S<0$, we obtain $\partial\left(w^F(t)-w^H(t)\right)/\partial\beta>0$. Therefore, we have $w^F(t)>w^H(t)$ for

$t \in (2\beta\bar{t}^F - \hat{t}_T^H,\, t_{\max}^F]$ when $\beta > 1$.

Proof for Proposition 7

We use $V^W(t)$ to represent the welfare under offshoring. Let $V^i(t)$ and $V^{Ti}(t)$, $i = H, F$, to represent the welfare of workers in country i under autarky and under free trade, respectively. Since the preference is homothetic, the indirect utility function satisfies $V(p_C, p_S, W(t)) = v(p_C, p_S)W(t)$. Because $V(p_C, p_S, W(t))$ is homogeneous of degree zero, we have

$$V(p, 1, w(t)) = \nu(p)w(t),$$

where $\nu(p) \equiv v(p, 1)$ and $w(t) = W(t)/p_S$. Using Euler's homogeneous function theorem, we have

$$\frac{d\nu}{dp} = -\frac{1}{p}\left[v_2(p, 1) + \nu(p)\right].$$

1. The effects of offshoring on welfare

We focus on the case that $p^H < p^W < p^F$.

(1) The effects of offshoring on Home welfare.

Since $p^H < p^W$ and offshoring leads to a decline in the nominal wages of workers with skills $t \in [t_{\min}, t_{HO})$, where $t_{HO} \in (\hat{t}^W, \hat{t}^H)$, we have $V^W(t) < V^H(t)$ for $t < t_{HO}$. For $t \in [\hat{t}^H, 2\bar{t}^W - \hat{t}^W]$, differentiating V^W with respect to δ yields

$$\begin{aligned}
\frac{\partial V^W}{\partial \delta} &= w^W(t)\frac{d\nu(p^W)}{dp}\frac{dp^W}{d\delta} + \nu(p^W)\frac{\partial w^W(t)}{\partial \delta} \\
&= w^W(t)\frac{d\nu(p^W)}{dp}\frac{dp^W}{d\delta} + \nu(p^W)\frac{\lambda_C t}{2}\frac{dp^W}{d\delta} \\
&= \left[\frac{d\nu(p^W)}{dp} + \frac{\nu(p^W)}{p^W}\right] w^W(t)\frac{dp^W}{d\delta} \\
&= \left[\frac{\nu(p^W)}{p^W} - \frac{1}{p^W}v_2(p^W, 1) - \frac{1}{p^W}\nu(p^W)\right] w^W(t)\frac{dp^W}{d\delta} \\
&= -w(t)v_2(p^W, 1)\frac{1}{p^W}\frac{dp^W}{d\delta} < 0.
\end{aligned}$$

It follows that $V^H(t) < V^W(t)$ for $t \in [\hat{t}^H, 2\bar{t}^W - \hat{t}^W]$.

For $t \in [t_{HO},\, \hat{t}^H)$, $V^H(t) - V^W(t)$ becomes

$$V^H(t) - V^W(t) = \nu(p^H)\left[\frac{p^H \lambda_C \hat{t}^H}{2} - \int_t^{\hat{t}^H} F_1^S(\tau, 2\bar{t}^H - \tau)d\tau\right] - \nu(p^W)\frac{p^W \lambda_C t}{2}.$$

Differentiating $V^H(t) - V^W(t)$ with respect to t yields

$$\frac{d}{dt}\left[V^H(t) - V^W(t)\right] = \nu(p^H)F_1^S(t, 2\bar{t}^H - t) - \nu(p^W)\frac{F^S(\hat{t}^W, 2\bar{t}^W - \hat{t}^W)}{2\bar{t}^W}$$

The second derivative of $V^H(t) - V^W(t)$ with respect to t is obtained as

$$\frac{d^2}{dt^2}\left[V^H(t) - V^W(t)\right] = \nu(p^H)\left[F_{11}^S(t, 2\bar{t}^H - t) - F_{12}^S(t, 2\bar{t}^H - t)\right] > 0,$$

implying that $V^H(t) - V^W(t)$ is a convex function. We know that

$$\lim_{t \to \hat{t}^H}\left[V^H(t) - V^W(t)\right] < 0 < \lim_{t \to t_{HO}}\left[V^H(t) - V^W(t)\right].$$

Therefore, there uniquely exists a $\tilde{t}_{HO} \in (t_{HO},\, \hat{t}^H)$ such that $V^H(t) > V^W(t)$ for $t \in [t_{HO},\, \tilde{t}_{HO})$ and $V^H(t) < V^W(t)$ for $t \in (\tilde{t}_{HO},\, \hat{t}^H)$.

Summarizing these results, we have

$$\begin{cases} V^H(t) > V^W(t), & t \in [t^H_{\min},\, \tilde{t}_{HO}) \\ V^H(t) < V^W(t), & t \in (\tilde{t}_{HO},\, 2\bar{t}^W - \hat{t}^W] \end{cases}$$

(2) The effects of offshoring on Foreign welfare.

We know that $p^F > p^W$ and offshoring leads to an increase in the nominal wages of workers with skill levels $t \in [t^F_{\min},\, t_{FO})$, where $t_{FO} \in (\hat{t}^F,\, \hat{t}^W)$. It follows that $V^F(t) < V^W(t)$

for $t < t_{FO}$. For workers who remain in the supermodular sector after offshoring, we have

$$\begin{aligned}\frac{\partial V^W}{\partial \beta} &= w^W(t)\frac{d\nu(p)}{dp}\frac{dp^W}{d\beta} + \nu(p)\frac{\partial w^W(t)}{\partial \beta}\\ &= w^W(t)\frac{d\nu(p)}{dp}\frac{dp^W}{d\beta} + \nu(p)\frac{\lambda_C t}{2}\frac{dp^W}{d\beta}\\ &= \left[\frac{d\nu(p)}{dp} + \frac{\nu(p)}{p}\right] w^W(t)\frac{dp^W}{d\beta}\\ &= \left[\frac{\nu(p)}{p} - \frac{1}{p^W}v_2(p^W,1) - \frac{1}{p^W}\nu(p)\right] w^W(t)\frac{dp^W}{d\beta}\\ &= -w(t)v_2(p,1)\frac{1}{p}\frac{dp}{d\beta} < 0.\end{aligned}$$

Therefore, we obtain $V^W(t) < V^F(t)$ for $t \in [\hat{t}^W, 2\bar{t}^F - \hat{t}^F]$.

For $t \in [t_{FO}, \hat{t}^W)$, we have

$$V^F(t) - V^W(t) = \nu(p^F)\frac{p^F\lambda_C t}{2} - \nu(p^W)\left[\frac{p^W\lambda_C\hat{t}^W}{2} - \int_t^{\hat{t}^W} F_1^S(\tau, 2\bar{t}^W - \tau)d\tau\right].$$

Differentiating $V^F(t) - V^W(t)$ with respect to t yields

$$\frac{d(V^F(t) - V^W(t))}{dt} = \nu(p^F)\frac{F^S(\hat{t}^F, 2\bar{t}^F - \hat{t}^F)}{2\bar{t}^F} - \nu(p^W)F_1^S(t, 2\bar{t}^W - t).$$

The second derivative of $V^F(t) - V^W(t)$ is obtained as

$$\frac{d^2(V^F(t) - V^W(t))}{dt^2} = -\nu(p^W)\left[F_{11}^S(t, 2\bar{t}^W - t) - F_{12}^S(t, 2\bar{t}^W - t)\right] < 0,$$

implying that $V^F(t) - V^W(t)$ is a concave function. Since we have

$$\lim_{t\to t_{FO}}\left[V^F(t) - V^W(t)\right] < 0 < \lim_{t\to \hat{t}^W}\left[V^F(t) - V^W(t)\right],$$

there uniquely exists a $\tilde{t}_{FO} \in (t_{FO}, \hat{t}^W)$ such that $V^F(t) < V^W(t)$ for $t \in [t_{FO}, \tilde{t}_{FO})$ and $V^F(t) > V^W(t)$ for $t \in (\tilde{t}_{FO}, \hat{t}^W]$.

Summarizing these results, we have

$$\begin{cases} V^F(t) < V^W(t), & t \in [t^F_{\min}, \tilde{t}_{FO}) \\ V^F(t) > V^W(t), & t \in (\tilde{t}_{FO}, 2\bar{t}^F - \hat{t}^F] \end{cases}$$

2. The effects of free trade on welfare.

From (5), we obtain

$$\frac{\partial \hat{t}}{\partial p} = \frac{\lambda_C \bar{t}}{F^S_1(\hat{t}, 2\bar{t} - \hat{t}) - F^S_2(\hat{t}, 2\bar{t} - \hat{t})} < 0.$$

Differentiating $w^H(t)$ and $w^F(t)$ with respect to p yields

$$\frac{\partial w^H}{\partial p} = \begin{cases} -\lambda_C(\bar{t}^H - \hat{t}^H) < 0, & t < \hat{t}^H \\ \dfrac{\lambda_C t}{2} > 0, & \hat{t}^H \leqq t \leqq 2\bar{t}^H - \hat{t}^H \\ \lambda_C(\bar{t}^H - \hat{t}^H) > 0, & 2\bar{t}^H - \hat{t}^H < t \end{cases}$$

$$\frac{\partial w^F}{\partial p} = \begin{cases} -\lambda_C(\bar{t}^F - \hat{t}^F) < 0, & t < \hat{t}^F \\ \dfrac{\lambda_C t}{2} > 0, & \hat{t}^F \leqq t \leqq 2\bar{t}^F - \hat{t}^F \\ \lambda_C(\bar{t}^F - \hat{t}^F) > 0, & 2\bar{t}^F - \hat{t}^F < t \end{cases}$$

To examine the effects of opening to trade on workers' welfare, we differentiate V with respect to p to obtain

$$\begin{aligned} \frac{\partial V^i(t)}{\partial p} &= \frac{d\nu(p)}{dp} w^i(t) + \nu(p) \frac{\partial w^i(t)}{\partial p} \\ &= -\frac{v_2(p,1)}{p} w^i(t) + \left[\frac{\partial w^i(t)}{\partial p} - \frac{w^i(t)}{p} \right] \nu(p), \quad i = H, F. \end{aligned}$$

For workers who remain in sector C after trade, we have

$$\frac{\partial V^i(t)}{\partial p} = -v_2(p,1) \frac{\lambda_C t}{2} > 0, \quad i = H, F.$$

Therefore, we obtain

$$
\begin{aligned}
V^H(t) &< V^{TH}(t), \quad \hat{t}^H \leqq t \leqq 2\bar{t}^H - \hat{t}^H, \\
V^F(t) &> V^{TF}(t), \quad \hat{t}_T^F \leqq t \leqq 2\bar{t}^F - \hat{t}_T^F.
\end{aligned}
$$

In Home (Foreign), opening to trade leads to a rise (decrease) in p, in turn resulting in a decline (an increase) in the nominal wages of low-skilled workers remaining in sector S after trade. Thus, we have

$$
\begin{aligned}
V^H(t) &> V^{TH}(t), \quad t < \hat{t}_T^H. \\
V^F(t) &< V^{TF}(t), \quad t < \hat{t}^F.
\end{aligned}
$$

For workers with $t \in [\hat{t}_T^H, \hat{t}^H)$, we have $V^{TH}(t) - V^H(t)$

$$
V^{TH}(t) - V^H(t) = \nu(p^T)\frac{p^T \lambda_C t}{2} - \nu(p^H)\left[\frac{p^H \lambda_C \hat{t}^H}{2} - \int_t^{\hat{t}^H} F_1^S\left(\tau, 2\bar{t}^H - \tau\right) d\tau\right].
$$

Differentiating $V^{TH}(t) - V^H(t)$ with respect to t yields

$$
\frac{d\left(V^{TH}(t) - V^H(t)\right)}{dt} = \nu(p^T)\frac{F^S\left(\hat{t}^H, 2\bar{t}^H - \hat{t}^H\right)}{2\bar{t}^H} - \nu(p^H)F_1^S(t, 2\bar{t}^H - t).
$$

Thus, the second derivative of $V^{TH}(t) - V^H(t)$ is obtained as

$$
\frac{d^2\left(V^{TH}(t) - V^H(t)\right)}{dt^2} = -\nu(p^H)\left[F_{11}^S(t, 2\bar{t}^H - t) - F_{12}^S(t, 2\bar{t}^H - t)\right] < 0,
$$

implying that $V^{TH}(t) - V^H(t)$ is a concave function. Since we have $V^{TH}(\hat{t}_T^H) - V^H(\hat{t}_T^H) < 0$ and $V^{TH}(\hat{t}^H) - V^H(\hat{t}^H) > 0$, there uniquely exists a $\tilde{t}^{TH} \in [\hat{t}_T^H, \hat{t}^H)$ such that $V^H(t) > V^{TH}(t)$ for $t \in [\hat{t}_T^H, \tilde{t}^{TH})$ and $V^H(t) < V^{TH}(t)$ for $t \in (\tilde{t}^{TH}, \hat{t}^H)$.

For workers with $t \in [\hat{t}^F, \hat{t}_T^F)$, we have

$$
V^{TF}(t) - V^F(t) = \nu(p^T)\left[\frac{p^T \lambda_C \hat{t}_T^F}{2} - \int_t^{\hat{t}_T^F} F_1^S\left(\tau, 2\bar{t}^F - \tau\right) d\tau\right] - \nu(p^F)\frac{p^F \lambda_C t}{2}.
$$

Differentiating $V^{TF}(t) - V^F(t)$ with respect to t yields

$$\frac{d\left(V^{TF}(t) - V^F(t)\right)}{dt} = \nu(p^T)F_1^S(t, 2\bar{t}^F - t) - \nu(p^F)\frac{F^S(\hat{t}^F, 2\bar{t}^F - \hat{t}^F)}{2\bar{t}^F}.$$

The second derivative of $V^{TF}(t) - V^F(t)$ is obtained as

$$\frac{d^2\left(V^{TF}(t) - V^F(t)\right)}{dt^2} = \nu(p^T)\left[F_{11}^S(t, 2\bar{t}^F - t) - F_{12}^S(t, 2\bar{t}^F - t)\right] > 0,$$

implying that $V^{TF}(t) - V^F(t)$ is a convex function. Due to $V^{TF}(\hat{t}^F) - V^F(\hat{t}^F) > 0$ and $V^{TF}(\hat{t}_T^F) - V^F(\hat{t}_T^F) < 0$, there uniquely exists $\tilde{t}^{TF} \in [\hat{t}^F, \hat{t}_T^F]$ such that $V^F(t) < V^{TF}(t)$ for $t \in [\hat{t}^F, \tilde{t}^{TF})$ and $V^F(t) > V^{TF}(t)$ for $t \in (\tilde{t}^{TF}, \hat{t}_T^F]$.

Summarizing these results, we have

$$\begin{cases} V^H(t) > V^{TH}(t), & t \in [t_{\min}^H, \tilde{t}^{TH}) \\ V^H(t) < V^{TH}(t), & t \in (\tilde{t}^{TH}, 2\bar{t}^H - \hat{t}^H] \end{cases},$$

$$\begin{cases} V^F(t) < V^{TF}(t), & t \in [t_{\min}^F, \tilde{t}^{TF}) \\ V^F(t) > V^{TF}(t), & t \in (\tilde{t}^{TF}, 2\bar{t}^F - \hat{t}_T^F] \end{cases}.$$

References

[1] Antras, Pol and Davin Chor (2013), "Organizing the Global Value Chain", *Econometrica,* Vol. 81, No. 6, pp. 2127–2204.

[2] Antras, Pol and Esteban Rossi-Hansberg (2009), "Organizations and Trade", *Annual Review of Economics*, Vol. 1, pp. 43-64.

[3] Antras, Pol, Luis Garicano, and Esteban Rossi-Hansberg (2006), "Offshoring in a Knowledge Economy", *Quarterly Journal of Economics,* Vol. 121, Issue 1, pp. 31-77.

[4] Artuc, Erhan and John McLaren (2015), "Trade policy and wage inequality: A structural analysis with occupational and sectoral mobility", *Journal of International Economics,* Vol. 97, Issue 2, pp. 278-294.

[5] Baldwin, Richard and Anthony J. Venables (2013), "Spiders and snakes: Offshoring and agglomeration in the global economy", *Journal of International Economics,* Vol. 90, Issue 2, pp. 245-254.

[6] Bernard. A, Moxnes. A, and Saito, Y (2019), "Production Networks, Geography, and Firm Performance", *Journal of Political Economy,* Vol. 127, No. 2, pp. 639–688.

[7] Bombardini, Matilde, Gallipoli, Giovanni, and Pupato, Germán (2012) "Skill Dispersion and Trade Flows," *American Economic Review*, Vol. 102, No. 5, pp. 2327-2348.

[8] Bombardini, Matilde, Gallipoli, Giovanni, and Pupato, Germán (2014) "Unobservable Skill Dispersion and Comparative Advantage," *Journal of International Economics*, Vol. 92, No. 2, pp. 317-329.

[9] Bougheas, Spiros and Raymond Riezman (2007), "Trade, and the Distribution of Human Capital", *Journal of International Economics,* Vol. 73, Issue 2, pp. 421-433.

[10] Bond, Eric W. (2001), "Commercial Policy in a 'Fragmented' World," *American Economic Review*, Vol. 91, No. 2, pp. 358-362.

[11] Chang, Pao-Li and Fali Huang (2014) "Trade and Divergence in Education Systems," *International Economic Review*, Vol. 55, No. 4, pp. 1251-1280.

[12] Costinot, Arnaud (2009), "An Elementary Theory of Comparative Advantage", *Econometrica*, Vol. 77, No. 4, pp. 1165-1192.

[13] Costinot, Arnaud and Jonathan Vogel (2010), "Matching and Inequality in the World Economy", *Journal of Political Economy*, Vol. 118, Issue 4, pp. 747-786.

[14] Costinot, Arnaud, Jonathan Vogel, and Su Wang (2013), "An Elementary Theory of Global Supply Chain", *Review of Economic Studies,* Vol. 80, Issue 1, pp. 109-144.

[15] Feenstra, Robert C and Gordon H, Hanson (1996), "Globalization, Outsourcing, and Wage Inequality," *American Economic Review*, vol. 86, No. 2, pp. 240-245.

[16] Feenstra, Robert C and Gordon H, Hanson (1997), "Foreign direct investment and relative wages: Evidence from Mexico's maquiladoras", *Journal of International Economics,* Vol. 42, Issue 3-4, pp. 371-393.

[17] Feenstra, Robert C and Gordon H, Hanson (1999), "The Impact of Outsourcing and High-Technology Capital on Wages: Estimates For the United States, 1979–1990", *Quarterly Journal of Economics,* Vol. 114, Issue 3, pp. 907-940.

[18] Grossman, Gene (2004) "The Distribution of Talent and the Pattern and Consequences of International Trade," *Journal of Political Economy*, Vol. 112, No. 1, pp. 209-239.

[19] Grossman, Gene and Esteban Rossi-Hansberg (2008), "Trading Task: A Simple Theory of Offshoring", *American Economic Review*, Vol. 98, No. 5, pp. 1978-1997.

[20] Grossman, Gene and Rossi-Hansberg (2012) "Task Trade between Similar Countries", *Econometrica*, Vol. 80, No. 2, pp. 593–629.

[21] Grossman, Gene and Maggi, Giovanni (2000) "Diversity and Trade," *American Economic Review*, Vol. 90, No. 5, pp.1255-1275.

[22] Hummels, David, Jun Ishii, and Kei-Mu Yi (2001), "The Nature and Growth of Vertical Specialization in World Trade", *Journal of International Economics,* Vol. 54, Issue 1, pp. 75-96.

[23] Hummels, David, Rasmus Jorgensen, Jakob R. Munch, and Chong Xiang (2014), "The Wage Effects of Offshoring: Evidence from Danish Matched Worker-Firm Data", *American Economic Review,* Vol. 104, No. 6, pp. 1597-1629.

[24] Jones, Ronld W. and Henryk Kierzkowski (1990), "The Role of Services in Production and International Trade: A Theoretical Framework", ch. 3 in Jones and Krueger, (eds.), *The Political Economy of International Trade* (Blackwell's), pp. 31-48.

[25] Mundell, Robert (1957),"International Trade and Factor Mobility", *American Economic Review,* Vol. 47, No. 3, pp. 321-335.

[26] Nocke, Volker and Stephen Yeaple (2008), "An Assignment Theory of Foreign Direct Investment", *Review of Economic Studies*, Vol. 75, Issue 2, pp. 529-557.

[27] Ohnsorge, Franziska and Daniel Trefler (2007), "Sorting It Out: International Trade with Heterogeneous Workers", Journal of Political Economy, Vol. 115, No. 5, pp. 868-892.

[28] Yeaple, Stephen (2005), "A Simple Model of Firm Heterogeneity, International Trade, and Wages", *Journal of International Economics*, Vol. 65,Issue 1, pp. 1-20.

[29] Yi, Kei-Mu (2003), "Can vertical specialization explain the growth of world trade?", *Journal of Political Economy*, Vol. 111, No. 1, pp. 52–102.

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