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Trend Inflation in the Japanese pre-2000s: A Markov-Switching DSGE Estimation*

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Abstract

In Japan, the inflation rate declined to near-zero, whereas the monetary policy faced a Zero Lower Bound (ZLB) in the 1990s. We examine whether trend inflation has fallen to near-zero *prior to* the ZLB. For this purpose, we estimate Japanese pre-2000 trend inflation developing a Markov-Switching New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model in which non-zero trend inflation is explicitly incorporated as a Markov chain state. Our estimation results indicate that the trend inflation remained broadly stable at 2.0–4.0 percent from the 1960s to the late 1970s when it fell somewhat. Up until 1997 when the ZLB was hit, the trend inflation hovered well above zero, mostly at near 1.0 percent.

JEL Classification: E31, E52, C54

Keywords: Trend Inflation, Markov-Switching Dynamic Stochastic General Equilibrium Model, Deflation, Monetary Policy, Lost Decade of Japan.

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1 Introduction

An underlying motivation for our work is to investigate the *lost decade* of Japan – a protracted stagnation of the economy since the 1990s. A seminal work in the literature is Hayashi and Prescott (2002), who argued that real and supply-side factors of the economy, specifically a slowdown in technological progress and reduction in labor hours, can broadly account for low GDP growth rates during the periods.[1] In contrast, Nishizaki, Sekine and Ueno (2014) identify multiple factors, including declines in inflation expectations. Debate remains ongoing over the major driving factors that gave rise to Japan's slow growth since the 1990s.

Looking at the nominal aspects of the economy, deflation in Japan began in the late 1990s. Figure 1 presents monthly year-on-year changes in the consumer price index (CPI). CPI inflation was 4.3 percent on average before 1960, 9.1 percent in the 1970s and fell to near one percent remaining low for 1980–2005. In the 2000s, CPI inflation hovered around zero. In the meantime, nominal interest rates declined, hitting the zero lower bound (ZLB) in the late 1990s.

We examine whether trend inflation has fallen to near-zero *prior to* ZLB. Although the data suggest possible declines in trend inflation pre-dating the 2000s, timing and rate of such declines remain issues for exploration. To this end, we focus on pre-2000s macroeconomic time series data, including high inflation eras since the 1960s, particularly the 1970s, during which CPI inflation reached 25 percent in the wake of global oil crises.

Specifically, we estimate a small-scale New Keynesian model using pre-2000s' Japanese data. However, with higher than 5 percent average inflation, linearizing the model around the zero-inflation steady-state could elicit significant estimation bias, as argued by Cogley and Sbordone (2008) and Ascari and Sbordone (2014, hereafter denoted as AS 2014). Because our study covers transition periods from high- to low-trend inflation eras, we estimate a Generalized New Keynesian (GNK) model that explicitly allows non-zero trend inflation as a Markov-switching unobserved state.

Our main results indicate that trend inflation remained broadly stable between 2.0–4.0 percent from 1960 to the late 1970s. From the late 1970s onward, trend inflation is likely to have declined to somewhere around one percent. There is, however, little evidence that trend

[1] See also Kaihatsu and Kurozumi (2014) for similar arguments based on an estimated structural macroeconomic model with financial market frictions. Kato and Nishiyama (2005) suggest the importance of monetary policy in the 1990s.

inflation fell near-zero percent, or even to the negative, prior to 1996, when the ZLB was hit in Japan. In addition, our results detect regime switches regarding monetary policy stance and supply-shock volatility. The estimated regime-switching monetary policy rule suggests that monetary policy stance has become "hawkish" in the sense that the short-term policy rate responded more to inflation since the 1980s than in the earlier periods. The results clearly show that the supply-shock volatility surged at the time of two global oil crises in 1973 and 1979 while remaining low and stable for the entire non-crisis periods.

Figure 1: Consumer Price Index in Japan

Our work is related to the three strands of literature. First, because we estimate a (G)NK dynamic stochastic general equilibrium (DSGE) model using data in which trend inflation is likely to be significantly above zero, we apply a non-zero trend inflation model proposed by AS (2014) rather than the standard NK models.[2] Second, regarding the estimation procedure, we apply a Markov-Switching Rational Expectations framework which has been developed in the literature of Bayesian inference for structural macroeconomic models. Specifically, we follow the framework introduced by Maih (2014) and later demonstrated by Bjønland, Larsen, and Maih (2018, hereafter denoted as BLM 2018). Third, many studies estimate NK-DSGE models using Japanese data, including Sugo and Ueda (2008), Aruoba, Cuba-Borda, and Schorfheide (2018), Abe, Fueki and Kaihatsu (2019), Hirose (2020), and Iiboshi, Shintani, and Ueda (2020), among others. Most of the earlier studies rely on non-regime-switching DSGE models.[3] The most closely related studies are Aruoba, Cuba-Borda and Schorfheide (2018) and Abe, Fueki and Kaihatsu (2019), both of which allow their models to regime-switch, but they do not consider the non-zero trend inflation discussed by AS (2014) and this paper.

The remainder of this paper is organized into four sections. Section 2 presents the GNK model applied to this study. Section 3 establishes the estimation procedure used. Section 4 details the results of the analyses and Section 5 concludes.

[2] See Woodford (1999) and Clarida, Galí, and Gertler (1999), among many others, for the standard NK models

[3] See also, Ichiue, Kurozumi and Sunakawa (2013), Kaihatsu and Kurozumi (2014) and Fueki et al. (2016). All these DSGE estimations use the data starting later than 1980, thus exlcuding the high inflation periods.

2 A New Keynesian model with non-zero trend inflation

As shown in Figure 1, given the fact that average inflation was quite high – far above two percent – before the 1970s, we consider a small-scale GNK-DSGE model developed by AS (2014), which explicitly incorporates non-zero trend inflation $\bar{\pi}_t$. Formally, trend inflation is defined in terms of the infinite horizon forecast,

$$\bar{\pi}_t = \lim_{j \to \infty} E_t \pi_{t+j}.$$

In the context of our GNK-DSGE model, trend inflation is the steady state inflation under the rational expectations equilibrium.

Our model is a log-linearized version of the full model developed by AS (2014). We first articulate the supply-side of the economy by presenting a generalized Phillips curve. The four equations of the generalized Phillips curve are derived from the log-linearized optimal conditions of the Calvo-price setting firms and the standard resource constraint. The model's set-up is presented in the Appendix.

2.1 Aggregate supply side: GNK Phillips curve

A standard small-scale NK model consists of three equations, that is, the NK Phillips curve, the consumption-output Euler equation, and a monetary policy rule. By contrast, in our model, the generalized Phillips curve is expressed by four separate equations. Let P_t be the price index of final goods and we define $\pi_t = P_t/P_{t-1}$ and $\bar{\pi}_t$ as gross inflation rate and the steady state π_t, i.e, trend inflation, respectively. In contrast to early studies, we allow $\bar{\pi}_t$ to vary over time, such that $\bar{\pi}_t = \bar{\pi}\left(\mathcal{S}_t^{\bar{\pi}}\right)$, in which $\mathcal{S}_t^{\bar{\pi}}$ will be defined later combined with other unobserved states. Further, y_t denotes output which is equal to consumption in this model, and z_t^{AS} indicates the aggregate supply shock. In the remainder of this paper, the variables with tilde denote log-deviations from steady-state values of the variable, i.e., $\tilde{x}_t = ln(x_t/\bar{x})$.

The four equations of the GNK Phillips curves are as follows:

$$
\begin{aligned}
\tilde{\pi}_t &= \frac{1-\Gamma_a(\bar{\pi}_t)}{\Gamma_a(\bar{\pi}_t)}(\tilde{\psi}_t - \tilde{\phi}_t) + \rho\tilde{\pi}_{t-1}, && (1)\\
\tilde{\psi}_t &= \{1-\beta\Gamma_b(\bar{\pi}_t)\}\left\{\chi\tilde{s}_t + (1+\chi)\left(\tilde{y}_t - z_t^{AS}\right)\right\} \\
&\quad +\beta\Gamma_b(\bar{\pi}_t)\left(E_t\tilde{\psi}_{t+1} + \varepsilon E_t\tilde{\pi}_{t+1} - \rho\varepsilon\tilde{\pi}_t\right), && (2)\\
\tilde{\phi}_t &= \{1-\beta\Gamma_a(\bar{\pi}_t)\}(1-\sigma)\tilde{y}_t \\
&\quad +\beta\Gamma_a(\bar{\pi}_t)\left\{E_t\tilde{\phi}_{t+1} + (\varepsilon-1)E_t\tilde{\pi}_{t+1} + \rho(1-\varepsilon)\tilde{\pi}_t\right\}, && (3)\\
\tilde{s}_t &= \Gamma_b(\bar{\pi}_t)\tilde{s}_{t-1} + \frac{\varepsilon\bar{\pi}_t\Gamma_a(\bar{\pi}_t)}{1-\Gamma_a(\bar{\pi}_t)}\tilde{\pi}_t - \varepsilon\rho\Gamma_b(\bar{\pi}_t)\tilde{\pi}_{t-1}, && (4)
\end{aligned}
$$

where $\tilde{\psi}_t$ and $\tilde{\phi}_t$ represent auxiliary variables inherited from the first-order condition of the Calvo-price setting firms. In (4), $\tilde{s}_t$ denotes the degree of the real distortion stemming from price dispersion. AS (2014) demonstrated that (i) this additional state variable $\tilde{s}_t$ shows up only in the "generalized" NK Phillips curve and (ii) plays an important role in creating richer dynamics if the trend inflation is away from zero.

Several remarks on notations are in order: β, σ and χ indicate the discount factor, the degree of risk aversion and inverse labor supply elasticity. $\varepsilon > 1$ denotes the elasticity of substitution among intermediate goods. $\rho \in [0, 1)$ represents the degree of inflation indexation by non-optimizing price-setting firms. In the model, $\Gamma_a(\bar{\pi}_t) \equiv \theta\bar{\pi}_t^{(\varepsilon-1)(1-\rho)}$ and $\Gamma_b(\bar{\pi}_t) \equiv \theta\bar{\pi}_t^{\varepsilon(1-\rho)}$ wherein θ is a Calvo-parameter representing the fraction of firms that cannot change prices in the period. We note that the model is linear in terms of variables whereas the parameters are nonlinear functions in trend inflation. It can be confirmed that setting $\bar{\pi}_t = \bar{\pi} = 1$ with $\rho = \chi = 0$ and $\sigma = 1$ in (1)–(4) eliminates $\tilde{\psi}_t$, $\tilde{\phi}_t$ and $\tilde{s}_t$, resulting in a standard NKPC, such that $\tilde{\pi}_t = \beta E_t\tilde{\pi}_{t+1} + \theta^{-1}(1-\beta\theta)(1-\theta)\left(\tilde{y}_t - z_t^{AS}\right)$.

2.2 Aggregate demand side and structural shocks

The model closes with the otherwise standard consumption-output Euler equation and a monetary policy rule, such that

$$
\begin{aligned}
\tilde{y}_t &= E_t\tilde{y}_{t+1} - \sigma^{-1}(\tilde{\imath}_t - E_t\tilde{\pi}_{t+1}) + z_t^{IS}, && (5)\\
\tilde{\imath}_t &= \rho^{MP}\tilde{\imath}_{t-1} + \left(1-\rho^{MP}\right)(\alpha_y\tilde{y}_t + \alpha_{\pi,t}\tilde{\pi}_t) + e_t^{MP}, && (6)
\end{aligned}
$$

where $\tilde{\imath}_t$ denotes nominal interest rate and α_π, α_y, and $\rho^{MP} \in [0,1)$ represent monetary policy response parameters. Following many early works, we allow α_π to regime-switch such that $\alpha_{\pi,t} = \alpha_\pi \left(\mathcal{S}_t^{MP}\right)$. The Markov-switching state $\mathcal{S}_t^{MP}$ is defined with others in subsection 2.3.

The model contains three structural shocks; that is, the aggregate supply shock z_t^{AS}, the IS shock z_t^{IS}, and the monetary policy shock e_t^{MP}. While monetary policy shock e_t^{MP} is distributed as $N(0,\sigma_{MP}^2)$, the other two follow stationary first-order autoregressive processes such as:

$$z_t^x = \rho^x z_{t-1}^x + e_t^x,\ e_t^x \sim N(0,\sigma_x^2),$$

where $\rho^x \in [0,1)$ and $x \in \{AS, IS\}$. The aggregate supply (AS) shock contains shocks affecting the supply-side of the economy, such as technology shocks and various markup shocks, including oil price shocks. IS shock could be a composite of preference shock, fiscal policy shock, and shocks to the natural rate of interest.

2.3 Markov-switching states

In the main model, we stipulate the following four Markov chains,

$$\begin{aligned} \mathcal{S}_t^{\bar{\pi}} &\in \{\text{High}, \text{Low}\}, && (7) \\ \mathcal{S}_t^{MP} &\in \{\text{Hawkish}, \text{Dovish}\}, && (8) \\ \mathcal{S}_t^{dy^*} &\in \{\text{High growth}, \text{Low growth}\}, && (9) \\ \mathcal{S}_t^{AS} &\in \{\text{High volatility}, \text{Low volatility}\}. && (10) \end{aligned}$$

First, we allow trend inflation $\bar{\pi}_t$ to change according to a Markov chain that moves among the two regimes $\{\text{High}, \text{Low}\}$ in the main model. Second, we allow two monetary policy regimes given by (8). We define a “hawkish” regime as the periods during which the Bank of Japan responds more sensitively to inflation. Specifically, α_π in (6) follows the monetary policy chain $\mathcal{S}_t^{MP}$. Third, we also allow the steady-state real per-capita GDP growth denoted by dy_t^* to follow the macroeconomic growth chain $\mathcal{S}_t^{dy^*}$ given by (9). We explicitly consider $\mathcal{S}_t^{dy^*}$ because our data include the 1950s–60s, the era known as the post-war Japanese “economic miracle.”[4] However, we note that $\mathcal{S}_t^{dy^*}$ does not affect any parameters in the model presented

[4] See Patrick and Rosovsky (1976) for discussions on the Japan’s rapid growth in the 1960s.

in this section, but dy_t^*, which appears only in the observation equation to be presented later.

We also allow the volatility of the supply shock σ_{AS}^2 to change according to a Markov chain $\mathcal{S}_t^{AS}$. In our small-scale NK model, σ_{AS}^2 includes shocks arising from global oil markets and we note that BLM (2018) emphasized the importance of considering the role of oil price volatility in accounting for business cycles. In the same spirit, we allow $\sigma_{AS}^2 = \sigma_{AS}^2(\mathcal{S}_t^{AS})$ to Markov-switch because (i) our sample periods include the two global oil crisis experiences in the 1970s and, (ii) Japan's economy was heavily dependent on imported crude oil during this period.

3 Estimation procedure

3.1 A Markov switching rational expectations framework

Our model can be cast in a general Markov-Switching DSGE (MS DSGE) framework expressed as,

$$E_t \sum p_{\mathcal{S}_t,\mathcal{S}_{t+1}} \mathbf{d}_{\mathcal{S}_t} \left[\mathbf{x}_{t+1}\left(\mathcal{S}_{t+1}\right), \mathbf{x}_t\left(\mathcal{S}_t\right), \mathbf{x}_{t-1}, \mathbf{e}_t\right] = 0, \tag{11}$$

where $\mathbf{d}_{\mathcal{S}_t}$ is an $n_d \times 1$ vector of functions with their arguments $\mathbf{x}_{t+1}\left(\mathcal{S}_{t+1}\right), \mathbf{x}_t\left(\mathcal{S}_t\right), \mathbf{x}_{t-1}$, and $\mathbf{e}_t$. Note that in our log-linearized model setting, function $\mathbf{d}_{\mathcal{S}_t}$ is cast as a set of linear functions. $\mathcal{S}_t = 1, 2, ...h$, is the regime at time t, $\mathbf{x}_t$ is an $n_x \times 1$ vector of all the endogenous variables. $\mathbf{e}_t$ is an $n_\varepsilon \times 1$ vector of Gaussian shocks with $\mathbf{e}_t \sim N(0, \mathbf{I}_{n_\varepsilon})$. $p_{\mathcal{S}_t,\mathcal{S}_{t+1}}$ is the transition probability for moving from regime $\mathcal{S}_t$ in period t to $\mathcal{S}_{t+1} = 1, 2, ...h$ in the next period such that $\sum_{\mathcal{S}_{t+1}=1}^{h} p_{\mathcal{S}_t,\mathcal{S}_{t+1}} = 1$. In our main model, we have $h = 16$. This number follows from the model, wherein we specified two trend inflation states (high and low), two monetary policy states (hawkish and dovish), two macroeconomic growth states (high and low), and two supply-shock volatility states (high and low), yielding a total of $2\times2\times2\times2 = 16$ possible regimes.

In general, no analytical solution to (11) exists, even though $\mathbf{d}_{\mathcal{S}_t}$ is linear. In this paper, we apply Maih's (2014) perturbation technique for solving MS DSGE models. For the stability condition of our MS DSGE models, we rely on the concept of mean-square stability (MSS) following Farmer, Waggoner, and Zha (2011) among others.[5]

[5] In implementing the Maih's (2014) perturbation method with the stability condition to solve (11), we use the RISE toolbox for Matlab, which is available at https://github.com/jmaih/RISE_toolbox/.

3.2 Data and Bayesian inference

Our dataset is quarterly frequency, consisting of real per-capita GDP (y_t), CPI, excluding imputed rent (P_t), and the official discount rate ($\log R_t$). The real GDP and CPI, excluding imputed rent, are taken from 68SNA and from CPI (2015 Base Year index). The official discount rate is taken from the Bank of Japan's Time Series Data Search site. The per-capita real GDP is computed as the real GDP divided by total labor force. Log differences of the real per-capita GDP and CPI are used for the estimation. All data, including the official discount rate, are shown in annualized rates.

Our estimation framework includes the following observation equations, summarized as,

$$\begin{bmatrix} 400\Delta \log y_t \\ 400\Delta \log P_t \\ 100 \log R_t \end{bmatrix} = \begin{bmatrix} dy_t^* \\ \pi_t^* \\ \pi_t^* + r_t^* \end{bmatrix} + \begin{bmatrix} 4\left(\tilde{y}_t - \tilde{y}_{t-1} + z_t^{AS}\right) \\ 4\tilde{\pi}_t \\ 4\tilde{\imath}_t \end{bmatrix} + \begin{bmatrix} \eta_t^y \\ \eta_t^\pi \\ 0 \end{bmatrix}, \tag{12}$$

where $r_t^* = 400\,(1/\beta - 1) + \sigma^{-1} dy_t^*$ and η_t^{obs} is a measurement error distributed as $N(0, Var(\eta_t^{obs}))$, where $obs \in \{y, \pi\}$. As noted in the previous section, we assume that the steady-state real per-capita GDP growth rate $dy_t^* = dy^*(\mathcal{S}_t^{dy^*})$ and trend inflation $\bar{\pi}_t = \bar{\pi}\left(\mathcal{S}_t^{\bar{\pi}}\right)$ change according to different Markov chains given by (9) and (8). For notational convenience, we redefine *net* annual trend inflation rate as $\pi_t^* = 100 \times (\bar{\pi}_t - 1)$. Accordingly, we will report $\pi_{high}^* = 100 \times \{\bar{\pi}\left(\mathcal{S}_t^{\bar{\pi}} = \text{High}\right) - 1\}$ and $\pi_{low}^* = 100 \times \{\bar{\pi}\left(\mathcal{S}_t^{\bar{\pi}} = \text{Low}\right) - 1\}$ in line with data counterpart $400\Delta \log P_t$ in (12). In the same spirit, we note $4\tilde{\imath}_t = 100 \log(R_t/\bar{R}_t)$ which connects the nominal interest rates in the model and in the data.[6]

The sample period of our dataset is from 1958Q2 to 1997Q1. Because our interest lies in the transition of the Japanese economy from high- to low-trend inflation eras, including the 1970s when the inflation was historically high, assessing this period is indispensable. The end of the sample period is chosen for the following three reasons. First, our key question is whether Japan's trend inflation declined before the ZLB. Nominal short-term interest rates, including the official discount rate and the Call rate, were cut to 0.5 percent in 1996, which were then considered the effective lower bound (ELB). Moreover, including data beyond 1997 can seriously distort the estimation results due to the nonlinearity arising from ZLB or ELB.[7]

[6] See A.4 in the appendix for greater details.

[7] A number of early studies, such as Aruoba, Cuba-Borda, and Schorfheide (2018), Inoue and Okimoto (2008), and Hirose (2020) argue that the ZLB/ELB gave rise to a structural break in Japan in the late 1990s.

The second reason arises from the hike of the consumption tax rate from three percent to five percent in April 1997. Reflecting this exogenous shock, CPI inflation reveals a blip in 1997Q2 in Figure 1, providing a ground for excluding the data after 1997Q2. Third, because we are estimating a regime-switching model, ensuring consistent and continuous time series data is critically important. One advantage of our dataset is that all variables remain continuous on the same official basis, and neither artificial discontinuation nor connection of different time series is found, such as revision of the base-year, sampling method, and data definitions. In particular, longitudinal time series of real GDP covering the 1950s to the 2010s do not exist due to base year changes and other statistical revisions.

Before discussing the priors of the parameters to be estimated, we calibrate two parameters for avoiding identification issues. We calibrated the discount factor $\beta = 0.999$ based on the sample medians of ex-post real interest rate and output growth rate. We set the inverse labor supply elasticity $\chi = 2$ following Hirose (2020). Table 1 summarizes the prior distributions of the parameters. Most of the priors for the structural parameters $(\varepsilon, \theta, \sigma, \rho)$ are taken from Hirose (2020). The main parameters of our interest are trend inflation, π^*_{high} and π^*_{low} of which priors are set to 3.0 and 1.0 percent, respectively. We note that, in contrast to gamma distribution for π^*_{high}, normal distribution is assumed for π^*_{low} not to over-restrict the domain of lower bound of trend inflation. Assuming normal distribution for π^*_{low} flexibly allows that negative trend inflation, if it is the case, could be estimated.

Regarding the priors for the Taylor rule parameters, we set $\alpha_y = 0.5$ and α_π (Hawkish) $=$ 1.5 and α_π (Dovish) $=$ 1.0. Finally, considering the sample sub-period averages, we set dy^*(High growth) $= 8$ and dy^*(Low growth) $= 2$ percents.

We compute the posteriors of the parameters combining the likelihood function of the model with the priors noted above. Exploiting the linear-Gaussian nature of the model, the likelihood function is evaluated based on Kalman filter. In the procedure, 10,000 draws from the posterior distribution are generated by the Metropolis-Hastings algorithm. Convergence of posterior distributions has been checked based on trace plots and Gelman et al. (2004) statistics.

Table 1: Prior Distributions of Parameters

4 Results

We report parameter estimates, state transition probabilities and point estimate of trend inflation based on the main model. In Section 4.4, we discuss additional results obtained from two alternative specifications in comparison with the main case.

4.1 Parameter estimates

Table 2 summarizes the parameter estimation results. The main interests of this study are the two values of trend inflation rates in each state. The posterior modes of high and low trend inflation states are 2.4, and 0.9 percent, respectively. Standard errors of each estimate are small enough, suggesting that each of the two states is well identified. Other important regime-switching parameters include the Taylor coefficients on inflation rate $\alpha_\pi(\mathcal{S}_t^{MP})$, the potential GDP growth rate $dy^*(\mathcal{S}_t^{dy^*})$ and the supply-shock volatility $\sigma_{AS}^2(\mathcal{S}_t^{AS})$. When monetary policy stance is hawkish, the Taylor coefficient is estimated at 1.8, whereas it is at 0.5 under the dovish state. The higher potential GDP growth rate is 7.7 percent compared with 3.0 percent in the lower state. The supply-shock volatility in terms of standard deviation $\sigma_{AS}(\mathcal{S}_t^{AS})$ in the high state is 1.0 while that in the low state is 0.1, each of which is translated into annual 4.0 and 0.4 percent, respectively.

The remainders are non-switching parameters and their standard errors are also small. Although the outright comparison is not appropriate because of differing sample periods, most of the posterior modes and means of the parameters are broadly similar to those in early studies using Japanese data. Relatively, the Calvo parameter θ and the inflation indexation ρ are estimated at lower values compared with early studies. The lower θ implies more flexible price adjustments. This is consistent with the higher average and volatility of actual inflation in our pre-2000 sample periods as shown in Figure 1. While the inflation indexation has been a controversial "structural" parameter in the literature, AS (2014) claim that applying the GNK model can eliminate estimation bias and yield lower estimates for ρ.

Table 2: Posterior Estimation Results

4.2 Smoothed state probabilities

Another important output of our estimation is the smoothed state probabilities. The upper panel in Figure 2 presents the probability for being in the high trend inflation state. Because there are only two states for each Markov chain, the flipside is the probability of being low. The shaded areas in the figure indicate recession periods identified by the Economic and Social Research Institute (ESRI) of the Japanese government. From the early 1960s to 1977, the high trend inflation state was likely to be dominant. In the late 1970s, the probability of the high trend inflation state declines and remains at zero up to 1996, except for a short bout in the late phase of the "bubble" period of 1989–1991.

The second panel of Figure 2 presents the probability for being in the hawkish monetary policy state. The figure indicates that monetary policy was dovish in the early 1960s and for most of the 1970s. From 1980 up until 1996, monetary policy stance remained hawkish. The high probability for being in the hawkish state since the 1980s is in line with the findings in the literature of Great Moderation in the U.S. context.[8] The third panel shows the probability for being in the high supply shock volatility state. In the figure, two spikes reaching 100 percent clearly identify the well known first and second global oil crisis episodes: the 1973–74 OPEC embargo and the 1978 Iranian revolution. This result suggests that the two recessions in the Japanese 1970s were precipitated by the elevated global oil price volatility.

Figure 2: Smoothed Transition Probabilities

4.3 Point estimate of the trend inflation

By combining the posterior modes of $\bar{\pi}(\mathcal{S}_t^{\bar{\pi}})$ and the smoothed probabilities for each state, the point estimate for trend inflation can be calculated over time. Figure 3 shows the point estimate of the trend inflation rate. The figure suggests that the Japanese trend inflation was stable, between 2.0–2.5 percent from 1958 to 1977. Even in the midst of the first oil crisis in 1973, trend inflation was notably stable at 2.4 percent while actual inflation reached 25 percent. Except for the previously noted occasion of the bubble period, the trend inflation is lower, at around one percent, in the 1980s and in the 1990s, and little evidence is found that it fell anywhere close to zero percent. Detecting that trend inflation declined to negative territory prior to the ZLB period is even harder.

[8] See Clarida, Galí, and Gertler (2000), among others.

Figure 3: Trend Inflation

4.4 Alternative specifications

We consider two alternative specifications for checking the robustness of the main estimation results. Specifically, we report the cases of (i) constant supply-shock volatility and (ii) three states in trend inflation.

4.4.1 Constant supply-shock volatility

Because $\mathcal{S}_t^{AS}$ is the only Markov-chain which affects the second moment of exogenous shocks, here we examine whether the estimation results noticeably change if the aggregate supply shock is assumed to be homoskedastic. Table 2 presents the posterior estimates of the model in the case that σ_{AS}^2 is non-switching. In this case, the estimated π_{high}^* and π_{low}^* are 2.3 and 1.4, respectively, in comparison to 2.4 and 0.9 in the main case. In the case of non-switching σ_{AS}^2, π_{low}^* is estimated even higher than one percent while π_{high}^* remains almost the same across the two specifications. Figure 4 compares the point estimate of the trend inflation. The dotted line shows the trend inflation in the case that σ_{AS}^2 is non-switching. The trend inflation moderately increases at the times of the first and second oil crises. This result is intuitive: Because σ_{AS}^2 is assumed to be constant, the times of oil crises are identified as the periods when the levels of trend inflation are elevated, instead of its volatility. Our overall assessment is that allowing σ_{AS}^2 to follow a Markov chain gives a better fit of the model to the data.

4.4.2 Allowing three states in trend inflation

A substantial extension is to allow $\mathcal{S}_t^{\bar{\pi}}$ to follow a Markov chain with three states,

$$\mathcal{S}_t^{\bar{\pi}} \in \{\text{High, Medium, Low}\}.$$

The outright motivation of this extension is to detect whether a more inflationary state would arise in or before the 1970s and thus, whether such an additional state may affect the levels of other states in $\mathcal{S}_t^{\bar{\pi}}$. As indicated by Figure 3, the deviation of actual inflation from the estimated trend inflation is the largest in pre-1980 and the direction of the deviation is upward. Reflecting this upward deviation, we set the priors $\left\{\pi^*_{high}, \pi^*_{medium}, \pi^*_{low}\right\} = \{5.0, 2.5, 1.0\}$ so that, if any, π^*_{high} would be additionally identified in the high inflation periods. The posteriors of the three-state π^* model are reported in the right column in Table 2. The posterior modes of the trend inflation states are $\left\{\pi^*_{high}, \pi^*_{medium}, \pi^*_{low}\right\} = \{3.9, 2.8, 0.8\}$ and each of the three levels is reasonably identified. We emphasize that other parameters are estimated at close values of those in the main case. In Figure 4, the thin solid line shows the point estimate of the three-state π^* model. Three observations can be noted; (i) trend inflation dipped in the mid-1970s and fell sharply in the following years, the same timing as that in the main case. (ii) The level of trend inflation before the sharp decline in the late 1970s is higher at 3–4 percent than under the main case. Finally, (iii) after the sharp decline, the trend inflation remains at its lowest slightly less than one percent and stable up to 1997.

Figure 4: Trend Inflation under Alternative Specifications

5 Concluding Remarks

Our estimation results broadly indicate that, in Japan, the trend inflation was unlikely to decline to a near- or even below-zero level prior to 1996 when the ZLB was hit. The fact is that, Japan's deflation began around the year 2000 and continued for protracted periods. One implication is that, even if trend inflation is well above zero—somewhere around one to two percent—it is possible for a central bank to quickly get caught by the ZLB; hence, it can lose some, if not all, control over inflation. This implication needs to be assessed with a few caveats. One is that the estimation is not based on the real-time data but on the historical data. As of 1997, which is the end of our sample period, policymakers were observing the real-time data which were later revised. Estimation using real-time data may elicit more nuanced implications. Another limitation of our work is that our estimation is based on a small-scale model. Obviously, medium-scale models including more variables, particularly financial sector and asset market variables, would provide richer information regarding trend inflation as well as for possible state transitions. These are remaining issues to be explored in future studies.

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A Set-up of the model with non-zero trend inflation

Our model is a version of the GNK DSGE model developed by Ascari and Sbordone (2014). This appendix provides the basic setup of our model.

A.1 Households

A representative household exhibits a utility function which is separable in consumption C_t and labor supply N_t:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - v\frac{N_t^{1+\chi}}{1+\chi},$$

where v is a constant parameter. With the period-by-period budget constraint given by,

$$P_t C_t + R_t^{-1} B_t = W_t N_t + D_t + B_{t-1}, \tag{13}$$

where B_t, R_t, W_t and D_t denote one-period bond holdings and its (gross) interest rate, nominal wage and distributed dividend, respectively, her utility maximization yields the first-order conditions as follows,

$$\beta E_t R_t \left(\frac{P_t}{P_{t+1}}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = 1, \tag{14}$$

$$v N_t^{\chi} C_t^{\sigma} = \frac{W_t}{P_t}. \tag{15}$$

The consumption-output Euler equation corresponds to (14), whereas (15) eliminates real wages in the firms' first order conditions.

A.2 Production

An intermediate goods producer i, has a linear production function in which labor is the only input:

$$Y_{i,t} = A_t N_{i,t},$$

where A_t denotes productivity that follows a stationary stochastic process. Then, the aggregate labor demand is

$$N_t = \int_0^1 N_{i,t} di = \underbrace{\int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} di}_{\equiv s_t} \frac{Y_t}{A_t} = \frac{s_t Y_t}{A_t}, \tag{16}$$

where s_t denotes the price dispersion arising from the Calvo pricing.

In the economy, the final good producer aggregates intermediate goods $Y_{i,t}$ according to,

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε represents the elasticity of substitution among intermediate goods.

A.3 Firms' pricing

In each period, a fraction $1-\theta$ of firms re-optimize their prices denoted as $P^*_{i,t}$. The rest of the firms index their prices to the previous period's inflation rate such that $P_{i,t} = \pi_t^\rho P_{i,t-1}$ where $\pi_t = P_t/P_{t-1}$ and ρ represents the degree of indexation. The profit maximization problem for the firms is given by,

$$\max_{P^*_{i,t}} : E_t \sum_{j=0}^{\infty} \mathfrak{D}_{t,t+j} \theta^j \left(\frac{P^*_{i,t}\Pi^\rho_{t-1,t+j-1}}{P_{t+j}} Y_{i,t} - \frac{W_{t+j}}{P_{t+j}} \frac{Y_{i,t+j}}{A_{t+j}}\right),$$

subject to the demand constraint,

$$Y_{i,t+j} = \left(\frac{P_{i,t}\Pi^\rho_{t-1,t+j-1}}{P_{t+j}}\right)^{-\varepsilon} Y_{t+j},$$

where $\mathfrak{D}_{t,t+j}$ is a stochastic discount factor and $\Pi_{t,t+j}$ indicates cumulative inflation from period t to $t+j$ such that,

$$\Pi_{t,t+j} = \frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}} \times \cdots \times \frac{P_{t+j}}{P_{t+j-1}},$$

for $t \geqq 1$.

Let $p^*_{i,t} = P^*_{i,t}/P_t$ and $w_t = W_t/P_t$. Then, the first-order condition for the firms' price-setting can be written as

$$p^*_{i,t} = \frac{\varepsilon}{\varepsilon - 1}\frac{\psi_t}{\phi_t},$$

where

$$\begin{aligned}
\psi_t &\equiv E_t \sum_{j=0}^{\infty} (\theta\beta)^j \frac{Y_{t+j}^{1-\sigma} w_t}{A_t} \left(\frac{\Pi^{\rho}_{t-1,t+j-1}}{\Pi_{t+j}} \right)^{-\varepsilon}, \\
\phi_t &\equiv E_t \sum_{j=0}^{\infty} (\theta\beta)^j Y_{t+j}^{1-\sigma} \left(\frac{\Pi^{\rho}_{t-1,t+j-1}}{\Pi_{t+j}} \right)^{1-\varepsilon},
\end{aligned}$$

which result in (1), (2), and (3) combined with (15) and s_t defined in (16).

A.4 Monetary policy

Recall that the (gross) official discount rate denoted by R_t in (12) is the data counterpart of the one-period risk-free interest rate in (13). Let $4\tilde{\imath}_t = 100 \log(R_t/\bar{R}_t)$. Then, in the model, monetary policy follows a standard Taylor rule given by

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho^{MP}} \left[\left(\frac{\pi_t}{\bar{\pi}_t} \right)^{\alpha_{\pi,t}} \left(\frac{Y_t}{Y^*_t} \right)^{\alpha_y} \right]^{1-\rho^{MP}} exp(e^{MP}_t),$$

which corresponds to (6).

Table 1: Calibration and Prior Distributions of Parameters

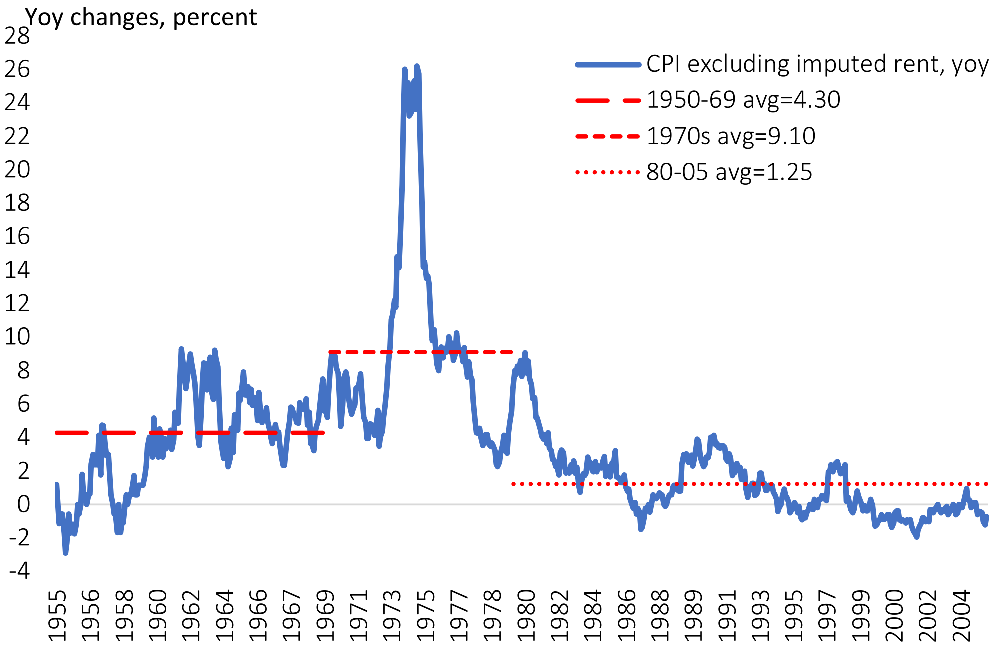
Parameter	Definition	Dist.	Mean	S.D.
ε	Elasticity of substitution among intermed. goods	G	8.00	1.50
θ	Calvo-Yun parameter	B	0.66	0.10
σ	Inverse intertemporal elasticity of substitution	G	3.00	1.00
α_y	MP reaction cofficient of output	G	0.50	0.30
ρ^{IS}	Persistence of IS shock	B	0.80	0.10
ρ^{AS}	Persistence of AS shock	B	0.80	0.10
ρ^{MP}	Interest rate smoothing	B	0.80	0.10
ρ	Inflation indexation	B	0.50	0.20
σ_{IS}	S.D. of IS shock (Q%)	IG	0.125	0.125
σ_{MP}	S.D. of MP shock (Q%)	IG	0.125	0.125
π^*_{high}	Trend inflation when $S_t^{\overline{\pi}} = \text{High}$	G	3.00	1.00
π^*_{low}	Trend inflation when $S_t^{\overline{\pi}} = \text{Low}$	N	1.00	1.00
$\alpha_\pi(S_t^{MP} = \text{Hawkish})$	Taylor coefficient when $S_t^{MP} = \text{Hawkish}$	G	1.50	0.50
$\alpha_\pi(S_t^{MP} = \text{Dovish})$	Taylor coefficient when $S_t^{MP} = \text{Dovish}$	G	1.00	0.30
$dy^*(S_t^{dy*} = \text{High growth})$	Potential GDP growth when $S_t^{dy*} = \text{High growth}$	G	8.00	2.00
$dy^*(S_t^{dy*} = \text{Low growth})$	Potential GDP growth when $S_t^{dy*} = \text{Low growth}$	G	2.00	1.00
$\sigma_{AS}(S_t^{oil} = \text{High volatility})$	S.D. of tech. shock when $S_t^{oil} = \text{High volatility}$	IG	1.00	1.00
$\sigma_{AS}(S_t^{oil} = \text{Low volatility})$	S.D. of tech. shock when $S_t^{oil} = \text{Low volatility}$	IG	0.25	0.25
p_{High,Low}	Transition prob. from High $\overline{\pi}$ to Low $\overline{\pi}$	B	0.05	0.025
p_{Low,High}	Transition prob. from Low $\overline{\pi}$ to High $\overline{\pi}$	B	0.05	0.025
p_{Dovish,Hawkish}	Transition prob. from Dovish to Hawkish	B	0.20	0.10
p_{Hawkish,Dovish}	Transition prob. from Hawkish to Dovish	B	0.20	0.10
p_{High growth,Low growth}	Transition prob. from High growth to Low growth	B	0.20	0.10
p_{Low growth,High growth}	Transition prob. from Low growth to High growth	B	0.20	0.10
p_{Low vol.,High vol.}	Transition prob. from Low vol. to High vol.	B	0.025	0.05
p_{High vol.,Low vol.}	Transition prob. from High vol. to Low vol.	B	0.20	0.10
$stderr\_\eta^y$	S.D. of measurement error for GDP growth	IG	0.50	0.50
$stderr\_\eta^\pi$	S.D. of measuremnt error for inlation	IG	0.10	0.10
β	Quarterly discount factor	Calib.	0.999	–
χ	Inverse labor supply elasticity	Calib.	2.00	–

NOTE: MP stands for monetary policy, S.D. stands for standard deviation, and Calib. stands for calibrated parameter. B, G, IG, and N stand for Beta, Gamma, inverse Gamma and normal distribution, respectively. Unit of trend inflation and potential GDP growth rate is annual percent.

Table 2: Posterior Estimation Results

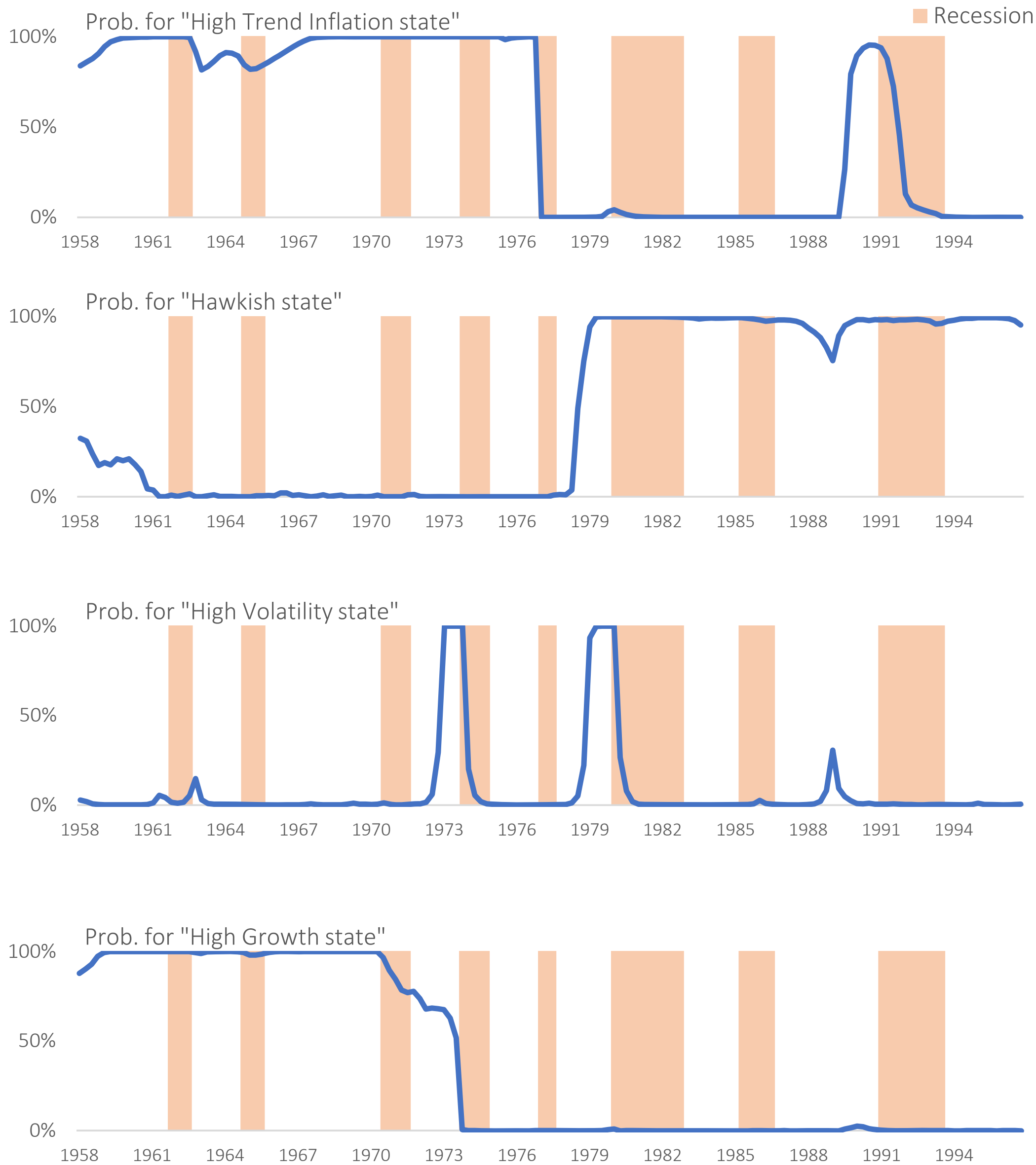
Parameter	Main Model			Constant σ_{AS} Model			3-state π^* Model		
	Mode	Mean	S.D.	Mode	Mean	S.D.	Mode	Mean	S.D.
ε	7.409	7.512	0.063	7.962	7.640	0.023	7.376	7.281	0.077
θ	0.437	0.529	0.017	0.789	0.827	0.014	0.475	0.455	0.009
σ	4.073	4.390	0.027	5.583	5.284	0.023	4.948	4.575	0.076
α_y	0.529	0.457	0.006	0.570	0.531	0.009	0.487	0.454	0.024
ρ^{IS}	0.915	0.941	0.006	0.843	0.849	0.013	0.888	0.918	0.008
ρ^{AS}	0.902	0.887	0.007	0.944	0.942	0.009	0.891	0.888	0.013
ρ^{MP}	0.881	0.864	0.010	0.860	0.846	0.005	0.880	0.897	0.004
ρ	0.205	0.203	0.020	0.022	0.020	0.007	0.247	0.209	0.030
σ_{IS}	0.021	0.019	0.003	0.051	0.050	0.009	0.026	0.030	0.005
σ_{AS}	–	–	–	0.143	0.165	0.021	–	–	–
σ_{MP}	0.091	0.083	0.007	0.044	0.040	0.006	0.090	0.105	0.006
π^*_{high}	2.402	2.333	0.042	2.343	2.228	0.012	3.949	3.559	0.012
π^*_{medium}	–	–	–	–	–	–	2.796	2.265	0.024
π^*_{low}	0.942	1.041	0.024	1.350	1.184	0.016	0.820	0.957	0.018
$\alpha_\pi(S_t^{MP} = \text{Hawkish})$	1.802	1.902	0.012	1.804	1.778	0.032	1.760	1.997	0.034
$\alpha_\pi(S_t^{MP} = \text{Dovish})$	0.504	0.288	0.027	0.351	0.285	0.020	0.394	0.504	0.010
$dy^*(S_t^{dy*} = \text{High growth})$	7.720	7.200	0.045	7.921	8.109	0.037	7.816	7.779	0.028
$dy^*(S_t^{dy*} = \text{Low growth})$	3.009	2.621	0.060	2.440	2.390	0.020	2.840	2.894	0.019
$\sigma_{AS}(S_t^{oil} = \text{High volatility})$	1.008	1.333	0.014	–	–	–	1.327	1.361	0.018
$\sigma_{AS}(S_t^{oil} = \text{Low volatility})$	0.102	0.116	0.039	–	–	–	0.178	0.137	0.008
p_{High,Low}	0.028	0.049	0.016	0.027	0.031	0.007	0	0	–
p_{High,Medium}	–	–	–	–	–	–	0.034	0.021	0.007
p_{Medium,High}	–	–	–	–	–	–	0.036	0.053	0.012
p_{Medium,Low}	–	–	–	–	–	–	0.021	0.032	0.013
p_{Low,Medium}	–	–	–	–	–	–	0.018	0.014	0.005
p_{Low,High}	0.026	0.024	0.004	0.027	0.013	0.007	0	0	–
p_{Dovish,Hawkish}	0.047	0.049	0.016	0.078	0.091	0.014	0.042	0.031	0.008
p_{Hawkish,Dovish}	0.054	0.033	0.009	0.040	0.039	0.007	0.038	0.046	0.011
p_{High growth,Low growth}	0.030	0.053	0.013	0.032	0.078	0.029	0.036	0.037	0.010
p_{Low growth,High growth}	0.020	0.041	0.012	0.021	0.020	0.006	0.024	0.039	0.018
p_{Low vol.,High vol.}	0.013	0.000	0.002	–	–	–	0.012	0.000	0.002
p_{High vol.,Low vol.}	0.196	0.160	0.015	–	–	–	0.188	0.195	0.012
$stderr\_\eta^y$	4.110	4.121	0.044	4.056	4.183	0.125	3.999	4.063	0.034
$stderr\_\eta^\pi$	0.050	1.338	0.021	2.526	2.703	0.063	0.434	0.420	0.035

NOTE: S.D. stands for standard deviation. Transition probability from High $\bar{\pi}$ to Low $\bar{\pi}$ and vice-versa are set to be zero in 3-state π^* model.



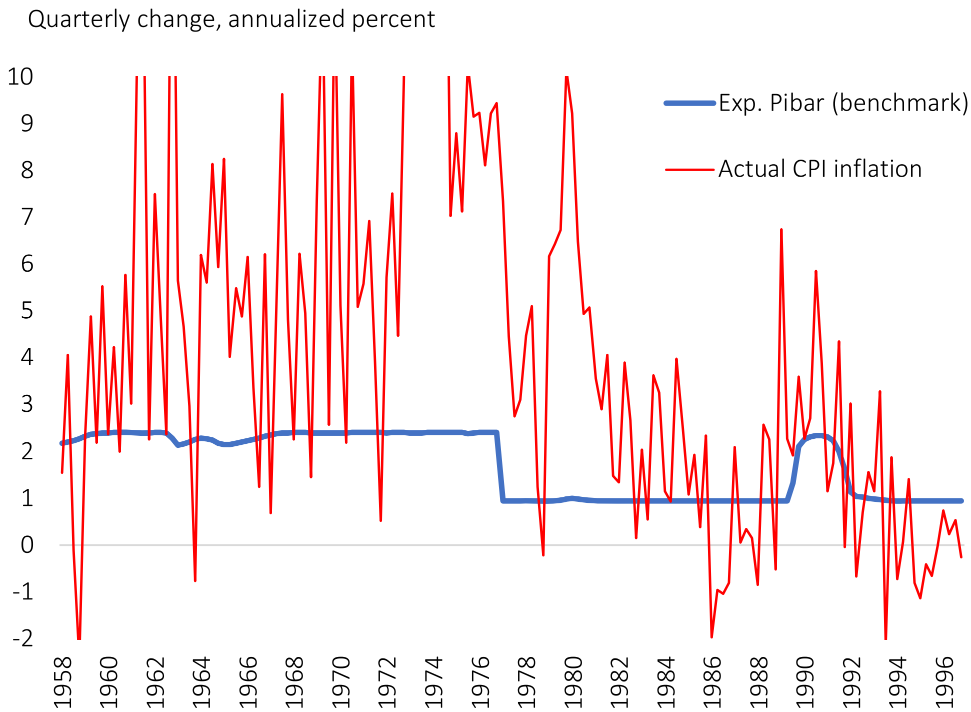
Note: Monthly year-over-year changes in Consumer Price Index, excluding imputed rent.

Figure 1



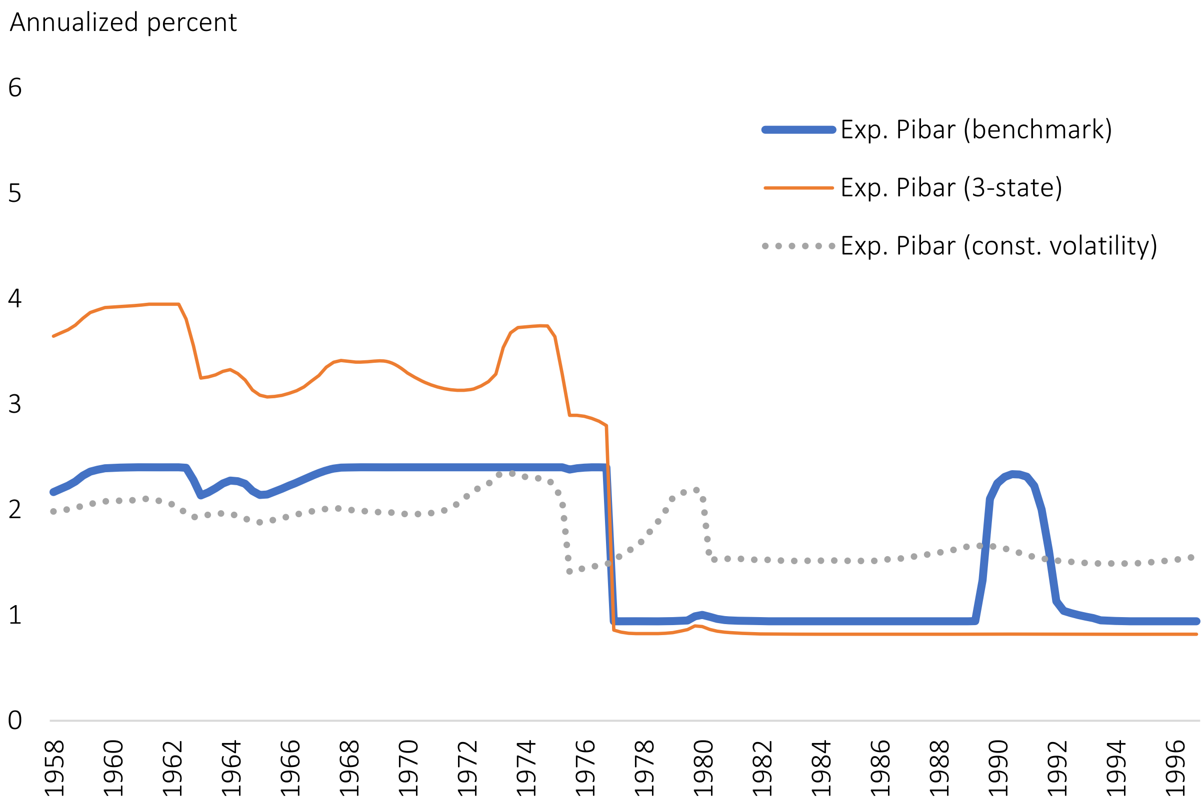
Notes: The figures graph the mode estimates. The shaded areas correspond to the dated ESRI recessions.

Figure 2



Note: The blue bold line is the estimated average trend inflation rate. The thin red line is the actual CPI inflation. Both are seasonally adjusted quarterly changes of price levels expressed as anualized percent rates.

Figure 3



Note: The blue bold line is the point estimate of trend inflation rate under benchmark model. Thin red line and dotted gray line are those under the 3-state model and the constant volatility model, respectively.

Figure 4