Vertical relationship with downstream production defects or corporate social responsibility

竹澤，秀平

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経済学専攻

指導教員　水野倫理

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(川下製品の欠陥あるいは企業の社会的責任を伴う垂直的関係)

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Chapter 1

Introduction

Firms' costs will fluctuate due to a variety of factors. For example, if a firm fails to manufacture a product or faces logistic inefficiency caused by double marginalization problem, the costs go up. Conversely, costs are reduced when innovation occurs in an industry or when upstream CSR activities are undertaken. The input price change is an important issue for every firms, and it is meaningful to analyze the interactive effects of the factors. In analyzing this issue, we focus on three areas in particular: production defects, R&D, and CSR activities. In addition, we evaluate the effects of factors on surpluses because increases or decreases in the costs of these firms are generally passed on to the prices of final goods.

First, we briefly explain the analysis of these factor in previous studies. Product defects naturally occur in any industry. There are several approaches to research dealing with production defects. Daughety and Reiganum (1995) consider a model with production defects where a firm can reduce negative impacts of production defects through R&D. Dada et al. (2005) consider a vertically related market and show that consumer surplus worsens when upstream production becomes unstable. Deo and Corbett (2009) analyze a model where entry occurs when yield uncertainty exists.

Next, CSR activities are of interest to many firms. KPMG's CSR survey conducted in 2017 shows that CSR activities are of interest to many firms in recent years (KPMG, 2017). There are many studies analyzing CSR activities in vertical markets. Brand

and Grothe (2015) analyze a model in which both upstream and downstream firms are performing CSR in a vertical market. Garcia et al. (2018) develop the model presented by Brand and Grothe (2015) analyze endogeneous timing of CSR activities.

Finally, R&D is also relevant for many firms in vertical relationships. Extant literature considers the effects of upstream R&D. Milliou and Pavlou (2013) analyze a horizontal integration between upstream firms when upstream firms engage in R&D. Pinopoulos (2020) shows that upstream investment can increase when there is no price discrimination in a two-part tariff with incomplete information. Inderst and Wey (2003) examine the impact of horizontal mergers and negotiations on investment in vertical markets.

This thesis investigates the occurrence of defects in production process and the impact of CSR activities on profits and society in vertical markets. Here, we provide a summary of main results in each chapter.

Chapter 2 analyzes a model in which downstream production defects occur in a vertical market and a upstream firm engages cost reducing R&D. Normally, when a defect occurs, a firm's profit margins are reduced because it makes the firm more inefficient. However, if the upstream firm conducts cost reducing R&D, when the degree of defects intensifies, input demand increases, which leads to actively R&D. Then, the upstream firm becomes more efficient, which lowers input price and reduces downstream marginal cost. As a result, downstream profits increase despite an increase in the degree of production defects.

In Chapter 3, we analyze a model in which downstream firms use technology with production defects and engage in CSR activities. When a firm engages in CSR activities, it will overproduce above the level at which it maximizes profit. If more defects are generated at this time, the degree of overproduction is eliminated by an increase in marginal cost but does not reach a level at which the firm's profits increase.

In Chapter 4, we will discuss a model where a downstream firm conducts CSR activities and an upstream firm engages cost reducing R&D. In general, CSR activities have a commitment effect of increasing outputs, which in turn reduces the profit margins of competing firms. At this time, the upstream firm expands its R&D investment because CSR activities yields larger input demand. Hence, the CSR activities have reduction effect of input price. As a result, the competing firms loosing market share can also face the efficient upstream firm, which may lead to higher profits of them.

The structure of this thesis is as follows: The second chapter examines the impact of downstream R&D when defects occur in the production process of downstream firms. In Chapter 3, we analyze the impact of downstream CSR activities when the downstream defects occur. In Chapter 4, we consider a model with downstream CSR activities and upstream cost reducing R&D. Chapter 5 concludes the thesis.

Chapter 2

Upstream R&D and downstream production defect

2.1 Introduction

Many laws protect consumers, so product defects that do not cause health problems do not diminish their utility.[1] For example, if a book we buy has a bad binding or missing pages, we can replace it for free.[2] Hence, the change in demand due to the presence of product defects is small, and firms owe the cost of product defects.

To consider the positive effects of product defects on firms, we build the following simple model. We consider a market with one upstream and one downstream firm. Using input sold by the upstream firm, the downstream firm produces the final product and sells it to the consumer. In downstream production, product defects occur with a certain probability. If a downstream firm sells a defective product, it must replace it free of charge. We also assume that the upstream firm invests in reducing its marginal cost.

As the defective product rate increases input demand, downstream production costs rise. In addition, an increase in the market for input encourages the upstream firm to invest in marginal cost reduction, which lowers input price. As a result, an increase in the defective product rate has a different effect on the profit of downstream firms.

[1] Japan's Civil Code requires the replacement of defective products.

[2] For research on warranty, see Glickman and Berger (1976), Chien (2008), Wu et al. (2009) and others.

If upstream investment technology is efficient, the latter positive impact dominates the former negative one. The increase in the defective product rate increases the profit of a downstream firm. As a market with an efficient upstream firm is socially desirable, increasing the defective product rate may increase the total surplus.

Daughety and Reiganum (1995) consider a model with production defects where a firm can reduce negative impacts of production defects through R&D. Takaoka (2005) considers a market where monopolists engage in cost-reducing R&D and product may harm both the monopolist and consumers. Dada et al. (2005) consider a vertically related market and show that consumer surplus worsens when upstream production becomes unstable. Deo and Corbett (2009) analyze a model where entry occurs when yield uncertainty exists. However, in the above studies, the situation where product defects happen in the downstream market and upstream firms invest in marginal cost reduction has not been analyzed.

Extant literature considers the effects of upstream R&D. Milliou and Pavlou (2013) analyze a horizontal integration between upstream firms when upstream firms engage in R&D. Pinopoulos (2020) shows that upstream investment can increase when there is no price discrimination in a two-part tariff with incomplete information. Inderst and Wey (2003) exmains the impact of horizontal mergers and negotiations on investment in vertical markets. However, the above studies do not consider upstream R&D and production defects.

The following is the organization of this paper. Section 2 presents the model. Section 3 calculates equilibrium outcomes. Section 4 gives the comparative statics results. Section 5 provides a case with linear inverse demand. Section 6 concludes this section.

2.2 The Model

We consider a market with an upstream firm and a downstream firm. The downstream firm faces an inverse demand function $p(q)$, where q is the downstream firm's output. We assume the inverse demand function is continuously differentiable and has constant curvature. The curvature of inverse demand function is denoted as $z \equiv -qp''(q)/p'(q)$, where $p'(q), p''(q)$ and $p'''(q)$ are the first, second and third derivatives, respectively. Precisely, the inverse demand function takes the following formula: $p(q) = a - bq^{1-z}/(1-z)$, where $a, b > 0$ and $z \leq 0$. The last inequality guarantees that positive output in equilibrium.[3] Note that $p'(q) = -bq^{-z} < 0$ and $p''(q) = -zp'(q)/q \leq 0$ (Ritz 2008).

A downstream firm's output contains a certain percentage of defective products. We denote the defective rate by $(\mu-1)/\mu$.[4] Hence, we assume that when the downstream firm produces q units of the final product, it requires μq units of input, where μ is constant and $\mu \geq 1$. As the defective rate monotonically increases with μ, μ is considered the degree of defectiveness (Shy 1995). The downstream firm has to buy more input as μ increases. The downstream firm only pays for inputs and no other costs. The input price is denoted as w. Then, the downstream firm's profit is

$$\pi_D \equiv [p(q) - \mu w]q.$$

The upstream firm produces input and sells to the downstream firm. Further, the upstream firm can invest in marginal cost-reducing R&D. The marginal cost of upstream firm is $c - x$ when the upstream firm pays the investment cost $f(x)$, where $c(> 0)$ is the initial marginal cost without investment (e.g., d'Aspremont and Jacquemin 1988). To guarantee positive marginal cost, we assume $c - x > 0$. In addition, we assume that $f(0) = 0$ and for any $x > 0$, $f'(x) > 0$ and $f''(x) > 0$. The upstream firm's profit is given by

$$\pi_U \equiv [w - (c - x)]\, \mu q - f(x).$$

Consumer surplus is $CS \equiv \int_0^q p(y)dy - p(q)q$; total surplus is $TS \equiv CS + \pi_D + \pi_U$.

The upstream and downstream firms engage in a two-stage game. In the first stage, the upstream firm chooses input price w and R&D level x [5]. In the second stage, the

[3]This inverse demand function contains the familiar shape of an inverse demand function. When $z = 0$, the inverse demand function is linear.

[4]Note that the defective rate is $(\mu q - q)/(\mu q) = (\mu - 1)/\mu$.

[5]Changing the timing of the game to determine x in the first stage and w in the second stage would not change the result. This is because upstream profit is a concave function with respect to investment and input prices.

downstream firm chooses output q. Using backward induction, we solve the game.

2.3 Calculating equilibrium

2.3.1 Downstream firm's decision

In the second stage, the downstream firm sets output q to maximize its profit π_D. From the first-order condition, $\partial\pi_D/\partial q = p(q) - qp(q) - \mu w = 0$, we have the following output.

$$q(w, \mu) = -\frac{p(q) - \mu w}{p'(q)}. \tag{2.1}$$

In addition, rearranging the first-order condition, we can show that price elasticity $\varepsilon \equiv -p(q)/[qp'(q)]$ is larger than one: $\varepsilon = p(q)/[p(q) - \mu w] > 1$.

Next, we consider the effects of w and μ on $q(w, \mu)$. We obtain the following comparative static result from the first and second derivatives of $q(w, \mu)$ with respect to w and μ.

Lemma 2.1 *The results of comparative statics are as follows.*

$$\begin{aligned}
\frac{\partial q(w,\mu)}{\partial w} &= -\frac{q\varepsilon\mu}{(2-z)p(q)} < 0, \\
\frac{\partial q(w,\mu)}{\partial \mu} &= -\frac{q\varepsilon w}{(2-z)p(q)} < 0, \\
\frac{\partial^2 q(w,\mu)}{\partial w^2} &= \frac{qz\varepsilon^2\mu^2}{(2-z)^2p(q)^2} > 0, \\
\frac{\partial^2 q(w,\mu)}{\partial \mu^2} &= \frac{qz\varepsilon^2 w^2}{(2-z)^2p(q)^2} > 0, \\
\frac{\partial^2 q(w,\mu)}{\partial w\partial \mu} &= -\frac{q\varepsilon[(2-z)p(q) - z\varepsilon\mu w]}{(2-z)^2p(q)^2} < 0.
\end{aligned}$$

Proof. First, we derive $p'''(q)$. Since the inverse demand function has constant curvature z, differentiating $z = -qp''(q)/p'(q)$ with respect to q and substituting $p''(q) = -zp'(q)/q$ into it, we get

$$p'''(q) = \frac{z(z+1)p'(q)}{q^2}. \tag{2.2}$$

Substituting $p'(q) = -p(q)/(\varepsilon q)$, $p''(q) = -zp'(q)/q$, and (2.2) into the first and second derivatives of $q(w,\mu)$ and solving for $\partial q(w,\mu)/\partial w$, $\partial q(w,\mu)/\partial \mu$, $\partial^2 q(w,\mu)/\partial w^2$, $\partial^2 q(w,\mu)/\partial \mu^2$, and $\partial^2 q(w,\mu)/\partial w \partial \mu$, we obtain this lemma. $\square$

The intuition behind Lemma 1 is as follows. Since the marginal cost of downstream firm is μw, an increase in w or μ reduces output $q(w,\mu)$; hence, we obtain $\partial q(w,\mu)/\partial w < 0$ and $\partial q(w,\mu)/\partial \mu < 0$. When marginal cost is large, the downstream output is small. Consequently, the output-reduction effects of w and μ become small. Hence, the second derivatives of $q(w,\mu)$ is positive: $\partial^2 q(w,\mu)/\partial w^2 > 0$ and $\partial^2 q(w,\mu)/\partial \mu^2 > 0$. An increase in μ by one unit raises the downstream firm's unit cost by w. Therefore, with large w, output reduction effect of μ is significant and $\partial^2 q(w,\mu)/\partial w \partial \mu < 0$.

2.3.2 Upstream firm's decision

Next, we consider the upstream firm's decision. Substituting (2.1) into the profit of upstream firm, we have $\pi_U(w,x) = [w-(c-x)]\,\mu q(w,\mu) - f(x)$. Substituting $p'(q) = -p(q)/(\varepsilon q)$ and the result of Lemma 1 into the first-order conditions $\partial \pi_U(w,x)/\partial w = 0$ and $\partial \pi_U(w,x)/\partial x = 0$, we obtain the followings.

$$\frac{\partial \pi_U(w,x)}{\partial w} = \mu q + [w-(c-x)]\mu \frac{\partial q(w,\mu)}{\partial w} = 0, \quad \frac{\partial \pi_U(w,x)}{\partial x} = \mu q - f'(x) = 0. \tag{2.3}$$

Solving the above equations and (2.1) for w, x, and $f'(x)$, we obtain the equilibrium outcomes.

$$w(\mu) = \frac{(\varepsilon-1)p(q)}{\varepsilon\mu}, \quad x(\mu) = c - \frac{(\varepsilon+z-3)p(q)}{\varepsilon\mu}, \quad f'(x) = \mu q. \tag{2.4}$$

As we assume the positive marginal cost of downstream firms, to guarantee $c - x > 0$,

Assumption 2.1 $\varepsilon + z - 3 > 0$.

2.3.3 Concavity of upstream firm's profit

The second-order necessary condition is shown in the first stage. As $\partial^2\pi_U(w,x)/\partial x^2 = -f''(x) < 0$, only the inequality is considered: $[\partial^2\pi_U(w,x)/\partial w^2][\partial^2\pi_U(w,x)/\partial x^2] - [\partial^2\pi_U(w,x)/\partial w\partial x]^2 > 0$. Substituting Lemma 2.1, (2.1), (2.4), and $p'(q) = -p(q)/(\varepsilon q)$ into the above inequality, the following inequality is obtained:

$$\frac{\partial^2\pi_U(w,x)}{\partial w^2}\frac{\partial^2\pi_U(w,x)}{\partial x^2} - \left[\frac{\partial^2\pi_U(w,x)}{\partial w\partial x}\right]^2 = \frac{q\varepsilon\mu^2\Phi}{(2-z)^2p(q)^2} > 0.$$

where $\Phi \equiv (2-z)^2p(q)f''(x) - q\varepsilon\mu^2$. As the above inequality depends only on the sign of Φ, we assume $\Phi > 0$ to satisfy the second-order necessary condition. Solving $\Phi > 0$ for $f''(x)$, the assumption is rewritten as follows.

Assumption 2.2 *To satisfy the second-order necessary condition, we assume $\Phi > 0$, which is equivalent to*

$$f''(x) > \frac{q\varepsilon\mu^2}{(2-z)^2p(q)} \equiv f^{SOC}.$$

According to this assumption, the investment cost function is sufficiently convex.

2.4 Comparative statics

2.4.1 Input price, upstream investment, and downstream marginal cost

The next step is to do comparative statics. Differentiate the first-order conditions of upstream firm, (2.3), with respect to μ, substituting the result of Lemma 2.1 and upstream outcomes (2.4), and solving them for $w'(\mu)$ and $x'(\mu)$, as follows:

$$w'(\mu) = \frac{p(q)[q\varepsilon(\varepsilon+z-3)\mu^2 - (2-z)(1+\varepsilon-z\varepsilon)p(q)f''(x)]}{\varepsilon\mu^2\Phi}, \tag{2.5}$$

$$x'(\mu) = \frac{(7-5z+z^2-\varepsilon)qp(q)}{\Phi}. \tag{2.6}$$

Then, we obtain Proposition 2.1

Proposition 2.1 *Input price $w(\mu)$ decreases with the degree of defectiveness μ if $\varepsilon < 7 - 5z + z^2$ or $f''(x) > f^w$, where*

$$f^w \equiv \frac{(7 - 5z + z^2 - \varepsilon)\varepsilon\mu^2 q}{(2-z)^2(1+\varepsilon - z\varepsilon)p(q)}.$$

Investment level $x(\mu)$ increases with the degree of defectiveness μ if $\varepsilon < 7 - 5z + z^2$.

Proof. As the denominators in (2.5) and (2.6) are positive, the sings of $w'(\mu)$ and $x'(\mu)$ only depend on those of numerators. The terms in the square brackets of $w'(x)$ is a linear function of $f''(x)$; the coefficient of $f''(x)$ takes a negative value. Hence, we have $w'(\mu) < 0$ if

$$f''(x) > \frac{(7 - 5z + z^2 - \varepsilon)\varepsilon\mu^2 q}{(2-z)^2(1+\varepsilon - z\varepsilon)p(q)} \equiv f^w.$$

From Assumption 2.2, we need to compare f^w with f^{SOC}.

$$f^{SOC} - f^w = \frac{(7 - 5z + z^2 - \varepsilon)\varepsilon\mu^2 q}{(2-z)^2(1+\varepsilon - z\varepsilon)p(q)}.$$

From $z \leq 0$ and $\varepsilon > 1$, the denominator is positive. Hence, we have $f^w < f^{SOC}$ if $\varepsilon < 7-5z+z^2$, which means that we always have $w'(x) < 0$ since we assume $f''(x) > f^{SOC}$. In the case with $\varepsilon \geq 7 - 5z + z^2$, we have $f^{SOC} \leq f^w$. Hence, $w'(x) < 0$ if $f''(x) > f^w$. Therefore, we obtain this proposition. □

The intuition behind this proposition is as follows. When input demand, μq, increases, the upstream firm will increase the amount of investment. From Lemma 2.1, for given input price, an increase in μ decreases downstream output because the marginal cost of downstream firm, μw, increases. Hence, if input demand increases with μ, then the reduction of downstream output must be small. First, we consider the case where the downstream firm faces less elastic demand, small ε. In this case, the reduction of downstream output is small because of less elastic demand. Then, investment level $x(\mu)$ increase with μ. Since the upstream firm with large investment is efficient, it chooses low input price.

Next, we consider the case with elastic demand where the upstream firm's investment decreases with μ. In this case, the upstream production becomes less efficient, and the upstream firm faces small input demand as μ increases. The inefficient upstream firm will choose high input prices, while small input demand forces the upstream firm to charge lower input prices. Hence, whether input price decreases depends on which effect dominates. If upstream investment technology is less efficient, that is large $f''(x)$. The decrease in investment as μ increases becomes less important, which in turn, the latter effect dominates the former. Therefore, with large ε, input price decreases with μ if upstream investment technology is inefficient.

Using Proposition 2.1, we show the effect of μ on the downstream marginal cost: $\mu w(\mu)$. Differentiating $\mu w(\mu)$ with respect to μ, we have $d[\mu w(\mu)]/d\mu = w + \mu w'(\mu)$. Substituting (2.4) and (2.5) into the derivative and using the definition of Φ, we have the following equation.

$$\frac{d[\mu w(\mu)]}{d\mu} = \frac{(2-z)p(q)[-q\varepsilon\mu^2 + (\varepsilon + z - 3)p(q)f''(x)]}{\varepsilon\mu\Phi}. \tag{2.7}$$

Solving $d[\mu w(\mu)]/d\mu < 0$ for $f''(x)$, we obtain the following lemma.

Lemma 2.2 *Marginal cost of downstream firm decreases with the degree of defectiveness if $\varepsilon < 7 - 5z + z^2$ and $f''(X) < f^{MC}$, where*

$$f^{MC} \equiv \frac{q\varepsilon\mu^2}{(\varepsilon + z - 3)p(q)}.$$

Proof. Since the denominator of (2.7) is positive, the sign of the derivative only depends on that of terms in the square brackets. Hence, solving $d[\mu w(\mu)]/d\mu < 0$ for $f''(x)$, we have the following inequality.

$$f''(x) < \frac{q\varepsilon\mu^2}{(\varepsilon + z - 3)p(q)} \equiv f^{MC}.$$

Comparing f^{MC} with f^{SOC} yields

$$f^{MC} - f^{SOC} = \frac{q(7 - 5z + z^2 - \varepsilon)\varepsilon\mu^2}{(2 - z)^2(\varepsilon + z - 3)p(q)}.$$

Since the denominator is positive, we have $f^{MC} - f^{SOC} > 0$ if $\varepsilon < 7 - 5z + z^2$. Summarizing the above results, we obtain this lemma. □

The intuition behind this lemma is as follows. If the downstream marginal cost decreases with μ, input price $w(\mu)$ must decrease with μ, since the derivative of downstream marginal cost is $d[\mu w(\mu)]/d\mu = w + \mu w'(\mu)$. From Proposition 2.1, we have two cases with $w'(\mu) < 0$. In the first case with elastic demand, $\varepsilon \geq 7 - 5z + z^2$, investment level $x(\mu)$ decreases with μ. Then, the marginal cost of the upstream firm increases, which shrinks input price reduction. Hence, for large ε, downstream marginal cost always increases with μ.

In the second case with less elastic demand, $\varepsilon < 7 - 5z + z^2$, the upstream firm increases its investment level as μ increases. Then, a magnification of the decline in input prices occurs. This input price reduction is significant when the change in the upstream investment level is large, which means that the upstream investment technology is efficient. Therefore, downstream marginal cost decreases with μ if $f''(x) < f^{MC}$.

2.4.2 Profits of downstream and upstream firms

To discuss the effect of degree of defectiveness on the profits of downstream and upstream firms, first the condition where the profit of downstream firm increases with μ is shown.

We denote the equilibrium profit of downstream firm by $\pi_D(\mu)$. Differentiating $\pi_D(\mu)$ with respect to μ and substituting $p'(q) = -p(q)/(\varepsilon q)$, the result of Lemma 2.1, and (2.4) into the derivative, we obtain the following equation.

$$\frac{d\pi_D(\mu)}{d\mu} = -\frac{(\varepsilon - 1)qp(q)}{\varepsilon\mu} - \mu q w'(\mu).$$

Hence, to increase the profit of downstream firm with μ, input price must decrease with

μ. From Proposition 2.1, either price elasticity must be small or downstream investment must be inefficient. Substituting (2.5) into $d\pi_D(\mu)/d\mu$, we have the following formula.

$$\frac{d\pi_D(\mu)}{d\mu} = \frac{(2-z)qp(q)[q\varepsilon\mu^2 - (\varepsilon + z - 3)p(q)f''(x)]}{\varepsilon\mu\Phi}. \tag{2.8}$$

Solving $d\pi_D(\mu)/d\mu > 0$, we obtain the following proposition.

Proposition 2.2 *The profit of downstream firm increases with the degree of defectiveness if* $\varepsilon < 7 - 5z + z^2$ *and* $f''(x) < f^{MC}$.

Proof. Since the denominator in (2.8) is positive, the sign of $d\pi_D(\mu)/d\mu$ only depends on the terms in square brackets. The function in the square brackets is a liner function of $f''(x)$ and the coefficient of $f''(x)$ is negative. Hence, solving $d\pi_D(\mu)/d\mu > 0$ for $f''(x)$, we have the following inequality.

$$f''(x) < \frac{q\varepsilon\mu^2}{(\varepsilon + z - 3)p(q)} \; (= f^{MC}).$$

Therefore, we obtain this proposition. □

The intuition behind this proposition is simple. When μ rises, the only change for downstream firms is that its marginal cost rises. Therefore, whether an increase in μ raises the profit of the downstream firm depends only on whether μ increases its marginal cost. Therefore, the condition of Proposition 2.2 is equivalent to that stated by Lemma 2.2.

Next, we consider the effect of μ on the profit of the upstream firm. We denote the equilibrium profit of upstream firm by $\pi_U(\mu)$. Since the profit of upstream firm in the first stage is $\pi_U(w,x) = [w - (c - x)]\,\mu q(w,\mu) - f(x)$, using envelop theorem, we obtain the effect of μ on the upstream firm's profit.

$$\frac{d\pi_U(\mu)}{d\mu} = \frac{\partial\pi_U(w,x)}{\partial\mu} = (w - c + x)q + (w - c + x)\mu\frac{\partial q(w,\mu)}{\partial\mu}.$$

Substituting the result of Lemma 2.1, we rearrange the above equation as follows.

$$\frac{d\pi_U(\mu)}{d\mu} = -\frac{(\varepsilon + z - 3)qp(q)}{\varepsilon\mu} < 0. \tag{2.9}$$

Therefore, we obtain the following proposition.

Proposition 2.3 *The profit of an upstream firm decreases with the degree of defectiveness.*

This proposition has a simple intuition. An increase in μ makes a downstream firm inefficient. Hence, an upstream firm facing a less efficient downstream firm always earns a small profit.

2.4.3 Consumer and total surpluses

Next, the effect of μ on consumers and total surpluses is discussed, where $CS(\mu)$ and $TS(\mu)$ denote equilibrium consumer and total surpluses, respectively. Differentiating $CS(\mu)$ with respect to μ leads to

$$\frac{dCS(\mu)}{d\mu} = -qp'(q)\left[\frac{\partial q(w,\mu)}{\partial w}w'(\mu) + \frac{\partial q(w,\mu)}{\partial \mu}\right].$$

Hence, consumer surplus increases with μ only if input price decreases with μ: $w'(\mu) < 0$. Substituting $p'(q) = -p(q)/(\varepsilon q)$, the results of Lemma 2.1, (2.4), and (2.5) into $dCS(\mu)/d\mu$, we obtain the followings.

$$\frac{dCS(\mu)}{d\mu} = \frac{qp(q)[q\varepsilon\mu - (\varepsilon + z - 3)p(q)f''(x)]}{\varepsilon\mu\Phi}. \tag{2.10}$$

The sign of $dCS(\mu)/d\mu$ is the same as tha of terms in the square brackets. From Assumption 2.1, the coefficient of $f''(x)$ is negative. Hence, solving $dCS(\mu)/d\mu > 0$ for $f''(x)$, we obtain the result of comparative statics.

Proposition 2.4 *Consumer surplus increases with the degree of defective if* $\varepsilon < 7-5z+z^2$ *and* $f''(x) < f^{MC}$.

Proof. Solving $dCS(\mu)/d\mu > 0$ for $f''(x)$, we obtain the followings.

$$f''(x) < \frac{q\varepsilon\mu^2}{(\varepsilon + z - 3)p(q)} \ (= f^{MC}).$$

We know that $f^{MC} > f^{SOC}$ if $\varepsilon < 7 - 5z + z^2$. Therefore, we obtain this proposition. □

The intuition behind this proposition is the same as that behind Lemma 2.2. When an increase in μ reduces downstream marginal cost, the downstream firm expands its output, increasing consumer surplus.

Finally, we consider the effect of μ on total surplus. Using (2.8), (2.9), (2.10), and the definition of Φ, we obtain the derivative of $TS(\mu)$ with respect to μ as follows.

$$\begin{aligned}\frac{dTS(\mu)}{d\mu} &= \frac{d\pi_D(\mu)}{d\mu} + \frac{d\pi_U(\mu)}{d\mu} + \frac{dCS(\mu)}{d\mu} \\ &= \frac{qp(q)[q\varepsilon - 2\mu^2 - (7 - 5z + z^2)(\varepsilon + z - 3)p(q)f''(q)]}{\varepsilon\mu\Phi}.\end{aligned}$$

Since the denominator of the above derivative is positive, the sing of $dTS(\mu)/d\mu$ only depends on terms in the square brackets. Solving it for $f''(x)$, we obtain Proposition 2.5.

Proposition 2.5 *Total surplus increases with the degree of defective if* $\varepsilon < 7 - 5z + z^2$ *and* $f''(x) < f^{TS}$, *where*

$$f^{TS} \equiv \frac{q\varepsilon^2\mu^2}{(7 - 5z + z^2)(\varepsilon + z - 3)p(q)}.$$

Proof. In the square brackets of $dTS(\mu)/d\mu$, the coefficient of $f''(x)$ is negative. Solving $dTS(\mu)/d\mu > 0$ for $f''(x)$, we obtain the following inequality.

$$f''(x) < \frac{q\varepsilon^2\mu^2}{(7 - 5z + z^2)(\varepsilon + z - 3)p(q)} \equiv f^{TS}.$$

Here, we compare f^{TS} with f^{SOC}.

$$f^{TS} - f^{SOC} = \frac{q(3 - z)(7 - 5z + z^2 - \varepsilon)\varepsilon\mu^2}{(2 - z)^2(7 - 5z + z^2)(\varepsilon + z - 3)p(q)}.$$

Since the denominator is positive, the sing of $f^{TS} - f^{SOC}$ only depends on that of $7 - 5z + z^2 + \varepsilon$. Hence, we have $f^{TS} > f^{SOC}$ if $\varepsilon < 7 - 5z + z^2$. Therefore, these results lead to this proposition. □

The intuition behind this proposition is as follows. When an increase in μ makes the downstream firm inefficient, this effect reduces total surplus. In addition, from Lemma 2.2, the downstream marginal cost drops down if a downstream demand is elastic and upstream investment technology is efficient. This effect partially solves the double marginalization problem. The latter effect becomes important if the upstream firm uses efficient investment technology. Therefore, total surplus increases with μ if $f''(x)$ is small, $f''(x) < f^{TS}$.

2.4.4 Comparison of threshold values for $f''(x)$

To summarizing the results of propositions, we compare the threshold values: f^{SOC}, f^w, f^{MC}, and f^{TS}. Then, we obtain the following threshold ranking.

$$\begin{aligned} f^w < f^{SOC} < f^{TS} < f^{MC} \quad &\text{if} \quad \varepsilon < 7 - 5z + z^2, \\ f^{MC} \leq f^{TS} \leq f^{SOC} \leq f^w \quad &\text{if} \quad \varepsilon \geq 7 - 5z + z^2. \end{aligned}$$

Figure 2.1 summarizes the results of comparative statics. The results of comparative statics differ depending on the value of price elasticity. The upper panel of Figure 2.1 shows the results for the case with less elastic demand, and the lower panel shows the results with elastic demand. For example, in the case with $\varepsilon < 7 - 5z + z^2$ and $f^{SOC} < f''(x) < f^{TS}$, we have $x'(\mu) > 0$, $w'(\mu) < 0$, $\pi'_D(\mu) > 0$, $\pi'_U(\mu) < 0$, $CS'(\mu) > 0$, and $TS'(\mu) > 0$. From this figure, we find that downstream firms have no incentive to improve the degree of defectiveness if the price elasticity is small and the investment efficiency of the upstream firm is high. In addition, this is desirable from the perspective of the consumer and total surpluses. Furthermore, we find that when the investment efficiency of an upstream firm is intermediate, $f^{TS} < f''(x) < f^{MC}$, an increase in the degree of defectiveness increases the profit of downstream firm and consumer surplus, but decreases total surplus.

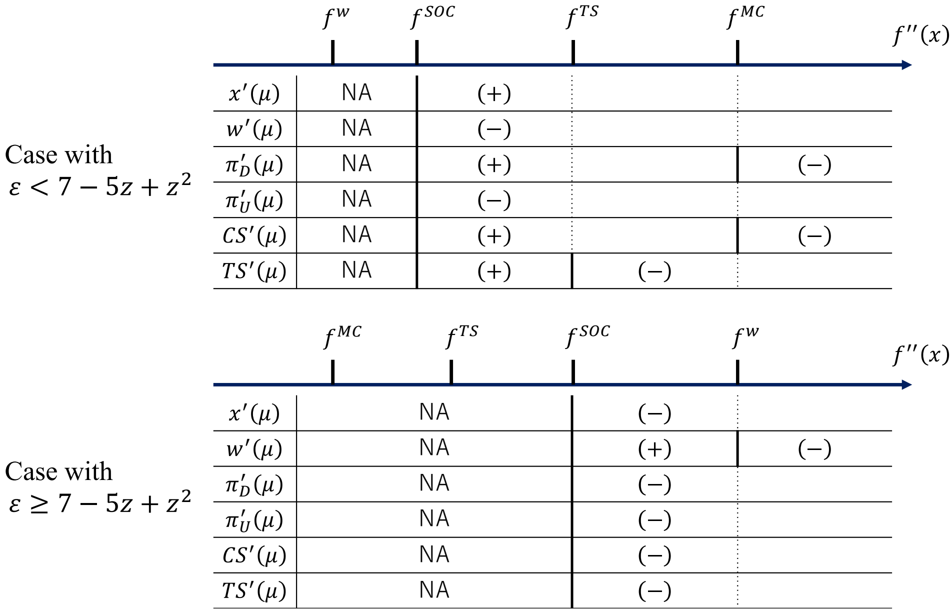

Figure 2.1: Summary of comparative statics results

2.5 Case with linear inverse demand and quadratic R&D cost

Here, we provide a case where inverse demand function is linear, $p = 1-q$, and investment cost function takes a quadratic form, $f(x) \equiv rx^2/2$, where to guarantee positive outcomes, we assume $r > \mu/(4c)$. Note that $f''(x) = r$. In addition, we assume $0 < c\mu < 1$. This condition implies that the upstream can sell its product to the downstream firm even when the upstream firm does not invest. The other setting is the same as in the previous section.

Solving this game by using backward induction, we obtain the following outcomes.

$$q_L = \frac{r(1-c\mu)}{4r-\mu^2}, \quad w_L = \frac{2r(1+c\mu)-\mu^2}{\mu(4r-\mu^2)}, \quad x_L = \frac{\mu(1-c\mu)}{4r-\mu^2},$$
$$\pi_{DL} = \frac{r^2(1-c\mu)^2}{(4r-\mu^2)^2}, \quad \pi_{UL} = \frac{r(1-c\mu)^2}{2(4r-\mu^2)},$$
$$CS_L = \frac{r^2(1-c\mu)^2}{2(4r-\mu^2)^2}, \quad TS_L = \frac{r(1-c\mu)^2(7r-\mu^2)}{2(4r-\mu^2)^2}.$$

Where subscript 'L' denotes the case with linear inverse demand and quadratic R&D cost. Differentiating the outcomes concerning μ, we obtain the following result.

Collorary 2.1 *Consider a case where the degree of defectiveness increases. Then, (i) upstream firm's profit decreases; (ii) downstream firm's profit and consumer surplus increase if $r < r^{MC}$; (iii) total surplus increases if $r < r^{TS}$, where*

$$r^{MC} \equiv \frac{\mu(2-c\mu)}{4c}, \quad r^{TS} \equiv \frac{\mu\left(10+c\mu+\sqrt{100-92c\mu+c^2\mu^2}\right)}{56c}.$$

Proof. Differentiating equilibrium outcomes concerning μ, we obtain the following.

$$\frac{\partial \pi_{UL}}{\partial \mu} = -\frac{r(4cr-\mu)(1-c\mu)}{(4r-\mu^2)^2} < 0, \quad \frac{\partial \pi_{DL}}{\partial \mu} = \frac{2r^2(1-c\mu)[-4cr+\mu(2-c\mu)]}{(4r-\mu^2)^3},$$
$$\frac{\partial CS_L}{\partial \mu} = \frac{r^2(1-c\mu)[-4cr+\mu(2-c\mu)]}{(4r-\mu^2)^3}, \quad \frac{\partial TS_L}{\partial \mu} = \frac{r(1-c\mu)[-28cr^2+r\mu(10+c\mu)-\mu^3]}{(4r-\mu^2)^3}.$$

The sign of $\partial\pi_{DL}/\partial\mu$ is same as that of $\partial CS_L/\partial\mu$. Solving $\partial\pi_{DL}/\partial\mu > 0$ and $\partial CS_L/\partial\mu > 0$ for r, we obtain $r < \mu(2-c\mu)/(4c) \equiv r^{MC}$.

As the sign of $\partial TS_L/\partial\mu$ only depends on the terms in square brackets and the coefficient of r^2 is negative, solving $\partial TS_L/\partial\mu > 0$ for r yields $r_0 < r < r^{TS}$, where

$$r_0 \equiv \frac{\mu\left(10+c\mu-\sqrt{100-92c\mu+c^2\mu^2}\right)}{56c}, \quad r^{TS} \equiv \frac{\mu\left(10+c\mu+\sqrt{100-92c\mu+c^2\mu^2}\right)}{56c}.$$

The lower threshold value r_0 is smaller than $\mu/(4c)$. This is shown as follow:

$$\frac{\mu}{4c} - r_0 = \frac{\mu\left(4-c\mu+\sqrt{100-92c\mu+c^2\mu^2}\right)}{56c} > 0.$$

It is assumed that $r > \mu/(4c)$; thus, we obtain $\partial TS_L/\partial\mu > 0$ if $r < r^{TS}$.□

As $f''(x) = r$, this result is consistent with Lemma 2.2 and Proposition 2.2–2.5. Hence, the intuition behind this result is the same as that in the previous section.

To visually interpret the results of Collorary 2.1, we obtain Figure 2.2 by depicting the thresholds, r^{MC}, r^{TS}, and $\mu/(4c)$, where $c = 1/2$. The horizontal axis is μ and the vertical axis is r. Figure 2.2 has three curves: The solid curve is $r = r^{MC}$; the dashed curve is $r = r^{TS}$; the dotted line is $r = \mu/(4c)$. From Figure 2.2, we can confirm the results in Collorary 2.1. For example, downstream firm's profit and consumer and social surpluses increase with the degree of defectiveness if the investment technology is efficient: $\mu/(4c) < r < r^{TS}$.

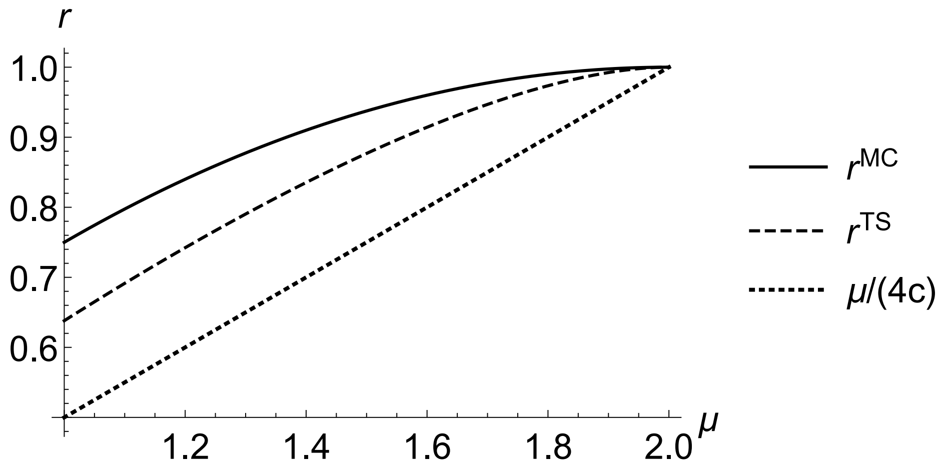

Figure 2.2: The threshold values of r at $c = 1/2$

2.6 Conclusion

This study considers a vertically related market with one upstream firm engaging cost-reducing R&D and one downstream firm and evaluates the impact of the downstream firm's product defects. We show that even though downstream product defects increase, the downstream firm's profit, consumers, and total surpluses may increase if the upstream firm's investment technology is efficient. Therefore, even if a downstream firm can reduce the degree of product defects with no cost, they may not do so.

However, this study has certain limitations. First, it is assumed that a downstream firm bears costs caused by product defects, but an upstream firm could also pay the costs. Second, to simplify the analysis, it is assumed that product defects do not affect demand. However, even with free replacements, demand may decrease with the degree of product defect, as product defects may reduce consumer utility. Finally, the rate of product defects is considered constant. However, the degree of product defects may reduce through R&D by a downstream firm. These gaps provide scope for future work in the field.

Chapter 3

Downstream production defects and corporate social responsibility

3.1 Introduction

Many laws protect consumers, so product defects that do not cause health problems do not diminish their utility.[1] For example, if a book we buy has a bad binding or missing pages, we can replace it for free.[2] Hence, the change in demand due to the presence of product defects is small, and firms owe the cost of product defects.

To consider the effects of product defects on firms, we build the following simple model. We consider a market with one upstream and one downstream firm. Using input sold by the upstream firm, the downstream firm produces the final product and sells it to the consumer. In downstream production, product defects occur with a certain probability. If a downstream firm sells a defective product, it must replace it free of charge. We also assume that the upstream firm invests in reducing its marginal cost.

As the defective product rate increases input demand, downstream production costs rise. In addition, an increase in the degree of CSR will increase downstream output. An increase in the defective product rate then brings the downstream output closer to the value that maximizes profit. If the downstream profit-increasing effect of an increase in

[1]Japan's Civil Code requires the replacement of defective products.

[2]For research on warranty, see Glickman and Berger (1976), Chien (2008), Wu et al. (2009) and others.

defective product rate exceeds the profit-reducing effect of an increase in marginal cost, then downstream profits will rise due to the increase in defects.

Daughety and Reiganum (1995) consider a model with production defects where a firm can reduce negative impacts of production defects through R&D. Their research shows that product defects are generally harmful to firms and consumers [3]. The occurrence of defects in vertical markets is analyzed by Fang and Shou(2015) and others [4]. Brand and Grothe (2015) analyze a model in which both upstream and downstream companies engage CSR activities. However, in the above studies, the situation with downstream production defects and downstream firm's CSR activities has not been analyzed.

The following is the organization of this paper. Section 2 presents the model. Section 3 calculates equilibrium outcomes. Section 4 gives the comparative statics results. Section 5 provides a case with linear inverse demand. Section 6 concludes this section.

3.2 The Model

3.2.1 Model

We consider a market with an upstream firm and a downstream firm. The downstream firm faces an inverse demand function $p(q)$, where q is the output of the downstream firm. We assume the inverse demand function is continuously differentiable and has constant curvature. The curvature of inverse demand function is denoted as $z \equiv -qp''(q)/p'(q)$, where $p'(q), p''(q)$ and $p'''(q)$ are the first, second and third derivatives, respectively. Precisely, the inverse demand function takes the following formula: $p(q) = a - bq^{1-z}/(1-z)$, where $a, b > 0$ and $z \leq 0$. The last inequality guarantees that positive output in equilibrium.[5] Note that $p'(q) = -bq^{-z} < 0$ and $p''(q) = -zp'(q)/q \leq 0$ (Ritz, 2008).

A downstream firm's output contains a certain percentage of defective products. We

[3] There are many developments of this model, such as Daughety and Reiganum (2005, 2008); Rössler and Friehe (2020)

[4] There are others such as Dada et al. (2005) consider a vertically related market and show that consumer surplus worsens when upstream production becomes unstable. Also, Deo and Corbett (2009) analyze a model where entry occurs when yield uncertainty exists.

[5] This inverse demand function contains the familiar shape of an inverse demand function. When $z = 0$, the inverse demand function is linear.

denote the defective rate by $(\mu - 1)/\mu$.[6] Hence, we assume that when the downstream firm produces q units of the final product, it needs μq units of input, where μ is constant and $\mu \geq 1$. Since the defective rate monotonically increases with μ, we refer to μ as the degree of defective (Shy, 1995). The downstream firm has to buy more input as μ increases. The downstream firm only has to pay for inputs and no other costs. We denote input price as w. Then, the downstream firm's profit is

$$\pi_D \equiv [p(q) - \mu w]q.$$

The downstream firm is also engaged in CSR. We denote consumer surplus by $CS \equiv \int_0^q p(y)dy - p(q)q$. The downstream firm focuses on consumer surplus at a rate of $\theta \in (0, 1)$, where θ is a parameter. At this time, the objective function of the downstream is

$$\Pi_D \equiv \theta CS + (1 - \theta)\pi_D.$$

Then if θ will be higher (lower), downstream firm more emphasize (disredard) CSR (see e.g. Kopel et al., 2014). We assume $\theta < 1/2$ to guarantee that equilibrium output is positive.

The upstream firm produces input with constant marginal cost c and sells it to the downstream firm. The upstream firm's profit is given by

$$\pi_U \equiv [w - c]\, \mu q.$$

Total surplus is $TS \equiv CS + \pi_D + \pi_U$.

The upstream and downstream firms engage in a two-stage game. In the first stage, the upstream firm chooses the input price w. In the second stage, the downstream firm chooses output q. Using backward induction, we solve the game.

[6] Note that the defective rate is $(\mu q - q)/(\mu q) = (\mu - 1)/\mu$.

3.3 Calculating equilibrium

3.3.1 Downstream firm's decision

In the second stage, the downstream firm sets output q to maximize its objective function Π_D. From the first-order condition, $\partial \Pi_D / \partial q = (q - 2\theta q)p'(q) + (1-\theta)p(q) - (1-\theta)\mu w = 0$, we have the following output.

$$q(w, \theta, \mu) = \frac{(1-\theta)(p(q) - \mu w)}{(2\theta - 1)p'(q)}. \tag{3.1}$$

In addition, rearranging the first-order condition, we can show that price elasticity $\varepsilon \equiv -p(q)/[qp'(q)]$ is larger than one:

$$\varepsilon = \frac{2\theta - 1}{\theta - 1} \frac{p(q)}{p(q) - \mu w} > 1. \tag{3.2}$$

Next, we consider the effects of w and μ on $q(w, \mu)$. We obtain the following comparative static result from the first and second derivatives of $q(w, \mu)$ with respect to w and μ.

Lemma 3.1 *The results of comparative statics are as follows.*

$$\begin{aligned}
\frac{\partial q(w,\theta,\mu)}{\partial w} &= \frac{(\theta-1)\mu q\varepsilon}{(2-3\theta+(2\theta-1)z)p(q)} < 0,\\
\frac{\partial q(w,\theta,\mu)}{\partial \theta} &= \frac{q\varepsilon(p(q)-\mu w)}{(1-2\theta)(2-3\theta+(2\theta-1)z)p(q)} > 0,\\
\frac{\partial q(w,\theta,\mu)}{\partial \mu} &= \frac{(\theta-1)qw\varepsilon}{(2-3\theta+(2\theta-1)z)p(q)} < 0,\\
\frac{\partial^2 q(w,\theta,\mu)}{\partial w^2} &= \frac{(1-\theta)(1-2\theta)\mu^2 qz\varepsilon}{(2-3\theta+(2\theta-1)z)^2 p(q)(p(q)-\mu w)} > 0,\\
\frac{\partial^2 q(w,\theta,\mu)}{\partial \theta^2} &= \frac{q\varepsilon(2(2-3\theta+(2\theta-1)z)+(2-z))(p(q)-\mu w)}{(1-\theta)(1-2\theta)(2-3\theta+(2\theta-1)z)^2 p(q)} > 0,\\
\frac{\partial^2 q(w,\theta,\mu)}{\partial \mu^2} &= \frac{(1-\theta)(1-2\theta)qw^2 z\varepsilon}{(2-3\theta+(2\theta-1)z)^2 p(q)(p(q)-\mu w)} > 0,\\
\frac{\partial^2 q(w,\theta,\mu)}{\partial w\partial \theta} &= -\frac{\mu q\varepsilon}{(2-3\theta+(2\theta-1)z)^2 p(q)} < 0,\\
\frac{\partial^2 q(w,\theta,\mu)}{\partial w\partial \mu} &= -\frac{qw\varepsilon}{(2-3\theta+(2\theta-1)z)^2 p(q)} < 0,\\
\frac{\partial^2 q(w,\theta,\mu)}{\partial \theta\partial \mu} &= -\frac{(1-\theta)q\varepsilon((2-3\theta+(2\theta-1)z)p(q)+(3\theta-2)\mu w)}{(2-3\theta+(2\theta-1)z)^2 p(q)(p(q)-\mu w)} < 0.
\end{aligned}$$

Proof. First, we derive $p'''(q)$. Since the inverse demand function has constant curvature z, differentiating $z = -qp''(q)/p'(q)$ with respect to q and substituting $p''(q) = -zp'(q)/q$ into it, we get

$$p'''(q) = \frac{z(z+1)p'(q)}{q^2}. \tag{3.3}$$

Substituting $p'(q) = -p(q)/(\varepsilon q)$, $p''(q) = -zp'(q)/q$, and (3.3) into the first and second derivatives of $q(w,\mu)$ and solving for $\partial q(w,\theta,\mu)/\partial w$, $\partial q(w,\theta,\mu)/\partial\theta$, $\partial q(w,\theta,\mu)/\partial\mu$, $\partial^2 q(w,\theta,\mu)/\partial w^2$, $\partial^2 q(w,\theta,\mu)/\partial\theta^2$, $\partial^2 q(w,\theta,\mu)/\partial\mu^2$, $\partial^2 q(w,\theta,\mu)/\partial w\partial\theta$, $\partial^2 q(w,\theta,\mu)/\partial w\partial\mu$, and $\partial^2 q(w,\theta,\mu)/\partial\theta\partial\mu$, we obtain this lemma. □

The intuition behind Lemma 1 is as follows. Since the marginal cost of downstream firm is μw, an increase in w or μ reduces output $q(w,\theta,\mu)$; hence, we obtain $\partial q(w,\theta,\mu)/\partial w < 0$ and $\partial q(w,\theta,\mu)/\partial\mu < 0$. Since θ is the degree of CSR, if θ is high, downstream firms will adopt a strategy more committed to output: $\partial q(w,\theta,\mu)/\partial\theta > 0$. When marginal cost is large, the downstream output is small. Consequently, the output-reduction effects of w and μ become small. Hence, the second derivatives of $q(w, \theta, \mu)$ is positive: $\partial^2 q(w, \theta, \mu)/\partial w^2 > 0$ and $\partial^2 q(w, \theta, \mu)/\partial \mu^2 > 0$. If the firm is CSR oriented, the output is large. Therefore, the output-rising effect will be large: $\partial^2 q(w, \theta, \mu)/\partial \theta^2 > 0$. A large θ has the output-rising effect, while an increase in marginal costs such as μ and w has a smaller effect: $\partial^2 q(w, \theta, \mu)/\partial w \partial \theta < 0, \partial^2 q(w, \theta, \mu)/\partial \theta \partial \mu < 0$. An increase in μ by one unit raises the downstream firm's unit cost by w. Therefore, with large w, output reduction effect of μ is significant and $\partial^2 q(w, \mu)/\partial w \partial \mu < 0$.

3.3.2 Upstream firm's decision

Next, we consider the upstream firm's decision. Substituting (3.1) into the profit of upstream firm, we have $\pi_U(w) = [w - c]\, \mu q(w, \theta, \mu)$. Substituting $p'(q) = -p(q)/(\varepsilon q)$ and the result of Lemma 3 into the first-order conditions $\partial \pi_U(w)/\partial w = 0$, we obtain the followings.

$$\frac{\partial \pi_U(w)}{\partial w} = \mu q + [w - c]\mu \frac{(\theta - 1)\mu q \varepsilon}{p(q)(2 - 3\theta + (2\theta - 1)z)} = 0. \tag{3.4}$$

Solving the above equations and (3.1) for w, we obtain the equilibrium outcomes.

$$w(\theta, \mu) = c + \frac{(2 - 3\theta + (2\theta - 1)z)p(q)}{\mu \varepsilon (1 - \theta)}. \tag{3.5}$$

3.3.3 Concavity of upstream firm's profit

We drive the second-order necessary condition in the first stage: $\partial^2 \pi_U(w)/\partial w^2 < 0$. Substituting Lemma 3.1, (3.1), (3.5), and $p'(q) = -p(q)/(\varepsilon q)$ into the above inequality, we obtain the following inequality.

$$\frac{\partial^2 \pi_U(w)}{\partial w^2} = -\frac{\mu q \Phi}{(1 - \theta)\varepsilon(w - c)(p(q) - \mu w)} < 0.$$

where $\Phi \equiv 2c(\theta - 1)\mu\varepsilon + p(q)[6\theta - 2\theta z + z - 2(\theta - 1)\varepsilon - 4]$. As the above inequality depends only on the sign of Φ, we assume $\Phi > 0$ to satisfy the second-order necessary condition. Solving $\Phi > 0$ for μ, the assumption is rewritten as follows.

Assumption 3.1 *To satisfy the second-order necessary condition, we assume* $\Phi > 0$*, which is equivalent to*

$$\mu < \frac{p(q)(-6\theta + (2\theta - 1)z + 2(\theta - 1)\varepsilon + 4)}{2c(\theta - 1)\varepsilon} \equiv \mu^{SOC}.$$

According to this assumption, the upstream profit function is strictly concave.

3.4 Comparative statics

3.4.1 Input price, upstream investment, and downstream marginal cost

The next step is to do comparative statics. Differentiate the first-order condition of upstream firm, (3.4), with respect to θ and μ, substituting the result of Lemma 3.1 and upstream outcomes (3.5), and solving them for $\partial w(\theta, \mu)/\partial\theta$ and $\partial w(\theta, \mu)/\partial\mu$, we obtain the followings.

$$\frac{\partial w(\theta, \mu)}{\partial\theta} = \frac{(p(q) - \mu w)[c(\theta - 1)\mu\varepsilon + p(q)(5\theta - 2\theta z + z - \theta\varepsilon + \varepsilon - 3)]}{(1 - \theta)(1 - 2\theta)\mu\Phi}, \tag{3.6}$$

$$\frac{\partial w(\theta, \mu)}{\partial\mu} = \frac{c\varepsilon(1 - \theta)(p(q) - \mu w) - \Phi w}{\mu\Phi}. \tag{3.7}$$

Then, we obtain Proposition 3.1

Proposition 3.1 *Input price* $w(\theta, \mu)$ *increases with the degree of CSR. Input price* $w(\theta, \mu)$ *decreases with the degree of defectiveness* μ *if* $\mu > \mu^w$*, where*

$$\mu^w \equiv \frac{p(q)(-5\theta + (2\theta - 1)z + (\theta - 1)\varepsilon + 3)}{c(\theta - 1)\varepsilon}.$$

Proof. As the denominators in (3.6) and (3.7) are positive, the sings of $\partial w(\theta, \mu)/\partial\theta$ and $\partial w(\theta, \mu)/\partial\mu$ only depend on those of numerators. The terms in the square brackets of $\partial w(\theta, \mu)/\partial\theta$ is a linear function of μ; the coefficient of μ takes a negative value. Hence, we have $\partial w(\theta, \mu)/\partial\theta < 0$ if

$$\mu > \frac{p(q)(-5\theta + (2\theta - 1)z + (\theta - 1)\varepsilon + 3)}{c(\theta - 1)\varepsilon} (\equiv \mu^w).$$

From Assumption 3.1, we need to compare μ^w with μ^{SOC}.

$$\mu^{SOC} - \mu^w = \frac{(1 - 2\theta)(2 - z)p(q)}{2c(1 - \theta)\varepsilon} > 0.$$

Hence, we obtain the second part of Proposition 3.1.

Next, we consider the first part of the proposition. The numerator of (3.7) is positive if

$$\mu < \frac{c(1 - \theta)\varepsilon p(q) - \Phi w}{c(1 - \theta)w\varepsilon} (\equiv \mu^{w\mu}).$$

From Assumption 3.1, we need to compare $\mu^{w\mu}$ with μ^{SOC}.

$$\mu^{SOC} - \mu^{w\mu} = \frac{2c(1 - \theta)\varepsilon(p(q) - \mu w) + 3\Phi w}{2c(\theta - 1)w\varepsilon} < 0.$$

Hence, we can ignore the threshold value $\mu^{w\mu}$. Therefore, we obtain this proposition. □

The intuition behind this proposition is as follows. When the degree of CSR, θ, increases, the downstream will expand its production. From Lemma 3.1, for given input price, an increase in θ increases downstream output. Then, a high μ makes the downstream firm inefficient, input demand decreases, and the upstream firm chooses a lower input price. Conversely, if μ is low, the downstream firm is efficient enough that when θ increases, input demand increases and the upstream firm chooses a higher input price. Also, as μ increases, the demand of input becomes larger. The upstream firm will then choose a higher input price.

Using Proposition 3.1, we show the effect of μ on the upstream demand: $\mu q(\theta, \mu)$. Differentiating $\mu q(\theta, \mu)$ with respect to μ, we have $\partial[\mu q(\theta, \mu)]/\partial\mu = q + \mu(\partial q(\theta, \mu)/\partial\mu)$. Substituting (3.5) and (3.7) into the derivative and using the definition of Φ, we have the following equation.

$$\frac{\partial[\mu q(\theta,\mu)]}{\partial\mu} = c. \tag{3.8}$$

Then, we obtain the following lemma.

Lemma 3.2 *Input demand increases with the degree of defectiveness.*

Upstream demand is constantly increasing due to the increase in μ. In other words, the direct increase effect of μ always exceeds the indirect effect of the decrease in downstream output.

3.4.2 Profits of downstream and upstream firms

To discuss the effect of degree of defectiveness on the profits of downstream and upstream firms, we show that the profit of downstream firm decreases with μ.

We denote the equilibrium profit of downstream firm by $\pi_D(\theta,\mu)$. Differentiating $\pi_D(\theta,\mu)$ with respect to μ and substituting $p'(q) = -p(q)/(\varepsilon q)$, the result of Lemma 3.1, and (3.5) into the derivative, we obtain the followings.

$$\frac{\partial \pi_D(\theta,\mu)}{\partial\mu} = \frac{q((p(q)-\mu w)(\theta-1)\varepsilon-(1-2\theta)(1-z)p(q)}{(1-\theta)\mu\varepsilon(w-c)}\left[\mu\frac{\partial w(\theta,\mu)}{\partial\mu}+w\right]. \tag{3.9}$$

We can confirm that the first part has a positive denominator and a negative numerator. From Proposition 3.1, the terms in the square brackets are positive. Then, we obtain the following proposition.

Proposition 3.2 *The profit of downstream firm decreases with the degree of defectiveness.*

The intuition behind this proposition is simple. When μ rises, from (3.7) and Lemma 3.2, the downstream firm's marginal cost and input price increase. Hence, profit margin for the downstream firm decreases.

Next, we consider the effect of μ on the profit of the upstream firm. We denote the equilibrium profit of upstream firm by $\pi_U(\theta,\mu)$. Since the profit of upstream firm in the first stage is $\pi_U(w) = (w - c)\mu q(w, \theta, \mu)$, using envelop theorem, we obtain the effect of μ on the upstream firm's profit.

$$\frac{\partial \pi_U(\theta, \mu)}{\partial \mu} = \frac{\partial \pi_U(w)}{\partial \mu} = (w - c)q + (w - c)\mu \frac{\partial q(w, \theta, \mu)}{\partial \mu}.$$

Substituting the result of Lemma 3.1, we rearrange the above equation as follows.

$$\frac{\partial \pi_U(\theta, \mu)}{\partial \mu} = -cq < 0. \tag{3.10}$$

Therefore, we obtain the following proposition.

Proposition 3.3 *The profit of the upstream firm decreases with the degree of defectiveness.*

This proposition has a simple intuition. An increase in μ makes a downstream firm inefficient. Hence, an upstream firm facing a less efficient downstream firm always earns a small profit.

3.4.3 Consumer and total surpluses

Next, the effects of μ on consumers and total surpluses are discussed. We denote equilibrium consumer and total surpluses by $CS(\theta, \mu)$ and $TS(\theta, \mu)$, respectively. Differentiating $CS(\theta, \mu)$ with respect to μ leads to

$$\frac{\partial CS(\theta, \mu)}{\partial \mu} = -qp'(q) \left[\frac{\partial q(w, \theta, \mu)}{\partial w} \frac{\partial w(\theta, \mu)}{\partial \mu} + \frac{\partial q(w, \theta, \mu)}{\partial \mu} \right].$$

Substituting $p'(q) = -p(q)/(\varepsilon q)$, the results of Lemma 3.1, (3.5), and (3.7) into $\partial CS(\mu)/\partial \mu$, we obtain the followings.

$$\frac{\partial CS(\theta, \mu)}{\partial \mu} = \frac{c(1 - \theta)qp(q)(\mu w - p(q))}{\mu \Phi(w - c)} < 0. \tag{3.11}$$

From Assumption 3.1, the numerator is negative and the denominator is positive. Hence, we obtain the result of comparative statics.

Proposition 3.4 *Consumer surplus decreases with the degree of defective.*

The intuition behind this proposition is the same as that behind Lemma 3.1. As the degree of defectiveness increases, the downstream firm becomes inefficient, and therefore its output decreases.

Finally, we consider the effect of μ on total surplus. Using (3.9), (3.10), (3.11), and the definition of Φ, we obtain the derivative of $TS(\theta, \mu)$ with respect to μ as follows.

$$\frac{\partial TS(\theta,\mu)}{\partial \mu} = \frac{\partial \pi_U(\theta,\mu)}{\partial \mu} + \frac{\partial \pi_D(\theta,\mu)}{\partial \mu} + \frac{\partial CS(\theta,\mu)}{\partial \mu} < 0$$

Since all terms are negative, we obtain Proposition 3.5.

Proposition 3.5 *Total surplus increases with the degree of defective.*

The intuition behind this proposition is simple. When an increase in μ makes the downstream firm inefficient, this effect reduces total surplus.

3.5 Case with linear inverse demand and quadratic R&D cost

Here, we provide a case where inverse demand function is linear, $p = 1 - q$. We assume $0 < c\mu < 1$ and $0 < \theta < 1/2$. The other setting is the same as in the previous section.

Using backward induction, we obtain the following outcomes.

$$\begin{aligned}
&q_L = \frac{(1-\theta)(1-c\mu)}{4-6\theta}, \quad w_L = \frac{c\mu+1}{2\mu},\\
&\pi_{DL} = \frac{(1-\theta)(1-2\theta)(c\mu-1)^2}{4(2-3\theta)^2}, \quad \pi_{UL} = \frac{(1-\theta)(1-c\mu)^2}{8-12\theta},\\
&CS_L = \frac{(1-\theta)^2(1-c\mu)^2}{8(2-3\theta)^2}, \quad TS_L = \frac{(1-\theta)(7-11\theta)(c\mu-1)^2}{8(2-3\theta)^2}.
\end{aligned}$$

Where subscript 'L' denotes the case with linear inverse demand. Differentiating the outcomes with respect to μ, we obtain the following result.

Collorary 3.1 *Consider a case where the degree of defectiveness increases. Then, (i)* *upstream firm's profit decreases; (ii) downstream firm's profit and consumer surplus decrease; (iii) total surplus decreases.*

Proof. Differentiating equilibrium outcomes with respect to μ, we obtain the followings.

$$\frac{\partial \pi_{UL}}{\partial \mu} = \frac{c(\theta-1)(c\mu-1)}{6\theta-4} < 0, \quad \frac{\partial \pi_{DL}}{\partial \mu} = \frac{c(\theta-1)(2\theta-1)(c\mu-1)}{2(2-3\theta)^2} < 0,$$
$$\frac{\partial CS_L}{\partial \mu} = \frac{c(\theta-1)^2(c\mu-1)}{4(2-3\theta)^2} < 0, \quad \frac{\partial TS_L}{\partial \mu} = \frac{c(\theta-1)(11\theta-7)(c\mu-1)}{4(2-3\theta)^2} < 0.$$

Hence, we obtain this result.□

This result is consistent with Lemma 3.1 and Proposition 3.2–3.5. Hence, the intuition behind this result is the same as that in the previous section.

3.6 Conclusion

We analyzed the relationship between downstream production defects and CSR. We have developed a model with an upstream firm and a downstream firm. This analysis showed that production defects are detrimental to profits and society.

This means that CSR activities cannot reverse the negative effects of production defects. If CSR activities are to ameliorate the negative effects of defects, there needs to be some element that reduces the increase in marginal cost due to defects. For example, a model that allows for the recycling of defective products might provide a socially desirable conclusion.

Several issues remain to be addressed in this analysis. First, we analyzed the case in which only downstream firms are responsible for production defects, but there may be cases in which upstream firms are responsible. For example, the downstream retailers are not responsible for defective products under the Product Liability Law(PL Law), but the upstream manufacturers are. Second, we assume that production defects have no direct impact on demand. This conclusion may not hold if consumers will no longer demand goods with defects as they are unreliable. Finally, we assume that only downstream firms are engaged in CSR. If upstream also engages in CSR, input prices are expected to fall more, which would help increase downstream profits.

Chapter 4

Downstream corporate social responsibility and upstream R&D

4.1 Introduction

Corporate social responsibility (CSR) is defined as corporate activities and their impacts on different social groups (Carter and Jennings, 2002). In recent years, CSR activities have come to play an important role for firms (KPMG 2017).

In general, CSR activities are known to reduce the profit margins of competing firms. This is because CSR activities increase production and reduce the market share of other firms. That is, CSR activities have the commitment effect of expanding production. Therefore, in a oligopolistic market, it occurs in equilibrium to ensure that both parties engage in some degree of CSR with each other.[1]

From the above, we might seem that CSR activities are harmful to rival firms. However, it is possible that a firm's CSR activities can bring a positive impact on its rivals.

To consider the positive effects of CSR activities on other firms, we build the following simple model. There is a market with one upstream and two downstream firms. Using input sold by the upstream firm, the downstream firms produce final product and sell it to consumer. Both the downstream firms will conduct CSR activities. We also assume

[1] Previous studies have also concluded that if firms could strategically determine the degree of CSR, they would engage in CSR with each other (e.g. Planer-Friedrich and Sahm (2020)).

that the upstream firm can make marginal cost reducing investment.[2].

An increase in the degree of one downstream firm's CSR leads to smaller output of rival downstream firms and larger aggregate output, which in turn large input demand. In addition, an increase in the input demand encourages the upstream firm to invest in marginal cost reduction, which lowers input price. As a result, an increase in the one downstream CSR rate has a different effect on the profit of another downstream firm. If upstream investment technology is efficient, the positive effect of upstream R&D dominates the negative one of rivals' output reduction. That is, the increase in the one downstream CSR rate may increase the profit of other downstream firms. As a market with an efficient upstream firm is socially desirable, increasing the one downstream CSR rate increase the total surplus.

Previous literature consider several models in which firms engage CSR and cost reduction R&D in vertical markets. Garcia et al. (2018) analyzed a model in which upstream and downstream firms engage CSR activity and the upstream firm makes R&D investment. Li and Zhou (2019) analyzed a model where upstream and downstream firms engage CSR activity and the downstream firm makes R&D investment. They assume a situation of bilateral monopoly and no analysis is made of downstream competition. Therefore, the above studies do not consider a vertical model with CSR activities, upstream R&D, and downstream competition.

The following is the organization of this chapter. Section 2 presents the model. Section 3 calculates equilibrium outcomes. Section 4 gives the comparative statics results. Section 5 provides a case with linear inverse demand and quadratic R&D cost. Section 6 concludes this section.

[2]Extant literature considers the effects of upstream R&D. Milliou and Pavlou (2013) analyze a horizontal integration between upstream firms when upstream firms engage in R&D. Pinopoulos (2020) shows that upstream investment can increase when there is no price discrimination in a two-part tariff with incomplete information. Hu et al. (2022) analyze downstream cross-holdings on upstream R&D.

4.2 The Model

4.2.1 Model

We consider a market with an upstream firm and two downstream firms. The downstream firms faces an inverse demand function $p(Q)$, where $Q = q_1 + q_2$ is aggregate output and q_i $(i = 1, 2)$ is output of downstream i. We assume the inverse demand function is continuously differentiable and has constant curvature. The curvature of inverse demand function is denoted as $z \equiv -qp''(Q)/p'(Q)$, where $p'(Q), p''(Q)$ and $p'''(Q)$ are the first, second and third derivatives, respectively. Precisely, the inverse demand function takes the following formula: $p(Q) = a - bQ^{1-z}/(1-z)$, where $a, b > 0$ and $z \leq 0$. The last inequality guarantees that positive output is in equilibrium.[3] Note that $p'(Q) = -bQ^{-z} < 0$ and $p''(Q) = -zp'(Q)/Q \leq 0$ (Ritz, 2008).

The downstream firms only have to pay for inputs, and no other costs. We denote input prices as w. Then, the downstream firm i's profit is

$$\pi_{D_i} \equiv [p(Q) - w]q_i.$$

Downstream firms are also engaged in CSR. Downstream firms focus on consumer surplus at a rate of $\theta_i \in (0, 1)$.[4] Then, the objective function of the downstream is

$$\Pi_{D_i} \equiv \theta_i CS + (1 - \theta_i)\pi_{D_i}.$$

If θ_i becomes higher (lower), downstream firms more emphasize (less emphasis) CSR. We now assume $3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1 > 0$ to guarantee that equilibrium output is positive. We can also assume $\theta_i \geq \theta_j$ without loss of generality.

The upstream firm produces input and sells to the downstream firms. In addition, the upstream firm can invest in marginal cost-reducing R&D. The marginal cost of the upstream firm is $c - x$ when the upstream firm pays the investment cost $f(x)$, where

[3] This inverse demand function contains the familiar shape of an inverse demand function. When $z = 0$, the inverse demand function is linear.

[4] This formulation is a standard way of modelling for CSR (see Kopel et al. (2014), Brand and Grothe (2015), Fanti and Buccella (2017)).

$c(> 0)$ is the initial marginal cost without investment (e.g., d'Aspremont and Jacquemin, 1988). To guarantee positive marginal cost, we assume $c - x > 0$ in equilibrium. In addition, we assume that $f(0) = 0$ and for any $x > 0$, $f'(x) > 0$ and $f''(x) > 0$. The upstream firm's profit is given by

$$\pi_U \equiv [w - (c - x)]\, Q - f(x).$$

Consumer surplus is $CS \equiv \int_0^Q p(y)dy - p(Q)Q$; total surplus is $TS \equiv CS + \pi_{D_1} + \pi_{D_2} + \pi_U$.

The upstream and downstream firms engage in the following three-stage game. In the first stage, the downstream firms choose CSR rate θ_i. In the second stage, the upstream firm chooses input price w and R&D level x. In the third stage, the downstream firms choose output q_i. Using backward induction, we solve the game.

4.3 Calculating equilibrium

4.3.1 Downstream firm's decision

In the second stage, each downstream firm sets output q to maximize its profit π_D. From the first-order condition, $\partial \Pi_{D_i}/\partial q_i = (1-\theta_i)q_i p'(Q) + (1-\theta_i)(p(Q) - w) - \theta_i Q p'(Q) = 0$, we have the following output.

$$q_i(w, \theta_i, \theta_j) = \frac{(\theta_i\theta_j - 2\theta_j + 1)(w - p(Q))}{(3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1)p'(Q)}, \tag{4.1}$$

$$Q(w, \theta_i, \theta_j) = \frac{2(1-\theta_i)(1-\theta_j)(w - p(Q))}{(3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1)p'(Q)}. \tag{4.2}$$

In addition, rearranging the first-order condition, we can show that price elasticity $\varepsilon \equiv -p(Q)/[Qp'(Q)]$ is positive:

$$\varepsilon = \frac{(3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1)}{2(1-\theta_i)(1-\theta_j)} \frac{p(Q)}{p(Q) - w} > 0. \tag{4.3}$$

Next, we consider the effects of w, θ_i and θ_j on $Q(w, \theta_i, \theta_j)$. From the first and second derivatives of $Q(w, \theta_i, \theta_j)$ with respect to w, θ_i and θ_j, we obtain the following comparative static result.

Lemma 4.1 *The results of comparative statics are as follows.*

$$\begin{aligned}
\frac{\partial q_i(w,\theta_i,\theta_j)}{\partial w} &= \frac{Q\varepsilon(\theta_i\theta_j - 2\theta_j + 1)}{\Phi p(Q)} < 0, \\
\frac{\partial q_i(w,\theta_i,\theta_j)}{\partial \theta_i} &= Q\frac{(z-2)(3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1) - 2(1-\theta_j)}{2(1-\theta_i)^2\Phi p(Q)} > 0, \\
\frac{\partial q_i(w,\theta_i,\theta_j)}{\partial \theta_j} &= Q\frac{(2-z)(3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1) - 2\theta_j(1-\theta_i)}{2(1-\theta_i)^2\Phi p(Q)} < 0, \\
\frac{\partial Q(w,\theta_i,\theta_j)}{\partial w} &= \frac{2(1-\theta_i)(1-\theta_j)Q\varepsilon}{\Phi p(Q)} < 0, \\
\frac{\partial Q(w,\theta_i,\theta_j)}{\partial \theta_i} &= -\frac{(1-\theta_i)Q}{(1-\theta_j)\Phi} > 0, \\
\frac{\partial^2 q_i(w,\theta_i,\theta_j)}{\partial w^2} &= \frac{4Qz(1-\theta_i)^2(1-\theta_j)^2\varepsilon^2}{\Phi^2 p(Q)^2} > 0, \\
\frac{\partial^2 q_i(w,\theta_i,\theta_j)}{\partial \theta_i^2} &= \frac{(1-\theta_i)Q(2\Phi - (2-z)(1-\theta_j))}{{\theta_i}^2\Phi^2} < 0, \\
\frac{\partial^2 q_i(w,\theta_i,\theta_j)}{\partial w \partial \theta_i} &= \frac{2(1-\theta_j)^2 Q\varepsilon}{\Phi^2 p(Q)} > 0, \\
\frac{\partial^2 q_i(w,\theta_i,\theta_j)}{\partial \theta_i \partial \theta_j} &= \frac{Q(2-z)}{\Phi^2} > 0.
\end{aligned}$$

where $\Phi \equiv (z-2)(3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1) - (1 - \theta_i\theta_j) < 0$.

Proof. First, we derive $p'''(Q)$. Since the inverse demand function has constant curvature z, differentiating $z = -Qp''(Q)/p'(Q)$ with respect to Q and substituting $p''(Q) = -zp'(Q)/Q$ into it, we get

$$p'''(Q) = \frac{z(z+1)p'(Q)}{Q^2}. \tag{4.4}$$

Substituting $p'(Q) = -p(Q)/(\varepsilon Q)$, $p''(Q) = -zp'(Q)/Q$, and (4.4) into the first and second derivatives of $Q(w, \theta_i, \theta_j)$ and solving for $\partial Q(w, \theta_i, \theta_j)/\partial w$, $\partial Q(w, \theta_i, \theta_j)/\partial \theta_i$, $\partial^2 Q(w, \theta_i, \theta_j)/\partial w^2$, $\partial^2 Q(w, \theta_i, \theta_j)/\partial \theta_i^2$, $\partial^2 Q(w, \theta_i, \theta_j)/\partial w \partial \theta_i$, and $\partial^2 Q(w, \theta_i, \theta_j)/\partial \theta_i \partial \theta_j$, we obtain this lemma. □

An intuition behind Lemma 4.1 is as follows. Since the marginal cost of downstream firm is w, an increase in w reduces output $Q(w, \theta_i, \theta_j)$; hence, we obtain $\partial q_i(w, \theta_i, \theta_j)/\partial w < 0$ and $\partial Q(w, \theta_i, \theta_j)/\partial w < 0$. When marginal cost is large, the downstream output is small. Then, the output-reduction effects of w become small. Hence, the second derivatives of $Q(w, \theta_i, \theta_j)$ is positive: $\partial^2 Q(w, \theta_i, \theta_j)/\partial w^2 > 0$. An increase in θ_i raises the downstream firm i's output q_i; $\partial q_i(w, \theta_i, \theta_j)/\partial \theta_i > 0$. Now, q_j decreases because of strategic substitution for output decision; $\partial q_j(w, \theta_i, \theta_j)/\partial \theta_i < 0$. Since the decreasing effect of q_j is always dominated at this time, an increase in θ_i increases the total output $Q(w, \theta_i, \theta_j)$; hence, we obtain $Q(w, \theta_i, \theta_j)/\partial \theta_i > 0$. When the downstream i's CSR rate is large, the total output is large. Then, the output increase effects of θ_i become small. Hence, the second derivatives of $Q(w, \theta_i, \theta_j)$ is negative: $\partial^2 Q(w, \theta_i, \theta_j)/\partial \theta_i^2 < 0$. An increase in θ_i raises the downstream firm's unit cost by w. Therefore, with large w, output reduction effect of θ_i is significant and $\partial^2 q(w, \theta_i, \theta_j)/\partial w \partial \theta_i > 0$. An increase in θ_i raises the total output Q. Hence, with large θ_i, output reduction effect of θ_j is significant. Therefore, we have $\partial^2 Q(w, \theta_i, \theta_j)/\partial \theta_i \partial \theta_j > 0$.

4.3.2 Upstream firm's decision

Next, we consider the upstream firm's decision. Substituting (4.2) into the profit of upstream firm, we have $\pi_U(w, x) = [w - (c - x)]\, Q(w, \theta_i, \theta_j) - f(x)$. Substituting $p'(Q) = -p(Q)/(\varepsilon Q)$ and the result of Lemma 4.1 into the first-order conditions $\partial \pi_U(w, x)/\partial w = 0$ and $\partial \pi_U(w, x)/\partial x = 0$, we obtain the followings.

$$\frac{\partial \pi_U(w, x)}{\partial w} = Q + [w - (c - x)] \frac{\partial Q(w, \theta_i, \theta_j)}{\partial w} = 0, \quad \frac{\partial \pi_U(w, x)}{\partial x} = Q - f'(x) = 0. \tag{4.5}$$

Solving the above equations and (4.1) for w, x, and $f'(x)$, we obtain the equilibrium outcomes.

$$w(\theta_i, \theta_j) = \frac{p(Q)\,(2s\varepsilon - 3\theta_i\theta_j + 2\theta_i + 2\theta_j - 1)}{2s\varepsilon}, \tag{4.6}$$

$$x(\theta_i, \theta_j) = c - \frac{p(Q)\,(2s\varepsilon + \Phi - 3\theta_i\theta_j + 2\theta_i + 2\theta_j - 1)}{2s\varepsilon}, \quad f'(x) = Q, \tag{4.7}$$

where $s \equiv (1-\theta_i)(1-\theta_j)$.

To guarantee $c - x > 0$, that is, positive marginal cost of upstream firm, we assume

Assumption 4.1 $2s\varepsilon + \Phi - (3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1) > 0$.

4.3.3 Concavity of upstream firm's profit

We consider the second-order necessary condition in the second stage. Since $\partial^2\pi_U(w,x)/\partial x^2 = -f''(x) < 0$, only the inequality is considered: $[\partial^2\pi_U(w,x)/\partial w^2][\partial^2\pi_U(w,x)/\partial x^2] - [\partial^2\pi_U(w,x)/\partial w\partial x]^2 > 0$. Substituting Lemma 4.1, (4.1), (4.6), (4.7), and $p'(q) = -p(q)/(\varepsilon q)$ into the above inequality, the following inequality is obtained:

$$\frac{\partial^2\pi_U(w,x)}{\partial w^2}\frac{\partial^2\pi_U(w,x)}{\partial x^2} - \left[\frac{\partial^2\pi_U(w,x)}{\partial w\partial x}\right]^2 = \frac{2sQ\varepsilon\,[\Phi(z-2)p(Q)f''(x) - 2sQ\varepsilon]}{\Phi^2 p(Q)^2} > 0,$$

As the above inequality depends only on $[\Phi(z-2)p(Q)f''(x) - 2sQ\varepsilon]$, we assume this term is positive to satisfy the second-order necessary condition. Solving this inequality for $f''(x)$, the assumption is rewritten as follows.

Assumption 4.2 *To satisfy the second-order necessary condition, we assume* $\Phi(z-2)p(Q)f''(x) - 2sQ\varepsilon > 0$, *which is equivalent to*

$$f''(x) > \frac{2sQ\varepsilon}{\Phi(z-2)p(Q)} \equiv f^{SOC}.$$

According to this assumption, the investment cost function is sufficiently convex.

4.4 Comparative statics

4.4.1 Input price, upstream investment, and downstream marginal cost

The next step is to do comparative statics. Differentiating the first-order conditions of upstream firm, (4.5), with respect to μ, substituting the result of Lemma 4.1 and upstream outcomes (4.6), (4.7), and solving them for $\partial w(\theta_i,\theta_j)/\partial\theta_i$ and $\partial x(\theta_i,\theta_j)/\partial\theta_i$, we have the followings:

$$\frac{\partial w(\theta_i,\theta_j)}{\partial \theta_i} = \frac{(1-\theta_j)Qp(Q)}{(1-\theta_i)\left(2sQ\varepsilon - \Phi(z-2)p(Q)f''(x)\right)}, \tag{4.8}$$

$$\frac{\partial x(\theta_i,\theta_j)}{\partial \theta_i} = \frac{(z-2)(1-\theta_j)Qp(Q)}{(1-\theta_i)\left(2sQ\varepsilon - \Phi(z-2)p(Q)f''(x)\right)}. \tag{4.9}$$

From the above derivatives, we obtain Proposition 4.1

Proposition 4.1 *Input price $w(\theta_i,\theta_j)$ decreases with the downstream i's degree of CSR θ_i Investment level $x(\theta_i,\theta_j)$ increases with the downstream i's degree of CSR θ_i.*

Proof. From Assumption 4.2, the denominator in (4.8) and (4.9) is negative. Then, the signs of $\partial w(\theta_i,\theta_j)/\partial\theta_i$ and $\partial x(\theta_i,\theta_j)/\partial\theta_i$ only depend on those of numerators. From assumptions, we have $\theta_i \in (0,1)$. Then, we can check the numerator of $\partial w(\theta_i,\theta_j)/\partial\theta_i$ is positive. Similarly, we can check the numerator of $\partial x(\theta_i,\theta_j)/\partial\theta_i$ is negative. Therefore, we obtain this proposition. □

The intuition behind this proposition is as follows. When input demand, Q, increases, the upstream firm will increase the amount of investment. From Lemma 4.1, for given input price, an increase in θ_i increases downstream outputs. Hence, if input demand increases with θ_i, upstream is more active R&D. Then, investment level $x(\theta_i,\theta_j)$ increase with θ_i. Since the upstream firm's production with a large investment is efficient, the upstream firm chooses a low input price.

4.4.2 Profits of downstream and upstream firms

To discuss the effect of rival downstream firm's CSR on the profits of downstream firm, we show the condition where the profit of downstream firm j increases with θ_i. We denote the equilibrium profit of downstream firm i by $\pi_{D_i}(\theta_i,\theta_j) \quad \forall i = 1,2$. Differentiating $\pi_{D_i}(\theta_i,\theta_j)$ and $\pi_{D_j}(\theta_i,\theta_j)$ with respect to θ_i and substituting $p'(Q) = -p(Q)/(\varepsilon Q)$, we obtain the following equation.

$$
\begin{aligned}
\frac{\partial \pi_{D_j}(\theta_i,\theta_j)}{\partial \theta_i} =& q_i \left(\frac{(\theta_j - 1)\, p(Q)}{\Phi \varepsilon\, (\theta_i - 1)} - \left(1 + \frac{2s}{\Phi}\right) \frac{\partial w(\theta_i,\theta_j)}{\partial \theta_i} \right), \\
\frac{\partial \pi_{D_j}(\theta_i,\theta_j)}{\partial \theta_i} =& -\frac{1}{Q\varepsilon} \frac{\partial w(\theta_i,\theta_j)}{\partial \theta_i} \left(q_j p(Q) \frac{\partial Q(w,\theta_i,\theta_j)}{\partial w} + Q\varepsilon (w - p(Q)) \frac{\partial q_j(w,\theta_i,\theta_j)}{\partial w} + Q\varepsilon q_j \right) \\
& + q_j p(Q) \frac{\partial Q(w,\theta_i,\theta_j)}{\partial \theta_i} + Q\varepsilon (w - p(Q)) \frac{\partial q_j(w,\theta_i,\theta_j)}{\partial \theta_i}.
\end{aligned}
$$

Substituting (4.8), the result of Lemma 4.1, (4.6), and (4.7) into the derivative $\partial \pi_{D_i}(\theta_i,\theta_j)/\partial \theta_i$ and $\partial \pi_{D_j}(\theta_i,\theta_j)/\partial \theta_i$, and solving it for $f''(x)$, we obtain the following proposition.

Proposition 4.2 *The profit of downstream firm i increases with the degree of downstream i's CSR if $f''(x) < f^{\pi_i}$ The profit of downstream firm j increases with the degree of downstream i's CSR if $f''(x) < f^{\pi_D}$, where*

$$
\begin{aligned}
f^{\pi_i} &\equiv \frac{Q\varepsilon}{(2 - z)p(Q)}, \\
f^{\pi_D} &\equiv \frac{2sQ\varepsilon[(p(Q) - w)Q\varepsilon - p(Q)q_j)]}{(z - 2)p(Q)[(p(Q) - w)Q\varepsilon\Phi_{\pi_D} - 2sp(Q)q_j]}, \\
\Phi_{\pi_D} &\equiv \Phi + (\theta_i\theta_j - 2\theta_i + 1).
\end{aligned}
$$

Proof. Differentiating $\pi_{D_i}(\theta_i,\theta_j)$ and $\pi_{D_j}(\theta_i,\theta_j)$ with respect to θ_i and substituting $p'(Q) = -p(Q)/(\varepsilon Q)$, the result of Lemma 4.1, (4.6), and (4.7) into the derivative, we obtain the following equation.

$$
\begin{aligned}
\frac{\partial \pi_{D_i}(\theta_i,\theta_j)}{\partial \theta_i} &= \frac{q_i\,(1 - \theta_j)\, p(Q)\, [(z - 2)p(Q)f''(x) + Q\varepsilon]}{\varepsilon\, (\theta_i - 1)\, (2Qs\varepsilon - \Phi(z - 2)p(Q)f''(x))}, \\
\frac{\partial \pi_{D_j}(\theta_i,\theta_j)}{\partial \theta_i} &= \frac{(z - 2)p(Q)((p(Q) - w)Q\varepsilon\Phi_{\pi_D} - 2sp(Q)q_j)f''(x)}{2(1 - \theta_i)^2 \varepsilon\, (2sQ\varepsilon - \Phi(z - 2)p(Q)f''(x)))} \\
&\quad - \frac{(z - 2)p(Q)(2sQ\varepsilon((p(Q) - w)Q\varepsilon + p(Q)q_j))}{2(1 - \theta_i)^2 \varepsilon\, (2sQ\varepsilon - \Phi(z - 2)p(Q)f''(x))}. \qquad (4.10)
\end{aligned}
$$

Since the $\partial \pi_{D_i}(\theta_i,\theta_j)/\partial \theta_i$'s denominator is positive from Assumption 4.2 and we assume $z \leq 0$, the sign of $\partial \pi_{D_j}(\theta_i,\theta_j)/\partial \theta_i$ only depends on the terms in the square brackets of numerator. The terms is a liner function of $f''(x)$ and the coefficient of $f''(x)$ is negative. Hence, solving $\partial \pi_{D_i}(\theta_i, \theta_j)/\partial\theta_i > 0$ for $f''(x)$, we have the following inequality.

$$f''(x) < \frac{Q\varepsilon}{(2-z)p(Q)} \ (\equiv f^{\pi_i}).$$

Here, we compare f^{π_i} with f^{SOC}.

$$f^{\pi_i} - f^{SOC} = \frac{Q\varepsilon(z-1)\left(3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1\right)}{\Phi(2-z)p(Q)}.$$

Since the numerator and the denominator is negative, we have $f^{\pi_D} - f^{SOC} > 0$.

Since the $\partial \pi_{D_j}(\theta_i, \theta_j)/\partial\theta_i$'s denominator is negative from Assumption 4.2 and we assume $z \leq 0$, the sign of $\partial \pi_{D_j}(\theta_i, \theta_j)/\partial\theta_i$ only depends on the terms in the square brackets of numerator. Since $\Phi_{\pi_D} \equiv \Phi + (\theta_i\theta_j - 2\theta_i + 1) = (z-2)\left(3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1\right) - 2\theta_i(1 - \theta_j) < 0$, the terms is a liner function of $f''(x)$ and the coefficient of $f''(x)$ is negative. Hence, solving $\partial \pi_{D_j}(\theta_i, \theta_j)/\partial\theta_i > 0$ for $f''(x)$, we have the following inequality.

$$f''(x) < \frac{2sQ\varepsilon[(p(Q) - w)Q\varepsilon - p(Q)q_j)]}{(z-2)p(Q)[(p(Q) - w)Q\varepsilon\Phi_{\pi_D} - 2sp(Q)q_j]} \ (\equiv f^{\pi_D}).$$

Here, we compare f^{π_D} with f^{SOC}.

$$f^{\pi_D} - f^{SOC} = \frac{2Qs\varepsilon\left((p(Q) - w)Q\varepsilon\left(\theta_i\theta_j - 2\theta_i + 1\right) + p(Q)q_j(1-z)\left(3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1\right)\right)}{\Phi(2-z)p(Q)\left((p(Q) - w)Q\varepsilon\Phi_{\pi_D} - 2sp(Q)q_j\right)}.$$

Since the numerator and the denominator is positive, we have $f^{\pi_D} - f^{SOC} > 0$. Summarizing the above results, we obtain this proposition. □

The intuition behind this proposition is as follows. From Lemma 4.1, when θ_i rises, the output of downstream i increases and that of downstream firm j drops down. In addition, the aggregate output Q rises. At this time, downstream i is in a state of overproduction, and increasing production reduces the profit. Also, for the downstream j's, profit margins have been reduced because of the loss of market share. However, since R&D is becoming more active, downstream profits may increase if R&D efficiency is above a certain level.

Next, we consider the effect of θ_i on the profit of the upstream firm. We denote the equilibrium profit of the upstream firm by $\pi_U(\theta_i, \theta_j)$. Since the profit of the upstream firm in the second stage is $\pi_U(w, x) = [w - (c - x)]\, Q(w, \theta_i, \theta_j) - f(x)$, using envelop theorem, we obtain the effect of θ_i on the upstream firm's profit.

$$\frac{\partial \pi_U(\theta_i, \theta_j)}{\partial \theta_i} = \frac{\partial \pi_U(w, x)}{\partial \theta_i} = (w - c + x)\frac{\partial Q(w, \theta_i, \theta_j)}{\partial \theta_i}.$$

Substituting the result of Lemma 4.1, we rearrange the above equation as follows.

$$\frac{\partial \pi_U(\theta_i, \theta_j)}{\partial \theta_i} = \frac{Qp(Q)}{2\varepsilon\,(1 - \theta_i)^2} > 0. \tag{4.11}$$

Therefore, we obtain the following proposition.

Proposition 4.3 *The profit of an upstream firm increases with the degree of downstream i's CSR.*

This proposition has a simple intuition. An increase in θ_i makes downstream total output large. Hence, the upstream firm facing large demand always earns large profit.

4.4.3 Consumer and total surpluses

Next, we discuss the effects of θ_i on consumers and total surpluses. We denote consumer and total surpluses by $CS(\theta_i, \theta_j)$ and $TS(\theta_i, \theta_j)$, respectively. Differentiating $CS(\theta_i, \theta_j)$ with respect to θ_i leads to

$$\frac{\partial CS(\theta_i, \theta_j)}{\partial \theta_i} = -Qp'(Q)\left[\frac{\partial Q(w, \theta_i, \theta_j)}{\partial w}\frac{\partial w(\theta_i, \theta_j)}{\partial \theta_i} + \frac{\partial Q(w, \theta_i, \theta_j)}{\partial \theta_i}\right].$$

Hence, consumer surplus increases with θ_i if input price decreases with θ_i: $\partial w(\theta_i, \theta_j)/\partial \theta_i < 0$. In particular, substituting $p'(Q) = -p(Q)/(\varepsilon Q)$, the results of Lemma 4.1, (4.6), and (4.8) into $\partial CS(\theta_i, \theta_j)/\partial \theta_i$, we obtain the followings.

$$\frac{\partial CS(\theta_i, \theta_j)}{\partial \theta_i} = \frac{Q(z-2)\,(1-\theta_j)\,p(Q)^2 f''(x)}{\varepsilon\,(1-\theta_i)\,(2Qs\varepsilon - \Phi(z-2)p(Q)f''(x))} > 0. \tag{4.12}$$

The numerator is negative and from Assumption 4.2, and the denominator is also negative. Hence, we obtain the result of comparative statics.

Proposition 4.4 *Consumer surplus increases with the degree of downstream i's CSR.*

The intuition behind this proposition is the same as that behind Lemma 4.1. Because of input price reduction, an increase in θ_i expands downstream aggregate outputs, which increases consumer surplus.

Finally, we consider the effect of θ_i on total surplus. Using (4.10), (4.11), (4.12), and the definition of Φ, we obtain the derivative of $TS(\theta_i)$ with respect to θ_i as follows.

$$\begin{aligned}\frac{\partial TS(\theta_i,\theta_j)}{\partial \theta_i} &= \frac{\partial \pi_{D1}(\theta_i,\theta_j)}{\partial \theta_i} + \frac{\partial \pi_{D2}(\theta_i,\theta_j)}{\partial \theta_i} + \frac{\partial \pi_{S}(\theta_i,\theta_j)}{\partial \theta_i} + \frac{\partial CS(\theta_i,\theta_j)}{\partial \theta_i} \\ &= \frac{Q(z-2)p(Q)^2 f''(x)\left(3\theta_i\theta_j - 2\theta_i - 2\theta_j + 1 - \Phi\right)}{2\varepsilon\left(1-\theta_i\right)^2\left(2Qs\varepsilon - \Phi(z-2)p(Q)f''(x)\right)} > 0.\end{aligned}$$

From Assumption 2.2, the denominator of the above derivative is negative and the numerator is also negative. Therefore, we obtain Proposition 4.5.

Proposition 4.5 *Total surplus increases with the degree of downstream i's CSR.*

The intuition behind this proposition is as follows. When an increase in θ_i yields larger downstream firms' outputs. This effect increases total surplus. In addition, from Lemma 4.1, the downstream firms' CSR activities make marginal cost low and upstream investment technology efficient. This effect partially solves the double marginalization problem. This effect also increases total surplus.

4.5 Optimal degree of CSR

We provide a case with a linear inverse demand function $p = 1 - q_1 - q_2$, and a quadratic investment cost function $f(x) \equiv rx^2/2$. To guarantee positive outcomes, we assume $0 \le \theta_i \le 1/2$ and $r > \frac{4\theta_i\theta_j - 3\theta_i - 3\theta_j + 2}{2c(8\theta_i\theta_j - 5\theta_i - 5\theta_j + 3)} (\equiv r^{positive})$. Note that $f''(x) = r$. In addition, we assume $0 < c < 1$. This condition implies that the upstream can sell its product to the downstream firms even when the upstream firm does not invest. The other setting is the same as in the previous section.

Solving this game by using backward induction, we obtain the following outcomes in the second stage.

$$\begin{aligned}
q_{i_L} =& \frac{(1-c)r(1-\theta_i)\,(1-2\theta_j)}{2r\,(8\theta_i\theta_j-5\theta_i-5\theta_j+3)-(4\theta_i\theta_j-3\theta_i-3\theta_j+2)},\\
w_L =& \frac{(1+c)r\,(8\theta_i\theta_j-5\theta_i-5\theta_j+3)-(4\theta_i\theta_j-3\theta_i-3\theta_j+2)}{2r\,(8\theta_i\theta_j-5\theta_i-5\theta_j+3)-(4\theta_i\theta_j-3\theta_i-3\theta_j+2)},\\
x_L =& \frac{(1-c)\,(4\theta_i\theta_j-3\theta_i-3\theta_j+2)}{2r\,(8\theta_i\theta_j-5\theta_i-5\theta_j+3)-(4\theta_i\theta_j-3\theta_i-3\theta_j+2)},\\
\pi_{D_iL} =& \frac{(1-c)^2r^2(1-\theta_i)\,(1-2\theta_i)\,(1-2\theta_j)^{\,2}}{(2r\,(8\theta_i\theta_j-5\theta_i-5\theta_j+3)-(4\theta_i\theta_j-3\theta_i-3\theta_j+2))^{\,2}},\\
\pi_{UL} =& \frac{(1-c)^2r\,(4\theta_i\theta_j-3\theta_i-3\theta_j+2)}{2\,(2r\,(8\theta_i\theta_j-5\theta_i-5\theta_j+3)-(4\theta_i\theta_j-3\theta_i-3\theta_j+2))},
\end{aligned}$$

where the subscript 'L' denotes the case with linear inverse demand and quadratic R&D cost. Then, we get Collorary 4.1.

Collorary 4.1 *Consider a case where the degree of downstream i's CSR increases. Then, (i) upstream firm's profit increases; (ii) downstream firm j's profit increase if* $r < 1/2$*; (iii) consumer and total surplus increases.*

Proof. See Appendix 4.7.

Because of $f''(x) = r$, this result is consistent with Lemma 4.1 and Proposition 4.2–4.5. Hence, the intuition behind this result is the same as that in the previous section.

To visually present the result (ii) in Collorary 4.1, we depict the threshold values, $r = 1/2$ and $r = r^{positive}$ at $c = 4/5$ and $\theta_j = 1/4$ in Figure 4.1. The horizontal axis is θ_i and the vertical axis is r. Figure 4.1 has two curves. The solid curve is $r = r^{positive}$; the dashed line is $r = 1/2$. From Figure 4.1, we can confirm the results in Collorary 4.1. The downstream firm j's profit increase with the degree of downstream i's CSR if the investment technology is efficient: $r^{positive} < r < 1/2$.

Now, we discuss the optimal degree of CSR. In the first stage, each downstream firm decides on the degree of CSR θ_i to maximize its profit π_{D_i}. We define the profit of downstream firm i in the first stage by $\pi_{D_i}(\theta_i, \theta_j)$. Then the maximization problem in

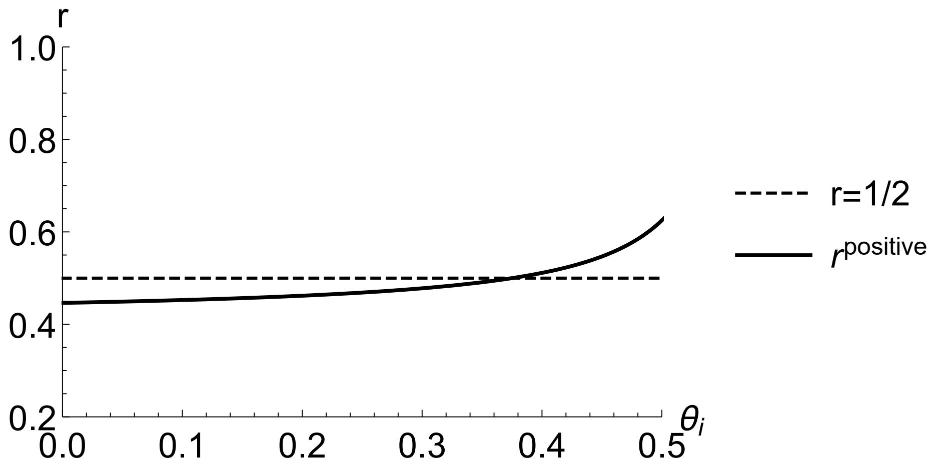

Figure 4.1: The threshold values of r at $c = 4/5$ and $\theta_j = 1/4$

the first stage is as follows.

$$\max_{\theta_i} \quad \pi_{D_i}(\theta_i, \theta_j).$$

From the first-order condition $\partial \pi_{D_i}(\theta_i, \theta_j)/\partial \theta_i = 0$, the best response of downstream firm i is as follows.

$$\theta_i(\theta_j) = \frac{(2r+1)\theta_j - 2r}{8r\theta_j - 6r + 1} \tag{4.13}$$

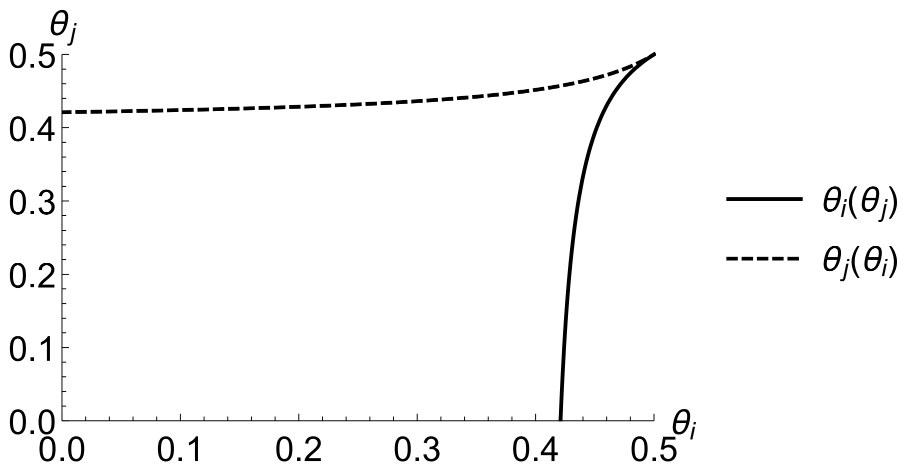

Figure 4.2: Optimal response of each downstream at $r = 4/5$

Because of symmetry between downstream firms, we substitute $\theta_i^* = \theta_j^* = \theta^*$ into the best response in (4.13). We also see from 4.2 that this equilibrium is a subgame perfect equiribilium. Then, we obtain $\theta_i^* = \theta_j^* = 1/2$, which leads to Proposition 4.6.

Proposition 4.6 *If an inverse demand function is linear and a R&D cost function is* *quadratic, the equilibrium degree of CSR is $\theta^* = 1/2$. In addition, it is independent from upstream R&D efficiency.*

From Lemma 4.1, θ_i increases the firm i's market share and reduces its marginal cost. Thus, each downstream firm acts to maximize its degree of CSR. Then, in equilibrium, they choose $\theta_i^* = \theta_j^* = 1/2$, which is independent from the efficiency of upstream R&D.

4.6 Conclusion

We analyzed the relationship between upstream cost-reducing R&D and downstream CSR. We have developed a model in which one upstream firm and tow downstream firms. We show that one downstream's CSR may increase the profit of the other downstream firm. In addition, the increase in the degree of CSR increases consumer and total surpluses.

Hence, the negative effect of a firm's CSR activities on other firms' profits may be small. If the upstream R&D investment is highly efficient, CSR may be desirable both for firms and society. Therefore, it is easy to recommend CSR in this market.

Several issues remain to be addressed in this analysis. First, we have not considered the case of upstream CSR. Assuming upstream CSR, one would expect the results of this study to be more likely to hold because input prices would be lower. Second, We made some limitations for R&D. We can consider a model in which downstream engage in R&D or in which R&D is stochastically successful. Finally, we do not consider competition in the upstream market. If upstream market is an oligopolistic, we might expect that upstream competition yields lower input price. Then, this conclusion is less likely to be obtained because the parameters are less sensitive.

4.7 Appendix

Proof of Collorary 4.1

Differentiating equilibrium outcomes with respect to θ_i, we obtain the followings.

$$
\begin{aligned}
\frac{\partial \pi_{UL}}{\partial \theta_i} &= \frac{(1-c)^2 r^2 \left(1-2\theta_j\right)^2}{\left(2r\left(8\theta_i\theta_j-5\theta_i-5\theta_j+3\right)-\left(4\theta_i\theta_j-3\theta_i-3\theta_j+2\right)\right)^2} > 0,\\
\frac{\partial \pi_{D_iL}}{\partial \theta_i} &= -\frac{(1-c)^2 r^2 \left(1-2\theta_j\right)^2 \left[\theta_i-\theta_j+2r\left(4\theta_i\theta_j-3\theta_i-\theta_j+1\right)\right]}{\left(2r\left(8\theta_i\theta_j-5\theta_i-5\theta_j+3\right)-\left(4\theta_i\theta_j-3\theta_i-3\theta_j+2\right)\right)^3},\\
\frac{\partial \pi_{D_jL}}{\partial \theta_i} &= \frac{2(1-c)^2 r^2 (1-2r)\left(1-2\theta_i\right)\left(1-\theta_j\right)\left(1-2\theta_j\right)^2}{\left(2r\left(8\theta_i\theta_j-5\theta_i-5\theta_j+3\right)-\left(4\theta_i\theta_j-3\theta_i-3\theta_j+2\right)\right)^3},\\
\frac{\partial CS_L}{\partial \theta_i} &= \frac{(1-c)^2 r^2 \left(1-2\theta_j\right)\left[\left(\theta_j-1\right)\left(\theta_i-\theta_j\right)-2r\left(\left(4\theta_i-5\right)\theta_j^2+\left(5-3\theta_i\right)\theta_j-1\right)\right]}{\left(2r\left(8\theta_i\theta_j-5\theta_i-5\theta_j+3\right)-\left(4\theta_i\theta_j-3\theta_i-3\theta_j+2\right)\right)^3},\\
\frac{\partial TS_L}{\partial \theta_i} &= \frac{(1-c)^2 r^2 \left(1-2\theta_j\right)\left[\left(\theta_j-1\right)\left(\theta_i-\theta_j\right)-2r\left(\theta_i\left(20\theta_j^2-19\theta_j+4\right)-13\theta_j^2+13\theta_j-3\right)\right]}{\left(2r\left(8\theta_i\theta_j-5\theta_i-5\theta_j+3\right)-\left(4\theta_i\theta_j-3\theta_i-3\theta_j+2\right)\right)^3}.
\end{aligned}
$$

Solving $\partial \pi_{D_jL}/\partial \theta_i > 0$ for r, we obtain $r < 1/2$.

As the sign of $\partial CS_L/\partial \theta_i$ and $\partial TS_L/\partial \theta_i$ only depend on the terms in square brackets and the coefficient of r are positive, solving $\partial CS_L/\partial \theta_i > 0$ and $\partial TS_L/\partial \theta_i > 0$ for r yields $r > r^{CS}$ and $r > r^{TS}$, where

$$
\begin{aligned}
r^{CS} \equiv& \frac{\left(\theta_j-1\right)\left(\theta_i-\theta_j\right)}{2\left(4\theta_i\theta_j^2-3\theta_i\theta_j-5\theta_j^2+5\theta_j-1\right)},\\
r^{TS} \equiv& \frac{\left(\theta_j-1\right)\left(\theta_i-\theta_j\right)}{2\left(4\theta_i+20\theta_i\theta_j^2-19\theta_i\theta_j-13\theta_j^2+13\theta_j-3\right)}.
\end{aligned}
$$

We can show that these threshold values are smaller than $r^{positive}$ as follow.

$$
\begin{aligned}
r^{positive}-r^{CS} =& \frac{1}{2}\left(\frac{\theta_i\left(4\theta_j-3\right)-3\theta_j+2}{c\left(\theta_i\left(8\theta_j-5\right)-5\theta_j+3\right)}-\frac{\left(\theta_j-1\right)\left(\theta_i-\theta_j\right)}{\left(4\theta_i-5\right)\theta_j^2+\left(5-3\theta_i\right)\theta_j-1}\right) > 0,\\
r^{positive}-r^{TS} =& \frac{1}{2}\left(\frac{\theta_i\left(4\theta_j-3\right)-3\theta_j+2}{c\left(\theta_i\left(8\theta_j-5\right)-5\theta_j+3\right)}-\frac{\left(\theta_j-1\right)\left(\theta_i-\theta_j\right)}{\theta_i\left(20\theta_j^2-19\theta_j+4\right)-13\theta_j^2+13\theta_j-3}\right) > 0.
\end{aligned}
$$

Therefore, we obtain $\partial CS_L/\partial \theta_i > 0$ and $\partial TS_L/\partial \theta_i > 0$.$\square$

Chapter 5

Conclusion

We analyzed the relationship between production defects, R&D, and CSR in vertical markets. In Chapter 2, we analyze the relationship between production defects and R&D, showing that product defects do not necessarily have a negative impact on firms or society. Chapter 3 shows that the negative effects of production defects cannot be remedied by CSR activities. Chapter 4 shows that the negative impacts of CSR activities on rival firm's profit can be resolved through R&D.

We can confirm key role of upstream R&D which mitigates or resloves negative impacts on firms' profits and society. This is because the negative impacts are caused from tougher competition. Therefore, the upstream R&D may be better promoted more aggressively. In the long run, it may also be meaningful for downstream and consumers to continue to keep input demand of downstream firms.

Several issues remain in this thesis. First, it is assumed that the factors addressed in each chapter are related only to either upstream or downstream. If both are involved, the conclusions of this thesis may be different. Next, we assume an upstream monopoly situation throughout the thesis. If there is more than one upstream firm, the competition effect will bring larger profits of downstream firms, which may have a positive social impact. Finally, we only consider a take-it-or-leave-it offer of linear contract between upstrem and downstream firms. Hence, non-linear contract or negotiation between them may provide some different results. We would like to leave these issues in the future.

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