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Fujinaga, Takashi

(Citation)
Japan Architectural Review,6(1):e12336

(Issue Date)
2023-01

(Resource Type)
journal article

(Version)
Version of Record

Japan Architectural Review Open Access

Architectural Institute of Japan

Original Paper

Ultimate flexural strength of rectangular concrete-filled steel tubular beam-columns using high-strength materials

Takashi Fujinaga

Research Center for Urban Safety and Security, Kobe University, Kobe, Japan

Correspondence
Takashi Fujinaga, Research Center for Urban Safety and Security, Kobe University, Kobe, Hyogo 657-8501, Japan.
Email: ftaka@kobe-u.ac.jp

Funding information
No funding information is provided

Received November 20, 2022; Accepted January 19, 2023

doi: 10.1002/2475-8876.12336

Abstract

The ultimate flexural strength of concrete-filled steel tubular (CFST) beam-columns is calculated using the full-plastic strength as per the "AIJ Recommendations for Design and Construction of Concrete Filled Steel Tubular Structures". However, the full-plastic strength might overestimate the ultimate strength of rectangular CFST beam-columns, when the width-to-thickness ratio is significant and/or high-strength materials are used. The calculation of the ultimate flexural strength when it exceeds the applicable range of AIJ CFT recommendations is based on a method that considers the stress reduction on the compression side of the steel tube or outermost edge strain of the concrete. However, both stress reduction and outermost edge strain should be considered when high-strength materials are used in CFST for accurate ultimate strength formulas. In this study, a simple evaluation method for the ultimate flexural strength of rectangular CFST beam-columns was proposed using the maximum strain at the extreme concrete compression fiber and stress reduction on the compression side of the steel tube. In addition, the appropriate strain at the extreme concrete fiber needed to accurately calculate the ultimate strength was discussed.

Keywords
concrete-filled steel tube, high tensile strength steel, high-strength concrete, strain at extreme fiber, width-to-thickness ratio

1. Introduction

In the "AIJ Recommendations for Design and Construction of Concrete Filled Steel Tubular Structures" (AIJ Recommendations),[1] the ultimate flexural strength of a concrete-filled steel tubular (CFST) beam-column is calculated using the full-plastic flexural strength assuming a rectangular stress block. The full-plastic flexural strength is also used for the ultimate flexural strength in other standards.[2–4] However, the flexural strength of a rectangular CFST calculated using the full plastic stress block may be overestimated when the width-to-thickness ratio is significant and/or when high-strength concrete and high-tensile-strength steel are used. In the AIJ Recommendations, the modified calculation method for the ultimate flexural strength has a criterion wherein a large width-to-thickness ratio of the thin-walled steel tube and high-strength concrete are beyond the scope of application.[1,5] The modified method is applied by considering the decrease in strength by the local buckling of the steel tube compression side. The rectangular stress distribution is still used as the stress block shape. However, the stress around the neutral axis typically remains within the elastic range. The effect of the stress distribution in the elastic range on the flexural strength increases when a high-tensile strength steel is used.

In contrast, another strength evaluation method uses the strain limit at the extreme concrete bending-compression fiber to calculate the ultimate flexural strength.[6–8] This method is useful because it considers the effect of the stress of the steel tube part in the elastic range as the strength, if the strain at the extreme concrete fiber is regulated. The elasto-plastic model is used for the stress–strain relationship of steel. In the "ACI Building Code Requirements for Structural Concrete (ACI318)",[6] the maximum strain at the extreme concrete compression fiber is assumed to be equal to 0.003. The regulated strain at the extreme concrete fiber is set based on the concrete compressive strain during concrete crushing. This value is acceptable for calculating the ultimate flexural strength of reinforced concrete sections. However, in the case of the CFST members, the filled concrete is confined by a steel tube. Therefore, the strength of the concrete does not suddenly deteriorate if the strain exceeds the compressive strain during concrete crushing.

In the "AIJ Design Standard for Steel Tube and Concrete Structures",[9] a strain value of 0.004 was used at the extreme concrete compression fiber for the filled and encased type steel tube and concrete columns. Kido and Tsuda[8] used 0.004 and 0.008 as the strains at the extreme concrete fiber for rectangular CFST columns. The ultimate flexural strength using the strain at the extreme concrete fiber and full-plastic flexural strength were compared. However, the appropriate strain at the extreme concrete compression fiber at the ultimate flexural strength was not examined.

The ultimate flexural strength of the current AIJ Recommendations is appropriate if the parameters are within the scope of application of the recommendations. However, there are many cases wherein high-strength materials out of the scope of application of the AIJ Recommendations are used, particularly in recently constructed CFST structure buildings. Moreover, in the future, the use of 800 MPa class high-tensile-strength steel and 150 MPa class high-strength concrete is expected. Therefore, the scope of application of the standards and/or recommendations should be expanded. If the use of high-strength materials becomes common, the effect of the strength of the materials on the strength of members cannot be ignored. Thus, the validity of the currently used design formulas must be re-examined. Based on this background, a calculation method for the ultimate flexural strength was proposed, considering both the strains at the extreme concrete compression fiber and the strength decreases caused by the local buckling at the compression side of the steel tube.

The experimental data obtained from the pure bending tests were compared with the calculated flexural strength to explain the basic structural performance and capacity of the CFST beam-columns. Because the experimental results of the flexure-shear loading tests include the strength increase due to the end restraint. However, the number of specimens by pure bending is limited.

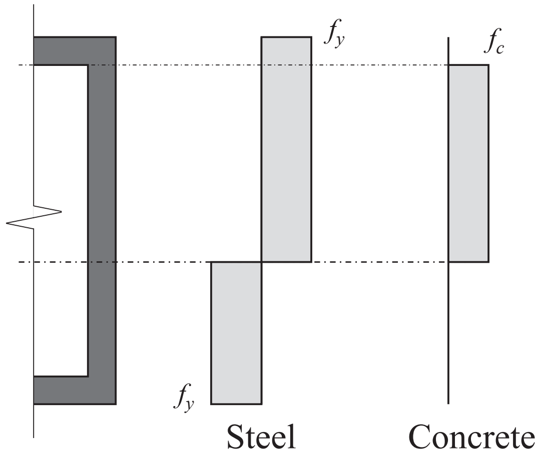

FIGURE 1. Stress distribution at the ultimate flexural strength using the AIJ CFT Recommendations

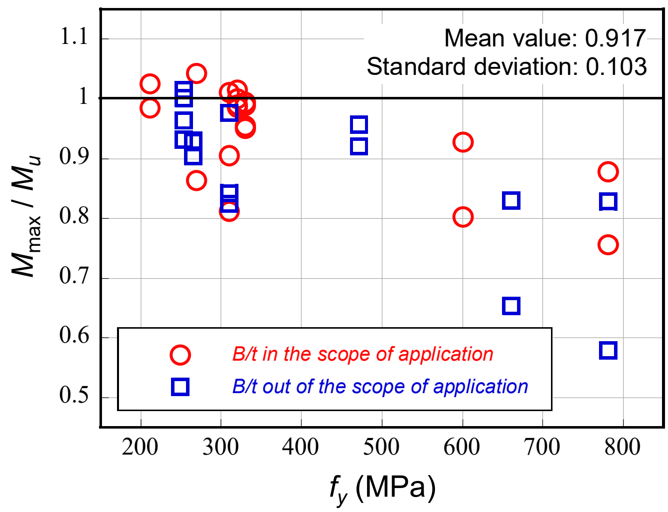

FIGURE 2. Comparison between the experimental maximum flexural strength and the ultimate flexural strength by AIJ Recommendations

2. Comparison Between the Experimental Data and AIJ Recommendation's Formula

In AIJ Recommendations Part 2 "The structural design and construction method", the ultimate flexural strength of the rectangular CFST beam-column is calculated using the generalized superposed strength (full-plastic flexural strength). It assumes the full-plastic stress distribution of the concrete and steel tube (see Figure 1). For the rectangular CFST, the strength increase owing to the confined effect is not considered in the design formulas in a visible form. However, 1.0 can be used as the strength reduction factor of the concrete (${}_c r_U$).

Figure 2 shows the comparison between the AIJ Recommendations' ultimate flexural strength and the maximum flexural strength measured by experiments on rectangular CFST beam-columns.[5,10–12] Data from 36 specimens of pure bending experiments of rectangular CFST beam-columns conducted in and out of Japan were used for comparison. The number of specimens is insufficient for the proposition of comprehensive design formulas. However, there is no problem with qualitative estimation and a certain degree of quantitative estimation. The ranges of the width-to-thickness ratio and material strength are relatively wide. Some specimen parameters exceed the scope of the application of the AIJ Recommendations. The limit width-to-thickness ratio for CFST beam-columns is 1.5 times the limit ratio for the hollow steel sections as shown in Equation 1. The calculated strength used in the comparison as the ultimate flexural strength was the full-plastic strength written in AIJ Recommendations Part 2.

$$\frac{B}{t} = 1.6\sqrt{\frac{E_s}{f_y}} \times 1.5 \tag{1}$$

The mean value of the strength ratio of the experimental maximum flexural strength and ultimate flexural strength of the AIJ Recommendations is 0.917. The calculated strengths tend to overestimate the experimental strengths: the standard derivation exceeds 0.1, and the variability of the data is large. In particular, the strength ratio is considerably small in the specimens using high-tensile-strength steel, which is outside the scope of application. Even in specimens with steel yield stresses of approximately 300 MPa, the flexural strength ratios were less than 1.0 in many cases. However, most of the specimens with strength ratios less than 1.0 were specimens using ultra-high-strength concrete. Their width-to-thickness ratios were larger than the width-to-thickness ratio limit in the AIJ Recommendations. Some specimens had flexural strength ratios of less than 1.0, although the specimen parameters were within the scope of application of the AIJ Recommendations.

In addition, for the case using the steel tube with a large width-to-thickness ratio and/or high strength concrete, which is out of the scope of application of the AIJ Recommendations, the modified calculation method for the ultimate flexural strength is specified in the AIJ Recommendations Part 1 "Structural performance of the concrete-filled steel tube members and frame".[1,5]

Figure 3 shows a comparison between the modified strengths of the AIJ Recommendations and the experimental data. The flexural strength ratios were approximately 1.0 and the variability of the data was decreased. Most specimens using high-tensile-strength steel with $f_y > 600$ MPa were overestimated. This tendency is observed in the relationship between the flexural strength ratio and yield stress of steel, where the strength ratio decreases when the yield strength of steel increases. The modified strength evaluation method is inappropriate when high-tensile-strength steels are used. This is because the effect of the strain distribution in the elastic range of the steel section around the neutral axis is not considered for the modified flexural strength of the AIJ Recommendations. The modified strength was proposed by focusing on the stresses of concrete and effect of the width-to-thickness ratio when high-strength concrete and thin-walled steel tube are used. The effect of the material strength of the steel is significant; further, the strength-related error becomes significant if high-tensile-strength steel is used.

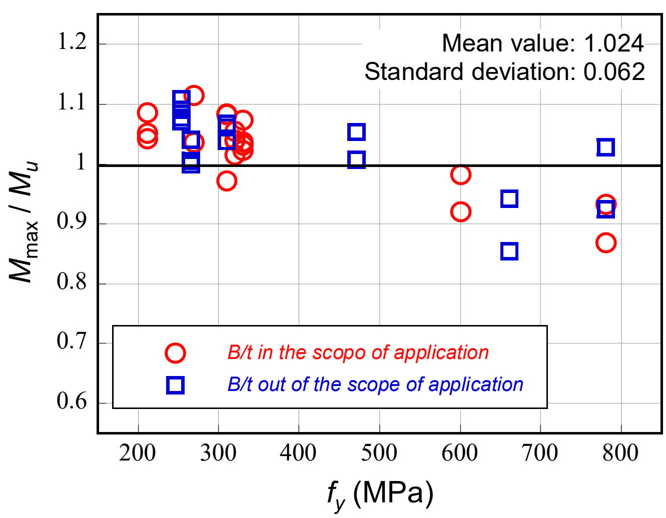

FIGURE 3. Comparison between the experimental maximum flexural strength and the modified ultimate flexural strength by AIJ Recommendations

3. Proposed Ultimate Flexural Strength for Rectangular CFST Beam-Columns

3.1 Strength considering the maximum strain at the extreme concrete compression fiber

The ultimate flexural strength was determined by regulating the strain at the extreme concrete compression fiber ${}_c\varepsilon_u$. The stress–strain relation of the steel part was assumed to be an elasto-plastic relation. For simplification purposes, the stress–strain relation of the concrete part was assumed to be an elasto-plastic model with a linear equation until the maximum stress point (${}_c\varepsilon_0$, f_c) and constant strength of f_c after the peak point. The tensile strength of concrete was neglected in the flexural and axial strength calculations.

Figure 4 shows the stress distribution of the ultimate flexural strength. Figure 5 shows a comparison between the ultimate flexural strength considering the strain at the extreme concrete fiber and the experimental maximum flexural strength. Here, the strengths were calculated using the strain at the extreme concrete fiber, ${}_c\varepsilon_u = 0.004$. The mean value of the strength ratio obtained from the experimental data was approximately 1.0. However, the standard deviation was still large. The strength ratios were approximately less than 10% of the errors. The ultimate strength considering the strain at the extreme concrete fiber showed a better evaluation than the formula from AIJ Recommendations. However, the calculated strength considerably overestimated the experimental maximum flexural strength when a high-tensile-strength steel larger than $f_y = 600$ MPa was used. The bearing stress at the compression side of the steel tube could be decreased owing to the occurrence of local buckling because the width-to-thickness ratio of these specimens is significant.

Expecting the yield stress of steel at the bending compression side of the steel tube might lead to the risk of overestimation when a high-tensile-strength steel is used and the width-to-thickness ratio of the steel tube is significant. However, the strength ratios were less than 1.0, even when the yield stresses of the steel tube were approximately 300 MPa when the AIJ Recommendations' ultimate flexural strength was used. The flexural strength ratio using the strain at the extreme concrete fiber became larger than that using the strength from AIJ Recommendations.

3.2 Strength considering the maximum strain at the extreme concrete compression fiber and the stress reduction on the compression side of the steel tube

In the previous section, the maximum flexural strength of a rectangular CFST was evaluated by regulating the strain at the extreme concrete compression fiber. This is true even if high-tensile-strength steel is used, which is outside the application of the AIJ Recommendations. However, the stress of the steel tube at the compression side may not reach the yield stress if a thin-walled steel tube is used because of the stress reduction owing to the occurrence of local buckling.

In this study, a method for evaluating the ultimate flexural strength of rectangular CFST columns was proposed. The reduction in the compressive stress of the steel tube owing to local buckling was considered in addition to the regulation of the strain at the extreme concrete compression fiber ${}_c\varepsilon_u$. Figure 6 shows the stress distribution of the steel part for the proposed ultimate flexural strength. The stress distribution for the concrete part was the same as that described in the previous section. As in the previous section, ${}_c\varepsilon_u = 0.004$ was used as the strain at the extreme concrete compression fiber. For

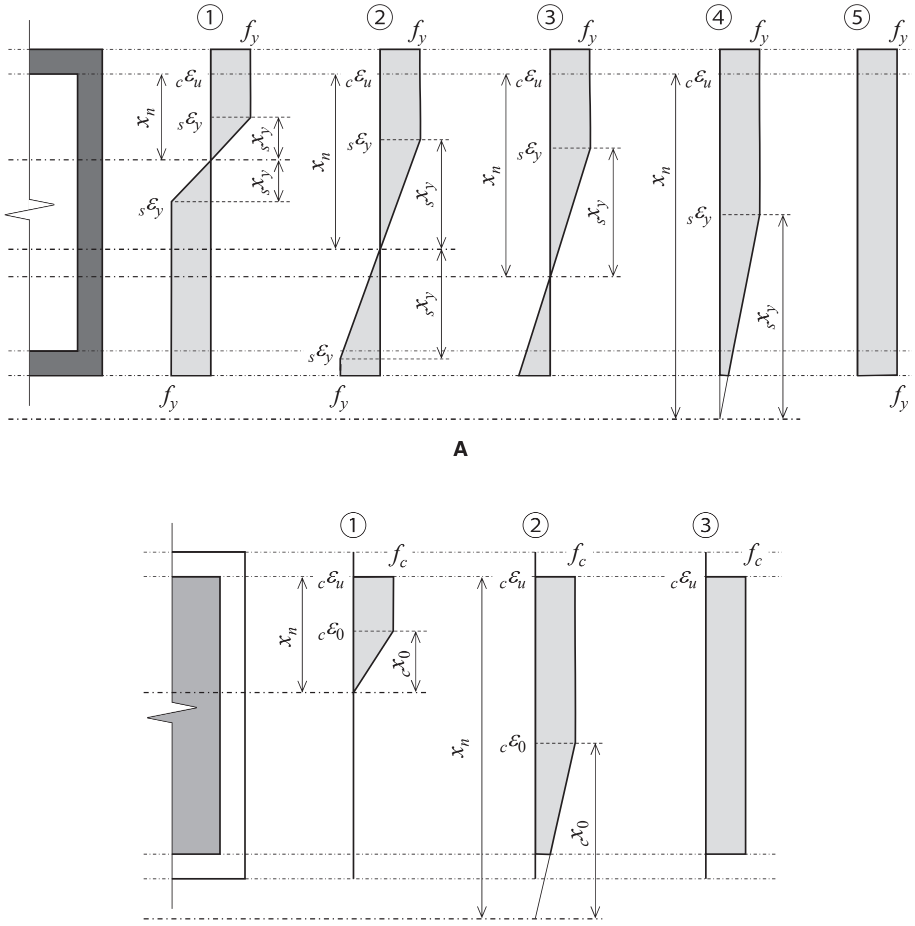

FIGURE 4. Stress distribution of ultimate strength considering the strain at the extreme concrete compression fiber: Stress distribution of (A) steel and (B) concrete sections

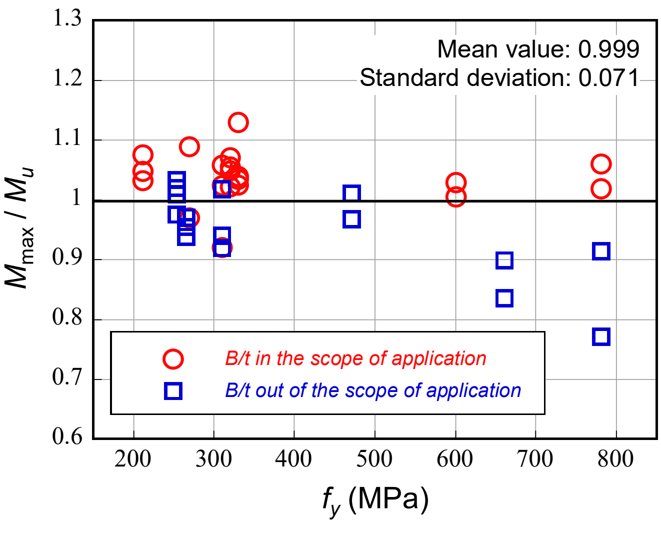

FIGURE 5. Comparison between the experimental maximum flexural strength and the calculated ultimate flexural strength considering the strain at the extreme concrete compressive fiber (${}_c\varepsilon_u$ = 0.004)

the reduction factor of steel, the stress at the compression side owing to the local buckling of the steel tube (Equation 2) was used. It was obtained using the compression test results of the rectangular hollow steel tube's compression test results.[13] The formulas proposed in this study are shown in Appendix A.

$$S = 1.51e^{-0.3\alpha}, \alpha = \frac{B}{t}\sqrt{\frac{f_y}{E_s}} \tag{2}$$

where α is the normalized width-to-thickness ratio, B is the width of the steel tube, t is the thickness of the steel tube, f_y is the yield stress of steel, and E_s is the Young's modulus of the steel.

Figure 7 shows a comparison between the experimental maximum flexural strength and the proposed ultimate flexural strength considering the strain at the extreme concrete fiber and the stress reduction at the compression side of the steel tube. Extreme concrete fiber strains of 0.004, 0.005, 0.006, and 0.008 were examined. The flexural strength calculated using ${}_c\varepsilon_u$ = 0.004 accurately predicted the experimental data and was approximately safe regardless of the yield stress of the steel tube and width-to-thickness ratio (see Figure 7A). The tolerance is small.

The strain at the extreme concrete compression fiber ${}_c\varepsilon_u$ = 0.004 used in this study was assumed to be the minimum limit for calculating the ultimate flexural strength of the rectangular CFST beam-column.

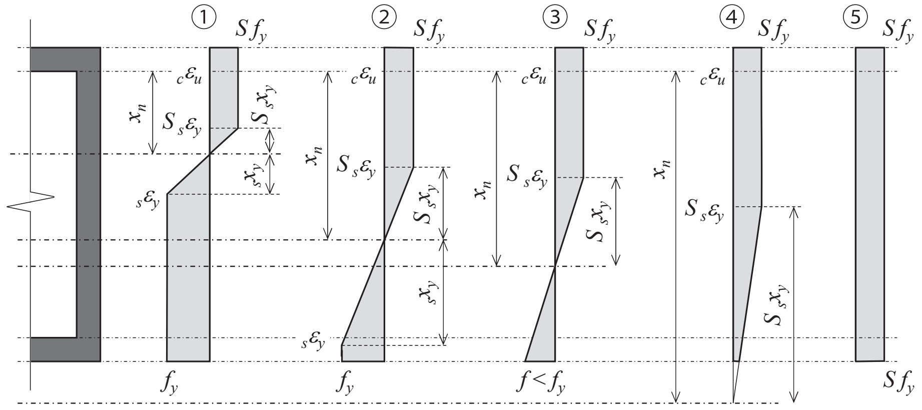

FIGURE 6. Stress distribution of proposed ultimate flexural strength (steel section)

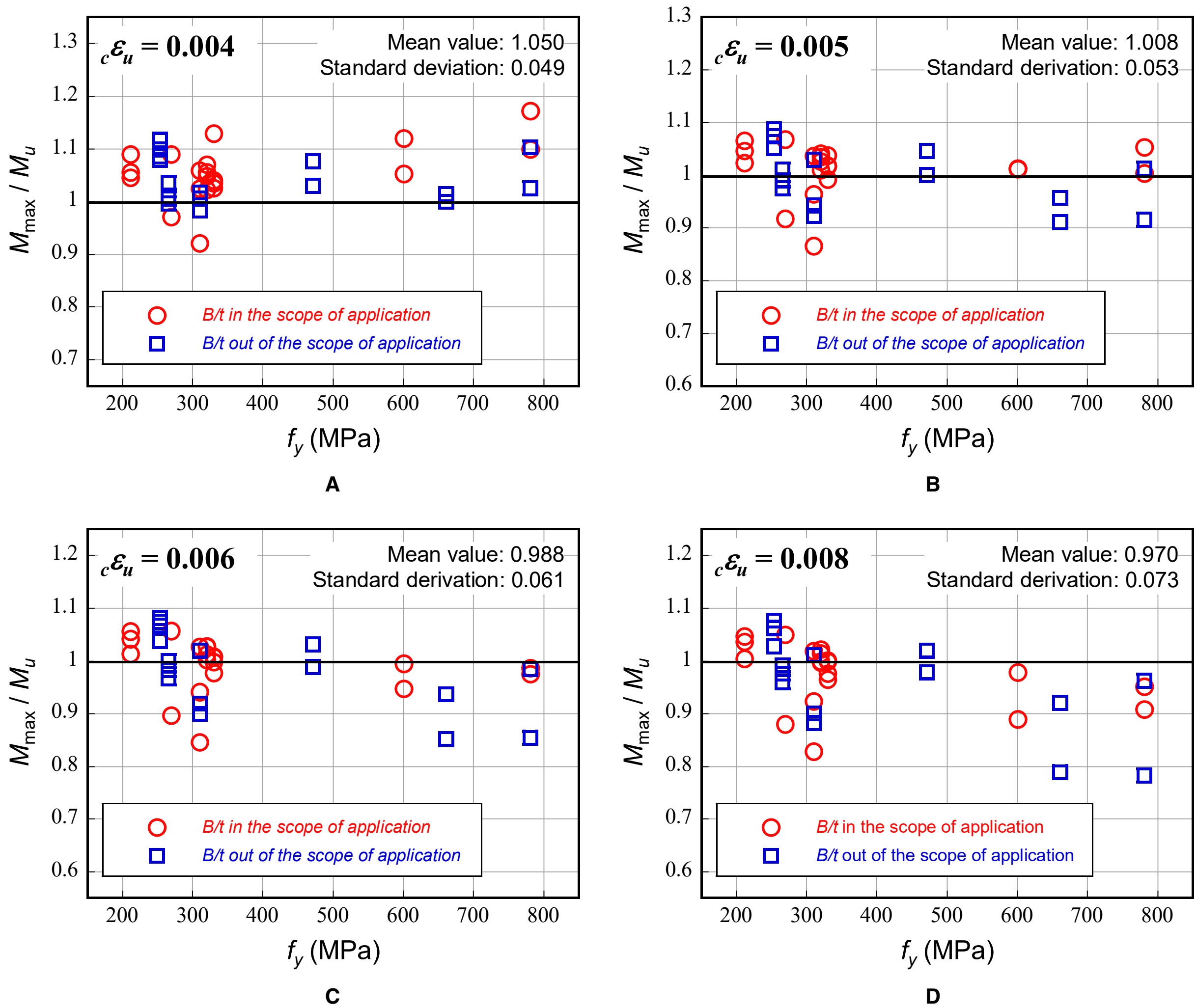

FIGURE 7. Comparison between the proposed ultimate flexural strength and the experimental data: The strain at extreme concrete fiber is (A) $_c\varepsilon_U = 0.004$, (B) $_c\varepsilon_U = 0.005$, (C) $_c\varepsilon_U = 0.006$, and (D) $_c\varepsilon_U = 0.008$

For the strain at the extreme concrete compression fiber $_c\varepsilon_u = 0.005$, the mean value of the strength ratio was 1.008. The standard deviation was 0.053. On average, the calculated ultimate strength accurately predicted the maximum strength. However, the tendency of the strength ratio decreases to the right with an increase in steel strength when a high-strength steel is used. Furthermore, the strength ratios of some of the specimens were less than 1.0.

Considering the strains at the extreme concrete compression fiber $_c\varepsilon_u = 0.006$ and 0.008, the strength ratio becomes approximately 1.0 for particular specimens. However, the flexural strength ratio for specimens using high-tensile-strength steel becomes significantly less than 1.0. The strains at the extreme concrete fibers of 0.006 and 0.008 were overestimated.

4. Concluding Remarks

In this study, an evaluation method for the accurate ultimate flexural strength of a rectangular CFST beam-column using materials with a wide range of strengths and steel tubes with large width-to-thickness ratios was proposed. The proposed ultimate flexural strength regulates the strain at the extreme concrete compression fiber and stress reduction of the steel tube at the compression side. The following conclusions were drawn from the comparison with the experimental data.

1 The proposed ultimate flexural strength can be evaluated well and safely even for a large width-to-thickness ratio using high-strength materials if the strain at the extreme concrete compression fiber is ${}_{c}\varepsilon_u = 0.004$.
2 For the strain at the extreme concrete compression fiber ${}_{c}\varepsilon_u = 0.005$, the calculated ultimate strength accurately predicts the average maximum strength.
3 For the strains at the extreme concrete compression fiber ${}_{c}\varepsilon_u = 0.006$ and 0.008, the flexural strength ratio is significantly less than 1.0, for the specimens using high-tensile-strength steel. The strains at the extreme concrete fibers, 0.006 and 0.008, were overestimated for a steel tube with a large width-to-thickness ratio and when high-strength materials are used.

Disclosures

The author has no conflict of interest to declare.

Data Availability Statement

The data that support the findings of this study are available from the author upon reasonable request.

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How to cite this article: Fujinaga T. Ultimate flexural strength of rectangular concrete-filled steel tubular beam-columns using high-strength materials. *Jpn Archit Rev*. 2023;6:e12336. https://doi.org/10.1002/2475-8876.12336

Appendix A

The formulas for the ultimate flexural strength of the rectangular CFST beam-columns proposed in this study, considering the maximum strain at the extreme concrete compression fiber and the stress reduction on the compression side of the steel tube, are as follows. The flexural strength of CFST beam-columns can be calculated as the sum of the strengths of the steel and concrete parts. The formulas for the strength of the steel part and the concrete part are listed in the following tables.

$$\begin{aligned} N_u &= {}_sN_u + {}_cN_u \\ M_u &= {}_sM_u + {}_cM_u \end{aligned} \tag{A1}$$

A. Steel tube part

(1) for $0 \le x_n < \frac{D-2t_f}{1+\varepsilon_y/{}_c\varepsilon_u}$

$$\begin{aligned} {}_sN_u &= 2t_w\left(2x_n - D + 2t_f\right)f_y - \left\{B\cdot t_f + 2t_w\left(x_n - {}_sx_y\right)\right\}(1-S)f_y - t_w\cdot {}_sx_y(1-S)^2 f_y \\ {}_sM_u &= B\cdot t_f\frac{D-t_f}{2}(1+S)f_y + t_w\left(D-2t_f-x_n+S\cdot{}_sx_y\right)\left(x_n - S\cdot {}_sx_y\right)S\cdot f_y \\ &\quad + t_w\left(D-2t_f-x_n-{}_sx_y\right)\left(x_n+{}_sx_y\right)f_y \\ &\quad + t_w\cdot S\cdot {}_sx_y\left(\frac{D}{2}-t_f-x_n+\frac{2}{3}S\cdot{}_sx_y\right)S\cdot f_y + t_w\cdot{}_sx_y\left(-\frac{D}{2}+t_f+x_n+\frac{2}{3}{}_sx_y\right)f_y \end{aligned}$$

(2) for $\frac{D-2t_f}{1+\varepsilon_y/{}_c\varepsilon_u} \le x_n < \frac{D-t_f}{1+\varepsilon_y/{}_c\varepsilon_u}$

$$\begin{aligned} {}_sN_u &= B\left\{(1+S)\cdot t_f - D + x_n + {}_sx_y\right\}f_y + t_w\left(2x_n - S\cdot{}_sx_y\right)S\cdot f_y - t_w\cdot\frac{\left(D-2t_f-x_n\right)^2}{{}_sx_y}f_y \\ &\quad - B\cdot\frac{{}_sx_y^2-\left(D-2t_f-x_n\right)^2}{2{}_sx_y}f_y \\ {}_sM_u &= B\cdot t_f\frac{D-t_f}{2}S\cdot f_y + t_w\left(D-2t_f-x_n+S\cdot{}_sx_y\right)\left(x_n-S\cdot{}_sx_y\right)S\cdot f_y \\ &\quad + t_w\cdot{}_sx_y\left(\frac{D}{2}-t_f-x_n+\frac{2}{3}S\cdot{}_sx_y\right)S^2\cdot f_y + t_w\cdot\frac{\left(D-2t_f-x_n\right)^2}{{}_sx_y}\cdot\frac{D-2t_f+2x_n}{6}\cdot f_y \\ &\quad + \frac{B}{2}\cdot\frac{{}_sx_y^2-\left(D-2t_f-x_n\right)^2}{{}_sx_y}\left\{{}_sx_y-\frac{D}{2}+t_f+x_n-\frac{{}_sx_y+2D-4t_f-2x_n}{3\left({}_sx_y+D-2t_f-x_n\right)}\left({}_sx_y-D+2t_f+x_n\right)\right\}f_y \\ &\quad + \frac{B}{2}\left(D-t_f-x_n-{}_sx_y\right)\left(t_f+x_n+{}_sx_y\right)f_y \end{aligned}$$

(3) for $\frac{D-t_f}{1+\varepsilon_y/{}_c\varepsilon_u} \le x_n < \frac{D-2t_f}{1-S\cdot\varepsilon_y/{}_c\varepsilon_u}$

$$\begin{aligned} {}_sN_u &= B\cdot t_f\cdot S\cdot f_y + 2t_w\left(x_n-S\cdot{}_sx_y\right)S\cdot f_y + t_w\cdot{}_sx_y\cdot S^2\cdot f_y - t_w\cdot\frac{\left(D-2t_f-x_n\right)^2}{{}_sx_y}f_y \\ &\quad - \frac{B}{2}\cdot\frac{2D-3t_f-2x_n}{{}_sx_y}\cdot t_f\cdot f_y \\ {}_sM_u &= B\cdot t_f\frac{D-t_f}{2}S\cdot f_y + t_w\cdot\left(x_n-S\cdot{}_sx_y\right)\left(D-2t_f-x_n+S\cdot{}_sx_y\right)S\cdot f_y \\ &\quad + t_w\cdot{}_sx_y\left(\frac{D}{2}-t_f-x_n+\frac{2}{3}S\cdot{}_sx_y\right)S^2\cdot f_y + t_w\cdot\frac{\left(D-2t_f-x_n\right)^2}{{}_sx_y}\cdot\frac{D-2t_f+2x_n}{6}\cdot f_y \\ &\quad + \frac{B}{2}\cdot\frac{2D-3t_f-2x_n}{{}_sx_y}\cdot t_f\left(\frac{D}{2}-\frac{3D-5t_f-3x_n}{3\left(2D-3t_f-2x_n\right)}\cdot t_f\right)f_y \end{aligned}$$

(4) for $\frac{D-2t_f}{1-S\cdot\varepsilon_y/{}_c\varepsilon_u} \le x_n < \frac{D-t_f}{1-S\cdot\varepsilon_y/{}_c\varepsilon_u}$

$$\begin{aligned} {}_sN_u &= {}_sA\times S\cdot f_y - \frac{B\left(S\cdot{}_sx_y-x_n+D-t_f\right)^2}{2{}_sx_y}f_y \\ {}_sM_u &= \frac{B\left(S\cdot{}_sx_y-x_n+D-t_f\right)^2}{2{}_sx_y}\left(\frac{D}{2}-\frac{S\cdot{}_sx_y-x_n+D-t_f}{3}\right)f_y \end{aligned}$$

(5) for $x_n \geq \frac{D-t_f}{1-S\cdot\varepsilon_y/{}_c\varepsilon_u}$ ${}_sN_u = {}_sA \times S \cdot f_y$, ${}_sM_u = 0$

B. Filled concrete part

(1) for $0 \leq x_n < {}_cD$

$${}_cN_u = {}_cB\left(x_n - \frac{{}_cx_0}{2}\right)f_c$$

$${}_cM_u = {}_cB\left(x_n - {}_cx_0\right)\left(\frac{{}_cD}{2} - \frac{x_n - {}_cx_0}{2}\right)f_c + \frac{{}_cB}{2}{}_cx_0\left(\frac{{}_cD}{2} - x_n + \frac{2}{3}{}_cx_0\right)f_c$$

(2) for ${}_cD \leq x_n < \frac{{}_cD}{1-{}_c\varepsilon_0/{}_c\varepsilon_u}$

$${}_cN_u = {}_cB\left(x_n - {}_cx_0\right)f_c + \frac{{}_cB}{2}\cdot\frac{{}_cx_0^2 - \left(x_n - {}_cD\right)^2}{{}_cx_0}f_c$$

$${}_cM_u = {}_cB\left(x_n - {}_cx_0\right)\left(\frac{{}_cD}{2} - \frac{x_n - {}_cx_0}{2}\right)f_c$$

$$-\frac{{}_cB}{2}\cdot\frac{{}_cx_0^2 - \left(x_n - {}_cD\right)^2}{{}_cx_0}\left(x_n - {}_cx_0 - \frac{{}_cD}{2} + \frac{{}_cx_0 + 2x_n - 2{}_cD}{3\left({}_cx_0 + x_n - {}_cD\right)}\left({}_cx_0 - x_n + {}_cD\right)\right)f_c$$

(3) for $x_n \geq \frac{{}_cD}{1-{}_c\varepsilon_0/{}_c\varepsilon_u}$ ${}_cN_u = {}_cA \cdot f_c$, ${}_cM_u = 0$

where B: Width of the steel tube, D: Depth of the steel tube, t_f: Thickness of the flange, t_w: Thickness of the web, ${}_cB$: Width of the concrete section (= $B - 2t_w$), ${}_cD$: Depth of the concrete section (= $D - 2t_f$), ${}_sA$: Cross-sectional area of the steel section, ${}_cA$: Cross-sectional area of the concrete, f_y: Yield strength of the steel tube, f_c: Compressive strength of the concrete, ε_y: Yield stress of the steel, ${}_c\varepsilon_0$: Strain at the compressive strength of the concrete (= $0.93\ f_c^{1/4}\ 10^{-3}$), ${}_c\varepsilon_u$: Strain at the extreme concrete compression fiber, S: Stress reduction factor of the steel, x_n: Distance from the neutral axis from the extreme concrete compression fiber, ${}_sx_y$: Distance from the neutral axis from the point steel yield, ${}_cx_0$: Distance from the neutral axis from the point where the strain reached the compressive strength of the concrete.

$${}_sx_y = \frac{\varepsilon_y}{{}_c\varepsilon_u}x_n \tag{A2}$$

$${}_cx_0 = \frac{{}_c\varepsilon_0}{{}_c\varepsilon_u}x_n \tag{A3}$$