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Superlative Modifiers as Concessive Conditionals^{*}

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Abstract. This paper offers a new compositional semantics of the superlative modifier *sukunakutomo* ‘at least’ in Japanese. The notable feature of this expression is that it does not utilize a superlative morpheme. How does *sukunakutomo* derive the superlative meaning, then? The main goal of this paper is to provide the answer to this question, while capturing the diverse behavior of superlative modifiers. Focusing on the decompositionality of *sukunakutomo*, we suggest that the meaning of this expression is essentially a concessive conditional with a focus to a contextually supplied degree. The analysis supports the view that *at least*-expressions have only one denotation, and moreover contributes to providing a strategy for deriving the superlative meaning.

Keywords: superlative modifiers · ignorance inference · at least · concessive conditionals · Japanese

1 Introduction

Sentences with superlative expressions such as *at least* are known to give rise to *ignorance inferences* of the speaker. In (1), for example, the speaker asserts that it is the case that John came to the party, but is uncertain as to who else came besides John.

- (1) A: Who came to the party?
B: At least John came.
 \rightsquigarrow the speaker is uncertain as to who else came besides John.

To explain the behavior of superlative modifiers, previous studies have proposed a variety of approaches. The modal analysis proposed by Geurts & Nouwen [13] utilizes a modal operator which is encoded in the lexical semantics of *at least*. Buring [5] alternatively proposes the disjunction analysis that attributes

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ignorance inferences of *at least* to a pragmatic reasoning: the inference could arise via Gricean reasoning from the disjunctive semantics of the expression (cf. Cummins & Katsos [10], Biezma [4]). Building on the idea of Buring, Coppock & Brochhagen [9] also offer the disjunctive semantics to the semantics of *at least* in terms of inquisitive semantics (Ciardelli, Groenendijk & Roelofsen [7]). Nouwen [23] classifies modified numerals into two separate classes (class A and B), and superlatives are in class B that denote a lower-bound degree, e.g., *from*, *minimally*.

Despite the existence of such a wide variety of approaches, we will not adopt any of the directions for our analysis. Alternatively, we offer a compositional approach of superlative modifiers by focusing on the Japanese *sukunakutomo*, which is sometimes considered to be the counterpart of *at least* (cf. Hirayama & Brasoveanu [16]).

- (2) Sukunakutomo Taro-ga kita yo.
 at.least T-NOM came DP
 ‘At least Taro came to the party.’

The notable feature of this expression is that, unlike *at least* (which is considered to be the spell-out of ‘*at* [*little*+*est*]’), it does not utilize a superlative morpheme or inflection.³ What does the superlative meaning of *sukunakutomo* stem from, then?

The goal of this paper is to provide the answer to this question, while capturing the diverse empirical facts of the superlative modifiers. Our analysis based on the compositional nature of *sukunakutomo* indicates that the meaning of *sukunakutomo* is essentially a concessive conditional in the sense that it contains *even* as its conditional antecedent. We argue that this proposal provides the most empirically successful account of *sukunakutomo* to date.

The rest of this paper is structured as follows. Section 2 provides an overview of existing studies on superlative modifiers and presents data on their empirical issues that arise when they are applied to *sukunakutomo*. Section 3 introduces the ingredients to be used in our analysis and then illustrates how the meaning of *sukunakutomo* is derived in a compositional way. Section 4 attempts to capture the core data at issue, and Section 5 provides further predictions that support our claim. Section 6 is the conclusion with theoretical and empirical implications for the study of superlative modifiers.

2 Previous Approaches

In this section, we provide brief backgrounds of the theories of superlative modifiers, which we believe are necessary to support our proposal. Here, for the

³ To the best of our knowledge, Chen [6] and Coppock [8] are the only works that attempt to offer a detailed decompositional analysis of English *at least*. In particular, Chen proposes that *at least* can be structurally decomposed into three morphological pieces: a quantity adjective *much*, a superlative *-est* and an existential operator (See also Coppock’s [8] treatment). Refer to Chen ([6], 238–243) to see how the composition goes on.

purpose of this study, we mainly focus on two approaches: the modal approach and the disjunction approach. We evaluate these two approaches in view of what they predict with regard to the data of *sukunakutomo*, and show that neither theory alone is sufficient to explain these data in a unified way.

2.1 Modal Approach

Geurts & Nouwen [13] give two lexical entries for *at least*: one modifies a proposition, and one modifies a modifier.

- (3) a. If α is of type t (a propositional argument),
 $\llbracket \text{at least } \alpha \rrbracket = \Box \alpha \wedge \exists \beta [\beta \triangleright \alpha \wedge \Diamond \beta]$,
 where \triangleright is a precedence relation.
- b. If α is of type $\langle a, t \rangle$ where a is any type (a predicative argument),
 $\llbracket \text{at least } \alpha \rrbracket = \lambda X [\Box \alpha(X) \wedge \exists \beta [\beta \triangleright \alpha \wedge \Diamond \beta(X)]]$ ([13]: 543)
- (4) *Taro had at least three beers.*
 $\Box \exists x [\#(x) = 3 \wedge \text{beer}(x) \wedge \text{had}(T, x)] \wedge \Diamond \exists x [\#(x) > 3 \wedge \text{beer}(x) \wedge \text{had}(T, x)]$

Crucially, they treat the ignorance effect of *at least* as part of its lexical semantics. The presence of an epistemic possibility modal explains the fact that sentences with *at least* (and *at most*) convey ignorance on the part of the speaker. On their theory, comparative modifiers, unlike superlative modifiers, do not signal epistemic possibility, which explains the contrast between superlative and comparative modifiers with respect to the ignorance inference.

Let us first suppose that *sukunakutomo* has the same semantics as *at least* proposed in Geurts & Nouwen, $\llbracket \text{at least} \rrbracket = \llbracket \text{sukunakutomo} \rrbracket$. As Geurts & Nouwen themselves admit, this line of analysis fails to account for conditionalized and negated examples, as shown in (5) and (6).

- (5) Sukunakutomo san-hai biiru-o nonde-tara, Hanako-wa
 at.least 3-CL beer-ACC drink-then H-TOP
 yottei-ta.
 get.drunk-PAST
 ‘If Hanako had had at least three beers, she would have been drunk.’
- (6) Hanako-wa biiru-o sukunakutomo san-hai(-wa) nom-anakat-ta.
 H-TOP beer-ACC at.least 3-CL(-TOP) drink-NEG-PAST
 ‘Hanako didn’t have at least three beers.’

The reading that Geurts & Nouwen’s theory predicts for (5) is “if it *must* be the case that Hanako had three beers, and it *may* be that she had more than three, then she would have been drunk,” which is not what the sentence means. Moreover, what the sentence in (6) implies is “it’s not the case that Hanako had three beers, and the amount was *at most* two beers.” However, the reading that their theory predicts for (6) is “it’s not the case that Hanako must have had three beers, and (at the same time) it’s not the case that she may have had

more than three beers.” Again, this goes against our intuition, and there is no way this line of analysis will capture the correct reading ([13], §10).

It is worth noting that in Geurts & Nouwen, the necessity operator is posited mainly in order to feed a process of modal concord. However, the need of such a process has been questioned in Büring [13] and Nouwen [23]. Geurts & Nouwen also motivate the necessity operator with a symmetry argument referring to *at least*’s negative partner *at most*, but Japanese *sukunakutomo* lacks such an explicit partner.⁴

Another study that makes use of a modal as the interpretation of superlative modifiers is the work of Nouwen [23], who proposes that *at least* stands for the lower bound modifier. He argues that an epistemic possibility modal for the speaker is introduced into the interpretation of sentences with superlative modifiers through a “reinterpretation” process. As Coppock & Brochhagen [9] point out, however, it is not clear how this reinterpretation process works or under what circumstances it applies. Moreover, since his analysis only deals with the numerical cases, it does not account for the non-numerical case like (2).

2.2 Disjunction Approach

Büring [13], building on Krifka [20], proposes that *at least* is interpreted as a disjunction operator over scalar alternatives. According to his proposal in (7), *at least* amounts to a disjunction between the prejacent and its higher-ranked alternatives.

- (7) a. $\llbracket \text{at least } \alpha \rrbracket = [\llbracket \alpha \rrbracket - \bigcup \text{ABOVE}(\alpha)] \vee \bigcup \text{ABOVE}(\alpha)$
 b. $\text{ABOVE}(\alpha) = \bigcup \{O' \mid \langle \alpha, O' \rangle \in \llbracket \alpha \rrbracket^A\}$,
 where $\llbracket \alpha \rrbracket^A$ is the alternative semantic value of α .
- (8) *Taro had at least three beers.*
 $[\llbracket \text{had}(T, 3 \text{ beers}) \rrbracket - \bigcup \text{ABOVE}(\llbracket \text{had}(T, 3 \text{ beers}) \rrbracket)] \vee$
 $\bigcup \text{ABOVE}(\llbracket \text{had}(T, 3 \text{ beers}) \rrbracket)$

Informally speaking, the example in (8) means “Taro had exactly three beers OR Taro had more than three beers.”

⁴ At first glance, *ookutomo* ‘at most’ in Japanese appears to be the counterpart to *sukunakutomo*, but it differs from *sukunakutomo* in that while *sukunakutomo* can be used in non-numerical contexts, *ookutomo* can be used only in the numerical contexts:

- (i) Sukunakutomo kare-wa isha-da. (Dakara kanemoti-da.)
 at.least he-TOP doctor-COP so rich-COP
 ‘He is at least a doctor. (So he is rich.)’
- (ii) *Ookutomo kare-wa tada-no isha-da. (Meii-de-wa nai.)
 at.most he-TOP only-GEN doctor-COP a.highly.skilled.doctor-COP-TOP NEG
 ‘He is only a doctor at best. (He’s not a highly skilled doctor.)’

The contrast may be derived from the difference in the lexical semantics of *sukunai* ‘little’ and *ooi* ‘many’ in Japanese, which will be a subject for future study.

Büring's proposal differs crucially from the one of Geurts & Nouwen in that his analysis derives all modal aspects of the meaning of *at least* from pragmatic implicatures. According to Büring, the particular conversational implicature in (9) arises when uttering sentences with *at least*. (9) expresses the Gricean intuition that one doesn't use a disjunction if one is certain of the truth of any individual conjunct.

- (9) If a speaker utters p or q , it is implied that (i) in all of the speaker's doxastic alternatives $p \vee q$ and (ii-a) not in all p , and (ii-b) not in all q .
Büring ([5]: 114)

Take the sentence in (8) for example. According to the schema in (9), the sentence implies that (i) the speaker is certain that Taro had three beers, and (ii-a) she is not certain that Taro had exactly three beers, and (ii-b) she is not certain that Taro had more than three beers.

A problem of Büring's approach is that sentences containing *at least* expressions are obviously not disjunctive on the level of surface form.⁵ For instance, the disjunction theory is problematic in a context where the speaker has perfect knowledge about the truth of alternatives.

- (10) (The speaker does not want the addressee to know exactly how many points the speaker scored, because she knows that the addressee is likely to have scored more than her.)
A: What was your score on yesterday's exam?
B: Sukunakutomo gookakuten-no rokuju-ten-wa tot-ta yo.
at.least passing.score-GEN 60-CL-TOP get-PAST DP
'I got at least a passing score of 60.'
B' #Gookakuten-no rokuju-ten-wa tot-ta ka, aruiwa motto
passing.score-GEN 60-CL get-PAST or either more
tot-ta ka da yo.
get-PAST or COP DP
'I scored a passing score of 60 or higher than that.'

If a sentence with *sukunakutomo* is interpreted as a disjunction in a surface level, the utterance by B' in (10) should be accepted, contrary to the fact that they are not. Then, what does it mean to say that *at least* amounts to a disjunction?

Coppock & Brochhagen [9] point out that it is not at the level of *denotation* that the sentence with *at least* is disjunctive for some conceptual reasons. While keeping the idea of Büring that disjunction is a core of the meaning of sentences with *at least* and that the ignorance effect is a pragmatic inference, Coppock & Brochhagen propose that a solution for the problems is to treat *at least* as a *proposition* including the prejacent and the higher-ranked possibilities in terms of inquisitive semantics (Ciardelli, Groenendijk & Roelofsen [7]). That

⁵ Coppock & Brochhagen [9], too, are aware of this problem ([9]:18), but do not provide clear counterexamples.

is, they argue that *at least* sentences have something in common with disjunctions, without resorting to the claim that there is any level of representation at which they *are* disjunctions. Technically, $at\ least(p)$ denotes the set containing all possibilities p' such that p' is at least as strong as p according to the pragmatic strength ranking over answers to the QUD (Question under Discussion; Roberts [24]).⁶ The denotation of *At least Ann snores* based on their analysis is represented as follows. (The detailed framework/notations and calculation process are discarded here.)

- (11) $\llbracket At\ least\ [Ann]_F\ snores \rrbracket^s = \{a, ab\}$,
 where s is a state, a is ‘Ann snores’, b is ‘Bill snores’, and ab is ‘Ann and Bill snore’.

In (11), since the focus-marking on *Ann* ensures that the QUD concerns who snores, we can assume that the possible answers are a , b , and $a\&b$. Here, the *at least* sentence entails ‘Ann snores’, because both a and $a\&b$ share the same informational content ‘Ann snores’. At the same time, the sentence is inquisitive in that it has a possibility of $a\&b$, which explains why the sentence gives rise to the ignorance implicature that it is possible that b .

The problem with their analysis is that they do not make clear how to distinguish between *at least* sentences and sentences containing disjunction expressions (e.g., *or*, *either*). More specifically speaking, the framework of inquisitive semantics translates the sentence with disjunctions “Ann snores OR both Ann and Bill snore” as $\{a, ab\}$, which is exactly the same as the one in (11), which ends up predicting that “At least Ann snores” and “Ann snores or both Ann and Bill snore” have the same meaning. Thus, their analysis requires a further explanation of how sentences with disjunctions can be analyzed in terms of inquisitive semantics, and how that analysis can capture the difference in behavior between *at least* sentences and sentences containing disjunctions.

2.3 Summary

From the discussion above, we conclude that the meaning of *sukunakutomo* should not be represented in terms of neither modals nor disjunctions. Let us summarize the fact to be accounted for in this paper as follows. (i) Ignorance inference: *sukunakutomo* gives rise to the ignorance inference, but the meaning should be represented without making use of modals. (ii) Embedding under conditionals and negations: again, the modalized interpretation of *sukunakutomo* goes against our intuition when embedded under conditionals and negations. (iii) Distribution: *sukunakuotmo* can modify a range of expressions, not just numerals, and are acceptable in contexts in which a speaker knows the truth of all relevant alternatives. Our proposal to be put forth in the next section is empirically successful in capturing these facts.⁷

⁶ We will not cover their detailed analysis here for reasons of space. Refer to Coppock & Brochhagen ([9], §3) for the relevant discussion.

⁷ Büring [13] and Coppock & Brochhagen [9] argue that (what they call) the speaker insecurity readings and the authoritative readings are also the data which should

3 Proposal

In this section, we propose a new analysis from a perspective that has not received much attention in previous studies on superlative modifiers. Instead of treating the Japanese superlative modifier *sukunakutomo* as a fully lexicalized expression, we highlight a compositionality of this expression and offer a compositional semantics which derives its meaning. Specifically, the denotation of *sukunakutomo* is morphologically broken down into *sukunai* ‘little’, a conditional morpheme *to*, and *mo* ‘even’: $\llbracket \text{sukunakutomo} \rrbracket = \llbracket \text{sukunai}_{\text{little}} \rrbracket + \llbracket \text{to}_{\text{conditional}} \rrbracket + \llbracket \text{mo}_{\text{even}} \rrbracket$. In this line of analysis, the core meaning of *sukunakutomo* is no longer the modal or the disjunctive semantics, but rather is a *concessive conditional* that composes of a conditional morpheme and *even*. We show in this section that the notion of concessive presupposition is the key of deriving the superlative meaning without superlative morphemes.

3.1 Ingredients

This section introduces the semantics of the individual pieces that compose *sukunakutomo* for the analysis.

First, *sukunai* is simply interpreted as *few* or *little* as in (12). For concreteness, we represent the interpretation of *sukunai* in the system of Kennedy [18], according to which the exact interpretation (here, *exactly* a ‘small’ amount) involves a degree quantifier incorporating the MAX-operator.

$$(12) \quad \llbracket \text{sukunai} \rrbracket^{c,w} = \lambda D_{\langle d,t \rangle}. \text{MAX}_d(D) = \mathbf{d}_\Delta,$$

where D is a set of degrees and \mathbf{d}_Δ is a small value relative to a context c .

$$(13) \quad \begin{aligned} \text{a. } & \textit{Taro-ga non-da sake-no ryoo-wa sukunai.} \\ & \text{‘The amount of sake that Taro drunk is “small”.’} \\ \text{b. } & \llbracket (13a) \rrbracket^{c,w} = \llbracket \text{sukunai} \rrbracket^{c,w}(\llbracket \text{the amount of beer that Taro drunk} \rrbracket^{c,w}) \\ & = [\lambda D. \text{MAX}_d(D) = \mathbf{d}_\Delta](\lambda d. \exists x. [\text{beer}(x) \wedge \text{drunk}_w(T, x) \wedge \mu(x) = d]) \\ & = \text{MAX}_d(\lambda d. \exists x. [\text{beer}(x) \wedge \text{drunk}_w(T, x) \wedge \mu(x) = d]) = \mathbf{d}_\Delta \end{aligned}$$

be explain. As Buring points out, the sentence with *at least* under deontic necessity modal is ambiguous:

- (i) The paper has to be at least 10 pages long.

The sentence has both the authoritative reading, on which it informs the interlocutor what the acceptable page lengths are, speaking as the authority on the subject, and the speaker insecurity reading, on which the speaker does not know what the required length of the paper is, but believes it to be over 10 pages. Looking at the data in Japanese, however, it is doubtful whether these readings are really a matter of the interaction between the superlative modifier and the modal. (We will not present a relevant example for the sake of space, but sentences with necessity modals seem to give both readings without *sukunakutomo*.) For this reason, we will not treat the data regarding to the two readings in this paper.

According to the semantics that we have in (12), the sentence in (13) conveys the meaning that the (maximal) amount of beer that Taro drunk is equal to the amount that we judge to be “small” in that context.

We assume a *to*-conditional as a prototypical conditional construction (Akatsuka [1]). Following the Kratzerean analysis of conditionals (Kratzer [19]), *to* introduces a covert necessity operator to derive the conditional meaning. The definition in (14) amounts to saying that in all the worlds (according to the conversational background f_c) in which p is true, q is true.

- (14) a. $\text{to}_{\text{conditional}}(p)(q) \rightsquigarrow \text{NEC}_w[p][q]$
 b. $\llbracket \text{NEC}_{to} \rrbracket^{c,w} = \lambda p. \lambda q. \forall w' \in \bigcap f_c^*(w) : q(w'),$
 where $f_c^*(w) = f_c(w) \cup \{\llbracket p \rrbracket\}$
- (15) a. *Sake-o nomu to, Taro-wa yopparau.*
 ‘If Taro drinks, he gets drunk.’
 b. $\llbracket (15a) \rrbracket^{c,w}$
 $= [\lambda p. \lambda q. \forall w' \in \bigcap f_c^*(w) : q(w')] (\llbracket \text{Taro drinks} \rrbracket) (\llbracket \text{Taro gets drunk} \rrbracket),$
 where $f_c^*(w) = f_c(w) \cup \{\llbracket p \rrbracket\}$
 $= \forall w' \in \bigcap f_c^*(w) : \llbracket \text{Taro gets drunk} \rrbracket(w'),$
 where $f_c^*(w) = f_c(w) \cup \{\llbracket \text{Taro drinks} \rrbracket\}$

What the example in (15a) says is that in all the best worlds in which ‘Taro drinks’ is true, ‘he gets drunk’ is true. Roughly speaking, when it is the case that Taro drinks, it must be the case that he gets drunk.

Following Nakanishi [21], the focus particle *mo* is assumed to be *even*, which ranks the alternatives by correlating them with a graded property which is salient in the context (Gianakidou [14]).⁸ *Mo* as *even*, which is defined as (16), is used to claim that the associated graded property w.r.t. the context holds to a degree that is lower than those of the alternatives.

- (16) $\llbracket \text{mo}_{\text{even}} \rrbracket^{c,w} = \lambda p. p(w) \wedge \partial [\forall q \in \text{Alt}_p : q \neq p \rightarrow p \prec_c q],$
 where ∂ is a presupposition operator (Beaver [3]) and \prec_c stands for ‘less than’ relation with respect to the contextually given scale.
- (17) a. $[\text{Roku-nin}]_F\text{-mo kita.}$ ‘Even six people came.’
 b. $\llbracket (17a) \rrbracket^{c,w}$
 $= [\lambda p. p(w) \wedge \partial [\forall q \in \text{Alt}_p : q \neq p \rightarrow p \prec_c q]] (\llbracket \text{six people came} \rrbracket^{c,w})$
 $= [\text{came}_w(\text{six people})](w) \wedge$
 $\partial [\forall q \in \text{Alt}_p : q \neq [\text{came}_w(\text{six people})] \rightarrow$
 $\quad [\text{came}_w(\text{six people})] \prec_c q],$
 where $\prec_c = \prec_{\text{likelihood}}.$
 c. $\text{Alt}_p = \{ \text{came}_w(\text{six people}), \text{came}_w(\text{five people}),$
 $\quad \text{came}_w(\text{four people}), \text{came}_w(\text{three people}), \dots \}$

⁸ Note that *mo* also corresponds to additive particles like *also* or *too*. This study exclusively assumes that *sukunakutomo* is a case where *mo* plays a role as *even*.

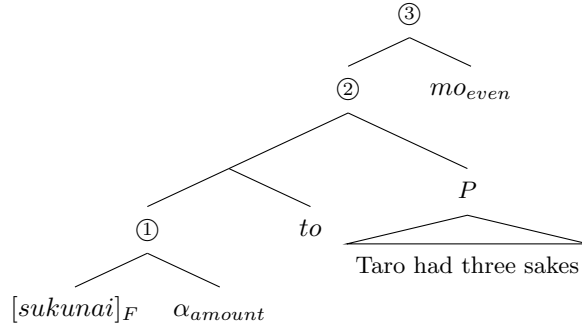
In (17), the relation \prec_c is resolved to the unlikelihood relation in terms of the number of people coming. (17) is true iff (i) it is the case that six people came, and (ii) it is presupposed that six people's coming is less likely than { five people's, four people's, three people's, ... } coming.

3.2 Deriving *At Least* without Superlative Morpheme

Given the semantic ingredients introduced in the last section, we are now in the position to derive the superlative meaning of *sukunakutomo*. We assume the simplified LF structure for the sentence with *sukunakutomo* in (18).

Before explaining the interpretation of this structure, it is necessary to add the following assumptions. First, *mo* is assumed to be a sentential operator that takes an entire sentence as its scope. This is motivated by the general fact that while a focus particle in Japanese (e.g., *mo*, *dake* 'only', *sae* 'even') appears as a postposition attached to a focused NP, the focus site can be wider than that NP (Aoyagi [2]).⁹

- (18) Simplified LF of “*Taro-wa sukunakutomo san-hai sake-o non-da.*” ‘Taro had at least three sakes.’



Second, the predicative *sukunai* in *sukunakutomo* takes a contextually determined (unpronounced) scalar anaphor α_{amount} , in (18), α_{amount} = ‘the amount of sake that Taro had’. This assumption may seem strange at first glance, but it is reasonable in light of the analysis of relevant previous studies. For instance, Kayne [17] proposes that the English minimizer *little* is an expression that modifies an unpronounced AMOUNT. In this account, “John has a little money” is interpreted as “John has a little AMOUNT of money”. Sawada [25] adopts this view and proposes that the Japanese *sukoshi*, the adverbial counterpart of *sukunai*, modifies an invisible AMOUNT predicate.

⁹ There would be at least two possible ways to explain this discrepancy between the interpretation in LF and the surface compositionality. One is to allow a LF-movement of focus particles (e.g., Futagi [12]). Another possibility is to assume that certain focus particles are associated with an alternative generating operator independently, and to assume that this operator takes a sentential scope (e.g., Tomioka [26]). Since what is important in this paper is that *mo* makes an alternatives with respect to the entire sentence of *sukunakutomo*, the reader may adopt either option.

The derivations represented in ①-③ in (18) are as follows. The intuition that we have for (18) is as follows. Suppose that we are considering how much sake Taro had and taking into account various cases where the amount of sake that he had is “the least,” “small,” “neither small nor large,” “large,” “the largest,” and so on. What (18) expresses is that in the case that the amount of sake that Taro had is ‘the least’, he had three sakes.

$$\begin{aligned}
 (19) \quad & \text{a. } \llbracket \textcircled{1} \rrbracket^{c,w} = \llbracket \text{sukunai} \rrbracket^{c,w}(\llbracket \alpha \rrbracket^{c,w}) \\
 & = [\lambda D_{\langle d,t \rangle}. \text{MAX}_d(D) = \mathbf{d}_\Delta](\lambda d. \exists x. [\text{sake}(x) \wedge \text{had}_w(T, x) \wedge \mu(x) = d]) \\
 & = \text{MAX}_d(\lambda d. \exists x. [\text{sake}(x) \wedge \text{had}_w(T, x) \wedge \mu(x) = d]) = \mathbf{d}_\Delta, \\
 & \text{where } \llbracket \alpha \rrbracket^{c,w} = \lambda d. \exists x. [\text{sake}(x) \wedge \text{had}_w(T, x) \wedge \mu(x) = d]. \\
 & \text{b. } \llbracket \textcircled{2} \rrbracket^{c,w} = \llbracket \text{to} \rrbracket^{c,w}(\llbracket \textcircled{1} \rrbracket^{c,w})(\llbracket P \rrbracket^{c,w}) \\
 & = \text{NEC}_w \left[\begin{array}{c} [\text{MAX}_d(\lambda d. \exists x. [\text{sake}(x) \wedge \text{had}_w(T, x) \wedge \mu(x) = d]) = \mathbf{d}_\Delta] \\ [\exists x. [\text{sake}(x) \wedge \text{had}_w(T, x) \wedge |x| = 3]] \end{array} \right] \\
 & \text{c. } \llbracket \textcircled{3} \rrbracket^{c,w} = \llbracket \text{mo} \rrbracket^{c,w}(\llbracket \textcircled{2} \rrbracket^{c,w}) \\
 & = [\lambda p. p(w) \wedge \partial[\forall q \in \text{Alt}_p : q \neq p \rightarrow p \prec_c q]] \\
 & \quad \left(\text{NEC}_w \left[\begin{array}{c} [\text{MAX}_d(\lambda d. \exists x. [\text{sake}(x) \wedge \text{had}_w(T, x) \wedge \mu(x) = d]) = \mathbf{d}_\Delta] \\ [\exists x. [\text{sake}(x) \wedge \text{had}_w(T, x) \wedge |x| = 3]] \end{array} \right] \right) \\
 & = \text{NEC}_w \left[\begin{array}{c} [\text{MAX}_d(\lambda d. \exists x. [\text{sake}(x) \wedge \text{had}_w(T, x) \wedge \mu(x) = d]) = \mathbf{d}_\Delta] \\ [\exists x. [\text{sake}(x) \wedge \text{had}_w(T, x) \wedge |x| = 3]] \end{array} \right] \\
 & \quad \wedge \partial[\forall q \in \text{Alt}_{\textcircled{2}} : q \neq \llbracket \textcircled{2} \rrbracket \rightarrow \llbracket \textcircled{2} \rrbracket \prec_c q], \\
 & \text{where } \prec_c \text{ is resolved to less-than relation.} \\
 & \text{d. } \text{Alt}_{\textcircled{2}} = \left\{ \begin{array}{c} \vdots \\ [\text{If the max-amount of sake that Taro had is } d_1 \prec_c \mathbf{d}_\Delta, \\ \text{then he had three sakes}], \\ [\text{If the max-amount of sake that Taro had is } d_2 = \mathbf{d}_\Delta, \\ \text{then he had three sakes}], \\ [\text{If the max-amount of sake that Taro had is } d_3 \succ_c \mathbf{d}_\Delta, \\ \text{then he had three sakes}], \\ \vdots \end{array} \right\}
 \end{aligned}$$

As an assertion, (19c) conveys that “if the amount of sake that Taro had is \mathbf{d}_Δ , which is a small value to the context c , he had three sakes”. Here, the truth of the consequent P “Taro had three sakes” is entailed; assuming that the antecedent exhausts all relevant possibilities, the assertion $\ulcorner mo_{\text{even}} \text{ if } [p]_F, P^\neg \urcorner$ will implicate the truth of the consequent P (von Fintel [11]: 147). At the same time, (19c) presupposes that “ \mathbf{d}_Δ is the ‘least’ value among the focus alternatives”. Since the focused element here is *sukunai*, the set of possible alternatives are calculated with respect to the amount of sake that Taro had, (19d). Putting the asserted and the presupposed meaning together, (19c) expresses that “if the amount of sake that Taro had is the least value, he had three sakes”. This successfully captures our intuition about the sentence with *sukunakutomo*. The crucial point here is that, thanks to the existence of *mo*, we can ensure that the amount of sake that Taro had is not merely ‘small’ but ‘the least’ amount, without using the superlative morpheme.

In our analysis, the ignorance inference is generated pragmatically via a typical rule of conversation (Grice [15]).¹⁰ For instance, in (19): (i) the speaker asserted that if the amount of sake that Taro had is the ‘least’ = ‘small’ amount, then he had three sakes; (ii) there are possible alternatives that the speaker could have made, i.e., [If the amount of sake that Taro had is more than the least (= small) amount, he had three sakes]. This intuitively means “we are considering some possibilities that Taro had more than the least amount, but in any case what the speaker is sure of is that he had three sakes”; (iii) from this, we can make an inference that there must be a reason for the speaker’s asserting only one of the possible alternatives and not asserting the remaining. For example, the reason could be “she does not know the truth about how much more sake Taro drank than that amount”, or “she just does not want to mention the other possibilities for personal reasons”, etc.

It should be noted that while this analysis has in common with Büring [13] and Coppock & Brochhagen [9] in that it derives the ignorance effect of superlative modifiers at the level of pragmatics, it departs from their analysis in that the starting point that signals the inference is not the disjunctive semantics but the conditional semantics.

4 Explaining the Data

Let us first illustrate how to derive the data of *sukunakutomo* in conditionals and negations that we have identified as problematic for the modal analysis.

As for the conditional example in (5), since our analysis does not assume the modal meaning to be encoded in the semantics of *sukunakutomo*, it does not cause the problem in the interpretation that occurs under the modal analysis. That is, the truth conditions of *sukunakutomo* are simply added to the truth conditions of the antecedent part of the conditional. Technically, (5) is true when in all the worlds (according to the conversational background) in which the antecedent “Hanako had *sukunakutomo* three beers” is true, the consequent “she would have been drunk” is true. More intuitively, (5) expresses “If it is the case that Hanako had drunk three beers, and the amount was considered to be ‘the least’, then she would have been drunk”.

The analysis also captures our intuition about the negated example in (6), whose interpretation is “the amount of beer that Hanako had is *at most two*”.

- (20) a. *Hanako-wa biiru-o sukunakutomo san-hai nomanakat-ta.*
 ‘Hanako didn’t have at least three beers.’
 b. LF: $mo [{}_P \text{ to } [sukunai_F \alpha], \neg[\text{Hanako had three beers}]]$,
 where $\llbracket \alpha \rrbracket^{c,w} = \lambda d. \exists x. [\text{beer}(x) \wedge \text{had}_w(H, x) \wedge \mu(x) = d]$.

¹⁰ The analysis here is inspired by Tomioka [26], who argues that the ignorance inference of sentences with the contrastive topic *wa* in Japanese is pragmatically derived by Gricean rules of conversation.

- c. $\llbracket (20a) \rrbracket^{c,w}$
 $= \text{NEC}_w \left[\begin{array}{l} [\text{MAX}_d(\lambda d. \exists x. [\text{beer}(x) \wedge \text{had}_w(H, x) \wedge \mu(x) = d]) = \mathbf{d}_\Delta] \\ [\neg \exists x. [\text{beer}(x) \wedge \text{had}_w(H, x) \wedge |x| = 3]] \end{array} \right]$
 $\wedge \partial[\forall q \in \text{Alt}_P : q \neq \llbracket P \rrbracket \rightarrow \llbracket P \rrbracket \prec_c q],$
 where \prec_c is resolved to the less-than relation with respect to the amount of beer that one **does not** drink.
- d. $\text{Alt}_P = \left\{ p \mid p = \begin{array}{l} \text{if the max-amount of beer that Hanako had is } d, \\ \text{she did not have three beers : } d \end{array} \right\}$

As represented above, the interpretation that our analysis predicts for (20a) is as follows. As an at-issue meaning, it conveys the conditional meaning that “if the amount of beer that Hanako had is ‘small’, she did not have three beers”.¹¹ This implicates the truth of the consequent “Hanako did not have three beers”, which implies that she did not have *more than* three beers. That is, the amount of beer that Hanako had is *at most* two beers. By the contribution of *mo*, the presupposition of the sentence ensures that the amount of beer that Hanako had is the least amount in terms of the amount of beer that one does not drink.

What about the cases of the example where the speaker has a perfect knowledge about the truth of alternatives (cf. (10))? Crucially, in our analysis, the ignorance inference induced by a sentence with *sukunakutomo* is not limited to the epistemic inference that the speaker does not know about the truth of the alternatives, which captures the fact that the use of *sukunakutomo* is fine in (10). In (10), the conversational inference that we can obtain from the utterance is that the speaker might have some personal reason for not mentioning about the alternative scores; here, she just does not want to tell the addressee the exact score that she got.

Thus far, we have dealt with examples with numerals. Consider the basic case in (21a) below, which dose not involve numerals. The interpretation here is that the speaker is certain that Taro came but is uncertain about whether people other than Taro came or not. The proposed analysis can easily derive this interpretation.

- (21) a. *Sukunakutomo Taro-ga kita.* ‘At least Taro came to the party.’
 b. LF: *mo* [_P to [*sukunai*_F α], [Taro came]]

¹¹ Under our analysis, since *sukunakutomo* is semantically a conditional and its preja-cent (e.g., in (20a) ‘Hanako didn’t have three beers’) is assumed to be the consequent clause, a negation in *sukunakutomo*(*p*) must apply to the consequent part in parallel with ordinary conditionals:

- (i) Asu-ga ame nara, soto-de asoba-**nai**. (ordinary conditional)
 tomorrow-NOM rain then outside-at play-NEG
 ‘If it rains tomorrow, I won’t spend my time outside.’
 \rightsquigarrow if it rains tomorrow, \neg (I will spend my time outside)
 \nrightarrow \neg (if it rains tomorrow, I will spend my time outside)

As shown in the truth conditions in (21c), (21a) asserts that if the amount of people who came is d_Δ , Taro came. In addition, d_Δ is presupposed to be the least among its alternatives. This in turn means that even when the amount of people who came is “1”, Taro came, which is equivalent to saying that the speaker is certain that Taro came. Hence, the current analysis can successfully generate the correct truth conditions of cases with proper nouns.

5.1 *Sukunakutomo* with \forall

(22) #Sukunakutomo zenin-ga kita.
at.least everyone-NOM came
‘[Int.] At least everyone came.’

¹² Note that the sentence (22) is acceptable when the associated scale is not numerical/plural, i.e. the context is “concessive”:

- In this case, the relevant set of alternative is represented with respect to the degree of success, rather than to the amount of people who came. Thus, the truth condition does not require that “if the amount of people who came is the least, everyone came”, but rather that “if the the degree of success is the least, everyone came”, which is possible to be true.

$$(23) \quad \llbracket (22) \rrbracket^{c,w} = \text{NEC}_w \left[\begin{array}{l} [\text{MAX}_d(\lambda d. \exists x. [\text{people}(x) \wedge \text{came}_w(x) \wedge |x| = d]) = \mathbf{d}_\Delta] \\ [\forall x. [\text{people}(x) \rightarrow \text{came}_w(x)]] \end{array} \right] \\ \wedge \partial[\forall q \in \text{Alt}_{p^0} : q \neq \llbracket p \rrbracket^0 \rightarrow \llbracket p \rrbracket^0 \prec_c q]$$

As shown above, the truth conditions require that if the amount of people who came is the least, everyone came. However, this is impossible, since the amount of people who came is the greatest if everyone came. Hence, these truth conditions cannot be satisfied, and the unacceptability arises.

5.2 *Mo* and Superlativity

Another empirical prediction is related to the presence of *mo* ‘even’. In our analysis, the existence of *mo* ensures that we are considering the least case among alternatives even when there is no superlative morpheme. An anonymous reviewer, however, points out that *sukunakute* ‘little + if’, which lacks *mo*, can also express the same meaning as *at least*, as shown in (24). Based on this, the reviewer doubts that *mo* plays a crucial role in deriving the superlative meaning.

- (24) Sukunakute san-nin-ga kita.
 little.if 3-CL-NOM came
 ‘[Int.] At least three people came.’

Although we agree with the reviewer’s view that *sukunakute* can express a similar meaning with *sukunakutomo*, there is still evidence in favor of our claim. As the following example indicates, the superlative expression *ichiban* ‘the most’ can be used with *sukunakute* but not with *sukunakutomo*:

- (25) Ichiban { sukunakute / #sukunakutomo } san-nin-ga kita.
 the.most { little.if / at.least } 3-CL-NOM came
 ‘At least three people came.’

The above contrast can be captured if *sukunakutomo* has the least meaning thanks to the existence of *mo* and the superlative expression *ichiban* is redundant.¹³

¹³ A problem with this line of analysis (in which *mo* ‘even’ contributes to adding the superlative meaning) is that another Japanese superlative expression *saitei-demo*, which consists of a superlative *sai*, *tei* ‘low’, and *demo* ‘even if’, may end up having a redundant meaning, since *saitei* ‘lowest’ itself denotes the superlative meaning (Yusuke Kubota, p.c.). Then the challenge for our account is to figure out the semantic contribution of *mo* in *saitei-demo*. We should identify, for instance, what difference there is between (i-a) and (i-b), namely *saitei* with and without *mo*:

- (i) a. Saitei-de-mo san-nin kuru.
 lowest-if-even 3-CL will come
 ‘At least three people came.’
 b. Saitei(-de) san-nin kuru.
 lowest-(if) 3-CL come
 ‘At least three people will come.’

6 Conclusion

Unlike English *at least*, Japanese *sukunakutomo* is morphologically broken down into *little*, a conditional morpheme, and *even*, which contribute to providing a strategy for deriving the meaning of superlative modifiers.

Much work is still needed to determine factors affecting the availability of the different readings. As is well known, Nakanishi & Rullmann [22] observe that sentences containing *at least* have not only an epistemic reading, but also a concessive reading (e.g., “Mary didn’t win a gold medal, but *at least* she won a silver medal.”). The analysis of the concessive reading of *sukunakutomo* will be our future task, but since our framework assumes that scales and alternatives related to the interpretation of *mo* are context/discourse dependent, it would not be difficult to capture the relevant data by utilizing the flexibility, although some refinement may be required.

If our analysis of *sukunakutomo* is on the right track, it would support the view of Biezma [4] that ‘at least’ expressions have only one denotation, not two different denotations (cf. Nakanishi & Rullmann [22]). We take this as an indication that the next interesting/crucial question about superlative modifiers is not simply “What do they denote?”, but rather “What are their semantic variations, and how are those variations distributed among natural languages?” (cf. Chen [6]).

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For us, intuitively (i-a) seems to have a stronger meaning than (i-b), but we don’t have any linguistic evidence for this intuition at this moment. We hope readers can help us figure out what is going on here.

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