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Was Japan's "rapid economic recovery" after WWII miraculous? : A theoretical explanation

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Abstract This paper is concerned with the existence of optimal recovery paths after an economy has been destroyed by natural disaster or war. A prominent example is Japan after World War II. Japan faced massive destruction of its infrastructure and production facilities and, in fact, starvation and severely high inflation. However, in a relatively short time, 10 years, the country achieved economic recovery. This historical fact indicates that there is an optimal recovery path after economic mass destruction. Under such extreme conditions, one of the key features is that 1) production technology is fixed, not selective, and 2) in view of two industry sectors; the capital goods sector and the consumer goods sector, the capital intensity of the former is much greater than that of the latter. To analyze such a situation, we return to the two-sector growth model with Leontief technologies studied by Shinkai and Corden in the 1960s and transform it into a continuous time optimal growth model with the Leontief technologies. In this study, we focus on nonlinearity and interior-point solutions, under the assumption that the capital intensity of the capital goods sector is extremely greater than that of the consumption goods sector, two critical points were derived. These two critical points indicated the unstable focus (or unstable point) and the saddle, respectively. The transversality conditions show that the saddle-point stable point is the only optimal solution and the optimal recovery path. Then, history-based parameter selection and numerical analysis showed that the initial per capita capital stock, given historically, must be close to the unstable focus and that government policy is needed to make the economy jump onto the trajectory which eventually converges to the optimal steady state path. In other words, Japan's economic recovery was miraculous. (280)

Key Words: Leontief technologies, two-sector optimal growth model, capital intensity condition, saddle-path stability, unstable focus, phase diagram

JEL: O22, O41, O53

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1. Introduction

Japan had to take the first steps toward economic recovery from a land devastated in both name and reality by its defeat in World War II. The loss of national wealth in World War II amounted to 25.4 percent, or just over a quarter of the country's total wealth. The factory production base was in a state of ruin, with particularly heavy losses in ships, industrial machinery and equipment, and buildings. Most of the production facilities that escaped damage were used for military production, and conversion to peaceful industry was not easy. The devastation caused by the war and the shortage of materials and labor resulted in a critical shortage of raw materials and, moreover, hyperinflation. In addition, the rapid increase in the birth rate, which was accompanied by the repatriation of military personnel and Japanese living abroad to the interior of the country, led to a rapid increase in the population and further accelerated the shortage of goods, especially food and clothing. A rationing system was put in place to distribute the scarce goods equally, but the imbalance between the supply and demand of goods was so great that black-market goods and black-market prices were rampant. This situation is vividly illustrated in Figure 1, where the 100 years of Japan's per capita real GDP is depicted.

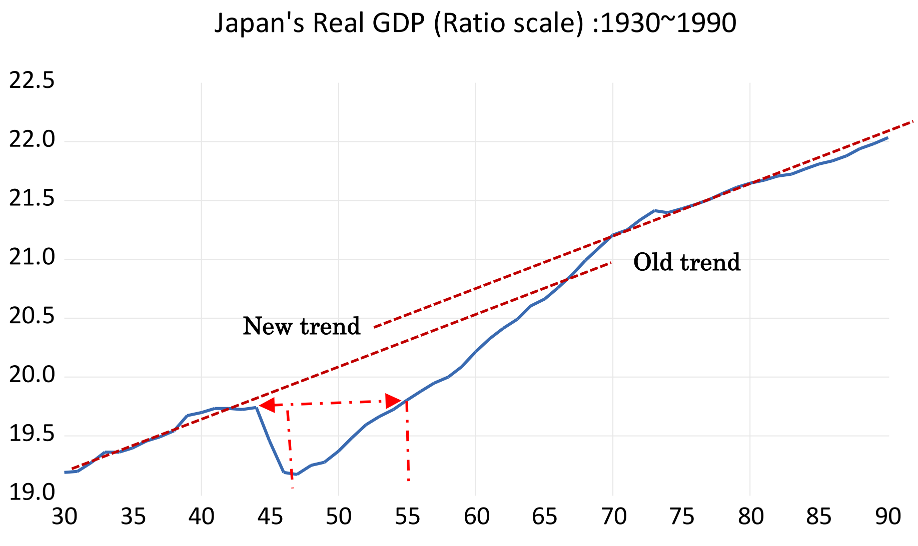


Figure 1. Postwar recovery period:1946 ~ 1955

From 1945 to 1946, GDP fell sharply due to World War II, returning to prewar levels in 1955. Furthermore, a steeply sloping trend line is observed from 1946 to 1975, intersecting the old trend line around 1964, when the Tokyo Olympics were held. The slope of the line indicates the average growth rate, which was about 10%. Such a fact would give the impression that Japan's rapid postwar economic recovery was inevitable and independent of government policies. Therefore, we will theoretically examine whether this impression is true by dating Japan's economic recovery from 1946 to 1955[2].

In order to set up a model to analyze Japan's rapid economic recovery, the following five characteristics must be met.

1) Due to the devastation of the war, the technology of choice was very limited, or the options were limited or fixed.

2) There was a very large difference in capital intensity when considering the two sectors: the capital goods sector and the consumption goods sector. The consumption goods sector had very low capital intensity compared to the capital goods sector, such as iron and steel manufacturing and shipbuilding, because the former imperial military personnel (7.6 million), military production workers (4 million) and repatriates from abroad (1.5 million) were mainly engaged in agriculture and self-employment in commerce.

3) Because of the large number of structurally unemployed, total labor input was determined by production technology.

4) The initial stock, which indicates the level of per-capita stock in 1946, is not zero, but very close to the lower limit.

5) In the early stage, the chaotic state immediately after the war, the optimal trajectory should be located in the unstable region, but in the later stage, it will converge to the optimal steady state and show a stable and high economic growth rate.

In this paper, the original Leontief two-sector model studied by Shinkai (1960) and Corden (1966) is adopted and transformed into a continuous time per capita optimal growth model with Leontief technologies. Shinkai (1960) and Corden (1966) studied a two-sector growth model with Leontief technologies and demonstrated that a balanced growth path is globally stable if the consumer goods sector is capital-intensive, as Uzawa (1961, 1963) showed in a two-sector growth model using neoclassical technology; Uzawa (1964) extended this property to the neoclassical two-sector optimal growth model. More recently, Fujio (2008, 2009) applied the geometric method developed in Kahn and Mitra (2007) to a discrete-time two-sector Leontief optimal growth model to study the properties of the optimal steady state.

To maintain differentiability, we restrict interior solutions. That is, only the equilibrium in which both goods are produced is considered. In doing so, we examine the local and global stability of the optimal steady state. In other words, as shown in Uzawa's

[2] Ch. 4 of Flath (2020) and Part 1 of Nakamura (1995) are recommended for a quick look at Japan's postwar reconstruction. Macroeconomic indicators and major events during the recovery period are shown in Table 1.

neoclassical two-sector optimal growth model, labor and capital grow in proportional to population growth.

The analytical results are as follows: 1) under the capital intensity condition, where the capital goods sector is extremely capital intensive than the consumer goods sector, two different optimal steady states are found to exist; 2) as indicated by both analytical and phase diagram analysis, one steady state is the saddle, and the other is the unstable focus or an unstable point. Therefore, only the saddle optimal steady state is the optimal steady state that can be achieved. In other words, there is a unique optimal steady state under this capital intensity condition. This result is in contrast to the standard result of optimal growth theory, first shown by Uzawa (1964) in his two-sector optimal growth theory, that the optimal steady state exhibits saddle-point stability when capital intensity in the consumption goods sector is higher than the capital goods sector.

Based on the analytical results, parameter values were selected to fit the historical facts, in particular, the annual growth rate of total labor input was chosen at 10% and the people's discount rate at 67%. Numerical analysis was then performed, and as expected from the analytical results, two types of solutions were found to exist: saddle point stable solutions and unstable central solutions. Applying the backward-shooting method, we found the optimal path with the initial point near the unstable center solution such that it eventually converges to the saddle-point stable solution. The saddle-point stable solution is the optimal steady state in which the level variable grows by 10%. There are two important points if this is the theoretical explanation for Japan's rapid economic recovery process shown in Figure 1: 1) the initial per capita capital stock is near the unstable solution point, and 2) some macroeconomic policies are needed to move the economy toward a stable solution and eventually a "stable arm" beyond the optimal steady state.

From these two perspectives, we can conclude that the main factors behind Japan's rapid economic recovery were: (1) the per capita stock at the end of the war happened to be close to the unstable steady state point, and (2) the government implemented appropriate policies, such as the "priority production system" and strong inflationary measures known as the "dodge line," to bring the economy to an optimal steady state. In other words, Japan's rapid economic recovery was truly a "miracle.

YEAR	GNP* GROWTH RATE (%)	INFLATION** RATE (%)	MAJOR EVENTS
1945	-	51.1	● Surrender of Japan (Aug.)
1946	-	364.5	● Financial asset freeze (Feb.), ● Breakup of the Zaibatsu
1947	8.6	195.9	● Land Reform, ● Labor Reform
1948	13.0	165.5	
1949	2.1	63.3	● Dodge Line (Feb.), ● The exchange rate pegged at 360 yen to the dollar
1950	11.0	18.2	● Korean War began (June)
1951	13.0	38.3	● Peace Treaty of San Francisco
1952	11.0	2.0	
1953	7.4	5.0	● Korean War Armistice Agreement (July)
1954	3.6	6.5	

Table 1. Macroeconomic conditions of Japan, 1945 – 1954

Source: *Bank of Japan Major Economic Statistics of Japan since the Meiji Era*

*) Gross National Expenditure in real terms (Table 011_9_1).

**) Wholesale Price Index (Table 018_2_1).

2. Notation and Model

We follow the two-sector model, consumption goods and capital goods sectors with Leontief technologies, which was studied originally by Shinkai (1960) and Corden (1966). Let us define some notation which are used in Coden (1966). The indices "*k*" and "*c*" indicate capital goods sector and consumption goods sector respectively.

(1) Capital coefficient: $K_k / Y_k, K_c / Y_c$,

(2) Outputs per person: $a_k = Y_k / L_k, a_c = Y_c / L_c$,

(3) Capital-labor ratio: $b_k = K_k / L_k, b_c = K_c / L_c$,

(4) Full employment conditions: $L = L_k + L_c$ and $K = K_k + K_c$,

(5) Total labor input growth: $L = L_0 e^{nt} = e^{nt}$,

Remark 1. As explained in Introduction, there exists a huge amount of structurally unemployed labor outside labor market, the technologies of both sectors determine the labor inputs *L*. The initial labor input is normalized as one.

(6) Capital Accumulation: $\dot{K} = Y_k - \delta K \Rightarrow \dot{k} = y_k - (n + \delta) k$

The following important assumptions are made:

Assumption 1. All the technological coefficients: a_c, a_k, b_c, b_k are constant.

Assumption 2. i) The capital goods sector is more capital intensive than the consumption goods sector ($b_c < b_k$) and ii) $b_c < k < b_k$.

In Assumptions 1and 2 ii), let's draw a familiar diagram consisting of the constraint lines of labor and capital. The slope of this line is indicative of the capital intensity of each sector. Based on the calculations below, a diagram like Figure 2 of Corden (1966) can be drawn.

$$\begin{cases} L = \left(\frac{1}{a_c}\right) Y_c + \left(\frac{1}{a_k}\right) Y_k \ (L - const.) \Rightarrow Y_c = -\left(\frac{a_c}{a_k}\right) Y_k + a_c L \\ \\ K = \left(\frac{b_c}{a_c}\right) Y_c + \left(\frac{b_k}{a_k}\right) Y_k \ (K - const.) \Rightarrow Y_c = -\left(\frac{b_k}{a_k}\right)\left(\frac{a_c}{b_c}\right) Y_k + \left(\frac{a_c}{b_c}\right) K \end{cases}$$

Assume that the total labor input and total capital stock are increasing at the rate of total labor input growth (n). Then, both constraint lines shift upward simultaneously, and the internal equilibrium point Q also shifts to point Q'. Let's trace only such a continuous shift.

From (1) through (4), we obtain the output ratio. Further rewriting in per-capita terms yields (7) derived as Eq. (3) in Corden (1966).

(7) $$\frac{y_k}{y_c} = \left(\frac{Y_k / L_k}{Y_c / L_c}\right) \frac{(K_c / L_c - K/L)}{(K/L - K_k / L_k)} \Rightarrow y_k = \left(\frac{a_k}{a_c}\right)\left(\frac{b_c - k}{k - b_k}\right) y_c$$

Remark 2. Note that (7) is an important function. It indicates that when k given and y_c chosen through the resource allocation between two sectors, y_k is automatically determined. Assumption 2 ii) guarantees that $y_k > 0$.

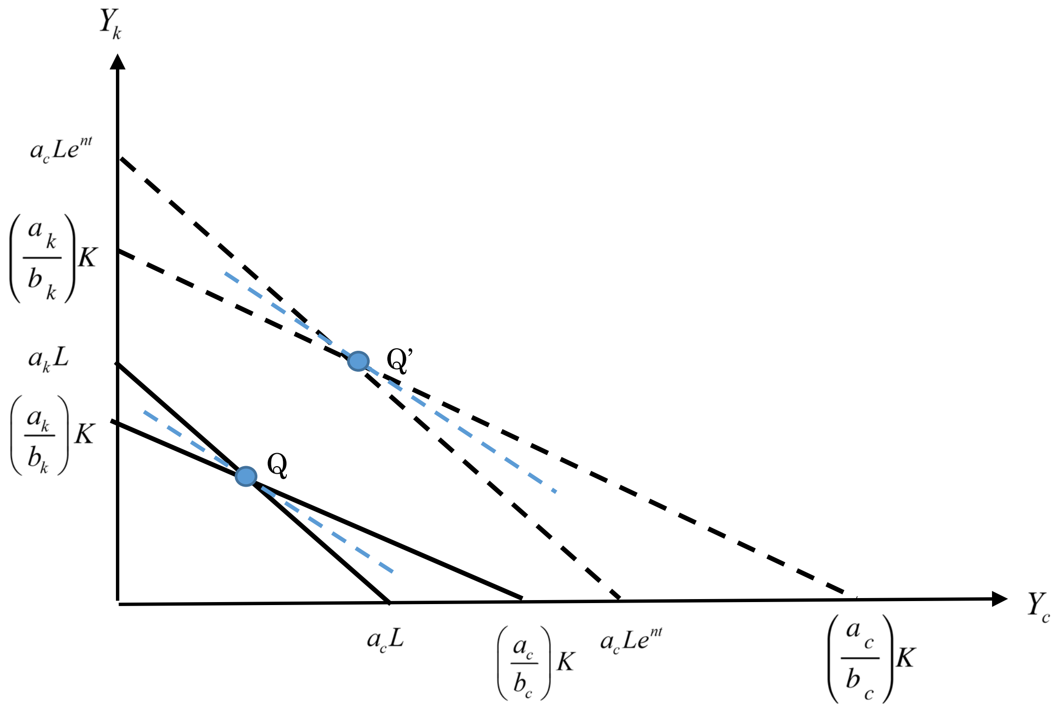


Figure 2.

3. Optimal growth problem

Now let us consider the problem that keeping (7), choose y_c so that the utility function lny_c is maximized. From (7), to choose y_c implies simultaneously to choose y_k. Due to accumulation equation (6) it increases per capita capital stock. In the next period, since the capital goods sector is more capital intensive than the consumption goods sector from *i*) of Assumption 2, y_k increases and y_c reduces by Rybczynski theorem[3]. In other words, to choose greater y_c implies that it directly increases today's utility, but it increases y_k more and reduces y_c in the next period. Thus, it has a negative effect to the utility in the next period. We have a trade-off between today's utility and tomorrow's utility to choose today's y_c. Thus, under the trade-off, the problem is to find out the proper path of y_c in the long run to maximize the objective function: lny_c .

We can set up the following intertemporal optimization problem to solve the problem:

[3] Rybczynski theorem implies that an increase in the factor endowment of a factor of production under a constant relative price of a good increases the output of the good produced using that factor of production in an intensive manner and decreases the output of other goods.

$$Max\int_0^\infty e^{-\rho t}e^{nt}\ln y_c\, dt \Rightarrow Max\int_0^\infty e^{(-\rho+n)t}\ln y_c\, dt$$

$$st$$

$$\dot{k}=\left(\frac{a_k}{a_c}\right)\left[\frac{b_c-k}{k-b_k}\right]y_c-(n+\delta)k,$$

$$k_0=\bar{k}$$

We make an extra assumption for making the objective function definable:

Assumption 3. $\rho > n$**.**

Then the Hamiltonian for the problem is defined as follows:

$$H=e^{(n-\rho)t}\ln y_c+\lambda\left\{\left(\frac{a_k}{a_c}\right)\left[\frac{b_c-k}{k-b_k}\right]y_c-(n+\delta)k\right\}$$

Then the first order conditions concerned with the Hamiltonian are obtained as follows:

(8) $$\frac{\partial H}{\partial \lambda}=\dot{k}\Rightarrow \dot{k}=\left(\frac{a_k}{a_c}\right)\left[\frac{b_c-k}{k-b_k}\right]y_c-(n+\delta)k$$

(9) $$-\frac{\partial H}{\partial k}=\dot{\lambda}\Rightarrow\dot{\lambda}=-\lambda\left\{\left(\frac{a_k}{a_c}\right)\left[\frac{b_k-b_c}{\left(k-b_k\right)^2}\right]y_c-(n+\delta)\right\}$$

$$\frac{\partial H}{\partial y_c}=0\Rightarrow e^{(n-\rho)t}\left(\frac{1}{y_c}\right)+\lambda\left(\frac{a_k}{a_c}\right)\left(\frac{b_c-k}{k-b_k}\right)=0\Rightarrow\ e^{(n-\rho)t}\left(k-b_k\right)+\lambda\left(\frac{a_k}{a_c}\right)\left(b_c-k\right)y_c=0,$$

(10) $$\Rightarrow y_c=-\frac{e^{(n-\rho)t}\left(k-b_k\right)}{\lambda\left(\frac{a_k}{a_c}\right)\left(b_c-k\right)}=-\left(\frac{1}{\lambda}\right)e^{(n-\rho)t}\frac{\left(k-b_k\right)}{\left(\frac{a_k}{a_c}\right)\left(b_c-k\right)}\Rightarrow-\left(\frac{1}{\lambda}\right)e^{(n-\rho)t}=\frac{\left(\frac{a_k}{a_c}\right)\left(b_c-k\right)}{\left(k-b_k\right)}y_c>0,$$

(11) $P(t)>0$ and $\lim_{t\to\infty}P(t)k(t)=0.$

Note that Eq. (11) indicates the transversality condition.

3.1 Solving the optimal problem

Due to the last relation in (10), it is convenient to define the following new variable:

Definition. $P\equiv-\left(\frac{1}{\lambda}\right)e^{(-\rho+n)t}>0$ **.**

Remark 3. It is worth noting that, by Rybczynski theorem, to increase Y_k has a negative impact on the objective function, as discussed in Remark 2, so the Lagrangian λ is negative, as shown in (10). Also note that P is considered the relative price of the consumption good measured in utility units.

Substituting the result of (10) into the equations (8) and (9) yields

$$\dot{k}=\left(\frac{a_k}{a_c}\right)\left[\frac{b_c-k}{k-b_k}\right]\left\{-\frac{e^{(n-\rho)t}\left(k-b_k\right)}{\lambda\left(\frac{a_k}{a_c}\right)\left(b_c-k\right)}\right\}-(n+\delta)k=\frac{-e^{(n-\rho)t}}{\lambda}-(n+\delta)k=P-(n+\delta)k$$

And

$$\begin{aligned}\dot{\lambda}&=-\lambda\left\{\left(\frac{a_k}{a_c}\right)\left[\frac{b_k-b_c}{\left(k-b_k\right)^2}\right]y_c-(n+\delta)\right\}\\&=-\lambda\left\{\left(\frac{a_k}{a_c}\right)\left[\frac{b_k-b_c}{\left(k-b_k\right)^2}\right]\left\{-\frac{e^{(n-\rho)t}\left(k-b_k\right)}{\lambda\left(\frac{a_k}{a_c}\right)\left(b_c-k\right)}\right\}-(n+\delta)\right\}=-\lambda\left\{-\frac{e^{(n-\rho)t}\left(b_k-b_c\right)}{\lambda\left(k-b_k\right)\left(b_c-k\right)}-(n+\delta)\right\},\end{aligned}$$

Note that due to ii) of Assumption 1, $\left(k-b_k\right)\left(b_c-k\right)\neq 0$.

We finally obtain:

$$-\frac{\dot{\lambda}}{\lambda}=\left\{-\frac{e^{(n-\rho)t}\left(b_k-b_c\right)}{\lambda\left(k-b_k\right)\left(b_c-k\right)}-(n+\delta)\right\}=P\frac{\left(b_k-b_c\right)}{\left(k-b_k\right)\left(b_c-k\right)}-(n+\delta).$$

Differentiating P w.r.t. time yields:

(11)
$$\begin{aligned}\dot{P}&=-\left(\frac{e^{(-\rho+n)t}}{\lambda}\right)'=-\frac{(-\rho+n)e^{(-\rho+n)t}\lambda-e^{(-\rho+n)t}\dot{\lambda}}{\lambda^2}\\&=-(n-\rho)\left(\frac{e^{(-\rho+n)t}}{\lambda}\right)+\left(\frac{1}{\lambda}\right)e^{(-\rho+n)t}\left(\frac{\dot{\lambda}}{\lambda}\right)\\&=(n-\rho)\left(-\frac{e^{(-\rho+n)t}}{\lambda}\right)+\left(-\frac{e^{(-\rho+n)t}}{\lambda}\right)\left(-\frac{\dot{\lambda}}{\lambda}\right)=(n-\rho)P+P\left(-\frac{\dot{\lambda}}{\lambda}\right)\end{aligned}$$

Furthermore, substituting $\left(-\frac{\dot{\lambda}}{\lambda}\right)$ into the above relation (11) yields,

$$\dot{P} = (n-\rho)P + P\left(-\frac{\dot{\lambda}}{\lambda}\right)$$

$$= (n-\rho)P + P\left[P\frac{\left(b_k - b_c\right)}{\left(k-b_k\right)\left(b_c - k\right)} - (n+\delta)\right],$$

$$= \frac{\left(b_k - b_c\right)}{\left(k-b_k\right)\left(b_c - k\right)}P^2 - (\rho+\delta)P$$

Then we have finally obtained the following dynamical system (*) constituted of two differential equations:

$$\textbf{(*)}\begin{cases}\dot{P} = P^2\left[\frac{\left(b_k - b_c\right)}{\left(k-b_k\right)\left(b_c - k\right)}\right] - (\delta+\rho)P = P\left\{P\left[\frac{\left(b_k - b_c\right)}{\left(k-b_k\right)\left(b_c - k\right)}\right] - (\rho+\delta)\right\}, \\ \dot{k} = P - (n+\delta)k\end{cases}$$

3.2 Multiple steady states

To obtain the optimal steady state, we need to solve the following simultaneous equations (**):

$$(**)\begin{cases}\dot{P} = 0 \Rightarrow 0 = P\left\{P\left[\frac{\left(b_k - b_c\right)}{\left(k-b_k\right)\left(b_c - k\right)}\right] - (\delta+\rho)\right\} \\ \qquad \Rightarrow (12)\ P = \frac{(\delta+\rho)}{\left(b_k - b_c\right)}\left(k-b_k\right)\left(b_c - k\right), \text{if } P \neq 0. \\ \dot{k} = 0 \Rightarrow (13)\ P = (n+\delta)k\end{cases}$$

The origin (0, 0) is one of the critical points of (*). The other two critical points are obtained as roots of the following quadratic equation:

$$\frac{(\rho+\delta)}{\left(b_k-b_c\right)}\left(k-b_k\right)\left(b_c-k\right)=(n+\delta)\,k$$

$$\Rightarrow f(k)\equiv k^2+\left\{\left(\frac{n+\delta}{\rho+\delta}\right)(b_k-b_c)-(b_k+b_c)\right\}k+b_kb_c=0$$

Note that $f(b_k)>0$ and $f(b_c)>0$. The discriminant of the equation: D turns to be as follows.

$$D=\left\{\left(\frac{n+\delta}{\rho+\delta}\right)(b_k-b_c)-(b_k+b_c)\right\}^2-4b_kb_c$$

We make the following assumption.

Assumption 4.

$$\left\{\left(\frac{n+\delta}{\rho+\delta}\right)(b_k-b_c)-(b_k+b_c)\right\}^2>4b_kb_c$$

Under Assumption 1, $D>0$ is established. Furthermore, it implies that the parabola expressed by (12) and the line (13) intersect each other as depicted in Figure 1 below. Therefore, we have two different roots α and β $(\alpha>\beta)$ that are possible optimal steady states, and they satisfy $b_c<\beta<\alpha<b_k$ because $f(b_c)>0$ and $f(b_k)>0$. This property contrasts with the one derived by Bruno (1966), where there exists a globally unique saddle optimal steady state. We also make an additional assumption regarding the parameters.

4. Local stability

Let us study local properties at each steady state point derived in Section 3. The Jacobian and the trace of the differential equation system (*) at two critical points k^* is obtained as follow:

$$J^* = \begin{pmatrix} \delta+\rho & -(\delta+\rho)^2 \dfrac{\left[(b_k+b_c)-2k^*\right]}{(b_k-b_c)} \\ & \\ 1 & -(\delta+n) \end{pmatrix}$$

$$\Rightarrow \det J^* = -(\delta+\rho)(\delta+n) + (\delta+\rho)^2 \frac{\left[(b_k+b_c)-2k^*\right]}{(b_k-b_c)}, \text{ and } Tr. = \rho - n.$$

Due to ii) of Assumption 2 and Assumption 4, it follows that

$$f\left(\frac{b_k+b_c}{2}\right) = \left(\frac{\delta+n}{\delta+\rho}\right)\left(\frac{b_k^2-b_c^2}{2}\right) - \frac{(b_k+b_c)^2}{4} + b_k b_c$$

$$= (b_k+b_c)\left\{\left(\frac{\delta+n}{\delta+\rho}\right)\left(\frac{b_k-b_c}{2}\right) - \frac{(b_k+b_c)}{4}\right\} + b_k b_c > 0,$$

We have $0 < b_c < \beta < \dfrac{b_k+b_c}{2} < \alpha < b_k$.

Proposition. At the critical value α denoted by Point A, the saddle-point stability is exhibited, while the critical value β denoted by Point B exhibits the unstable focus or unstable point.

Proof. Substituting α into $\det J^\star$ yields

$$\det J^* = -(\delta+\rho)(\delta+n) + (\delta+\rho)^2 \frac{\left[(b_k+b_c)-2\alpha\right]}{(b_k-b_c)} < 0$$

from $\dfrac{(b_k+b_c)}{2} < \alpha$.

The fact that $\det J^* < 0$ and $Tr^* > 0$ implies a saddle point at $k^* = \alpha$ denoted by Point A.

To show the proprietary of Point B, we need to explicitly derive the solution β, which is expressed as follows.

$$\beta = \frac{-\left[\left(\frac{n+\delta}{\rho+\delta}\right)\left(b_k-b_c\right) - \left(b_k+b_c\right)\right] - \sqrt{D}}{2}$$

Substitute this into the Jacobian to be evaluated,

$$\det J^* = -(\delta+\rho)(\delta+n) + (\delta+\rho)^2 \frac{\left[(b_k+b_c)-2\beta\right]}{(b_k-b_c)}$$

$$= -(\delta+\rho)(\delta+n) + (\delta+\rho)^2 \left\{ \left(\frac{b_k+b_c}{b_k-b_c}\right) + \frac{\left[\left(\frac{\delta+n}{\delta+\rho}\right)\left(b_k-b_c\right)-\left(b_k+b_c\right)\right]+\sqrt{D}}{\left(b_k-b_c\right)} \right\}$$

$$= -(\delta+\rho)(\delta+n) + (\delta+\rho)^2 \left[\left(\frac{\delta+n}{\delta+\rho}\right) + \frac{\sqrt{D}}{\left(b_k-b_c\right)}\right] = \frac{(\delta+\rho)^2\sqrt{D}}{\left(b_k-b_c\right)} > 0.$$

det. $J^* > 0$ and $Tr^* > 0$ implies that the following two cases occur.

$$\begin{cases} 4\det.\ J^* > Tr^* \ \Rightarrow \ \text{Unstable focus at Point B}, \\ 4\det.\ J^* < Tr^* \ \Rightarrow \ \text{Unstable at Point B}. \end{cases}$$

Point A has a "stable arm" indicated by arrows c-A and d-A in Figure 1, and any path on the stable arm converges to the steady state indicated by point A, which satisfies the transversality condition and is the optimal steady state. Conversely, the paths in the vicinity of point B are divergent and violate the transversality condition and are not optimal.∎

To reconfirm our results, let's draw a phase diagram and analyze the global dynamic behavior of the system (*). To do this, we need to identify the loci $\dot{k}=0$ and $\dot{P}=0$. The locus of $\dot{k}=0$ is a straight line: $P=(n+\delta)k$. The following differential equations should be analyzed to derive the locus of $\dot{P}=0$.

$$\frac{\dot{P}}{P} = P\left[\frac{\left(b_k-b_c\right)}{\left(k-b_k\right)\left(b_c-k\right)}\right] - (\delta+\rho) = 0, \text{if } P \neq 0$$

$$\Rightarrow P = \frac{(\delta+\rho)}{\left(b_k-b_c\right)}\left(k-b_k\right)\left(b_c-k\right) = \frac{(\delta+\rho)}{\left(b_c-b_k\right)}\left[k^2-(b_k+b_c)k+b_kb_c\right]$$

$$\Rightarrow P = \left(\frac{\delta+\rho}{b_c-b_k}\right)\left[k-\left(\frac{b_k+b_c}{2}\right)\right]^2 + \frac{(\delta+\rho)\left(b_k-b_c\right)}{4}$$

The last relation shows that the locus of $\dot{P}=0$ is a quadratic equation with the vertex

$\left(b_k + b_c/2, (\delta+\rho)\left(b_k - b_c\right)/4\right)$. Now we can draw the following phase diagram below:

Phase portraits in Figure 1 below are drawn based on the following properties.

$$\begin{cases} \dot{k} > (<)\, 0 \Leftrightarrow P > (<)\, (n+\delta)\, k, \\ and \\ \dfrac{\dot{P}}{P} > (<)\, 0 \Leftrightarrow P > (<) \left(\dfrac{\delta+\rho}{b_c - b_k}\right)\left[k - \left(\dfrac{b_k + b_c}{2}\right)\right]^2 + \dfrac{(\delta+\rho)\left(b_k - b_c\right)}{4} \end{cases}$$

Points A and B represent two different possible optimal steady-state solutions α and β respectively. As shown in Figure 3, the possible steady-state points A and B represent the saddle and the unstable focus (or instability) respectively. This phase diagram contrasts with the figure shown as Figure 4 in Bruno (1966), where the optimal steady state is unique and saddled-point stable in the same region as ours. Our model, on the other hand, has complex dynamics, even though there is only a single fixed production technology for each sector.

5. Numerical analysis[4]

In Section 4, the properties of the optimal steady state of the dynamical system (*) were qualitatively examined. To demonstrate that the analytical results obtained in the previous section can explain the miracle of Japan's postwar recovery, a numerical analysis was performed using values of parameters selected to be in line with historical facts.

5.1 Historical parameter values

For this purpose, each parameter should be carefully selected as follows.

[4] Mathematica 10.3 was used here. The Mathematica source codes will be provided upon request.

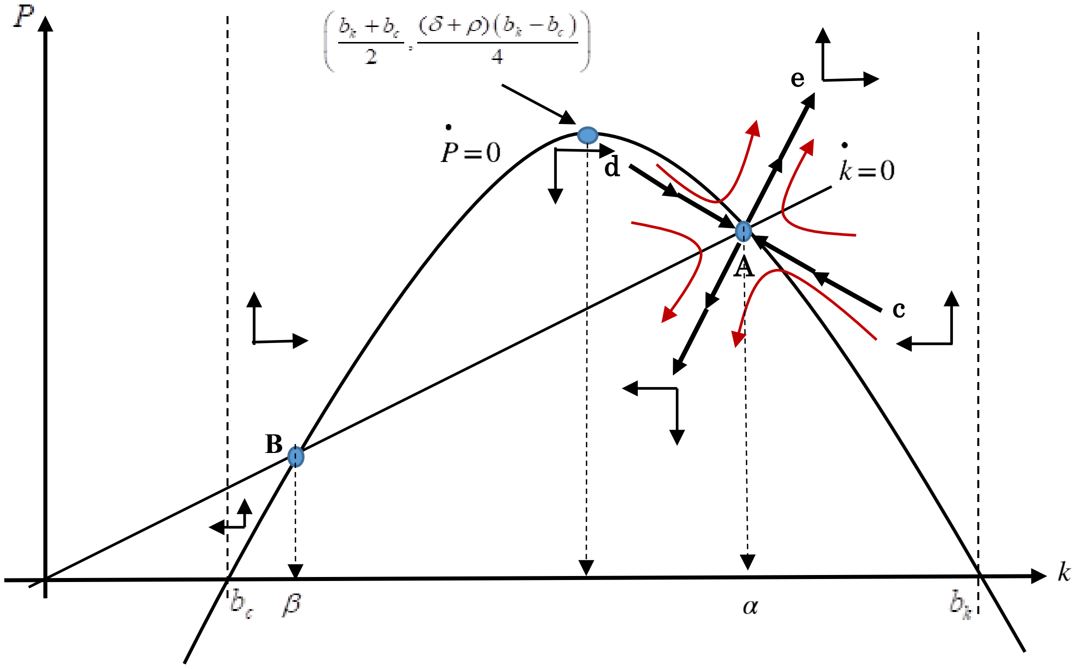


Figure 3: Phase Diagram

- b_k and b_c : Takahashi et al. (2012) measured the capital intensity of two sectors in Japan from 1955 to 1995 based on the I-O tables[5] and reported that the capital intensity of the consumption goods sector in 1955 was 1.14 (million/person) and that of the investment goods sector was 1.18 (million/person). The value of capital intensity varies with monetary units. Therefore, the value of capital intensity is arbitrarily chosen in monetary units. In other words, only the capital intensity ratio and the differential are meaningful. In fact, in 1955 it was 1.14/1.18 ≅ 0.97. Although the ratio was close to one in 1955, but it can be inferred that during early stage of the postwar reconstruction period from 1946 to 1955, there was a significant difference in the level of capital intensity between the two sectors. The reasons for this are as follows. After the war, 7.61 million troops went to war and 1.5 million civilians were repatriated from abroad. This meant that more than a quarter of the labor force was structurally unemployed. Many of them worked in agriculture or self-employment in the service sector. On the other hand, there were also workers who repaired machinery used for munitions and produced peacetime supplies. For example, producing merchant ships rather than combat vessels. Assuming that the capital intensity of the consumption sector is normalized as 1, the capital intensity of

[5] The 1955 I-O table is the earliest postwar table.

the capital goods sector is measured as a ratio. For example, assigning 6 to the capital goods sector implies that the capital goods sector is six times more capital intensive than the consumption sector. We assign here 35 to the capital intensity ratio.

- $k(0)$: The initial per capita stock must satisfy Assumption 2: $b_c < k(0) < b_k$. Furthermore, because of the catastrophic destruction of the capital stock due to the war, the initial stock is assumed to have been near the lower bound b_c.

- n: Since along the optimal steady state, all the level variables grow at n, we may be cautioned to assign the value. According to Okawa and Shinohara's data reported in Table 4.1 of Flath (2013), GDP per capita grew at a rate of almost 10% from 1946 to 1955. Therefore, the growth rate of total labor input, n, is set to 0.1.

- ρ: After World War II, people were busy with their day-to-day lives, and they probably had fairly myopic preferences. The values here range from 0.2 to 0.9. Since n is substituted for 0.1, the future discount rate of the objective function varies from 90% to 45%.

- δ: No data are available. The standard peacetime value is about 0.15, but it is considered relatively high during the reconstruction period. The main reason for this is that it was very difficult to find undamaged machinery and the resources to replace it during the economic reconstruction period, when the war had caused extensive damage. Here we assume 0.20. This means that the depreciation would be almost complete in five years.

Determining the discount rate during the restructuring period is difficult, but according to Table 2, 90% is very optimistic, and 55% is too pessimistic. Assume a discount rate of 67% and a capital intensity ratio of 35%.

Future discount factor (%)	**90**	**81**	**74**	**67**	**60**	**55**	**50**	**45**
$n-\rho$	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8

Table 2. Future discount factors

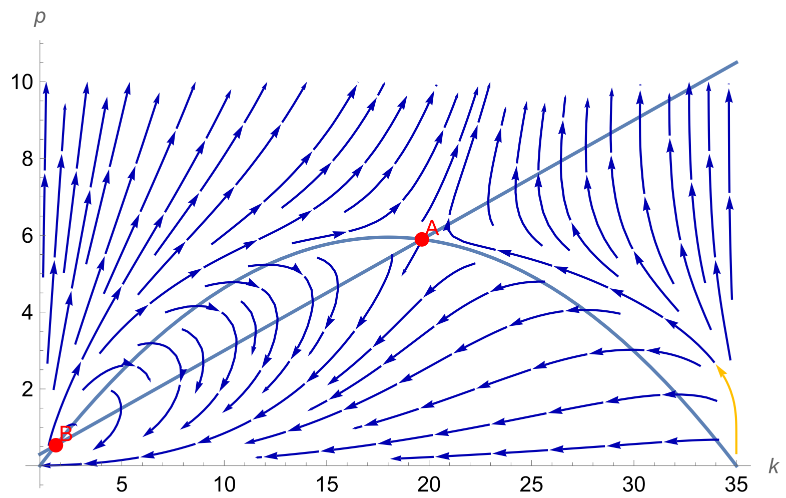
Now, we implement the chosen scenario as indicated in Table 3.

Parameters	b_k	b_c	ρ	n	δ
Values	**35**	**1**	**0.5**	**0.1**	**0.20**

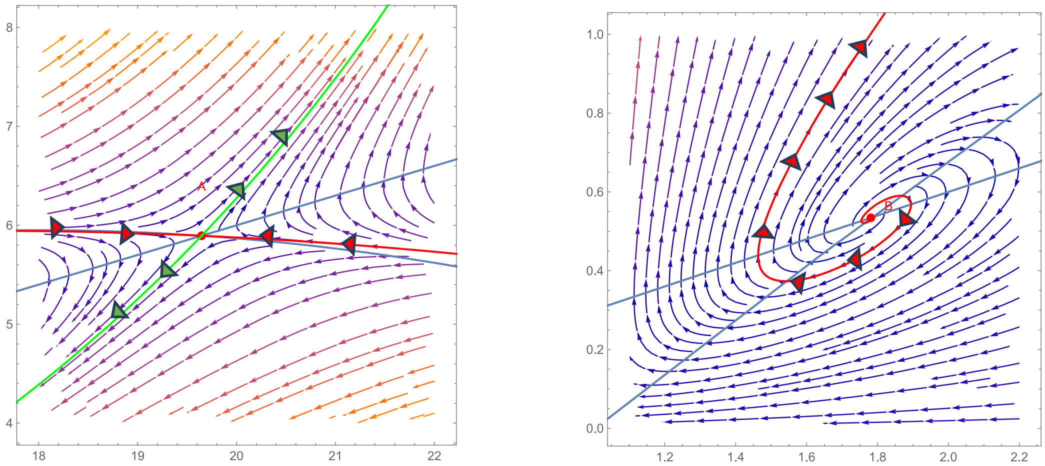
Table 3. Parameter values of the chosen scenario

In this scenario, the discount rate of the objective function is about 0.67 from Table 1. It means that people discount future heavily. This is reasonable under the post war situation because people were just trying to survive day by day during this period and they couldn't think about future. The difference in capital intensity by sector indicates that the capital goods sector was 35 times more capital intensive than the consumption sector. In fact, during this period, the so-called "priority production scheme" was implemented by the Reconstruction Finance Bank, which provided subsidized loans to heavy industries such as coal mining, electricity, steel, and chemical fertilizers, in accordance with the Japanese government's economic policy. This policy clearly further widened the gap in capital intensity.

Numerical analysis yielded the results shown in the three figures in Figure 3 below. It can be seen that there are two solutions, A:(19.6471, 5.8941) and B:(1.7814, 0.5344), indicated by the red dots in Figure 3 a). To
clarify the local dynamics near points A and B, we examined the dynamics in the neighborhood of the two solutions. The results are shown in Figures 3 b) and 3 c).



a)



b) Around the Point A **c) Around the Point B**

*) Stable arms are shown in red and unstable arms in green[6].

Figure 3. Local dynamics

As the theory presented in Section 4 indicates, Point A exhibits saddle-point stability and Point B exhibits an unstable focus. Figure 3 a) shows that every trajectory violates the transversality condition given by equation (10), except that the trajectories converge at point A.

It is important to note the simultaneous existence of saddle point stability and unstable focus points. Global numerical analysis applying the backward shooting method reveals the existence of interesting trajectories, indicated in red in Figure 4. This trajectory starts from the neighborhood of point B, shown in red in Figure 3 b), and eventually converges to the optimal stationary point A. This trajectory is optimal because it satisfies the transversality condition, and exhibits the recovery process of the Japanese economy after World War II. As mentioned before, the initial per capita stock just after the war was near Point B. Due to chaotic economic situations after the war, Japan experienced hyper-inflation as indicated Table 1, especially from 1945 to 1948. However, after Doge Line was implemented in 1949, it ended by 1952. After that the Japanese economy grew steadily under the stable price changes.

[6] The stable arm is obtained by starting from a neighborhood of the OSS and solving the system of differential equations backward. The unstable arm is obtained by solving the system of differential equations forward.

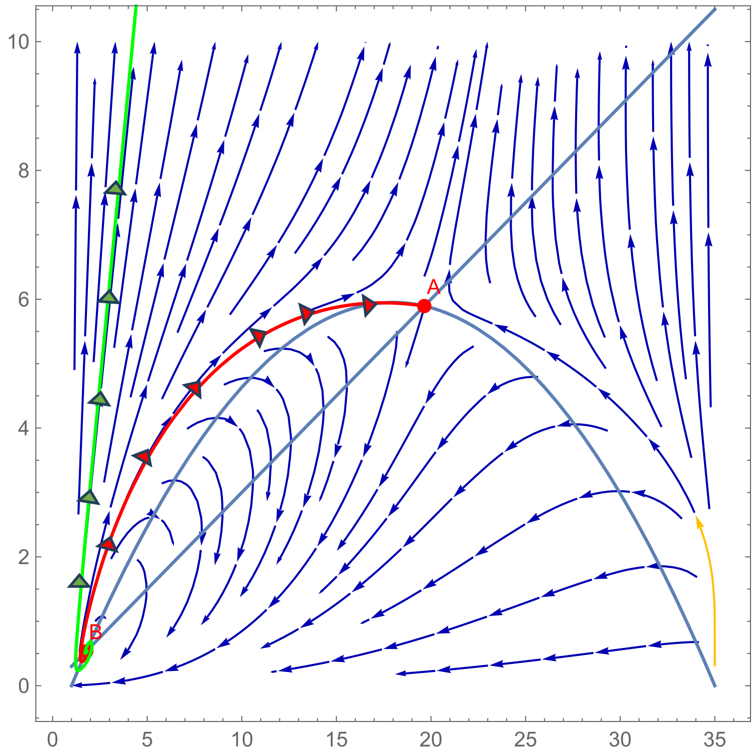


Figure 4. An optimal trajectory

5. Conclusion

Our two main results are follows.

1) According to standard optimal growth theory, there is no stable steady state when capital intensity in the capital goods sector is higher than in the consumer goods sector. However, the Leontief two-sector optimal growth model shows that if capital intensity in the capital goods sector is extremely higher than in the consumer goods sector, and the discount rate and depreciation rate are appropriately given, two different steady states exist: a saddle-point steady state and an unstable steady state.

2) According to the numerical results and their discussion, two miracles occurred during Japan's postwar recovery period. One was that the initial per capita stock remaining at the end of the war happened to be close to the unstable focus point, and the other was that the trajectory eventually converged to a steady-state path as a result of appropriate government policies. In other words, the rapid postwar recovery of the Japanese economy was truly a miracle.

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