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Watanabe, Minoru

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Minoru Watanabe

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Robot tax, unemployment and endogenous fertility in an overlapping generations model

Minoru Watanabe[1,2]

Abstract

Increasing unemployment and declining fertility rates are serious economic issues in many developed countries. This brief article constructs a simple overlapping generations model incorporating involuntary unemployment, fertility choice, and automation capital, with the assumption that automation capital is a perfect substitute for labor inputs. It is shown that robot tax imposed on automation capital improves employment and fertility as well as per capita income in the long run.

JEL Classification: H50, J13, and J51

Keywords: Automation capital, Robot tax, unemployment, endogenous fertility

[1] Hokusei Gakuen University, 2-3-1, Ohyachi-Nishi, Atsubetsu-ku, Sapporo 004-8631, Japan.
E-mail: m-watanabe@hokusei.ac.jp

[2] Research Fellow, Graduate School of Economics, Kobe University

1. Introduction

According to Fanti and Gori (2010) and Wang (2015), many developed countries face a combination of declining fertility and higher unemployment rates. However, few studies have made policy proposals to improve fertility and employment. As noted by Fanti and Gori (2010), unemployment and fertility choices have been discussed separately in previous studies.

The aim of this study is to provide solutions for the declining fertility, and increasing unemployment, rates. Fanti and Gori (2010) constructed a standard overlapping generations model, revealing that imposing a child tax enhances unemployment and fertility. Wang (2015) extended Fanti and Gori (2010) by incorporating social security measures, such as pension benefits and child allowances. Compared with Fanti and Gori (2010) and Wang (2015), this study incorporates two additional variables: automation capital and robot tax. First, automation capital is considered a perfect substitute for labor inputs, in line with Prettner (2019), Gasteiger and Prettner (2022), and Zhang, Palivos, and Liu (2022). Gasteiger and Prettner (2022) noted that automation and its potential impacts have attracted the attention of economists in recent years. Frey and Osborne (2017) estimated that 47% of all jobs in the USA will be replaced with automation in the coming two decades. Hence, automation is a factor that can potentially increase future unemployment. Second, we consider robot tax, which is levied on automation capital in line with Gasteiger and Prettner (2022) and Zhang, Palivos, and Liu (2022).

The combination of low fertility and high unemployment rates is a serious economic issue in many developed countries. This study develops a standard overlapping generations model that incorporates unemployment, fertility choices, and automation. A higher robot tax improves unemployment, fertility, and per capita income in the long run.

The remainder of this paper is organized as follows. Section 2 describes our proposed model, and Section 3 concludes the study.

2. Model

2.1 Households

The framework used in this study is a standard overlapping generations model. The study considers identical households, which experience two periods: young and old. They derive utility from consumption and the number of children. Following Fanti and Gori (2010) and Wang (2015), the utility function is expressed as,

$$\log c_t + \beta \log d_{t+1} + \gamma \log n_t \qquad (1)$$

c_t and d_{t+1} denote consumption in young and old periods, respectively, n_t is the fertility rate, $\beta < 1$ is the discount factor and, $\gamma > 0$ is the desire for children. Suppose N_t is the population size in period t, and the evolution of the population is $N_{t+1} = n_t N_t$.

Households are endowed with one unit of time, which they supply inelastically to the labor market when they are young. The labor market is imperfect because of the presence of a minimum wage. Employed households earn working income and split it into consumption, savings, and childcare in the young period. The budget constraints are expressed as,

$$c_t + s_t + \varepsilon n_t = w_{m,t}(1 - u_t), \tag{2}$$

$$d_{t+1} = R_{t+1} s_t, \tag{3}$$

where s_t is the savings, $\varepsilon > 0$ is the child rearing cost, u_t is the unemployment rate, $w_{m,t}$ is the minimum wage, and R_{t+1} is the gross interest rate. Following Fanti and Gori (2010) and Wang (2015), the unemployment rate is defined as the unemployed time. Optimal allocation is expressed as,

$$\frac{s_t}{n_t} = \frac{\beta\varepsilon}{\gamma}, \tag{4}$$

$$n_t = \frac{\gamma w_{m,t}(1 - u_t)}{(1 + \beta + \gamma)\varepsilon} \tag{5}$$

2.2 Firms

Identical firms use capital and labor to produce final goods in a competitive market. Following Prettner (2019), Gasteiger and Prettner (2022), and Zhang, Palivos, and Liu (2022), we assume two types of capital: traditional and automation capital. Automation capital and labor are perfect substitutes. The production technology is expressed as,

$$Y_t = AK_t^{\alpha}(L_t + P_t)^{1-\alpha},\ A > 0,\ 0 < \alpha < 1 \tag{6}$$

where Y_t is the total output, L_t is the total labor input, K_t is the total traditional physical capital, and P_t is the total automation capital (e.g., robots and artificial intelligence). The total labor input is as follows,

$$L_t = (1 - u_t)N_t \tag{7}$$

Assuming full depreciation, the total profit is expressed as,

$$AK_t^{\alpha}(L_t + P_t)^{1-\alpha} - w_{m,t}L_t - R_t^k K_t - (1 + \tau)R_t^p P_t \tag{8}$$

where R_t^k and R_t^p are the gross rental prices for traditional and automated capital, respectively, and τ is the robot tax, as indicated by Gasteiger and Prettner (2022) and Zhang, Palivos, and Liu (2022). The factor demands are expressed as,

$$w_{m,t} = A(1-\alpha)\left(\frac{k_t}{1-u_t+p_t}\right)^{\alpha}, \tag{9}$$

$$R_t^k = A\alpha\left(\frac{1-u_t+p_t}{k_t}\right)^{1-\alpha}, \tag{10}$$

$$(1+\tau)R_t^p = A(1-\alpha)\left(\frac{k_t}{1-u_t+p_t}\right)^{\alpha} \tag{11}$$

Here, $k_t \equiv K_t/P_t$ and $p_t \equiv P_t/N_t$ are the per capita traditional and automation capital, respectively. Suppose $y_t \equiv Y_t/N_t$, as the per capita output, and $y_t \, as \, y_t = Ak_t^{\alpha}(1-u_t+p_t)^{1-\alpha}$ from Equation (6). If $p_t = 0$, y_t simplifies to $y_t = Ak_t^{\alpha}(1-u_t)^{1-\alpha}$, and this per capita production technology is consistent with that of Fanti and Gori (2010) and Wang (2015). As noted above, the economy has a minimum wage. We denote $w_{c,t}$ as the competitive wage that satisfies $u_t = 0$. Following Irmen and Wigger (2006) and Fanti and Gori (2011), we assume that minimum wage is proportional to competitive wage. Thus, the minimum wage is expressed as,

$$w_{m,t} = \mu w_{c,t} \tag{12}$$

where $\mu > 1$ is a constant markup. Substituting $u_t = 0$ into Equation (9), the competitive wage is expressed as

$$w_{c,t} = A(1-\alpha)\left(\frac{k_t}{1+p_t}\right)^{\alpha} \tag{13}$$

From Equations (9) and (13), the relative wage between the minimum and competitive wage, under given k_t and p_t, is expressed as

$$\frac{w_{m,t}}{w_{c,t}} = \left(\frac{1+p_t}{1-u_t+p_t}\right)^{\alpha} \tag{14}$$

By combining Equations (12) and (14), we obtain the following unemployment rate:

$$u_t = (1+p_t)\left(1-\mu^{\frac{-1}{\alpha}}\right) \tag{15}$$

In this equation, an increase in automation capital increases unemployment. Furthermore, unemployment increases with the markup. If we omit automation capital, that is, $p_t = 0$, equilibrium unemployment rate is expressed by $u_t = 1 - \mu^{-1/\alpha}$ from Equation (15). This rate is consistent with that reported by Fanti and Gori (2011). Fanti and Gori (2010) and Wang (2015) assumed that a constant minimum wage leads to unemployment. In contrast, Zhang, Palivos, and Liu (2022) assume that matching frictions cause unemployment.

According to Gasteiger and Prettner (2022) and Zhang, Palivos, and Liu (2022), the no-arbitrage condition between traditional and automation capital is expressed as follows:

$$R_t^k = R_t^p \tag{16}$$

From Equations (10), (11), and (16), we derive,

$$k_t = \frac{\alpha(1+\tau)[1-u_t+p_t]}{1-\alpha} \tag{17}$$

Note that $R_t^k = R_t^p = R_t$ holds in equilibrium.

2.3 Government

The government levies taxes on automation capital to finance consumption with a balanced budget. Thus, we obtain:

$$\tau R_t^p P_t = G_t, \tag{18}$$

where G_t is the government consumption. We assume that G_t does not contribute to welfare or productivity. The present study does not incorporate social security, such as unemployment benefits. If all young households are unemployed, savings and capital accumulation would be impossible. Thus, the economy is unsustainable if $u_t = 1$ holds.

2.4 Equilibrium

The dynamic in this economy is expressed as:

$$k_{t+1} = \frac{s_t}{n_t} - p_{t+1} \tag{19}$$

From Equations (4), (9), (10), (11), (15), (16), (17) and (19), we obtain the following long run per capita automation capital:

$$p = \begin{cases} \dfrac{(1-\alpha)\frac{\varepsilon\beta}{\gamma} - \alpha(1+\tau)\mu^{\frac{-1}{\alpha}}}{1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}} & if \quad (1-\alpha)\dfrac{\varepsilon\beta}{\gamma} > \alpha(1+\tau)\mu^{\frac{-1}{\alpha}}, \\ 0 & if \quad (1-\alpha)\dfrac{\varepsilon\beta}{\gamma} \leq \alpha(1+\tau)\mu^{\frac{-1}{\alpha}} \end{cases} \tag{20}$$

We assume $(1-\alpha)\frac{\varepsilon\beta}{\gamma} > \alpha(1+\tau)\mu^{\frac{-1}{\alpha}}$ to ensure the interior solutions of automation capital in the rest of paper. Differentiating Equation (20) with respect to τ, we have:

$$\frac{dp}{d\tau} = \frac{-\alpha(1-\alpha)\mu^{\frac{-1}{\alpha}}\left(1+\frac{\varepsilon\beta}{\gamma}\right)}{\left[1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}\right]^2} < 0 \tag{21}$$

A higher robot tax reduces the long-run per capita automation capital. This is because robot taxation causes firms to switch from automation to traditional physical capital, as noted by Gasteiger and Prettner (2022). From Equations (15) and

(20), the equilibrium unemployment rate under the generic form of robot tax is expressed as,

$$u = u\{p(\tau)\} \tag{22}$$

Differentiating this equation with respect to τ, we obtain:

$$\frac{du}{d\tau} = \underbrace{\overbrace{\frac{du}{dp}}^{+}\overbrace{\frac{dp}{d\tau}}^{-}}_{-}. \tag{23}$$

If automation capital exists in the long run, a higher robot tax reduces unemployment, which can be explained as follows: A higher robot tax reduces automation capital, which raises the relative wage between minimum and competitive wages, as shown in Equation (14). Recall that we assume a constant relative wage in Equation (12). Unemployment declines with a higher robot tax, in order to maintain a constant relative wage.

By substituting Equation (20) into Equation (15), we obtain the following unemployment rate:

$$u = \frac{(1-\alpha)\left(1+\frac{\varepsilon\beta}{\gamma}\right)\left(1-\mu^{\frac{-1}{\alpha}}\right)}{1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}} > 0 \tag{24}$$

As previously noted, the economy is unsustainable if $u = 1$. To ensure $u < 1$, the following conditions are imposed[3]:

$$(1-\alpha)\left(1+\frac{\varepsilon\beta}{\gamma}\right)\left(1-\mu^{\frac{-1}{\alpha}}\right) < 1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}} \tag{25}$$

Differentiating Equation (24) with respect to τ, we obtain:

$$\frac{du}{d\tau} = -\frac{\alpha(1-\alpha)\left(1+\frac{\varepsilon\beta}{\gamma}\right)\left(1-\mu^{\frac{-1}{\alpha}}\right)\mu^{\frac{-1}{\alpha}}}{\left[1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}\right]^2} < 0 \tag{26}$$

Next, we investigate how robot taxation affects minimum wage levels. From Equations (4), (15), (19), and (20), we obtain the long-run per-capita traditional capital as

$$k = \frac{\alpha\left(1+\frac{\varepsilon\beta}{\gamma}\right)(1+\tau)\mu^{\frac{-1}{\alpha}}}{1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}} \tag{27}$$

From Equation (27), we have:

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[3] Kunze and Schuppert (2010) introduced parameter restriction to ensure positive unemployment rate.

$$\frac{dk}{d\tau} = \frac{\alpha(1-\alpha)\left(1+\frac{\varepsilon\beta}{\gamma}\right)(1+\tau)\mu^{\frac{-1}{\alpha}}}{\left[1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}\right]^2} > 0 \tag{28}$$

As previously explained, a higher robot tax leads to a shift from automation to traditional capital. Thus, an increase in robot tax promotes long-term per capita traditional capital. Substituting Equation (17) into Equation (9), we obtain the following long-run minimum wage[4]:

$$w_m = A(1-\alpha)\left(\frac{k}{1-u+p}\right)^{\alpha} = A\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+\tau)^{\alpha} \tag{29}$$

Differentiating this equation with respect to τ, we obtain:

$$\frac{dw_m}{d\tau} = A\alpha^{1+\alpha}(1-\alpha)^{1-\alpha}(1+\tau)^{\alpha-1} > 0 \tag{30}$$

An increase in robot taxes reduces automation capital and, hence, increases traditional physical capital, which increases wages. By contrast, an increase in the robot tax improves employment, as shown in Equation (26). This effect moderates the wage levels. The first effect dominates the second. Thus, a higher robot tax increases wages compared with a lower one. Note that Zhang, Palivos, and Liu (2022) demonstrated that robot taxation increases both, wages and employment.

We now analyze the impact of a robot tax on long-term fertility. Based on Equations (5), (24), and (29), long-run fertility under the generic form of the robot tax is expressed as,

$$n = n\{u(\tau), w_m(\tau)\} \tag{31}$$

From Equations (5), (26), and (30), we derive:

$$\frac{dn}{d\tau} = \underbrace{\overbrace{\frac{\partial n}{\partial u}}^{-}\overbrace{\frac{\partial u}{\partial \tau}}^{-}}_{+} + \underbrace{\overbrace{\frac{\partial n}{\partial w_m}}^{+}\overbrace{\frac{\partial w_m}{\partial \tau}}^{+}}_{+} \tag{32}$$

We demonstrate that an increase in robot taxation increases employment and wages. Therefore, a higher robot tax rate improves fertility. Substituting Equations (24) and (29) into Equation (5), long-run fertility is expressed as,

$$n = \frac{\gamma w_m(1-u)}{(1+\beta+\gamma)\varepsilon}, \tag{33}$$

[4] Note that the long-run competitive wage, that is w_c, is denoted by $w_c = \frac{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+\tau)^{\alpha}}{\mu}$ from Equations (12) and (27).

$$= \frac{\gamma A \alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+\tau)^{\alpha}}{(1+\beta+\gamma)\varepsilon}\left\{1 - \frac{(1-\alpha)\left(1+\frac{\varepsilon\beta}{\gamma}\right)\left(1-\mu^{\frac{-1}{\alpha}}\right)}{1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}}\right\}.$$

Recall that we assume $u < 1$ in Equation (25). Therefore, the sign in parentheses in Equation (33) is positive. By differentiating this equation with respect to τ, we obtain,

$$\frac{dn}{d\tau} = \frac{\gamma A \alpha^{1+\alpha}(1-\alpha)^{1-\alpha}(1+\tau)^{\alpha-1}}{(1+\beta+\gamma)\varepsilon}\left\{1 - \frac{(1-\alpha)\left(1+\frac{\varepsilon\beta}{\gamma}\right)\left(1-\mu^{\frac{-1}{\alpha}}\right)}{1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}}\right\}$$
$$+ \frac{\gamma A \alpha^{1+\alpha}(1-\alpha)^{2-\alpha}(1+\tau)^{\alpha}\left(1+\frac{\varepsilon\beta}{\gamma}\right)\left(1-\mu^{\frac{-1}{\alpha}}\right)\mu^{\frac{-1}{\alpha}}}{(1+\beta+\gamma)\varepsilon\left[1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}\right]^2} > 0 \tag{34}$$

Overall, we obtain the following proposition:

Proposition 1

A rise in robot tax improves not only employment, but also fertility.

Finally, we investigate how robot taxes affect per capita output. From Equation (6), the long-run per capita output under the generic form of the robot tax is expressed as,

$$y = y\{k(\tau), u(\tau), p(\tau)\} \tag{35}$$

Differentiating this equation with τ, we derive:

$$\frac{dy}{d\tau} = \underbrace{\overbrace{\frac{\partial y}{\partial k}}^{+}\overbrace{\frac{\partial k}{\partial \tau}}^{+}}_{+} + \underbrace{\overbrace{\frac{\partial y}{\partial u}}^{-}\overbrace{\frac{\partial u}{\partial \tau}}^{-}}_{+} + \underbrace{\overbrace{\frac{\partial y}{\partial p}}^{+}\overbrace{\frac{\partial p}{\partial \tau}}^{-}}_{-} \tag{36}$$

An increase in the robot tax has opposite effects on the long-run per capita output. First, a higher robot tax promotes both, traditional capital accumulation and employment, contributing to an increase in per capita output. Second, a higher robot tax reduces automation capital, thereby reducing long-run per capita output. If the former effect dominates the latter, a higher robot tax increases long-run per capita output. Using Equations. (17) and (27), y can be expressed as,

$$y = Ak^{\alpha}(1-u+p)^{1-\alpha},$$
$$= \frac{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(1+\frac{\varepsilon\beta}{\gamma}\right)(1+\tau)^{\alpha}\mu^{\frac{-1}{\alpha}}}{1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}} \tag{37}$$

Differentiating Equation (35) with respect to τ, we obtain:

$$\frac{dy}{d\tau}=\frac{A\alpha^{1+\alpha}(1-\alpha)^{2-\alpha}\left(1+\frac{\varepsilon\beta}{\gamma}\right)\mu^{\frac{-1}{\alpha}}(1+\tau)^{\alpha-1}\left[1-(1+\tau)\mu^{\frac{-1}{\alpha}}\right]}{\left[1-\alpha+\alpha(1+\tau)\mu^{\frac{-1}{\alpha}}\right]^2} \tag{38}$$

An increase in the robot tax increases per capita output if $1>(1+\tau)\mu^{\frac{-1}{\alpha}}$ holds. Therefore, we obtain the following proposition.

Proposition 2

If $1>(1+\tau)\mu^{\frac{-1}{\alpha}}$ holds, the long run per capita output increases with a higher robot tax.

Figures 1-4 present numerical examples[5]. We investigate the impact of τ on per capita automation capital, unemployment, fertility and long run per capita outputs, respectively. The parameters are set as follows: $\alpha=0.33, \beta=0.35, \gamma=0.1, \varepsilon=0.2, \mu=1.03$, and $A=16$.

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[Figure 1 here]
[Figure 2 here]
[Figure 3 here]
[Figure 4 here]

These figures indicate that a higher robot tax promotes not only employment and fertility, but also per capita output in the long run, if the robot tax is sufficiently small.

3. Conclusion

A combination of a higher unemployment rate and a decline in the fertility rate has been reported in many developed countries. This study constructs a simple overlapping generations model that incorporates automation and robot taxes. Our results demonstrate that robot taxation is effective in improving unemployment, fertility, and per-capita income.

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[5] Note that the equilibrium gross interest rate is denoted by $R=A\alpha\left[\frac{1-\alpha}{\alpha(1+\tau)}\right]^{1-\alpha}$. We assume A to be large enough to ensure that $R>1$.

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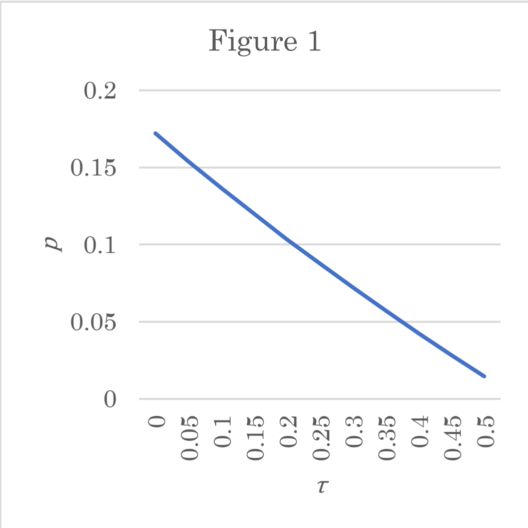
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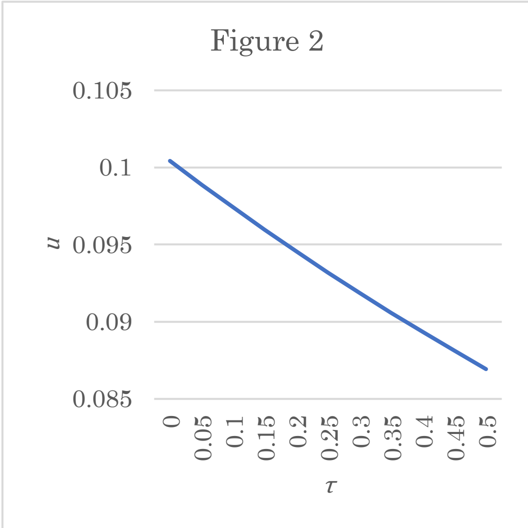
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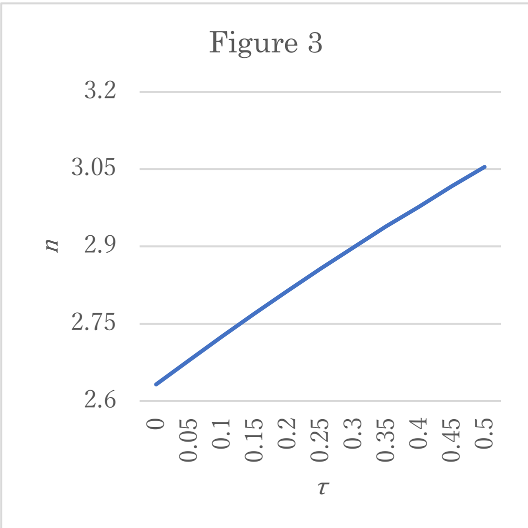
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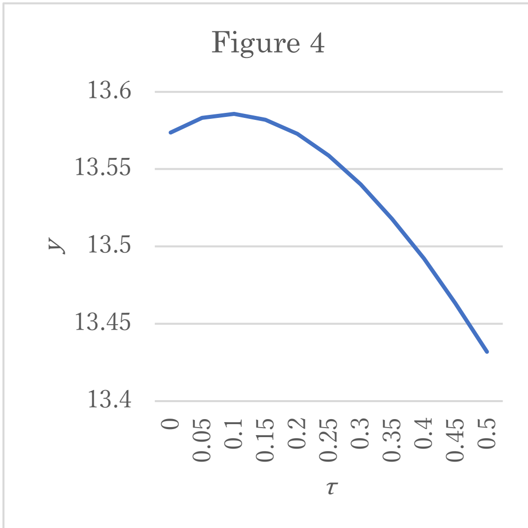
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