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Differential Tail Dependence between Crude Oil and Forex Markets in Oil-Importing and Oil-Exporting Countries during Recent Crisis Periods

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Abstract: The relationship between foreign exchange rates and crude oil prices holds significant importance in comprehending the dynamics of oil markets and their implications for diverse economies. This study utilizes the time-varying copula to examine the interrelationships between foreign exchange rates (FX) and West Texas Intermediate (WTI) crude oil prices, with a focus on time-varying tail dependence and time-varying linear correlation. We found that the tail dependence between foreign exchange rates (FX) and WTI crude oil prices is higher for oil-exporting countries compared to oil-importing countries. Moreover, the COVID-19 pandemic has further amplified the tail dependence for oil-exporting countries while simultaneously increasing the correlation of FXs–WTI for oil-importing countries. However, the 2022 Russian–Ukrainian conflict has exerted a significant receding effect on both the tail dependence and linear correlation of FXs–WTI, reaching or even surpassing levels comparable to those witnessed during the 2008 financial crisis. These results facilitate policymakers, investors, and market participants in making well-informed decisions and developing effective risk management strategies.

Keywords: foreign exchange rate; crude oil; dependence structure; tail dependence; COVID-19; Russian–Ukrainian war; time-varying copula; oil importing; oil exporting; value at risk; expected shortfall; risk hedge

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1. Introduction

The COVID-19 pandemic, which emerged in 2020 has had a profound influence on global economic development and the various world markets. One notable impact has been the significant turbulence in oil prices as a direct consequence of the pandemic, a fact that has greatly affected financial markets, including the foreign exchange (FX) market.

Numerous academic investigations have explored the intricate relationship between oil prices and foreign exchange rates. Noteworthy scholars such as [1–21] have employed a wide variety of methodologies to investigate this subject.

For instance, the work of Basher et al. [3] employed the Markov switching approach to analyze the impact of crude oil shocks on foreign exchange rates. Chang [6] utilized a symmetrized copula framework to examine the correlations between oil and foreign exchange rate returns, revealing the existence of a positive and symmetrical relationship. Wu et al. [21] employed copula-based GARCH models to evaluate the economic value of co-movement between oil prices and foreign exchange rates, substantiating the existence of a tail dependence structure.

Turhan et al. [18] adopted a consistent dynamic conditional correlation model that provides compelling evidence of an intensified correlation between oil prices and foreign exchange rates during the 2000–2013 period, resulting in the detection of a significant negative correlation. Reboredo and Rivera-Castro [15] used wavelet multi-resolution analysis and found strong evidence of contagion and negative dependence between foreign exchange rates and West Texas Intermediate (WTI) crude oil prices following the 2008 Global Financial Crisis, with independence observed during the pre-crisis period. Villarreal-Samaniego [20] examined the correlation between oil prices and foreign exchange rates in five emerging economies during the first quarter of 2020, revealing a negative correlation between the foreign exchange rates of these economies and oil prices. Baek [2] identified the COVID-19 pandemic as a crucial factor influencing the asymmetric dependence of oil prices on the South Korean won exchange rate against the U.S. dollar, both in the short and long term.

However, in addition to the significance of relationship analysis, analyzing tail dependence, especially time-varying tail dependence holds significant importance for a variety of reasons. Firstly, in risk management, understanding tail dependence allows market participants to quantify joint risks during extreme market conditions, aiding in the development of effective strategies and ensuring portfolio or financial system resilience. Secondly, tail dependence analysis assists investors and portfolio managers in evaluating the diversification benefits of different assets and contributes to systemic risk assessment in financial systems and interconnected markets. By comprehending the tail dependence relationship, they can determine whether crude oil prices offer distinct risk–return characteristics that can enhance portfolio diversification or serve as a hedge against extreme events in the foreign exchange market. Thirdly, tail dependence analysis assists economists and policymakers in comprehending the transmission channels via which changes in crude oil prices impact exchange rates. Consequently, it informs policy decisions pertaining to exchange rate management, energy policy, and macroeconomic stability.

Many researchers have endeavored to differentiate between oil exporters and oil importers when exploring the relationship between oil prices and foreign exchange rates. Noteworthy contributions by scholars such as [12,22–30] have shed light on this distinction. For instance, the research of Lizardo and Mollick [26] employed vector autoregressive methodologies to analyze the correlation between oil prices and US foreign exchange rates, revealing a significant appreciation of the currencies of oil-exporting countries in response to oil price increases. Chen and Chen [23] utilized Johansen's panel cointegration technique, unveiling a positive relationship between oil prices and foreign exchange rates in oil-importing countries. Reboredo [27] adopted copula models, showcasing stronger connections between foreign exchange rates and oil prices in oil exporters, with the strength of their relationship undergoing a notable increase after the 2008 financial crisis. Similar to the research of Reboredo [27], Beckmann et al. [22] applied static and dynamic copula methodologies using a sample from 2003 to 2013, yielding results that indicate that the variation in dependence between oil prices and foreign exchange rates varies over time, with tail dependence intensifying during extreme events. Yang et al. [30] discovered a negative link between the foreign exchange rates of oil-exporting countries and oil prices. Huang et al. [24] focused on the dynamic link between crude oil prices and foreign exchange rates under the distinction between fully floating and managed floating regimes of foreign exchange rates.

Additionally, reviewing the previous literature provides a comprehensive overview of the diverse array of scholarly investigations that have employed time-varying copula methodologies to effectively capture evolving tail dependence across various domains. The studies conducted by [31–38] contribute to our understanding of this tail dependence estimation approach. Rajwani et al. [31] embarked on an investigation into equity market contagion, examining the interconnections between Asian and American stock markets. Using a multivariate dynamic framework, Goel et al. [32] explored the contagion effect by analyzing the shifting dependence structure among prominent stock markets, employing a multivariate dynamic approach. Ji et al. [33] utilized six time-varying copula models to investigate the dynamic reliance between WTI crude oil and the currency rates of the United States and China, taking into account structural changes in dependence. Mensah et al. [34] investigated the tail dependence structure and evolution for three foreign exchange rates against the GHS by analyzing daily foreign exchange data. Hanif et al. [36] explored the potential nonlinear, symmetric, or asymmetric reliance dynamics of US and Canadian large-capitalized energy equity portfolios. Tiwari et al. [35] delved into the dependence structure and time-varying correlations between gold and oil prices. Echaust et al.'s [37] investigation examined the relationship between variations in the crude oil volatility index (OVX) and the extreme returns of WTI crude oil prices, as well as the accuracy of the OVX's estimation of the value at risk for crude oil.

However, in contrast to the previous literature, this study notably extends and advances prior research by delving deeper into the analysis of time-varying tail dependence between oil prices and exchange rates. Our investigation spans a more comprehensive time frame than previous studies, encompassing the period from 2004 to March 2023. This expanded scope allows us to capture the impact of significant events such as the COVID-19 pandemic and the 2022 Russian–Ukrainian conflict. Furthermore, this study places particular emphasis on discerning the distinguishing characteristics between oil exporters and importers. Via meticulous comparative analysis, we aim to shed light on the nuanced differences in the tail dependence patterns exhibited by these two groups of countries.

Therefore, this study aims to examine the dependence between foreign exchange rates and WTI prices for six countries, categorized as either crude oil importers or exporters, utilizing four copula models to estimate both constant and dynamic dependence. In order to capture the evolving nature of dependence, the copula models are combined with the generalized autoregressive score (GAS) model. The primary objective is to investigate the time-varying tail dependence and linear correlation between foreign exchange rates and WTI prices, while our secondary aim is to assess potential differences in FX–WTI tail dependence between oil-exporting and oil-importing countries. Notably, the study seeks to explore these relationships in the context of significant events such as the outbreak of the COVID-19 pandemic in 2020 and the Russian–Ukrainian conflict in 2022. Finally, this study employs the obtained estimation results to evaluate portfolio risk management, specifically focusing on the estimation of value at risk (VaR) and expected shortfall (ES). By utilizing the findings from our analysis, we aim to provide intriguing insights for risk measures in quantifying and managing portfolio risk in the context of foreign exchange rates and oil prices, which are crucial for market participants, portfolio managers, and risk analysts seeking to optimize their risk management strategies and make informed decisions regarding portfolio allocation and hedging strategies.

The novelty of this study lies in the fact that it is the first to use the copula models in combination with generalized autoregressive score approaches to investigate the time-varying tail dependence and dynamic correlation between the foreign exchange rates and crude oil prices, taking into consideration the difference between oil-exporting and oil-importing countries. This study provides insights into the dynamics of FXs–WTI tail dependence and time-varying linear correlation, emphasizing the role of global events and market conditions in shaping these relationships. The observed peaks in tail dependence during periods of oil market turbulence highlight the sensitivity of foreign exchange rates to fluctuations in oil prices and various other factors, including geopolitical tensions and disruptions in oil production. The higher tail dependence of oil-exporting countries underscores the economic significance of oil revenues and their vulnerability to economic uncertainty. The COVID-19 pandemic and geopolitical events have further acted to amplify the complexities of these interdependencies. By enhancing our understanding of these dynamics, policymakers, investors, and market participants can make more informed decisions and develop effective risk management strategies in the realm of oil markets.

The primary findings of this study are summarized as follows: Firstly, the analysis of FX–WTI tail dependence reveals prominent peaks in the tail dependence of oil-exporting countries during two periods: 2012–2013 and 2015–2016. These peaks coincide with significant events such as global oil price fluctuations, geopolitical tensions, disruptions in oil production, the Eurozone debt crisis, and OPEC's production decisions. Secondly, the tail dependence between foreign exchange rates and WTI prices is found to be higher for oil-exporting countries compared to that which exists for oil-importing countries. This implies that the foreign exchange rates of oil-exporting countries are more susceptible to declines in oil prices. Thirdly, during the COVID-19 pandemic from 2020 to 2022, notable variations were observed in the FX–WTI tail correlation. Oil-exporting countries exhibited significant increases in tail dependence, particularly during the oil price collapses in 2020, indicating a higher probability of simultaneous extreme declines in foreign exchange rates and crude oil prices. Lastly, the analysis of time-varying linear correlation reveals a gradual increase in the correlation between foreign exchange rates and crude oil prices after the occurrence of the 2008 financial crisis, with this relationship reaching its peak during the period of 2010–2012. However, a significant decline in correlation was observed during the oil price downturn from 2014 to 2016, followed by a rapid recovery thereafter. All three oil-importing countries exhibited an upward trend in their FX–WTI linear correlations during this unprecedented global financial crisis, i.e., the COVID-19 pandemic, suggesting an increased alignment between their FX–WTI correlations. Nonetheless, the Russian–Ukrainian conflict in 2022 resulted in significant collapses in FX–WTI correlations for both oil-exporting and oil-importing countries, signifying a temporary interruption in their dependence on WTI. Additionally, most correlations during this geopolitical risk event shifted towards exhibiting negative values.

The subsequent sections of this paper are organized as follows: Section 2 elaborates on the data employed and introduces the methodology adopted in this study. Section 3 presents and analyzes the empirical findings. Lastly, Section 4 offers concluding remarks, summarizing the key insights and implications of the research.

2. Data and Methodology

2.1. Data

The primary objective of this research endeavor is to scrutinize the evolutions in linear dependence and tail correlation between foreign exchange rates and crude oil prices. Furthermore, this study aims to explore the dissimilarities in the oil–FX dependencies between the predominant oil-exporting and oil-importing countries.

To achieve these goals, this study uses daily foreign exchange rates collected from the online Invest.com database, covering the period from 6 January 2004 to 1 March 2023, resulting in a total of 4838 daily observations. Based on the lists of countries provided by oil importers and exporters in 2022 as reported on Wikipedia and in accordance with the approach of Yang et al. [30], we focus on three oil-importing countries and three oil-exporting countries. Specifically, Russia, Canada, and Mexico are selected to represent the oil-exporting countries, while the European Union (EU), India, and South Korea are chosen as the oil-importing countries.

WTI (West Texas Intermediate) holds a paramount position as a benchmark for oil prices due to its immense significance in the United States and North American oil markets, as well as its widespread recognition across the globe. Given our utilization of foreign exchange rates against the U.S. dollar as an integral component of our investigation, the adoption of WTI as a key factor in our research becomes imperative. The dataset consists of the WTI crude oil spot price, the Russian ruble (RUB), the Mexican peso (MXN), the Canadian dollar (CAD), the Euro (EUR), the Indian rupee (INR), and the South Korean won (KRW) against the U.S. dollar. The daily returns are computed using the first-order logarithmic difference based on the collected data. Table 1 offers an elaborate account of the descriptive statistics for the six foreign exchange returns and WTI crude oil returns.

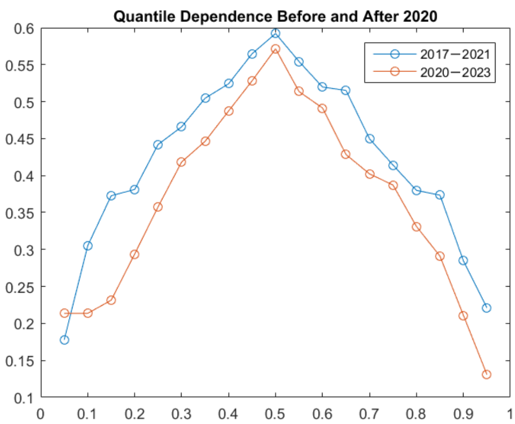
Table 1. Descriptive statistics of the returns.

	Mean	Variance	Skewness	Kurtosis	Correlation with WTI (Linear/Rank)		JB	p Value
RUB	−0.0200	1.0929	−2.6087	65.5400	0.2600	0.3358	781,290	0.0000
MXN	−0.0109	0.7799	−0.9215	14.4520	0.2244	0.2329	26,958	0.0000
CAD	−0.0014	0.5757	−0.1378	5.7763	0.3717	0.3982	1549	0.0000
EUR	−0.0039	0.5897	0.0096	5.2078	0.1632	0.1945	967	0.0000
INR	−0.0123	0.4442	−0.1516	9.5251	0.1356	0.1401	8422	0.0000
KRW	−0.0028	0.6797	0.0324	35.6190	0.1455	0.1578	210,280	0.0000
WTI	0.0132	2.7639	−1.1095	36.8600	-	-	230,720	0.0000

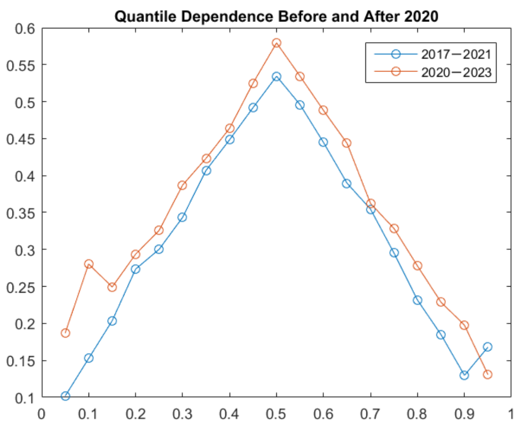
Notes: JB [39] stands for the test of normality, and the p value is the probability value of the Jarque-Bera test. RUB, MXN, CAD, EUR, INR, KRW, and WTI denote the Russian ruble, Mexican peso, Canadian dollar, Euro, Indian rupee, South Korean won, and Crude Oil West Texas Intermediate (WTI) Cushing (US dollar/bbl), respectively. The RUB, MXN, and CAD are from oil-exporting countries; the EUR, INR, and KRW are from oil-importing countries.

Analyzing the correlation with WTI (linear or rank) values, we observe that the oil-exporting countries generally exhibit higher constant correlations with WTI compared to the oil-importing countries during the period of 2004–2023. The Jarque–Bera test [39] results indicate that all considered p values are significant at 1%, suggesting that none of the returns follow a significant non-normal distribution.

In this study, we employ quantile regression to investigate whether there were any changes in the dependence structure between foreign exchange rates and crude oil prices. As shown in Figure 1, with the exception of the Russian ruble (RUB), the currencies of both oil-exporting and oil-importing countries have broadly demonstrated an overall increase in interdependence following the outbreak of the COVID-19 pandemic in 2020 compared to the preceding period of 2017–2020. Notably, there was a discernible shift in lower tail dependence across all countries after 2020. This observation implies a heightened synchronization or interconnectedness of variables during periods of extreme market conditions. Furthermore, our analysis uncovered significant shifts in both the magnitude and structure of dependence. The period spanning the 2020–2021 COVID-19 pandemic was characterized by an elevated level of dependency, along with a more asymmetric and pronounced lower-tail dependency structure.

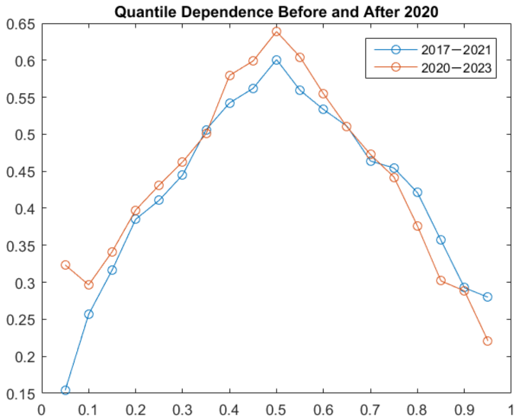


(**a**) RUB–WTI

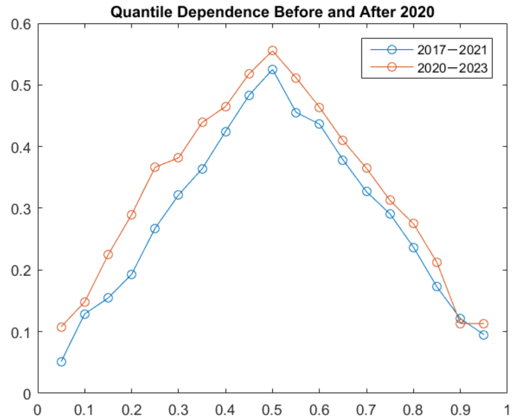


(**b**) MXN–WTI

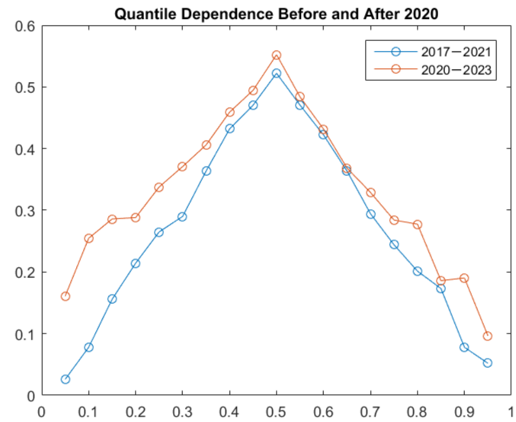
Figure 1. *Cont.*



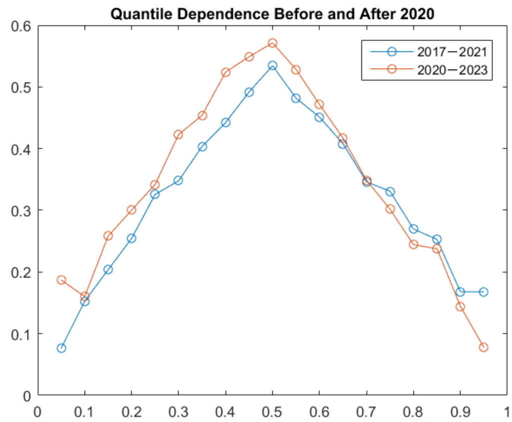
(**c**) CAD–WTI



(**d**) EUR–WTI



(**e**) INR–WTI



(**f**) KRW–WTI

Figure 1. Quantile dependence between the returns of FXs–WTI before and after 2020. Notes: (**a**–**f**) represent the quantile dependence before and after 2020 for RUB–WTI, MXN–WTI, CAD–WTI, EUR–WTI, INR–WTI, and KRW–WTI, respectively. RUB, MXN, CAD, EUR, INR, KRW, and WTI denote the Russian ruble, Mexican peso, Canadian dollar, Euro, Indian rupee, South Korean won, and Crude Oil West Texas Intermediate (WTI) Cushing (US dollar/bbl), respectively. The RUB, MXN, and CAD are from oil-exporting countries; the EUR, INR, and KRW are from oil-importing countries.

2.2. Methodology

2.2.1. Models for Marginal Distributions

The first task in the estimation of the dependence model is the construction of marginal distribution models. Firstly, this study considers model returns by utilizing the ARMA-GJR-GARCH model with a conditional mean and variance structure. This is expressed in Equations (1) and (2) below. The reasons for choosing the GJR-GARCH model are that it offers a leverage effect due to its indicator function and that it accounts for fat-tail distribution better than the GARCH model, both factors that could help to enhance estimation accuracy when applied to financial time series [40,41]. The orders for the ARMA model and the GJR-GARCH (1, 1) model are selected by the BIC (Bayesian information criterion). Y_t denotes the daily returns of a given variable, ε_t represents the return shocks, and σ_t stands for the square root of the conditional variance:

$$Y_t = \phi_0 + \Sigma_{i=1}^{p} \phi_i Y_{t-i} + \Sigma_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t,\ \varepsilon_t = \sigma_t \eta_t,\ \eta_t \sim F(0,1), \tag{1}$$

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + (\alpha + \delta I_{t-1}) \varepsilon_{t-1}^2, \tag{2}$$

$$\textit{where the indicator function},\ I_{t-1} = \begin{cases} 0\ if \varepsilon_{t-1} \geq 0 \\ 1\ if \varepsilon_{t-1} < 0 \end{cases}. \quad (3)$$

Secondly, we calculate the estimated standardized residuals $\hat{\eta}_t$ using the residuals obtained via the conditional mean model divided by the square root of the conditional variance $\hat{\sigma}_t$.

Thirdly, before using the standardized residuals in the copula model's estimation to explore the dependence, the practitioner must transform the standardized residuals into a uniform distribution (0, 1). Following Patton [42], this study considers parametric and semiparametric models to construct the distribution of standardized residuals for each variable. For the parametric model, instead of using the widely applied Student's t-distribution, we utilize the flexible skewed t-distribution introduced by Hansen [43] (see more detailed results for this distribution in Jondeau and Rockinger [44]), which involves the asymmetrical skewness that cannot be captured by the Student's t distribution and also take into consideration the fatted tail. In many real-world scenarios, financial or economic data often exhibit skewness and excess kurtosis, which deviate from the assumptions of Student's t distribution. The skewed t distribution provides a more flexible and realistic framework for modeling such data, leading to improved goodness-of-fit. The skewed t distribution also allows for the modeling of asymmetric or skewed distributions of residuals. This can be useful when dealing with data that exhibit significant skewness that cannot be adequately captured by the symmetric nature of the Student's t distribution. The distribution under consideration encompasses two crucial parameters governing its shape, providing insights into its asymmetry and tail behavior. The first parameter, known as the skewness parameter λ, assumes values within the interval of $\lambda \in (-1, 1)$ and regulates the degree of asymmetry observed in the distribution. The second parameter, denoted as the degrees of freedom parameter ν, assumes values within the range of $\nu \in (2, \infty)$ and regulates the thickness or heaviness of the tails exhibited by the distribution. With respect to the semiparametric model, this study adopts the empirical distribution function (EDF) for estimating the $\hat{F}(\eta)$:

$$\hat{F}(\eta) \equiv \frac{1}{T+1} \sum\nolimits_{t=1}^{T} 1\{\hat{\eta}_t \leq \eta\}. \quad (4)$$

2.2.2. Constant Copula Models

According to Sklar's [45] theory, any joint distribution of n random variables can be decomposed into two parts to describe the dependence between n random variables: n marginal distributions for each variable, which describe the randomness of each variable, and copula function C, which maps the univariate marginal distributions to the joint distribution.

There are various copula models that have been widely used to capture the dependency structures. In general, there are two major types of bivariate copula models: elliptical families, such as the Gaussian and Student's t copula models, and Archimedean families, e.g., the Gumbel and Clayton copula models. This study uses the Gaussian, Clayton, Student's t, and rotated Gumbel constant copula models to estimate the unconditional dependence. The Gaussian (normal) copula model can be used to capture the correlation coefficients in order to compare the degrees of correlation in each period rather than the tail dependence, performing well as the benchmark model. Because the normal distribution could not detect tail dependence, we considered using the fat-tailed Student's t distribution. The Clayton and the rotated Gumbel copula models can capture the tail dependence in the lower tail, which means when the distribution of two random variables follows these two specific copulas, it indicates a substantial likelihood that the two variables exhibit a simultaneous decrease in the same direction. Owing to this feature, we chose the Clayton and the rotated Gumbel copula models to capture the changes in the dependence structure of joint declines. Typically, financial asset returns are more likely to have joint negative extremes (sharp declines) than joint positive extremes (rapid surges).

Assuming that u and v are transformed into a uniform distribution, the copula models are elaborated as follows:

The normal copula function is expressed by Equation (8), where δ denotes the correlation coefficient between random variables, $\phi(u)$ and $\phi(v)$ are the standard normal distribution, and θ corresponds to the linear correlation:

$$C^{norm}(u,v) = \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\theta^2}} \exp\left\{-\frac{x^2-2\theta xy+y^2}{2(1-\theta^2)}\right\} dxdy. \quad (5)$$

The Clayton copula distribution function is written as Equation (11):

$$C^C(u,v) = \left(u^{-\gamma}+v^{-\gamma}-1\right)^{-\frac{1}{\gamma}}, \gamma \in (0,\infty), \tau = 2^{-\frac{1}{\gamma}}. \quad (6)$$

The Gumbel copula [46] focuses on upper-tail dependence and has a zero lower-tail dependence structure. Financial time-series returns are more prone to joint negative extremes than joint positive extremes. Hence, the rotated Gumbel function is introduced as follows:

$$C^{rG}(1-u,1-v) = \exp\left\{-\left[(-\ln(1-u))^{\gamma}+(-\ln(1-v))^{\gamma}\right]^{\frac{1}{\gamma}}\right\},\ \gamma \in (1,\infty),\ \tau = 2-2^{-\frac{1}{\gamma}}. \quad (7)$$

The Student's t copula function is written as Equation (9), where the parameter ν represents the degree of freedom and $t_\nu^{-1}(u)$ and $t_\nu^{-1}(v)$ are the Student's t distribution:

$$C^{St}(u,v) = \int_{-\infty}^{t_\nu^{-1}(u)} \int_{-\infty}^{t_\nu^{-1}(v)} \frac{1}{2\pi\sqrt{1-\theta^2}} [1+\frac{x^2-2\theta xy+y^2}{\nu(1-\theta^2)}]^{-\frac{\nu+2}{2}} dxdy, \quad (8)$$

where the upper and lower tail dependences are calculated using the following formula:

$$g_T = 2 \times F_{student}\left(-\sqrt{(\hat{\nu}+1)\frac{\hat{\theta}-1}{\hat{\theta}+1}}, \hat{\nu}+1\right). \quad (9)$$

2.2.3. Generalized Autoregressive Score (GAS) in Time-Varying Copula

In order to analyze time-varying dependence, this study adopts the rotated Gumbel copula and Student's t copula models combined with the generalized autoregressive score (GAS) model proposed by Creal et al. [47]. This latter technique was extensively employed to determine the time-varying parameters in copula models.

The GAS model introduces the concept of a time-varying copula parameter, denoted as δ_t, which is dynamically determined based on the lagged copula parameter, and a "forcing variable", denoted as $I_t^{-1/2}s_t$, which is linked to the standardized score of the copula log-likelihood, serving as a key determinant in capturing the temporal dynamics and dependencies within the model. The outline of the GAS model is explained as follows:

$$f_t = h(\delta_t) \leftrightarrow \delta_t = h^{-1}(f_t), \quad (10)$$

$$where\ f_{t+1} = \omega + \beta f_t + \alpha I_t^{-1/2} s_t, \quad (11)$$

$$\boldsymbol{s}_t \equiv \frac{\partial}{\partial\delta} log\boldsymbol{c}(u_t, v_t; \delta_t), \quad (12)$$

$$I_t \equiv E_{t-1}\left[s_t s_t'\right] = I(\delta_t). \quad (13)$$

3. Empirical Results

This section provides an in-depth analysis of the empirical estimation outcomes concerning various aspects, including the estimation results for the time-varying copula models, the investigation of time-varying tail-dependence, and the linear dependence between foreign exchange rates and WTI. Additionally, we present the evaluation results for the estimated models and explore their application in the context of portfolio value at risk and expected shortfalls. Furthermore, we shed light on the difference between the oil-exporting and oil-importing countries, aiming to offer valuable insights into their distinct joint behavior and their diverse implications for risk management in portfolios.

3.1. Constant Copula Results

We first estimated the marginal distribution using the ARMA-GJR-GARCH model. Then, the standardized residuals for each foreign exchange rate and WTI return undergo a transformation into a uniform marginal distribution. This transformation is accomplished via the utilization of two distinct probability integral transforms, namely the skewed t model and the empirical distribution function, which correspond to the parametric and nonparametric models.

Utilizing the transformed data, we proceed to estimate the parameters of several copula models. The constant copula models employed in this study encompass the normal copula, Clayton copula, rotated Gumbel copula, and Student's *t* copula. In conclusion, the estimation results of the student's *t* copula yield the highest log-likelihood values, while the Clayton copula exhibits the lowest log-likelihood values. (See the detailed estimation results for the marginal distributions in Appendix A and the detailed estimation results for the constant copula models in Appendix B.)

3.2. Time-Varying Copula Results

3.2.1. Time-Varying Copula Parameter Estimation Results

For the purpose of examining the presence of time-varying tail dependence, this study employs the rotated Gumbel copula and Student's *t* copula in conjunction with the GAS model to construct time-varying copula models. The estimation results of the time-varying copula parameters between the foreign exchange rates and WTI prices are presented in Table 2. Among the four constant copula models considered, we select the Student's *t* copula over the normal copula to capture the dependence on both tails more accurately. Additionally, our estimation results also indicate that the rotated Gumbel copula exhibits superior performance compared to the Clayton copula, although they both identify the lower tail dependence. Therefore, we focus on presenting the results obtained from the time-varying rotated Gumbel copula and the time-varying Student's *t* copula in this subsection.

The findings from these tables reveal that, in general, the log-likelihood values for the time-varying Student's *t* copula model exceed those of the time-varying rotated Gumbel copula model. Furthermore, the log-likelihood values of the semi-parametric model are higher compared to those of the parametric model.

Table 2. Time-varying copula models between the RUB and WTI.

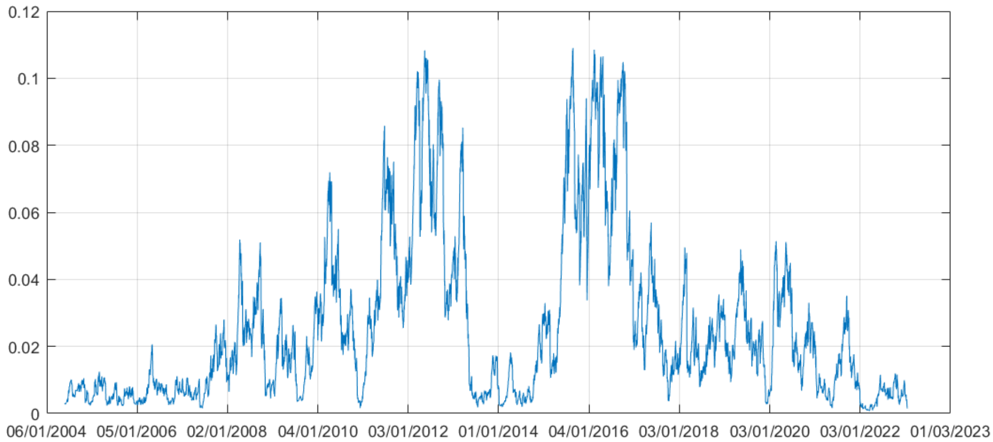
	RUB and WTI		MXN and WTI		CAD and WTI		EUR and WTI		INR and WTI		KRW and WTI	
	Para	**Semi**	**Para**	**Semi**	**Para**	**Semi**	**Para**	**Semi**	**Para**	**Semi**	**Para**	**Semi**
	Rotated Gumbel copula											
$\hat{\omega}$	−0.0075	−0.0073	−0.0912	−0.0854	−0.0087	−0.0088	−0.0027	−0.0969	−0.1006	−0.0975	−0.0150	−0.0139
s.e.	0.0046	0.0000	0.0020	0.0014	0.0005	0.0116	0.0006	0.0043	0.0019	0.0041	0.0177	0.0048
$\hat{\alpha}$	0.0650	0.0667	0.1988	0.2041	0.0525	0.0531	0.0474	0.1739	0.1951	0.1972	0.0580	0.0553
s.e.	0.0166	0.0001	0.0153	0.0194	0.0120	0.0285	0.0079	0.0332	0.0321	0.0284	0.0298	0.0208
$\hat{\beta}$	0.9956	0.9957	0.9545	0.9569	0.9927	0.9926	0.9988	0.9603	0.9600	0.9619	0.9938	0.9943
s.e.	0.0028	0.0007	0.0002	0.0016	0.0029	0.0098	0.0002	0.0020	0.0012	0.0025	0.0057	0.0011
$\log L$	339.23	345.98	161.34	164.18	432.02	434.17	124.35	108.75	66.093	66.554	67.861	70.042
	Student's copula											
$\hat{\omega}$	0.0045	0.0046	0.0030	0.0030	0.0094	0.0095	0.0013	0.0013	0.0009	0.0008	0.0016	0.0016
s.e.	0.0060	0.0012	0.0008	0.0005	0.0008	0.0019	0.0001	0.0007	0.0001	0.0001	0.0033	0.0003
$\hat{\alpha}$	0.0444	0.0466	0.0315	0.0315	0.0397	0.0401	0.0259	0.0258	0.0196	0.0199	0.0151	0.0151
s.e.	0.0153	0.0079	0.0073	0.0061	0.0072	0.0572	0.0125	0.0069	0.0022	0.0114	0.0350	0.0283
$\hat{\beta}$	0.9928	0.9927	0.9935	0.9933	0.9892	0.9889	0.9964	0.9964	0.9967	0.9965	0.9950	0.9950
s.e.	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0001
$\hat{\nu}^{-1}$	0.0714	0.0746	0.0527	0.0537	0.0693	0.0696	0.0784	0.0791	0.0365	0.0365	0.0493	0.0493
s.e.	0.0872	0.0120	0.0081	0.0099	0.0272	0.0000	0.0587	0.0207	0.0040	0.0163	0.0865	0.0633
$\log L$	379.70	382.11	175.79	176.66	475.14	475.25	154.89	154.35	82.572	82.847	79.017	78.939

Note: This table reports the estimation results of the time-varying copula. "Para" and "Semi" denote the parametric (skewed *t* distribution) and semi-parametric (empirical distribution function) models, respectively. RUB, MXN, CAD, EUR, INR, KRW, and WTI denote the Russian ruble, Mexican peso, Canadian dollar, Euro, Indian rupee, South Korean won, and Crude Oil West Texas Intermediate (WTI) Cushing (US dollar/bbl), respectively. The RUB, MXN, and CAD are from oil-exporting countries; the EUR, INR, and KRW are from oil-importing countries.

3.2.2. Time-Varying Tail and Linear Dependence Estimation from the Time-Varying Copula Model

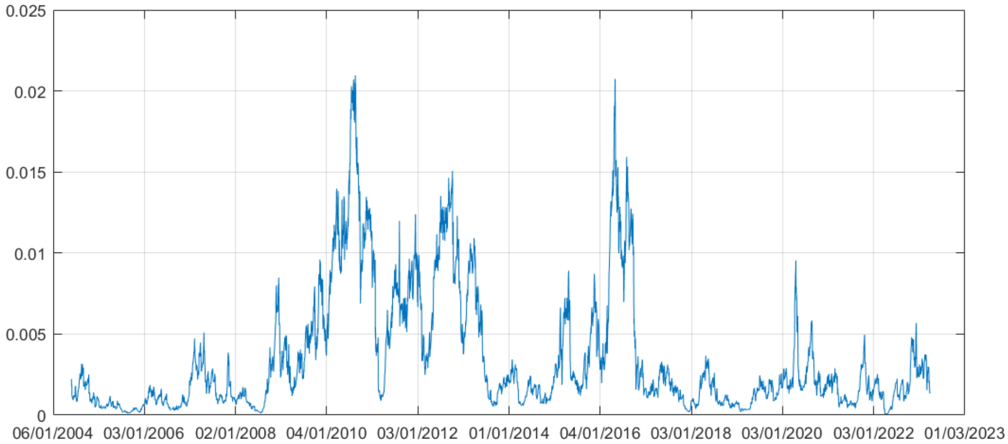
Consistent with the findings of Patton [42], our estimation results also indicate that the Student's *t* copula exhibits superior performance in terms of log-likelihood values compared to the rotated Gumbel copula. Therefore, we focus on presenting the results obtained from the time-varying student's *t* copula in this subsection.

Figure 2 presents the innovation time path of the FXs–WTI tail dependence implied by the time-varying Student's *t*–GAS copula model, which helps us to intuitively obtain the dynamics of the tail correlation development and compare the level of tail dependence between oil-importing and oil-exporting countries.

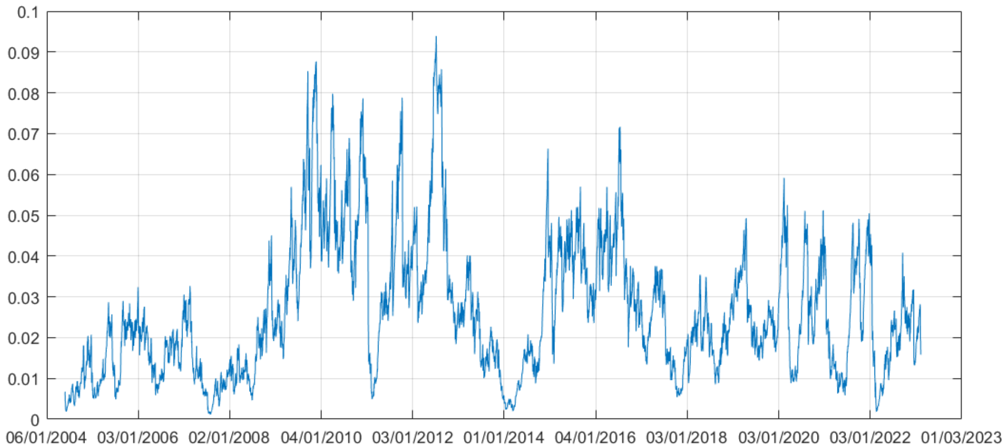


(**a**) Tail Dependence of RUB–WTI

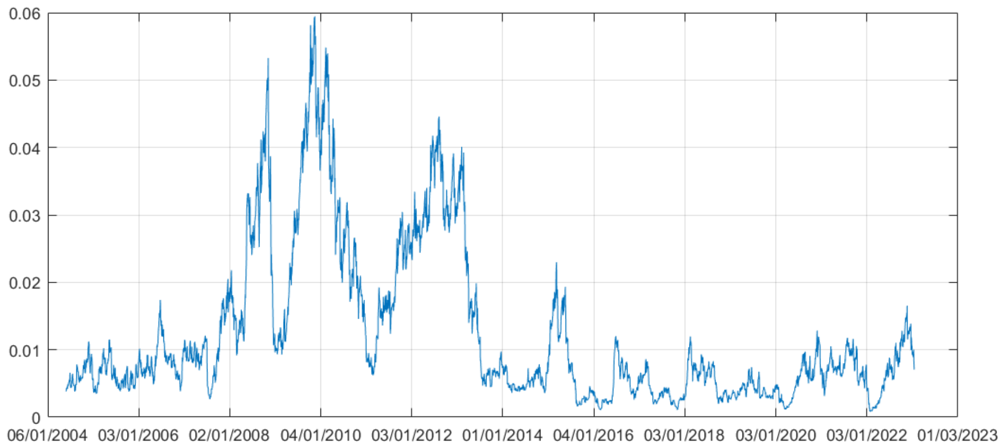
Figure 2. *Cont.*



(**b**) Tail Dependence of MXN–WTI

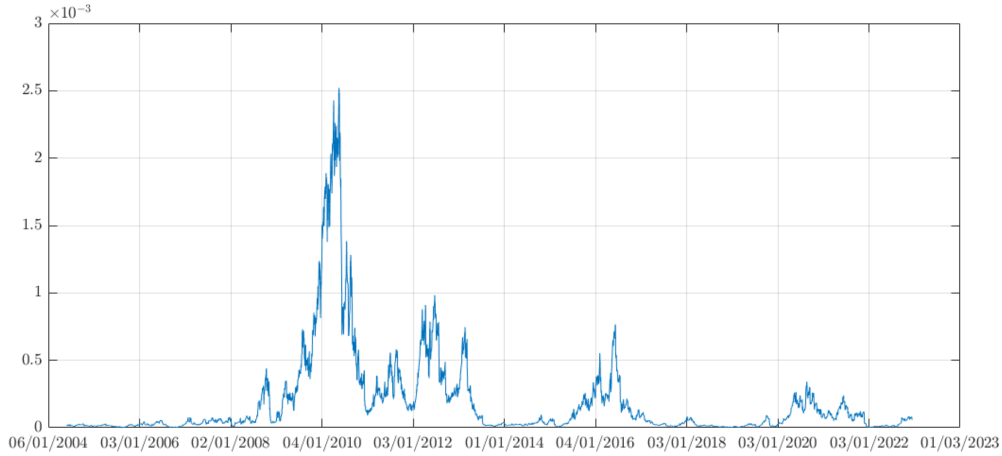


(**c**) Tail Dependence of CAD–WTI

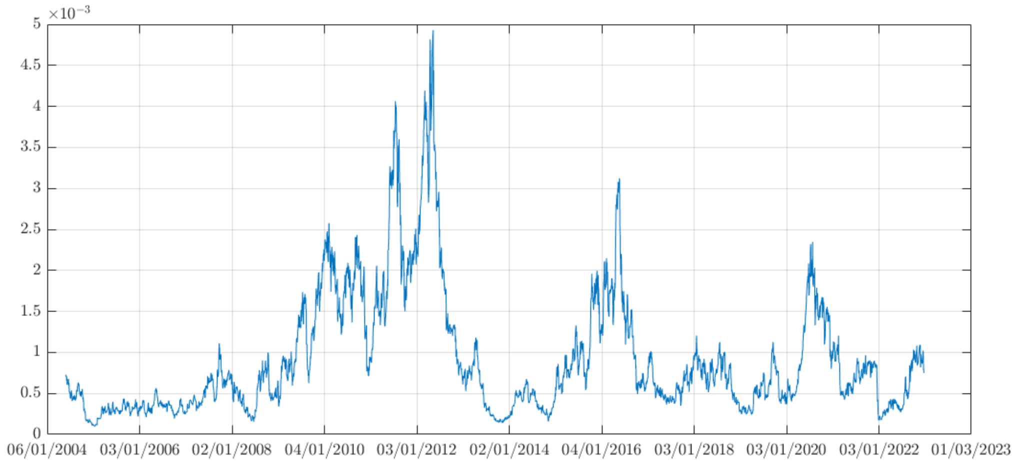


(**d**) Tail Dependence of EUR–WTI

Figure 2. *Cont*.



(**e**) Tail Dependence of INR–WTI



(**f**) Tail Dependence of KRW–WTI

Figure 2. Dynamic tail dependence estimates of FXs–WTI from the time-varying Student's *t* GAS copula. Notes: (**a**–**f**) denote the tail dependence of RUB–WTI, MXN–WTI, CAD–WTI, EUR–WTI, INR–WTI, and KRW–WTI, respectively. RUB, MXN, CAD, EUR, INR, KRW, and WTI represent the Russian ruble, Mexican peso, Canadian dollar, Euro, Indian rupee, South Korean won, and Crude Oil West Texas Intermediate (WTI) Cushing (US dollar/bbl), respectively. The RUB, MXN, and CAD are from oil-exporting countries; the EUR, INR, and KRW are from oil-importing countries.

Concerning the oil-exporting countries' FXs–WTI tail dependence innovation graph, we can observe that there are two major peaks, one of which is around 2012–2013, with the other surge corresponding to the period of 2015–2016.

In 2012, this period coincided with fluctuations in global oil prices, which held particular relevance for the major oil-exporting countries. Conversely, geopolitical tensions in oil-producing regions and disruptions in oil production or transportation infrastructure could have affected supply expectations and contributed to price fluctuations. Furthermore, the Eurozone debt crisis and OPEC's production decisions also influenced oil prices during this period. OPEC member countries implemented various production strategies to manage global oil prices, and changes in production levels or disagreements among member countries regarding production quotas could have impacted oil prices throughout this period.

During this period of 2012–2013, similar surges could also be observed in other countries' FXs–WTI tail dependence, where the oil-exporting countries' tail dependence had higher values than those of oil-importing countries overall.

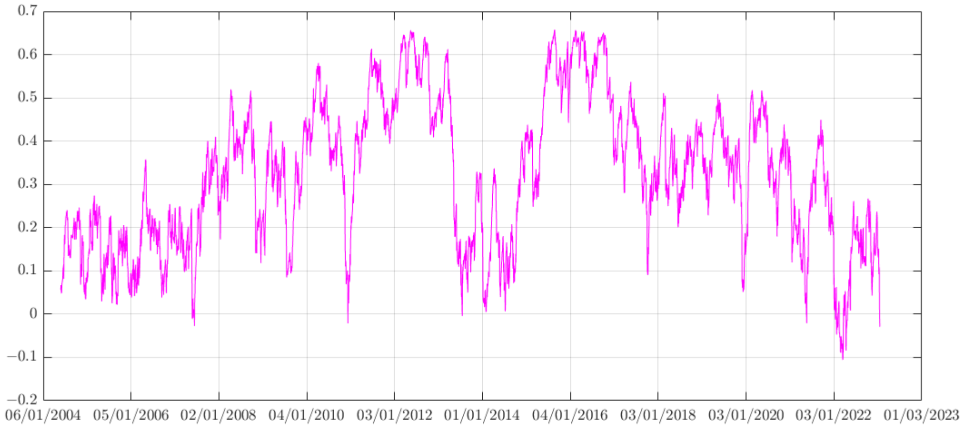
For the period of 2015–2016, 2016 was marked by significant events that had a profound impact on oil prices and the RUB. The oil price collapse, which began in mid-2014 and extended into 2016, exerted downward pressure on global oil prices and posed challenges for oil-exporting countries' foreign exchange due to the key role oil revenues play in the country's budget. Additionally, ongoing geopolitical tensions, particularly the conflict in Eastern Ukraine, as well as Brexit, both added to the complexities, influencing investor sentiment and the value of currencies. On the other side, the oil price collapse observed during this period was primarily attributed to supply side factors, such as booming U.S. oil production, reduced geopolitical tensions, and the evolving policies of OPEC. Nevertheless, during the subsequent period from mid-2015 to early 2016, the diminishing prospects for oil demand also contributed to the downward pressure on prices.

A comparable increase in the tail dependence between foreign exchange rates and WTI prices can be observed across various countries during the same time frame. Notably, the magnitude of this increase in tail dependence tends to be higher for countries that are oil exporters compared to those that are oil importers. This suggests that the foreign exchange rates of oil-exporting countries exhibit greater sensitivity to declines in oil prices than those of oil-importing countries.

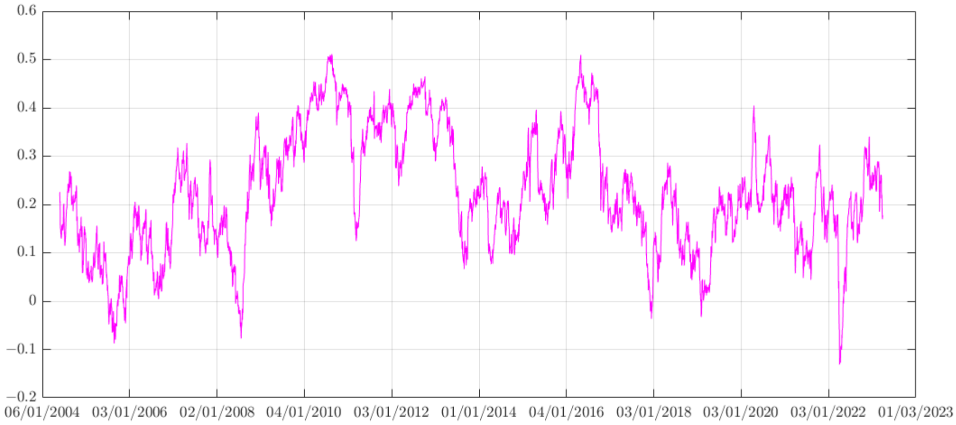
The notable increase observed in the tail dependence figure of RUB-WTI, MXN-WTI, CAD-WTI, EUR-WTI, and INR-WTI can be attributed to sovereign debt concerns and the prevalence of economic uncertainty in the year 2010. During this period, there were significant apprehensions regarding the sustainability of the global economic recovery. It is worth noting that 2010 was part of the post-crisis recovery phase following the Global Financial Crisis of 2008. Numerous countries were still contending with the aftermath of the crisis, grappling with challenges such as elevated debt levels, sluggish economic growth, and heightened volatility in financial markets. These conditions exerted downward pressure on currencies as investors sought refuge in safer assets and displayed a tendency toward risk aversion.

During the period encompassing the COVID-19 epidemic from 2020 to 2022, notable variations were observed regarding the FX–crude oil tail correlation. For oil-exporting countries, it is apparent that there were several significant increases in tail dependence for oil-exporting countries during the oil price collapses of 2020, suggesting that the COVID-19 epidemic heightened the probability of simultaneous extreme declines in both the foreign exchange rates and crude oil prices of oil-exporting nations.

Figure 3 illustrates the dynamic nature of the time-varying linear correlation between foreign exchange and WTI returns. Notably, a discernible pattern emerged after the 2008 financial crisis, whereby the correlation between foreign exchange rates and WTI exhibits a gradual increase, reaching its peak generally around the period of 2010–2012. However, 2011 witnessed a sharp plummet in the linear correlation, which is speculated to be caused by the surge in crude oil prices caused by the Arab Spring and the Libyan civil war. Subsequently, during the oil price downward phase of 2014–2016, the correlation experienced a sharp decline, where the potential causes could be postulated to be factors such as global oil production oversupply. Intriguingly, throughout the COVID-19 pandemic, all three oil-importing countries witnessed an upward trend in the correlation between their respective foreign exchange rates and WTI. However, it is noteworthy that both oil-exporting and oil-importing countries experienced substantial collapses in their FX–WTI correlations during the Russian–Ukrainian conflict in 2022. This indicates a temporary interruption in the dependence on WTI, with most of the correlations shifting towards negative values during this geopolitical risk event.



(**a**) Linear Dependence of RUB–WTI



(**b**) Linear Dependence of MXN–WTI

(**c**) Linear Dependence of CAD–WTI

Figure 3. *Cont.*

(**d**) Linear Dependence of EUR–WTI

(**e**) Linear Dependence of INR–WTI

(**f**) Linear Dependence of KRW–WTI

Figure 3. Dynamic linear dependence estimates of FXs–WTI from the time-varying Student's *t* GAS copula. Notes: (**a**–**f**) denote the linear dependence of RUB–WTI, MXN–WTI, CAD–WTI, EUR–WTI, INR–WTI, and KRW–WTI, respectively. RUB, MXN, CAD, EUR, INR, KRW, and WTI represent the Russian ruble, Mexican peso, Canadian dollar, Euro, Indian rupee, South Korean won, and Crude Oil West Texas Intermediate (WTI) Cushing (US dollar/bbl), respectively. The RUB, MXN, and CAD are from oil-exporting countries; the EUR, INR, and KRW are from oil-importing countries.

3.3. Value-at-Risk and Expected Shortfall in the Time-Varying Copula Model

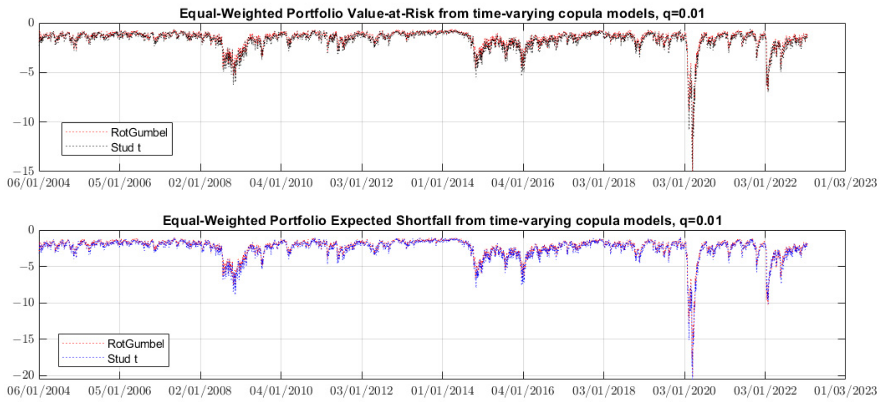
In this subsection, this study applies the time-varying copula model estimation results to risk management. The two key measures of risk are value at risk, defined as

$VaR_t^q \equiv F_t^{-1}(q)$, and expected shortfall, defined as $ES_t^q \equiv E\left[Y_t \middle| F_{t-1},\ Y_t \leq VaR_t^q\right]$, where $q \in (0,\ 1)$.

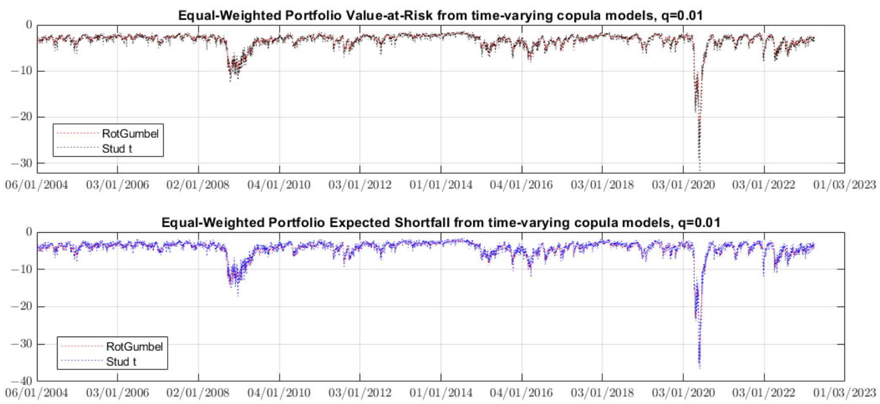
The q% value at risk represents the qth percentile of the conditional distribution, capturing the extreme tail behavior. Similarly, the expected shortfall corresponds to the expected value of the variable Y_t given that it falls below its VaR.

Simulation is a straightforward method of estimating the VaR and ES of a portfolio consisting of variables modeled using a copula-based approach. At each time point in the sample, we simulate S observations from the multivariate model, combine them to form the portfolio return, and then utilize the empirical distribution of these simulated portfolio returns to estimate VaR and ES.

Figure 4 illustrates the conditional 1% VaR (upper panel) and VaR (lower panel) for a portfolio with equal weights, utilizing the time-varying rotated Gumbel and Student's t copula models. Remarkably, the figure reveals two conspicuous downturns that command attention: the first being the 2008 financial crisis, while the second coincides with the unprecedented collapse of crude oil prices amid the ravages of the 2020 COVID-19 pandemic. Strikingly, the latter plunge surpassed the former by a significant margin, underscoring the immense levels of value at risk (VaR) and expected shortfall (ES) experienced in the equal-weight portfolio during the 2020 COVID-19 pandemic. Indeed, the magnitude and likelihood of losses during this period reached unparalleled historical levels. Subsequent to the 2022 Russian–Ukrainian war, we can observe some fluctuations; however, they pale in comparison to the magnitude of changes witnessed during the 2008 financial crisis and the 2020 COVID-19 pandemic.

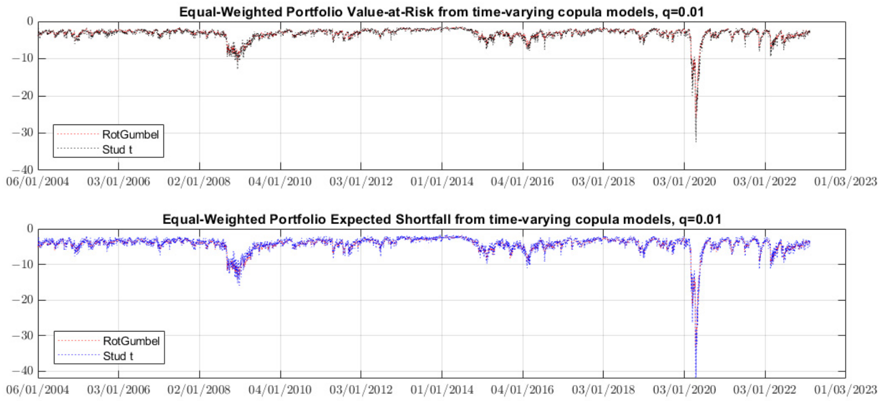


(**a**) RUB–WTI

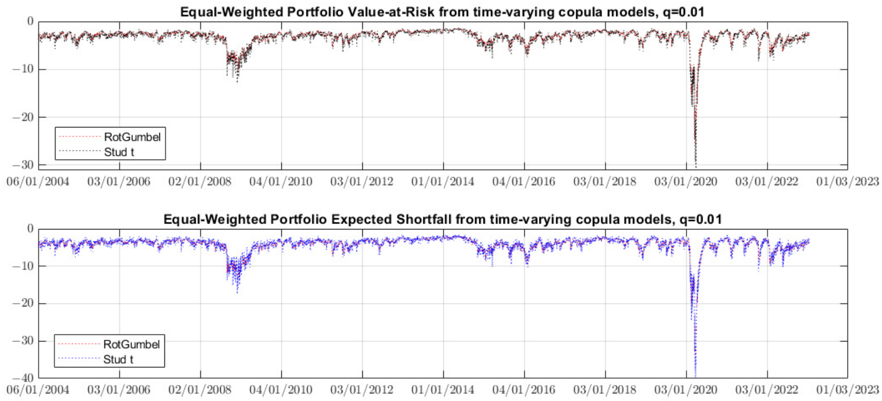


(**b**) MXN–WTI

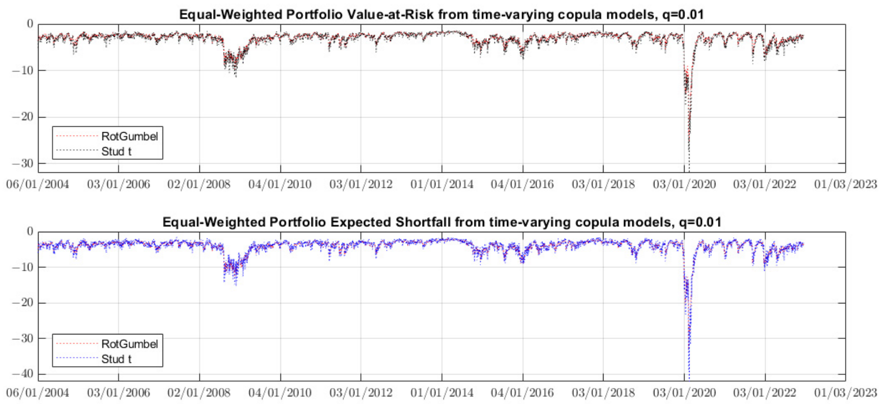
Figure 4. *Cont*.



(**c**) CAD–WTI

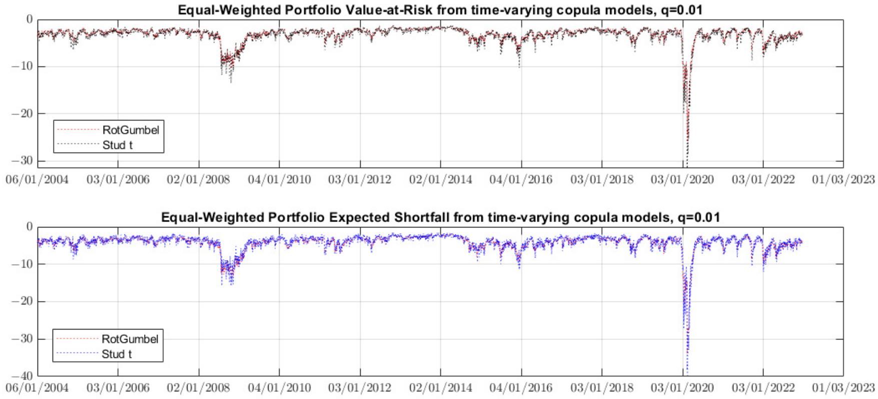


(**d**) EUR–WTI



(**e**) INR–WTI

Figure 4. *Cont*.



(**f**) KRW–WTI

Figure 4. VaR and ES for an equal-weight portfolio utilizing the time-varying copula models. Notes: (**a**–**f**) denote the VaR and ES for RUB–WTI, MXN–WTI, CAD–WTI, EUR–WTI, INR–WTI, and KRW–WTI, respectively. This illustration showcases the conditional 1% value at risk (VaR) in the upper panel and the expected shortfall (ES) in the lower panel for a portfolio with equal weights. The estimation is based on the utilization of the time-varying Student's *t* copula and rotated Gumbel models, employing 5000 simulations for each. RotGumbel stands for the rotated Gumbel copula model, while the Stud t represents the Student's *t* copula model. RUB, MXN, CAD, EUR, INR, KRW, and WTI denote the Russian ruble, Mexican peso, Canadian dollar, Euro, Indian rupee, South Korean won, and Crude Oil West Texas Intermediate (WTI) Cushing (US dollar/bbl), respectively. The RUB, MXN, and CAD are from oil-exporting countries; the EUR, INR, and KRW are from oil-importing countries.

4. Conclusions

This study focuses on analyzing the dependence between foreign exchange rates and WTI prices for six crude oil-importing and exporting countries based on four copula models in order to estimate constant dependence. Additionally, these copula models are combined with the generalized autoregressive score (GAS) model to capture the evolution of dynamic dependence. The primary objective of this study is to investigate the tail dependence between foreign exchange rates and WTI prices, while its secondary objective is to determine any differences in FXs–WTI tail dependence between oil-exporting and oil-importing countries, particularly after the outbreak of the COVID-19 pandemic in 2020 and the Russian–Ukrainian conflict in 2022.

In this study, we employed time-varying copula methodologies, as proposed by Patton [42,48]. Specifically, the following copula models were utilized: the normal copula, Clayton copula, rotated Gumbel copula, Student's *t* copula, time-varying rotated Gumbel GAS copula, and time-varying Student's *t* GAS copula. These copula models were chosen to capture the dynamic nature of the dependence structure between the variables under investigation. These methodologies were applied to estimate the correlation and tail dependence of foreign exchange rates from oil-exporting and -importing countries on WTI prices. Furthermore, they were deployed to analyze the difference between oil-exporting and -importing countries over the period from 6 January 2004 to 1 March 2023.

This study presents pioneering applications of the time-varying copula approach in conjunction with the generalized autoregressive score methodologies to comprehensively examine the conditional dependence and dynamic tail dependence between foreign exchange rates and WTI crude oil prices, differentiating the oil exporters from the importers. This study also contributes to the expanding literature on the correlation between foreign exchange rates and crude oil prices. Our results can provide insights for investors engaged in foreign exchange and crude oil markets.

The principal findings of this study can be summarized as follows: First, the analysis of FX–WTI tail dependence graphs reveals two notable peaks in the tail dependence of oil-exporting countries during 2012–2013 and 2015–2016. These peaks align with significant global events, including fluctuations in oil prices, geopolitical tensions, disruptions in oil production, the Eurozone debt crisis, and OPEC's production decisions. During periods of pronounced oil price volatility or shocks, currencies of oil-exporting nations exhibit heightened sensitivity. The sharp fluctuations in oil prices can profoundly impact their fiscal balances, trade positions, and foreign exchange reserves, given their relatively higher economic reliance on oil revenues. Additionally, investor sentiment plays a substantial role in FX–WTI tail dependence. During times of heightened market uncertainty or geopolitical instability in oil-producing regions, investors may perceive the currencies of oil-exporting countries as riskier assets, resulting in an increased correlation with oil prices. Consequently, FX–WTI tail dependence is more pronounced for these nations during turbulent periods. Second, the tail dependence between foreign exchange rates and WTI prices is greater for oil-exporting countries compared to their oil-importing counterparts. Typically, oil-exporting nations rely more heavily on oil exports for government revenue and foreign exchange earnings. This suggests that the foreign exchange rates of oil-exporting countries are more susceptible to declines in oil prices, as their economies are closely tethered to oil price movements. This heightened sensitivity increases the likelihood of simultaneous sharp declines during extreme events, in line with previous research by Tiwari et al. [16], Reboredo [27], and Kim and Jung [25]. Third, during the COVID-19 pandemic from 2020 to 2022, notable fluctuations were observed in the FX–crude oil tail correlation. Oil-exporting countries experienced significant increases in tail dependence, particularly during the oil price collapses in 2020. This points to an elevated probability of simultaneous extreme declines in foreign exchange rates and crude oil prices. The severe economic impact of the oil price collapse in early 2020, driven by reduced global demand and a price war among major oil-producing nations, was acutely felt by these countries. Given their direct economic ties to oil prices, their currencies became more responsive to oil price fluctuations, leading to increased tail dependence. Additionally, the pandemic's global impact on economic activity, trade, and financial markets exacerbated the economic challenges faced by oil-exporting nations, aligning with findings from previous studies by Lizardo and Mollick [26] and Ahmad and Moran Hernandez [1]. Fourth, the analysis of time-varying linear correlation reveals a gradual increase after the 2008 financial crisis, peaking around 2010–2012. However, during the oil price downturn from 2014 to 2016, the correlation sharply declined, rebounding swiftly thereafter. During the COVID-19 pandemic, all three oil-importing countries experienced an upward trend in their FX–WTI linear correlations. This suggests that, during the pandemic characterized by global economic uncertainty and oil price volatility, the movements of currencies in oil-importing countries became more aligned with crude oil, likely due to the overall downward pressure on the global economy. However, during the Russian–Ukrainian conflict in 2022, both oil-exporting and oil-importing countries witnessed substantial collapses in their FX–WTI correlations. This indicates a temporary interruption in their dependence on WTI, with most correlations shifting towards negative values during this period of geopolitical risk. The shift towards negative correlations implies that currency movements during the conflict increasingly moved in the opposite direction of WTI oil price movements, suggesting that other factors, such as geopolitical risks, became more dominant drivers of currency movements during the conflict.

In this paper, the empirical evidence may provide some insights for economists, policymakers, and investors. Firstly, our empirical analysis demonstrates that tail dependence and linear correlations between crude oil and foreign exchange markets exhibit significant variability over time. For investors building global portfolios that encompass these markets, it's essential to adopt dynamic hedging techniques. This recognizes the ever-changing interconnectedness among these factors, requiring nimble responses to unfolding events. Such adaptability enables investors and policymakers to fine-tune portfolio strategies swiftly, effectively mitigating risks and safeguarding domestic foreign exchange markets. Secondly, the identification of pronounced peaks in tail dependence during specific periods underscores the significance of comprehending the tail behavior of financial markets. Recognizing these heightened risks during these timeframes becomes imperative. For monetary authorities such as central banks, close attention to significant events such as geopolitical tensions and disruptions in oil production is warranted. The higher tail dependence observed in oil-exporting countries compared to their oil-importing counterparts signifies that foreign exchange rates in the former exhibit greater susceptibility to declines in oil prices. This emphasizes the imperative for economists and policymakers in oil-exporting nations to prioritize economic diversification strategies to reduce reliance on oil revenues and manage these vulnerabilities effectively. Thirdly, the observed variations in FX–crude oil tail correlations during the COVID-19 pandemic shed light on the unique and unprecedented nature of this global crisis. The study underscores the importance of recognizing the potential for significant shifts in financial market dynamics during such extraordinary periods, necessitating astute risk management practices and preparedness for unforeseen shocks. Lastly, the substantial collapses in FX–WTI correlations during the 2022 Russian–Ukrainian conflict underscore the pivotal role of geopolitical risks in shaping financial markets. It is incumbent upon investors and policymakers alike to be vigilant regarding abrupt changes in market dynamics during geopolitical events, including shifts toward negative correlations. For investors, these findings can be instructive for enhancing risk management strategies, emphasizing diversification across asset classes and geographic regions to mitigate the impact of extreme events and fluctuations in correlations. Furthermore, policymakers in oil-exporting countries may need to consider implementing proactive policies that reduce economic dependence on oil prices and enhance overall resilience to oil market fluctuations, including fiscal reforms, economic diversification initiatives, and prudent financial reserve management.

Despite these findings, this study has limitations. Due to the limited availability of data, it was not possible to consider a balanced sample of countries to examine the change in dependence after the 2020 COVID-19 outbreak compared to the pre-2020 phase for an adequate sample period. This paper conducted a quantitative analysis employing the dynamic copula model. Nonetheless, it is worth noting that a qualitative analysis such as the Delphi method involving questionnaires or analogous methodologies merits consideration as a subject for future research endeavors. The non-floating exchange rate system of China prevents the inclusion of the currencies of the two countries with the largest crude oil imports, namely the Chinese RMB (CNY) and the US dollar (USD).

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1 presents a comprehensive display of the summary statistics and estimation results for the marginal distributions. Specifically, the coefficients obtained from the ARMA and GJR-GARCH (1, 1) models for each variable are summarized in the table. Furthermore, we employed the skewed t model to conduct estimations for the skewness parameter λ and the degree of freedom parameter ν. Notably, all computed values for the skewness parameter λ demonstrate negativity, suggesting the presence of longer or fatter tails on the left side of the return distribution. In order to assess the goodness-of-fit of our models, we conducted the Kolmogorov–Smirnov (KS) and Cramer-von Mises (CvM) tests [49], renowned measures for evaluating the adequacy of the models. Significantly, the insignificant p values observed in both the KS and CvM tests indicate a failure to reject the null hypothesis at the significant level of 5%. The GOF test presents an intriguing null hypothesis that suggests the data follow the estimated skewed t-distribution. On the other hand, the alternative hypothesis is that the data deviate from the specified skewed t distribution.

Table A1. Marginal distributions.

	RUB	**MXN**	**CAD**	**EUR**	**INR**	**KRW**	**WTI**
				Conditional mean			
ϕ_0	−0.0070	−0.0109	−0.0014	−0.0039	−0.0123	−0.0023	0.0099
ϕ_1	0.6439					1.0352	0.2658
ϕ_2	−0.0038					−0.8926	0.3274
ϕ_3	−0.1157						−0.7940
ϕ_4	0.1262						
θ_1	−0.5764					−1.0080	−0.2449
θ_2						0.8563	−0.3791
θ_3						−0.0157	0.7632
θ_4							0.0972
				Conditional variance			
ω	0.0020	0.0090	0.0024	0.0010	0.0033	0.0031	0.1108
β	0.8832	0.8794	0.9492	0.9654	0.8956	0.9253	0.8898
α	0.0678	0.0581	0.0309	0.0201	0.0701	0.0407	0.0456
δ	0.0980	0.0959	0.0236	0.0229	0.0391	0.0521	0.0921
				Skew t density			
ν	6.4138	9.3960	12.6340	11.5400	4.8923	6.9903	8.4159
λ	−0.0647	−0.1643	−0.0367	−0.0317	−0.0720	−0.0564	−0.1220
				GoF tests			
KS p value	1.00	1.00	0.98	0.99	1.00	1.00	0.99
CvM p value	0.90	0.80	0.26	0.06	0.25	0.58	0.56

Notes: The presented table provides a comprehensive overview of the coefficient estimates derived from the ARMA and GJR-GARCH (1, 1) models for each variable under investigation. In addition, we have estimated the skewness parameter λ and the degrees of freedom parameter ν within the framework of the skewed t model. To assess the goodness-of-fit of the models, we have employed the Kolmogorov–Smirnov (KS) and Cramer-von Mises (CvM) tests [49] (based on 100 simulations), widely recognized measures of statistical adequacy. RUB, MXN, CAD, EUR, INR, KRW, and WTI denote the Russian ruble, Mexican peso, Canadian dollar, Euro, Indian rupee, South Korean won, and Crude Oil West Texas Intermediate (WTI) Cushing (US dollar/bbl), respectively. The RUB, MXN, and CAD are from oil-exporting countries; the EUR, INR, and KRW are from oil-importing countries.

Appendix B

Table A2 presents the estimated parameters, standard errors, and log-likelihood values for these four copula models. The normal copula model allows us to ascertain the correlation between the two data series, thereby revealing the level of association between foreign exchange rates and WTI returns. Conversely, the Clayton copula and rotated Gumbel copula models provide insights into the dependence observed in the lower tails of the data series. Additionally, the Student's t copula demonstrates symmetrical dependence structures in both the lower and upper tails. The utilization of these copula models provides a deeper understanding of the nature and extent of the dependence relationships observed between foreign exchange rates and WTI returns.

Table A2. Constant copula models between the RUB and WTI.

	RUB and WTI		MXN and WTI		CAD and WTI		EUR and WTI		INR and WTI		KRW and WTI	
	Para	**Semi**	**Para**	**Semi**	**Para**	**Semi**	**Para**	**Semi**	**Para**	**Semi**	**Para**	**Semi**
	Normal Copula											
$\hat{\theta}$	0.3264	0.3269	0.2271	0.2276	0.3945	0.3940	0.1861	0.1867	0.1376	0.1378	0.1568	0.1574
s.e.	0.0120	0.0123	0.0134	0.0134	0.0114	0.0114	0.0138	0.0138	0.0141	0.0142	0.0140	0.0140
log L	267.7100	269.0900	127.2800	127.8600	404.0000	402.9400	83.9010	84.4350	45.2500	45.4100	59.0310	59.4550
	Clayton copula											
$\hat{\gamma}$	0.3945	0.4166	0.2737	0.2795	0.5121	0.5178	0.2016	0.2031	0.1610	0.1626	0.1735	0.1777
s.e.	0.0214	0.0220	0.0200	0.0201	0.0230	0.0230	0.0196	0.0196	0.0185	0.0186	0.0186	0.0188
log L	229.9800	237.0200	123.1300	125.9900	337.1400	339.6700	67.5390	68.0620	47.3730	47.6260	55.4550	56.3300
	Rotated Gumbel copula											
$\hat{\gamma}$	1.2394	1.2492	1.1550	1.1564	1.3170	1.3183	1.1208	1.1211	1.1000	1.1000	1.1000	1.1000
s.e.	0.0131	0.0134	0.0117	0.0117	0.0144	0.0144	0.0113	0.0113	0.0108	0.0109	0.0107	0.0107
log L	261.6900	267.4400	126.6700	129.8000	385.3400	387.6000	81.2330	81.6260	46.6580	46.9570	56.6140	58.9110
	Student's copula											
$\hat{\theta}$	0.3266	0.3333	0.2304	0.2300	0.4008	0.4002	0.1942	0.1942	0.1391	0.1397	0.1578	0.1581
s.e.	0.0132	0.0134	0.0141	0.0141	0.0122	0.0123	0.0148	0.0148	0.0147	0.0148	0.0147	0.0147
$\hat{v}^{-1}$	0.0909	0.0979	0.0569	0.0582	0.0836	0.0869	0.0966	0.0980	0.0481	0.0494	0.0502	0.0507
s.e.	0.0148	0.0157	0.0145	0.0153	0.0152	0.0154	0.0157	0.0160	0.0153	0.0165	0.0145	0.0163
log L	292.6100	293.8300	135.6700	136.1300	423.8700	423.5300	108.2900	108.0900	50.8310	50.8230	65.1420	65.1210

Note: This table reports the estimation results of the constant copula. "Para" and "Semi" denote the parametric (skewed t distribution) and semi-parametric (empirical distribution function) models, respectively. RUB, MXN, CAD, EUR, INR, KRW, and WTI denote the Russian ruble, Mexican peso, Canadian dollar, Euro, Indian rupee, South Korean won, and Crude Oil West Texas Intermediate (WTI) Cushing (US dollar/bbl), respectively. The RUB, MXN, and CAD are from oil-exporting countries; the EUR, INR, and KRW are from oil-importing countries.

Several insightful conclusions can be drawn from the estimation results pertaining to the dependence between the foreign exchange rates of crude oil exporters and oil importers, as well as WTI, as presented in Table A2. Firstly, the constant copula estimation outcomes provide evidence that the correlation between the foreign exchange rates of oil-exporting countries and WTI surpasses that of oil-importing regions. This suggests that there was a higher degree of dependence for oil-exporting countries on WTI compared to oil-importing countries throughout the entire sample period. These findings align with the research conducted by Reboredo [27] and Tiwari et al. [16], thereby lending further support to the existing literature. Additionally, it is noteworthy that the dependency degree is observed to be the highest for CAD and the lowest for INR.

Secondly, the results derived from the Clayton copula and rotated Gumbel copula models indicate that the tail dependence among oil-exporting countries is generally more pronounced than that among oil-importing countries. This implies a greater likelihood of simultaneous extreme decreases in the foreign exchange rates of oil-exporting countries and WTI. Notably, CAD and RUB exhibit a higher susceptibility to decline alongside WTI prices owing to their elevated tail dependence values.

Lastly, when considering all estimation models, the log-likelihood values associated with the semi-parametric model consistently surpass those of the parametric model, thus highlighting the superiority of the empirical distribution function method. Specifically, the estimation results of the Student's t copula yield the highest log-likelihood values, while the Clayton copula exhibits the lowest log-likelihood values.

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