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Phenomenological motivation for gravitational positivity bounds: A case study of dark sector physics

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Positivity bounds on scattering amplitudes provide a necessary condition for a low-energy effective field theory to have a consistent ultraviolet completion. Their extension to gravity theories has been studied in the past years aiming at application to the swampland program, showing that positivity bounds hold at least approximately even in the presence of gravity. A theoretical issue in this context is how much negativity is allowed for a given scattering process. We show that this issue is relevant to physics within the scope of ongoing experiments, especially in the context of dark sector physics. A detailed analysis of dark photon scenarios is provided as an illustrative example. Our results do not merely show the phenomenological importance of the theoretical study of gravitational positivity bounds but also open up an exciting possibility of exploring the nature of quantum gravity via an experimental search of the dark sector.

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I. INTRODUCTION

The concept of effective field theory (EFT) and ultraviolet (UV) completion is a key framework for connecting various scales in nature. When applied to gravity, it will open up a road toward quantum gravity phenomenology. The swampland program [1] is aiming at this direction by exploring UV constraints on gravitational EFTs and their implications for particle physics and cosmology. See Refs. [2–4] for review articles.

Positivity bounds on scattering amplitudes is one of the interesting approaches to such UV constraints on EFTs. While the bounds were originally formulated in nongravitational theories with a mass gap [5–7], their extension to gravity theories has been studied intensively, having application to the swampland program in mind [8–17]. A finding there is that positivity bounds hold at least approximately even in gravity theories.

For concreteness, we consider an s - u symmetric scattering amplitude $\mathcal{M}(s, t)$ of the process $AB \rightarrow AB$ up to $\mathcal{O}(M_{\text{Pl}}^{-2})$ and perform its IR expansion as

$$\mathcal{M}(s, t) = \sum_{n,m=0}^{\infty} \frac{c_{n,m}}{n!m!} \left(\frac{s-u}{2}\right)^n t^m + (s, t, u\text{-poles of graviton exchange}), \quad (1)$$

where (s, t, u) are the standard Mandelstam variables and they satisfy $s + t + u = 2(m_A^2 + m_B^2)$. Then, the positivity bound implies

$$c_{2,0} \geq \frac{\lambda}{M_{\text{Pl}}^2 M^2} (\lambda = \pm 1), \quad (2)$$

where the sign λ and the scale M depend on details of UV completion of gravity. See the next section for the definitions of λ and M in terms of gravitational Regge amplitudes at high energies and also the assumptions used to derive the bound (2).

When the threshold energy m_{th} , i.e., the mass of the lightest intermediate on-shell state, is below the UV cutoff scale Λ of the EFT, it is convenient to define [18,19]

$$B^{(2)}(\Lambda) := c_{2,0} - \frac{4}{\pi} \int_{m_{\text{th}}^2}^{\Lambda^2} ds \frac{\text{Im} \mathcal{M}(s, 0)}{(s - m_A^2 - m_B^2)^3}, \quad (3)$$

which is calculable within the EFT. The bound (2) on $c_{2,0}$ is now sharpened as

$$B^{(2)}(\Lambda) \geq \frac{\lambda}{M_{\text{Pl}}^2 M^2}. \quad (4)$$

In the past years, the bounds (2) and (4) have been used in the swampland program, and it turned out that the sign λ and the scale M are crucial for deriving the swampland conditions. For example, the sign λ is crucial in the context of the weak gravity conjecture [20] for black holes, which implies that the charge-to-mass ratios of extremal black holes are increased by higher-derivative corrections.

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In particular, negativity of $c_{2,0}$ for a certain helicity amplitudes of light-by-light scattering implies violation of the conjecture. Hence, it is crucial to understand for which scattering process and in what class of UV completion of gravity negativity of $c_{2,0}$ is allowed. See, e.g., Refs. [8,9,12,21–31] for recent discussion.

The scale M is also important to use the bound (4) for constraining the particle spectrum in gravitational EFTs [10,15,32–37]. As we explain later, if the scale M is at a UV scale $M \gtrsim \Lambda$, one may derive nontrivial constraints generically. On the other hand, if the sign λ is negative and the scale M is sufficiently low, the bounds are trivially satisfied without giving any constraint on IR physics. Interestingly, a recent paper [15] pointed out that for a certain helicity amplitude of graviton-photon scattering the sign λ has to be negative and also the scale M has to be an IR scale for compatibility with the sum rule. Since the sign λ and the scale M depend on each scattering process and details of its UV completion, it is of great interest how generic this feature is beyond the graviton-photon scattering.¹

In this paper, we address importance of this rather technical issue by demonstrating that it is relevant to physics within the scope of ongoing experiments, especially in the context of dark sector physics. This implies not only the phenomenological importance of further studies on gravitational positivity bounds but also the exciting possibility of exploring the nature of quantum gravity via experimental search of the dark sector. In this study, we analyze dark photon scenarios as an illustrative example.

II. GRAVITATIONAL POSITIVITY BOUNDS

We begin with a brief review of positivity bounds in the presence of gravity. A set of sufficient conditions for deriving the bounds (2) and (4) are the following:

- (i) *Unitarity*.—The imaginary part of \mathcal{M} is non-negative, $\text{Im}\mathcal{M}(s \geq m_{\text{th}}^2, 0) \geq 0$.
- (ii) *Analyticity*.— \mathcal{M} is analytic on the physical sheet of the complex s -plane except for poles and discontinuities on the real axis demanded by unitarity.
- (iii) *Mild behavior in the Regge limit*.—The high-energy behavior of $\mathcal{M}(s, t)$ is sufficiently mild to satisfy $\lim_{|s| \rightarrow \infty} |\mathcal{M}(s, t < 0)/s^2| = 0$.
- (iv) *Regge behavior*.—The s^2 bound (iii) is satisfied as a consequence of Reggeization by an infinite tower of higher-spin states.

The properties (i)–(iii) are satisfied in the standard UV complete theory with a mass gap. Also, they are satisfied by construction in perturbative string theory (which emerged

in the S -matrix theory context). Even though it is nontrivial if these properties should hold in general gravity theories,² we assume that (i)–(iii) hold for a small negative $t < 0$ up to $\mathcal{O}(M_{\text{Pl}}^{-2})$. By contrast, the property (iv) is naturally derived from the properties (i)–(iii) in the process with massless spin-2 exchange. For our purpose, it is convenient to parametrize the imaginary part of the Regge amplitude up to $\mathcal{O}(M_{\text{Pl}}^{-2})$ as

$$\text{Im}\mathcal{M}(s, t) \simeq f(t)(s/M_*^2)^{\alpha(t)} + \cdots (s \geq M_*^2). \quad (5)$$

Here M_* is the Reggeization scale, which is well above the mass scale of the higher-spin states, e.g., the string scale M_s in perturbative string theory. Also, we suppressed terms irrelevant to Reggeization of graviton exchange.

Now, the twice-subtracted dispersion relation for $t < 0$ reads

$$c_{2,0} + \mathcal{O}(t) = \frac{2}{M_{\text{Pl}}^2 t} + \frac{4}{\pi} \int_{M_*^2}^{\infty} ds \frac{f(t)(s/M_*^2)^{\alpha(t)}}{(s + t/2 - m_A^2 - m_B^2)^3} + \frac{4}{\pi} \int_{m_{\text{th}}^2}^{M_*^2} ds \frac{\text{Im}\mathcal{M}(s, t)}{(s + t/2 - m_A^2 - m_B^2)^3}, \quad (6)$$

where the singular graviton $1/t$ -pole must be canceled with the second term on the right-hand side (RHS). The cancellation fixes the value of $f(0)$. Also, evaluating the $\mathcal{O}(t^0)$ term gives [11]

$$B^{(2)}(\Lambda) = \frac{4}{\pi} \int_{\Lambda^2}^{M_*^2} ds \frac{\text{Im}\mathcal{M}(s, 0)}{(s - m_A^2 - m_B^2)^3} + \frac{\lambda}{M_{\text{Pl}}^2 M^2} \quad (7)$$

with $\lambda (= \pm 1)$ and $M (> 0)$ defined by

$$\frac{\lambda}{M^2} = - \left[\frac{2f'(0)}{f(0)} - \frac{\alpha''(0)}{\alpha'(0)} \right]. \quad (8)$$

Then, $\text{Im}\mathcal{M}(s, 0) \geq 0$ implies the bounds (2) and (4).

III. GENERAL CONSIDERATION

We discuss general implications of the bound (4) for the dark sector physics. For earlier discussions along this line of considerations, see, e.g., Refs. [15,33]. Let us consider the following model in which the Standard Model Lagrangian \mathcal{L}_{SM} and the Lagrangian of the dark sector \mathcal{L}_{DS} are included in addition to the Einstein-Hilbert term:

$$\mathcal{L} = \frac{M_{\text{Pl}}^2}{2} R + \mathcal{L}_{\text{SM}}[A_b, \cdots] + \mathcal{L}_{\text{DS}}[X, \cdots]. \quad (9)$$

Here, A_b and X represent an ordinary photon and a hidden particle in the dark sector, respectively. We define \mathcal{L}_{DS} such

¹For example, if negativity of $c_{2,0}$ cannot be ruled out in light-by-light scattering eventually, this provides a counterexample for the black hole weak gravity conjecture. So far, there is no known counterexample in string theory, so this issue would tell us if the conjecture is specific in string theory or more generic in quantum gravity.

²See, e.g., Ref. [38] for recent discussion on high-energy behavior of gravitational scattering.

that direct interactions between the dark sector and the Standard Model, if they exist, are included in \mathcal{L}_{DS} . In this section, we assume that the matter sector described by $\mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DS}}$ is renormalizable and neglect nonrenormalizable terms for simplicity. We will add nonrenormalizable terms to the matter Lagrangian and discuss the implications in the next section. We discuss the implication of (4) for the scattering amplitude $\mathcal{M}(s, t)$ of the process $\gamma X \rightarrow \gamma X$. In Sec. IV, we will later consider the case $X = \gamma'$, the dark photon as an illustrative example.

We can calculate $B^{(2)}(\Lambda)$ by evaluating the amplitude \mathcal{M} within EFT in Eq. (3). We decompose diagrams up to $\mathcal{O}(M_{\text{Pl}}^{-2})$ into two parts: the nongravitational diagrams in which gravitons are absent and those with a graviton exchange. We refer to the contributions from the former diagrams and the latter diagrams as $B_{\text{non-grav}}^{(2)}(\Lambda)$ and $B_{\text{grav}}^{(2)}(\Lambda)$, respectively. Because the matter sector itself is renormalizable, the nongravitational EFT amplitude $\mathcal{M}_{\text{non-grav}}$ satisfies the twice-subtracted dispersion relation, leading to

$$B_{\text{non-grav}}^{(2)}(\Lambda) = \frac{4}{\pi} \int_{\Lambda^2}^{\infty} ds \frac{\text{Im} \mathcal{M}_{\text{non-grav}}(s, 0)}{(s - m_X^2)^3}, \quad (10)$$

where m_X is the mass of the particle X . $\mathcal{M}_{\text{non-grav}}(s, 0)$ at $s > \Lambda^2$ is the nongravitational EFT amplitude extrapolated to high-energy regimes $s > \Lambda^2$ by analytic continuation.

By contrast, the GR amplitude does not satisfy the twice-subtracted dispersion relation because of the nonrenormalizable nature of GR. An electron one-loop correction to the diagram with graviton t -channel exchange gives rise to the Λ -independent $B_{\text{grav}}^{(2)}(\Lambda) \supset -\mathcal{O}(e^2 M_{\text{Pl}}^{-2} m_e^{-2})$, while the corresponding s , u -channel diagrams give $\mathcal{O}(e^2 M_{\text{Pl}}^{-2} \Lambda^{-2})$ terms. Then, $B_{\text{grav}}^{(2)}(\Lambda)$ is given by a negative constant in a good approximation at $\Lambda \gg m_e$ ³:

$$B_{\text{grav}}^{(2)}(\Lambda) = -\mathcal{O}\left(\frac{e^2}{M_{\text{Pl}}^2 m_e^2}\right). \quad (11)$$

As a result, the bound (4) reads

$$B_{\text{non-grav}}^{(2)}(\Lambda) \geq \mathcal{O}(e^2 M_{\text{Pl}}^{-2} m_e^{-2}) - \lambda M_{\text{Pl}}^{-2} M^{-2}. \quad (12)$$

The RHS depends on the unknown scale M given in (8): *a priori*, even the sign of the RHS is undetermined. Implications of (12) crucially depend on the value of M . If M is given by an IR scale as $M \lesssim m_e/e$, the RHS of (12) may become negative. In this case, the bound (12) is satisfied

³Strictly speaking, loop corrections to the XXh vertex could also contribute to $B_{\text{grav}}^{(2)}$. The values of such contributions depend on the details of \mathcal{L}_{DS} , the dark sector model. However, as long as X is a stable particle, the sign of such contributions would be negative as far as we know. Hence, adding such terms will not change the essence of the discussion.

in any renormalizable unitary matter theories coupled to gravity because of the condition $B_{\text{non-grav}}^{(2)}(\Lambda) > 0$, which follows from the formula (10).

If M is given by the scale of UV physics such as the mass of higher-spin states and satisfies the condition $M \gg m_e/e$, we can then ignore the second term on the RHS of (12), leading to

$$B_{\text{non-grav}}^{(2)}(\Lambda) \geq -B_{\text{grav}}^{(2)}(\Lambda) = \mathcal{O}\left(\frac{e^2}{M_{\text{Pl}}^2 m_e^2}\right). \quad (13)$$

This gives a nontrivial bound on the EFT, even though the matter sector is renormalizable. In particular, this implies an upper bound on Λ in terms of parameters in the model such as coupling constants, because $B_{\text{non-grav}}^{(2)}(\Lambda \rightarrow \infty) \rightarrow 0$ as explicitly seen from Eq. (10).

Using the formula (10) and the relation $\text{Im} \mathcal{M}_{\text{non-grav}}(s, 0) \simeq s \sigma_{\gamma-X}^{\text{non-grav}}(s)$ at $s \gg m_X^2$, Eq. (13) becomes⁴

$$\frac{4}{\pi} \int_{\Lambda^2}^{\infty} ds \frac{\sigma_{\gamma-X}^{\text{non-grav}}(s)}{s^2} \geq -B_{\text{grav}}^{(2)} = \mathcal{O}\left(\frac{e^2}{M_{\text{Pl}}^2 m_e^2}\right). \quad (14)$$

Here, $\sigma_{\gamma-X}^{\text{non-grav}}(s)$ is a total cross section of the γ - X scattering evaluated from the nongravitational EFT amplitude $\mathcal{M}_{\text{non-grav}}$ at the center-of-mass energy \sqrt{s} . Therefore, the bound (13) implies that a particle X in the dark sector must nongravitationally couple to photon to satisfy the lower bound (14) on $\sigma_{\gamma-X}^{\text{non-grav}}$, meaning that the dark sector cannot be too dark.⁵ This is reminiscent of typical dark sector scenarios in extra dimensions, where decoupling of the two sectors is realized by taking a large volume limit and so it cannot be achieved as long as the volume is finite to make the four-dimensional Planck mass M_{Pl} finite. In the next section, we study implications of (13) and (14) more concretely for dark photon models.

For completeness, let us point out that, even if M is given by the IR scale, there remains a possibility that we get nontrivial bounds similar to (13) and (14); for instance, it is possible that the RHS of (12) is still given by some positive quantity of $\mathcal{O}(e^2 M_{\text{Pl}}^{-2} m_e^{-2})$ and consequently (13) and (14) are derived even when we have $M \sim m_e/e$. It is also possible that the first term on the RHS of (7), which must

⁴Based on analogous discussions, we can derive nontrivial bounds on processes with external Higgs bosons. See [10,37] for discussions of gravitational positivity bounds on scalar scatterings.

⁵In the standard renormalizable theories, the cross section is consistent with the Froissart-Martin bound. The integral in Eq. (14) may be then dominated by its lower end. Assuming this, Eq. (14) implies a lower bound on $\sigma_{\gamma-X}^{\text{non-grav}}(\Lambda^2)$ as

$$\sigma_{\gamma-X}^{\text{non-grav}}(\Lambda^2) \gtrsim \mathcal{O}\left(\frac{e^2 \Lambda^2}{M_{\text{Pl}}^2 m_e^2}\right). \quad (15)$$

This could also be useful for understanding the implication of the bound (13) for given models in terms of the total cross section.

be positive thanks to the unitarity, plays a role to give the bounds (13) and (14).

It is important to study the scale M and (13) and (14) in explicit quantum gravity scenarios. For instance, Ref. [39] pointed out an interesting string compactification example in which a resultant four-dimensional effective theory has $N = 2$ supersymmetry and states charged under $U(1)$ gauge symmetry. The charged states can be arbitrarily light so that the bounds (13) and (14) are not obviously satisfied. It would be interesting to study such a nontrivial setup in light of gravitational positivity bounds.

IV. DARK PHOTONS

The dark photon is the gauge boson A_a^μ of an extra $U(1)$ symmetry in the dark sector to which we refer as $U(1)_d$ [40] (see [41] for a review). The kinetic mixing of A_a and the ordinary photon A_b is renormalizable, so there is no reason to exclude it. Indeed, it appears naturally if there exist heavy fields which are charged under both of the electromagnetic $U(1)$ and the hidden $U(1)_d$. Also, A_a is massive in general. We then consider the massive dark photon model described by (9) with

$$\mathcal{L}_{\text{DS}} = -\frac{1}{4}(F_{a,\mu\nu})^2 - \frac{1}{2}m_{A'}^2(A_{a,\mu})^2 - \frac{\epsilon}{2}F_{a,\mu\nu}F_b^{\mu\nu} + \mathcal{L}_{\text{hm}}, \quad (16)$$

where \mathcal{L}_{hm} denotes Lagrangian of matters in the dark sector.⁶ The kinetic terms are diagonalized in terms of new fields (A, A') which are defined by

$$\begin{pmatrix} A_a \\ A_b \end{pmatrix} = \begin{pmatrix} 1/\sqrt{1-\epsilon^2} & 0 \\ -\epsilon/\sqrt{1-\epsilon^2} & 1 \end{pmatrix} \begin{pmatrix} A' \\ A \end{pmatrix}. \quad (17)$$

The rotation (17) generates tiny couplings between A' and the Standard Model: they are included in $\mathcal{L}_{\text{SM}}|_{A_b \rightarrow A - (\epsilon/\sqrt{1-\epsilon^2})A'}$. By contrast, couplings between A and hidden matters are not induced by this rotation. Below, we call A and A' photon and the dark photon, respectively, and analyze the photon-dark photon scattering $\gamma\gamma' \rightarrow \gamma\gamma'$.

There are two possibilities for the photon-dark photon scattering: one where the dominant diagrams involve Standard Model particles and another where the dominant diagrams involve particles from the dark sector. We consider implications of gravitational positivity bounds on each of these two possibilities respectively.

The amplitude depends on the helicity configurations in general. For the process with transversely polarized dark photon, we consider

⁶To be precise, the dark photon kinetically mixes with the hypercharge gauge boson. In this case, the dark photon couples to the weak neutral current as well. However, this coupling is suppressed as $\mathcal{O}(m_{A'}^2/m_Z^2)$ and irrelevant in the parameter space studied in the present paper.

$$\mathcal{M}_T(s, t) := \frac{1}{4}[\mathcal{M}(1^+2^+3^+4^+) + \mathcal{M}(1^+2^-3^+4^-) + \mathcal{M}(1^-2^-3^-4^-) + \mathcal{M}(1^-2^+3^-4^+)],$$

where 1 and 2 (3 and 4) are the ingoing (outgoing) photon and dark photon, respectively and the superscript \pm denotes the helicity. For the process with longitudinal modes, we consider

$$\mathcal{M}_L(s, t) := \frac{1}{2}[\mathcal{M}(1^+2^L3^+4^L) + \mathcal{M}(1^-2^L3^-4^L)],$$

where L means the longitudinal polarization. For these amplitudes, the crossing symmetry implies the $s \leftrightarrow u$ permutation invariance.

Below, we refer to $(B_{\text{non-grav}}^{(2)}, B_{\text{grav}}^{(2)})$ for \mathcal{M}_T and \mathcal{M}_L as $(B_{\text{T,non-grav}}^{(2)}, B_{\text{T,grav}}^{(2)})$ and $(B_{\text{L,non-grav}}^{(2)}, B_{\text{L,grav}}^{(2)})$, respectively.

The one-loop diagrams are calculated by using the *Mathematica* packages FeynRules [42–44], FeynArts [45], FeynCalc [46–48], and Package-X [49].

A. Simplest model

To begin with, let us consider the simplest setup where the contributions of heavy particles including those in the dark sector to $B^{(2)}(\Lambda)$ are negligibly small. In this setup, the scattering process $\gamma\gamma' \rightarrow \gamma\gamma'$ is described by the Lagrangian (16) with $\mathcal{L}_{\text{hm}} = 0$, and it is mediated only by the Standard Model particles. Since new particles coupled to photon below 1 TeV have not been found, we choose $\Lambda \geq 1$ TeV.

The form of interactions between A' and the Standard Model particles is the same as that for A , except that the coupling strength is suppressed by a factor $\epsilon/\sqrt{1-\epsilon^2} \simeq \epsilon$. As a result, the calculations of \mathcal{M}_T and \mathcal{M}_L are similar to those of the amplitude of the light-by-light scattering which has been done in [36]. As it is the case for the light-by-light scattering, in our case, the dominant contributions to $B_{\text{non-grav}}^{(2)}(\Lambda)$ at $\Lambda \geq 1$ TeV are given by the W-boson loop, whereas the dominant term in $B_{\text{grav}}^{(2)}$ arises from the electron, the lightest charged particle in the Standard Model. The results are

$$B_{\text{T,non-grav}}^{(2)}(\Lambda) \simeq \frac{32\alpha^2\epsilon^2}{m_W^2\Lambda^2}, \quad B_{\text{L,non-grav}}^{(2)}(\Lambda) \simeq \frac{8\alpha^2\epsilon^2 m_{A'}^2}{\Lambda^2 m_W^4},$$

$$B_{\text{T,grav}}^{(2)}(\Lambda) \simeq B_{\text{L,grav}}^{(2)}(\Lambda) \simeq -\frac{11\alpha}{180\pi m_e^2 M_{\text{Pl}}^2}, \quad (18)$$

where $\alpha \simeq 1/137$ denotes the fine-structure constant. Then, Eq. (13) implies a lower bound on ϵ : from \mathcal{M}_T , we obtain

$$\epsilon \geq \sqrt{\frac{11}{5760\pi\alpha}} \frac{m_W\Lambda}{m_e M_{\text{Pl}}} \simeq 1.9 \times 10^{-11} \times \left(\frac{\Lambda}{1 \text{ TeV}}\right), \quad (19)$$

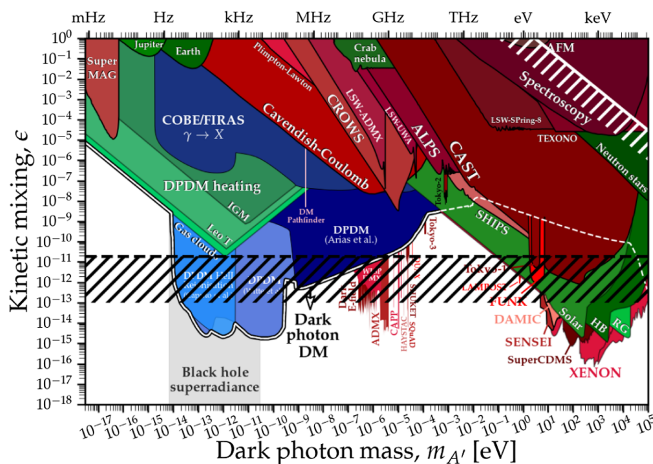


FIG. 1. The upper bounds (19) and (20) for $\Lambda = 1$ TeV are plotted in the $(m_{A'}, \epsilon)$ -plane by the black dotted line and the white line, respectively. The figure is made based on the data given in [50,51]. We find that the bounds are so strong and can be tested against experiments. This also implies the possibility of experimentally exploring general properties of quantum gravity S-matrix by testing the bounds.

while we obtain a stronger constraint from \mathcal{M}_L as

$$\begin{aligned} \epsilon &\geq \sqrt{\frac{11}{1440\pi\alpha}} \frac{m_W^2 \Lambda}{m_{A'} m_e M_{\text{Pl}}} \\ &\simeq 3.0 \times 10^{-3} \times \left(\frac{\Lambda}{1 \text{ TeV}} \right) \left(\frac{1 \text{ keV}}{m_{A'}} \right). \end{aligned} \quad (20)$$

These bounds (19) and (20) are plotted in Fig. 1. The bound (20) also shows the existence of a lower bound on $m_{A'}$ for given ϵ and Λ . Note that (20) only applies to the massive dark photon case. The origin of the lower bound on $m_{A'}$ is the suppression factor $(m_{A'}/m_W)^2$ contained in $B_{\text{L,non-grav}}^{(2)}$. This factor appears because longitudinal modes of dark photons are decoupled from W-bosons in the limit $m_{A'} \rightarrow 0$.

We have neglected QCD contributions so far. Since the QCD scale is lower than the electroweak scale, it may give relevant contributions to $B_{\text{non-grav}}^{(2)}$. Indeed, QCD effects, particularly the Pomeron exchange, provide the leading contribution to $B_{\text{non-grav}}^{(2)}$ in the Standard Model analysis of [36]. If we extend the analysis to transversely polarized dark photons simply by multiplying the factor ϵ to the coupling between photon and Pomeron, the bound (19) is slightly relaxed as $\epsilon \gtrsim 10^{-13}$ for $\Lambda \sim 1$ TeV.

It would be interesting to investigate how much the bound (20) is relaxed when QCD is taken into account. In general, the QCD contributions to \mathcal{M}_L will be suppressed by the dark photon mass $m_{A'}$ at least as long the mass is the Stückelberg mass. We thus expect that the existence of a lower bound on $m_{A'}$ like (20) is a robust result, leaving further studies for future work.

B. Remarks on higher-dimensional operators

In the above analysis, we have ignored contributions from heavy particles. In general, however, we can also consider dark photon models in which new particles other than γ' are also present whose mass scales are well below the quantum gravity scales such as M_{Pl} or M_s . Let us discuss how our bounds (19) and (20) on the kinetic mixing can be changed once new particles are introduced.

Let us introduce a new particle Y with mass m_Y for illustrations. Below the mass scale m_Y , we can take into account the contribution from Y to the process $\gamma\gamma' \rightarrow \gamma\gamma'$ by adding the higher-dimensional operators to the Lagrangian; operators which are relevant for the discussion of positivity bounds at the leading order are

$$\frac{C_1}{m_Y^4} F^2 F'^2, \quad \frac{C_2}{m_Y^6} F_{\mu\nu} F^\nu{}_\rho \partial^\mu F'_{\alpha\beta} \partial^\rho F'^{\alpha\beta}. \quad (21)$$

Here, Wilson coefficients C_1 and C_2 are typically given by the couplings between Y and (γ, γ') . The contributions of the leading higher-dimensional operators (21) to $B_{\text{T,non-grav}}^{(2)}(\Lambda)$ and $B_{\text{L,non-grav}}^{(2)}(\Lambda)$ are estimated as

$$B_{\text{T,non-grav}}^{(2)}(\Lambda)|_{\text{operators (21)}} \sim \frac{C_1}{m_v^4}, \quad (22a)$$

$$B_{\text{L,non-grav}}^{(2)}(\Lambda)|_{\text{operators (21)}} \sim \frac{C_2 m_{A'}^2}{m_V^6}, \quad (22b)$$

where we have $\Lambda < m_Y$ because the cutoff scale cannot be heavier than the mass of Y which is integrated out. When these contributions are larger than those from particles in the Standard Model (18), the implications of the positivity bound (13) will be different from (19) and (20).⁷ According to Eqs. (18) and (22), the contributions from higher-dimensional operators to $B_{\text{T,non-grav}}^{(2)}(\Lambda)$ and $B_{\text{T,non-grav}}^{(2)}(\Lambda)$ are negligible when the following conditions are satisfied, respectively:

$$m_Y \gtrsim \left(\frac{|C_1|}{\epsilon^2 \alpha^2} \right)^{\frac{1}{4}} \sqrt{m_W \Lambda}, \quad (23a)$$

$$m_Y \gtrsim \left(\frac{|C_2|}{\epsilon^2 \alpha^2} \right)^{\frac{1}{6}} (m_W^2 \Lambda)^{\frac{1}{3}}, \quad (23b)$$

where we impose $(m_W <) \Lambda < m_Y$.

When Y carries only electromagnetic $U(1)$ charge, C_1 and C_2 are suppressed by the kinetic mixing:

$$C_1, C_2 \lesssim \mathcal{O}(\epsilon^2). \quad (24)$$

⁷The contribution from Y to $B_{\text{T,grav}}^{(2)}$ and $B_{\text{L,grav}}^{(2)}$ will be typically $\mathcal{O}(M_{\text{Pl}}^{-2} m_Y^{-2})$, which is negligible compared to those from electron (18).

As a result, the conditions (23) are always satisfied. Hence, the bounds (19) and (20) are robust against the addition of heavy particles charged only under $U(1)$.

On the other hand, C_1 and C_2 are not suppressed by ϵ in general if Y is charged under both of $U(1)$ and $U(1)_d$. In particular, we can consider the scenario in which $|C_1|, |C_2| \gg \epsilon^2$ such that the conditions (23) are violated even for very heavy m_Y . Therefore, it will be possible to construct models which are consistent with the positivity bound (13) regardless of the value of ϵ , by introducing heavy bicharged particles.

C. Adding bicharged particles

Now, we construct a model in which both of the bound (13) and the current observational bound on ϵ are satisfied simultaneously. For simplicity, we set $\epsilon = 0$ and instead add to the model two bicharged vector bosons (V_1^μ, V_2^μ) with the same charges ($q_{\text{em}}, q_{\text{em}}$) under $U(1)$ but the opposite charges ($q_d, -q_d$) under $U(1)_d$. We choose $q_{\text{em}} = q_d = 1$ and write the $U(1)_d$ gauge coupling as e' below. We assume that their masses m_{V_1} and m_{V_2} are approximately identical: $m_V := m_{V_1} \simeq m_{V_2}$. With this choice, these particles generate a tiny kinetic mixing suppressed by a factor $|m_{V_1}^2 - m_{V_2}^2|/m_V^2 \ll 1$ so that the model can be consistent with the current observations. Since bicharged particles are coupled to photon with coupling strength e , they must be sufficiently heavy to be consistent with experiments; we then impose $m_V \gtrsim 1 \text{ TeV} \gg m_e$.

In this model, $B_{\text{T,nongrav}}^{(2)}(\Lambda)$ and $B_{\text{L,nongrav}}^{(2)}(\Lambda)$ at $\Lambda \gg m_V$ are evaluated up to one-loop level as

$$B_{\text{T,nongrav}}^{(2)}(\Lambda) \simeq \frac{4e^2 e'^2}{\pi^2 m_V^2 \Lambda^2}, \quad (25)$$

$$B_{\text{L,nongrav}}^{(2)}(\Lambda) \simeq \frac{e^2 e'^2 m_{A'}^2}{\pi^2 m_V^4 \Lambda^2}. \quad (26)$$

Bicharged particles also contribute to $B_{\text{grav}}^{(2)}$, but it is negligible compared to the electron contribution as long as $m_V \gg m_e$ and $e' \lesssim 1$. The bound (13) is⁸

$$\begin{aligned} \Lambda &\leq 24 \sqrt{\frac{5}{11}} \frac{m_e}{m_V} e' M_{\text{Pl}} \\ &\simeq 2.0 \times 10^{13} \text{ GeV} \times e' \left(\frac{1 \text{ TeV}}{m_V} \right) \end{aligned} \quad (27)$$

for \mathcal{M}_T , while for \mathcal{M}_L , it reads

⁸Note that, even if we add bi-charged particles with spin smaller than one, the bound will not be relaxed comparing with (27) and (28) because its contributions to $B_{\text{T,nongrav}}^{(2)}$ and $B_{\text{L,nongrav}}^{(2)}$ are proportional to Λ^{-4} and $m_{A'}^2 \Lambda^{-6}$, respectively.

$$\begin{aligned} \Lambda &\leq 12 \sqrt{\frac{5}{11}} \frac{m_e m_{A'}}{m_V^2} e' M_{\text{Pl}} \\ &\simeq 10 \text{ TeV} \times e' \left(\frac{m_{A'}}{1 \text{ keV}} \right) \left(\frac{1 \text{ TeV}}{m_V} \right)^2. \end{aligned} \quad (28)$$

Equation (27) shows that, by adding sufficiently light bicharged spin-1 particles appropriately, we can make massless dark photon models with a tiny kinetic mixing ϵ consistent with (13) even for large cutoff $\Lambda \gg 1 \text{ TeV}$. When the dark photon is massive $m_{A'} > 0$, the scattering of longitudinal modes gives a stronger constraint (28). The constraint implies $m_{A'} \gtrsim e' \text{ keV}$ because of the condition $\Lambda, m_V \gtrsim 1 \text{ TeV}$. As discussed below (20), this is due to the suppression of couplings between longitudinal modes and matters at small $m_{A'}$.

We conclude that dark photon cannot be too light, at least in models we discussed. It would be interesting to explore models that accommodate a very light dark photon without violating the bound (13), leaving it for future work.

V. CONCLUSION

In this paper, we addressed importance of determining the sign λ and the scale M in the gravitational positivity bound (4). If λ is negative and M is as small as the electron mass scale, the bound is trivially satisfied. By contrast, when M is sufficiently large, the bound gives a nontrivial constraint on IR physics even if the sign λ is negative. This gives a general bound (14) on the total cross section of the photon–dark particle scattering, which implies that the dark sector cannot be too dark.

We also provided more detailed analysis of dark photon scenarios. A general finding is that the scattering of longitudinal modes of dark photons gives a stronger constraint than that of transverse modes. In particular, dark photons cannot be too light. It would be interesting to explore its possible connection to the swampland argument on photon masses [52]. Besides, we observed that kinetic mixing with massive dark photons favors UV completion by bicharged vector bosons whose masses are bounded by (28), which is reminiscent of gauge unification.

We stress that the bounds obtained under the assumptions (i)–(iv) and $M \gg m_e/e$ are within the scope of ongoing dark photon searches.⁹ This implies not only the phenomenological importance of further studies on gravitational positivity bounds in four-dimensional space-time but also the exciting possibility of testing general properties of quantum gravity S-matrix via experimental search of dark sector physics.

⁹Notice that, even if the energy scale M is an IR scale $M \sim m_e/e$, nontrivial bounds may remain depending on the $\mathcal{O}(1)$ coefficient of the Regge parameters; see also discussions at the last paragraph of Sec. III.

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