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**(Citation)**

神戸大学経済学研究科 Discussion Paper, 2317:1-26

**(Issue Date)**

2023-10

**(Resource Type)**

technical report

**(Version)**

Version of Record

**(URL)**

<https://hdl.handle.net/20.500.14094/0100485230>



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**October 2023**

**Discussion Paper No. 2317**

**GRADUATE SCHOOL OF ECONOMICS**

**KOBE UNIVERSITY**

**ROKKO, KOBE, JAPAN**

# Unprofitable Common Ownership with Asymmetric Distribution Channels\*

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October 10, 2023

## Abstract

Common ownership has been observed in many industries and is believed to have a moderating effect on competition and tends to increase the profits of firms in which shares are held. This study challenges this common ownership characteristic. We consider a market with one upstream and two downstream firms. One downstream firm sells its products in two independent markets, while the other sells its products in only one of the two markets. The relationship between common ownership and input prices changes in the presence of channel asymmetry. In other words, an increase in the degree of common ownership can lead to an increase in input prices. Thus, common ownership may reduce downstream firms' profits, consumer surplus, and total surplus. We also investigate whether this result is robust to several extensions.

**JEL codes:** L13, D43, G32

**Keywords:** supply chain management, common ownership, vertical relationship, asymmetric distribution channel

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\*The authors gratefully acknowledge the financial support of the Japan Society for the Promotion of Science (JSPS), KAKENHI Grant Numbers JP20K13618, JP22H00043, and JP23H00764. The usual disclaimer applies.

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# 1 Introduction

Common ownership refers to institutional investors owning shares in a large number of firms within a market and is observed in various industries.<sup>1</sup> For example, Vanguard Group owns shares in Apple and Microsoft, which are competitors in the technology industry. Other examples of common ownership include the airline, pharmaceutical, and banking industries (Azar et al. [3]). The effect of common ownership has been studied for some time, and previous theoretical studies have analyzed the effect of common ownership on competition and found situations where common ownership may reduce competition (Chen et al. [5]; López and Vives [16]; Rotemberg [19]).<sup>2</sup>

From the perspective of managing a firm, the existence of common ownership may be welcomed, since less competition is desirable. However, several empirical studies show that common ownership does not necessarily increase profits. For example, Koch et al. [11] empirically show that common ownership does not have a robust positive effect on industrial profitability.<sup>3</sup>

Following the empirical findings, we focus on understanding a negative relationship between common ownership and profits. More specifically, we introduce a new factor that raises input prices due to common ownership, which reduces firms' profits despite the existence of common ownership. Our analysis provides a better understanding of input price determination and the impact of input price changes on profits in decentralized supply chains with common ownership. Our analysis allows us to answer the following questions: (i) How should input prices change as the degree of common ownership increases?; and (ii) Who benefits from common ownership?

To address these questions, we consider a vertically related market with one upstream and two downstream firms. Each downstream firm buys inputs from the upstream firm

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<sup>1</sup>Torshizi and Clapp [22] report on the price effects of common ownership in the seeds sector.

<sup>2</sup>A recent study in the operations management literature that analyzes the effects of common ownership is that of Avinadav and Shamir [1].

<sup>3</sup>See Dennis et al. [7], Kini et al. [10], and Lewellen and Lowry [13] for studies that reexamine the anticompetitive effects of common ownership. See Backus et al. [4] for a survey on common ownership.

with a linear contract and sells its products to consumers. One downstream firm sells its products to two spatially separated markets (e.g., the U.S. and Mexico), and the other downstream firm sells its products to only one of these markets (e.g., the U.S. only). Institutional investors partially own shares in the two downstream firms, and each downstream firm, depending on the degree of common ownership, is concerned with the profits of its rivals.

One example of the fit of our model is the supermarket industry. For example, Costco and Dollar Tree are giant retailers, and Vanguard owns shares in both. In the U.S., Costco and Dollar Tree compete. However, in Mexico, only Costco has stores.<sup>4</sup> Both retailers buy products from a common upstream firm (e.g., PepsiCo). This market structure is consistent with our basic model.

Under the above model, we analyze the effects of an increase in the degree of common ownership on equilibrium outcomes. First, we find that whether the input price increases with the degree of common ownership depends on the heterogeneity in the size of downstream markets. If the market size in which two downstream firms compete is smaller than the market size in which one downstream firm is the sole monopolist, input prices increase with the degree of common ownership. Since a higher input price increases the marginal cost to downstream firms, the price of the final product also increases. If the effect of higher input prices dominates the effect of relaxed competition among downstream firms, the profits of downstream firms will decline, even as the degree of common ownership increases. Moreover, because the reduction in competition among downstream firms due to common ownership is undesirable for the upstream firm and consumer, the profit of the upstream firm and the consumer surplus always decrease with the degree of common ownership.

We also investigate the robustness of our results. First, we consider the case where

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<sup>4</sup>For the number of Costco warehouses, see Costco's website (<https://investor.costco.com/company-profile/default.aspx>). For the number of Dollar Tree stores, see the 2022 Annual Report (<https://corporate.dollartree.com/investors/financial-information/annual-reports-proxies>).

the inverse demand function is nonlinear. More precisely, we assume an inverse demand function with constant curvature (Hu et al. [9]; López and Vives [16]). Since in the basic model, the condition for obtaining the main results is the small size of the market in which the two downstream firms compete, we limited our analysis to a situation in which the equilibrium output of the market supplied by the two downstream firms is positive but sufficiently close to zero. Consequently, the results obtained using the basic model are robust.

Next, we analyzed the case in which the downstream firms have symmetric distribution channels. Specifically, we analyzed two cases: (i) both downstream firms sell their products to two markets, and (ii) each downstream firm exclusively sells its products in a different market. The main results of the basic model are not obtained when the downstream firms' sales channels are symmetric. Therefore, we find that it is important for the sales channel to be asymmetric so that common ownership reduces the profits of downstream firms.

Furthermore, we analyzed the case in which each downstream firm produces a differentiated product in a market in which the two downstream firms compete. The analysis shows that, as in the basic model, when the size of the market in which the two downstream firms compete is small, the profits of the downstream firms decrease with the degree of common ownership. Thus, product differentiation does not alter the main results qualitatively.

This study is related to two strands of literature: the literature on the relationship between competition and input prices, and the literature on non-controlling ownership in vertical markets.<sup>5</sup>

In the literature on the relationship between competition and input prices, the study most relevant to ours is Yenipazarli [25]. He analyzed the effect of downstream entry

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<sup>5</sup>Our research may be related to studies analyzing the relationship between input prices and competition in the literatures on industrial organization or management. For these studies, see Cho [6], Greenhut and Ohta [8], Mukherjee [17], Pinopoulos [18], and Salinger [20].

on wholesale price and profits. Before entry, the two downstream firms sell goods exclusively to two different markets. Yenipazarli [25] proposed a condition in which the wholesale price decreases when one of the downstream firms enters the other market. Furthermore, he showed that if this wholesale price reduction effect exceeds the effect of increased competition due to entry, then entry increases downstream firms' profits. One difference between Yenipazarli [25] and our study is that Yenipazarli [25] used entry to capture changes in competition in the downstream market, whereas we used the degree of common ownership. Furthermore, Yenipazarli [25] analyzed under a linear inverse demand function, whereas we analyzed not only under a linear inverse demand function but also under an inverse demand function with a constant curvature. Therefore, our study complements the findings of Yenipazarli [25].

A seminal study analyzing the relationship between downstream competition and input prices was conducted by Tyagi [23], who showed that if the elasticity of the slope of the inverse demand function is constant, input prices are independent of the degree of downstream competition. Koulamas and Kyparisis [12] incorporated the effect of lower marginal costs associated with downstream entry into Tyagi's [23] model and showed that competition in downstream markets can increase input prices. These studies considered downstream market entry, but not common ownership in the downstream market.

Our study also contributes to the literature on non-controlling ownership in vertical markets, which is mainly in the field of economics.<sup>6</sup> Li and Shuai [14] considered a situation in which one of the downstream firms owns shares in a competitor, but the competitor does not own shares in the downstream firm (i.e., one-way shareholding). They showed that one-way shareholding reduces a holding firm's input price. Shuai et al. [21] also considered a situation with one-way shareholding. They showed that as the degree of shareholding increases, the input prices for downstream shareholding firms decrease. Hu et al. [9] showed that cross-holdings among downstream firms raise

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<sup>6</sup>One of the few studies to consider downstream shareholdings in supply chain management is that of Aviv and Shamir [2]. However, their model is significantly different from ours.

input prices when an upstream firm engages in marginal cost-reducing R&D. Lømo [15] showed that, under a general inverse demand function, when there is common ownership among downstream firms, input prices depend on the degree of common ownership if the curvature of the inverse demand function is not constant. Chen et al. [5] considered horizontal and vertical shareholding and noted that higher input prices can be realized as the degree of shareholding increases because the price elasticity of demand for upstream firms decreases.

The remainder of this paper is organized as follows. The basic model is described in Section 2. Section 3 presents the primary results. Section 4 discusses the robustness of our main results. Finally, Section 5 concludes the paper.

## 2 Basic Model

We consider a decentralized supply chain with one upstream and two downstream firms. The upstream firm  $U$  produces input at no cost and sells it to the downstream firms ( $D1$  and  $D2$ ) at input price  $w$ .

To produce one unit of the final product, each downstream firm uses one unit of input and does not bear any costs except for payments for inputs. Downstream firm  $D2$  supplies its product to two spatially separated markets, markets  $A$  and  $B$ , while downstream firm  $D1$  supplies its product only to market  $A$ . We assume that markets  $A$  and  $B$  are independent of each other. Additionally, each downstream firm supplies a homogeneous product in market  $A$ . For this assumption, in Section 4.3, we consider the situation in which both downstream firms produce differentiated products in market  $A$ .

We denote downstream firm  $D1$ 's output in market  $A$  by  $q_{A1}$  and downstream firm  $D2$ 's outputs in markets  $A$  and  $B$  by  $q_{A2}$  and  $q_{B2}$ , respectively. We assume that the inverse demand functions in markets  $A$  and  $B$  are  $p_A \equiv \alpha - q_{A1} - q_{A2}$  and  $p_B \equiv \beta - q_{B2}$ , respectively. We denote the ratio of the intercept of the inverse demand function of market  $A$  to that of market  $B$  as  $r \equiv \alpha/\beta$ . The operating profits of the downstream



firms are as follows:

$$\pi_{D1} \equiv (p_A - w)q_{A1}, \quad \pi_{D2} \equiv (p_A - w)q_{A2} + (p_B - w)p_{B2}.$$

The upstream firm's profit is as follows:

$$\pi_U \equiv w(q_{A1} + q_{A2} + q_{B2}).$$

We assume that an institutional investor holds shares in both downstream firms. Following the literature on common ownership (López and Vives [16]; Vives [24]), each downstream firm has the following objective function:

$$V_{Di} \equiv \pi_{Di} + \theta\pi_{Dj},$$

where  $i, j = 1, 2$  and  $i \neq j$  and  $\theta \in (0, 1)$  is the degree of common ownership. We assume  $r_{min} \equiv (3 + \theta)/[2(5 + \theta)] < r < (11 + \theta)/4 \equiv r_{max}$ , which guarantees positive outcomes in equilibrium.<sup>7</sup>

The consumer, producer, and total surpluses are, respectively, as follows:

$$CS \equiv \frac{(q_{A1} + q_{A2})^2}{2} + \frac{q_{B2}^2}{2}, \quad PS \equiv \pi_U + \pi_{D1} + \pi_{D2}, \quad TS \equiv CS + PS.$$

The timing of the game is as follows. In the first stage, the upstream firm  $U$  chooses the input price  $w$ . In the second stage, the downstream firm  $D1$  chooses  $q_{A1}$  and the downstream firm  $D2$  chooses  $q_{A2}$  and  $q_{B2}$ . We solved this game using backward induction.

## 3 Analysis

### 3.1 Calculating equilibrium

In the second stage, the first-order conditions of the downstream firms,  $\partial V_{D1}/\partial q_{A1} = 0$ ,  $\partial V_{D2}/\partial q_{A2} = 0$ , and  $\partial V_{D2}/\partial q_{B2} = 0$ , lead to the following outputs:

$$q_{A1}(w, \theta) = q_{A2}(w, \theta) = \frac{\alpha - w}{3 + \theta}, \quad q_{B2}(w) = \frac{\beta - w}{2}. \quad (1)$$

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<sup>7</sup>When  $r < r_{min}$  or  $r > r_{max}$ , equilibrium outputs in market  $A$  or  $B$  become zero.

By substituting the outputs into the upstream firm's profit, solving the first-order condition  $\partial\pi_U/\partial w = 0$  and using  $r = \alpha/\beta$ , we obtain the equilibrium input price.

$$w^* = \frac{\beta(3 + \theta + 4r)}{2(7 + \theta)},$$

where the superscript '\*' denotes equilibrium outcomes. Then, by using  $r = \alpha/\beta$ , we obtain the following outcomes:

$$\begin{aligned} q_{A1}^* &= q_{A2}^* = \frac{\beta[2r(5 + \theta) - (3 + \theta)]}{2(3 + \theta)(7 + \theta)}, & q_{B2}^* &= \frac{\beta(11 + \theta - 4r)}{4(7 + \theta)}, \\ p_A^* &= \frac{\beta[3 + \theta + r(11 + 8\theta + \theta^2)]}{(3 + \theta)(7 + \theta)}, & p_B^* &= \frac{\beta(17 + 3\theta + 4r)}{4(7 + \theta)}, \\ \pi_U^* &= \frac{\beta^2(3 + \theta + 4r)}{8(3 + \theta)(7 + \theta)}, & \pi_{D1}^* &= \frac{\beta^2(1 + \theta)[3 + \theta - 2r(5 + \theta)]^2}{4(3 + \theta)^2(7 + \theta)^2}, \\ \pi_{D2}^* &= \frac{\beta^2(1 + \theta)[3 + \theta - 2r(5 + \theta)]^2}{4(3 + \theta)^2(7 + \theta)^2} + \frac{\beta^2(11 - 4r + \theta)^2}{16(7 + \theta)^2}, \\ CS^* &= \frac{\beta^2 \left[ (3 + \theta)^2(137 + 22\theta + \theta^2) + 16r^2(109 + 46\theta + 5\theta^2) - 8r(219 + 139\theta + 25\theta^2 + \theta^3) \right]}{32(3 + \theta)^2(7 + \theta)^2}, \\ PS^* &= \frac{\beta^2 \left[ (3 + \theta)^2(171 + 50\theta + 3\theta^2) + 16r^2(101 + 96\theta + 25\theta^2 + 2\theta^3) - 8r(33 + 65\theta + 27\theta^2 + 3\theta^3) \right]}{16(3 + \theta)^2(7 + \theta)^2}, \\ TS^* &= \frac{\beta^2 \left[ (3 + \theta)^2(479 + 122\theta + 7\theta^2) + 16r^2(311 + 238\theta + 55\theta^2 + 4\theta^3) - 8r(285 + 269\theta + 79\theta^2 + 7\theta^3) \right]}{8r(285 + 260\theta + 79\theta^2 + 7\theta^3)}. \end{aligned}$$

### 3.2 Comparative statics

We discuss the effects of common ownership on the equilibrium outcomes. First, we consider the effect of common ownership on pricing. By differentiating  $p_A^*$ ,  $p_B^*$ , and  $w^*$  with respect to  $\theta$ , we obtain the following derivatives:

$$\frac{\partial p_A^*}{\partial \theta} = \frac{\beta[2r(29 + 10\theta + \theta^2) - (3 + \theta)^2]}{(3 + \theta)^2(7 + \theta)^2} > 0, \quad \frac{\partial p_B^*}{\partial \theta} = \frac{\beta(1 - r)}{(7 + \theta)^2}, \quad \frac{\partial w^*}{\partial \theta} = \frac{2\beta(1 - r)}{(7 + \theta)^2},$$

where the first inequality is satisfied because we assume  $r_{min} < r < r_{max}$ . Subsequently, we obtain the main results.

**Proposition 1** *The price in market A always increases with the degree of common ownership. An increase in the degree of common ownership increases the price in market B and the input price if the size of market A is smaller than that of market B, that is,  $r < 1$ .*

To illustrate and intuition behind Proposition 1, we explain the price elasticity of the demand for inputs. From the outputs in the second stage (1), we obtain the price elasticities of demand for inputs in markets A and B.

$$\begin{aligned}\varepsilon_A(w) &\equiv -\frac{\partial[q_{A1}(w, \theta) + q_{A2}(w, \theta)]}{\partial w} \cdot \frac{w}{q_{A1}(w, \theta) + q_{A2}(w, \theta)} = \frac{w}{\alpha - w}, \\ \varepsilon_B(w) &\equiv -\frac{\partial q_{B2}(w)}{\partial w} \cdot \frac{w}{q_{B2}(w)} = \frac{w}{\beta - w}.\end{aligned}$$

Since the price elasticity of the aggregate demand for inputs is equal to the weighted average of the price elasticities for inputs, we obtain the following equation:

$$\begin{aligned}\varepsilon(w, \theta) &\equiv \frac{q_{A1}(w, \theta) + q_{A2}(w, \theta)}{q_{A1}(w, \theta) + q_{A2}(w, \theta) + q_{B2}(w)} \cdot \varepsilon_A(w) + \frac{q_{B2}(w)}{q_{A1}(w, \theta) + q_{A2}(w, \theta) + q_{B2}(w)} \cdot \varepsilon_B(w) \\ &= \frac{w(7 + \theta)}{\beta(3 + 4A + \theta) - w(7 + \theta)}.\end{aligned}$$

For  $\varepsilon(w, \theta)$ , the coefficient of  $\varepsilon_A(w)$  decreases with the degree of common ownership  $\theta$  and that of  $\varepsilon_B(w)$  increases with  $\theta$ . This is because common ownership weakens competition only in market A, such that output in market A falls, but output in market B remains unchanged. Thus, as  $\theta$  increases  $\varepsilon(w, \theta)$  approaches  $\varepsilon_B(w)$ . Therefore, if  $\varepsilon_B(w)$  is greater than  $\varepsilon_A(w)$ ,  $\varepsilon(w, \theta)$  increases, and vice versa.

To explain an intuition behind Proposition 1, we need only explain the change in the price elasticity of demand for inputs as the degree of common ownership increases. More specifically, it is sufficient to explain that when the size of market A is smaller than that of market B, the price elasticity of demand for input decreases with the degree of common ownership. First, when the market is large, the price elasticity of demand for inputs decreases. Thus, if  $\alpha < \beta$ , which is equivalent to  $r < 1$ , we have  $\varepsilon_A(w) > \varepsilon_B(w)$ . Since

an increase in  $\theta$  brings  $\varepsilon(w, \theta)$  closer to  $\varepsilon_B(w)$ , a large  $\theta$  decreases  $\varepsilon(w, \theta)$ . Therefore, as the degree of common ownership increases, the input demand becomes less elastic, leading to a high input price:  $\partial w^*/\partial\theta > 0$  if  $r < 1$ .

Intuitively, the other results for Proposition 1 are simple. Since downstream firm  $D2$  is a monopolist in market  $B$  and the degree of common ownership has no effect on competition in market  $B$ , the price of the final product moves in the same direction as the input price. Finally, an increase in  $\theta$  reduces competition in market  $A$ . Consequently, the total output in market  $A$  decreases, leading to higher prices for the final product in market  $A$ .

Next, we consider the effects of common ownership on the profits of the upstream and downstream firms.

$$\begin{aligned}\frac{\partial \pi_U^*}{\partial \theta} &= \frac{\beta^2(3 + \theta + 4r)[(3 + \theta) - 2r(5 + \theta)]}{2(3 + \theta)^2(7 + \theta)^2} < 0, \\ \frac{\partial \pi_{D1}^*}{\partial \theta} &= \frac{\beta^2[2r(5 + \theta) - (3 + \theta)][2r(47 - 7\theta - 7\theta^2 - \theta^3) - (5 - \theta)(3 + \theta)^2]}{4(3 + \theta)^3(7 + \theta)^3}, \\ \frac{\partial \pi_{D2}^*}{\partial \theta} &= \frac{\beta^2 \left[ 4r^2(181 - \theta^4 - 14\theta^3 - 60\theta^2 - 42\theta) + 2r(3\theta^4 + 40\theta^3 + 174\theta^2 + 256\theta + 39) - (3\theta + 17)(\theta + 3)^3 \right]}{4(3 + \theta)^3(7 + \theta)^3},\end{aligned}$$

where the sign of the first inequality is satisfied because we assume  $r_{min} < r < r_{max}$ . By showing the signs of the derivatives, we obtain the following proposition:

**Proposition 2** *The upstream firm's profit always decreases with the degree of common ownership. The operating profits of the downstream firms  $D1$  and  $D2$  decrease with the degree of common ownership if  $r < r_{D1}$  and  $r < r_{D2}$ , respectively, where  $r_{D1} < r_{D2}$  and*

$$\begin{aligned}r_{D1} &\equiv \frac{(5 - \theta)(3 + \theta)^2}{2(47 - 7\theta - 7\theta^2 - \theta^3)}, \\ r_{D2} &\equiv \frac{(3 + \theta) \left[ (7 + \theta)\sqrt{757 - 3\theta^4 - 44\theta^3 - 170\theta^2 + 36\theta} - 3\theta^3 - 31\theta^2 - 81\theta - 13 \right]}{4(181 - \theta^4 - 14\theta^3 - 60\theta^2 - 42\theta)}.\end{aligned}$$

**Proof** See Appendix.

An intuition behind this proposition is straightforward. From Proposition 1, if market  $A$  is smaller than market  $B$ , an increase in the degree of common ownership increases the input price. In other words, when  $r$  is small, common ownership increases the marginal costs for downstream firms. This tendency strengthens as  $r$  decreases. Therefore, if the marginal cost-raising effect dominates the competition-relaxing effect of common ownership, common ownership is harmful to downstream firms. Furthermore, since market  $B$  is a monopoly, relaxed competition due to common ownership does not occur in market  $B$ . Thus, if the input prices increase with common ownership, the cost of increased common ownership is greater for downstream firm  $D2$ . Therefore, if input prices increase because of common ownership, the cost of common ownership will be greater for downstream firm  $D2$ . Therefore, in Proposition 2, the threshold value for downstream firm  $D2$  is greater than that for downstream firm  $D1$ :  $r_{D2} > r_{D1}$ .

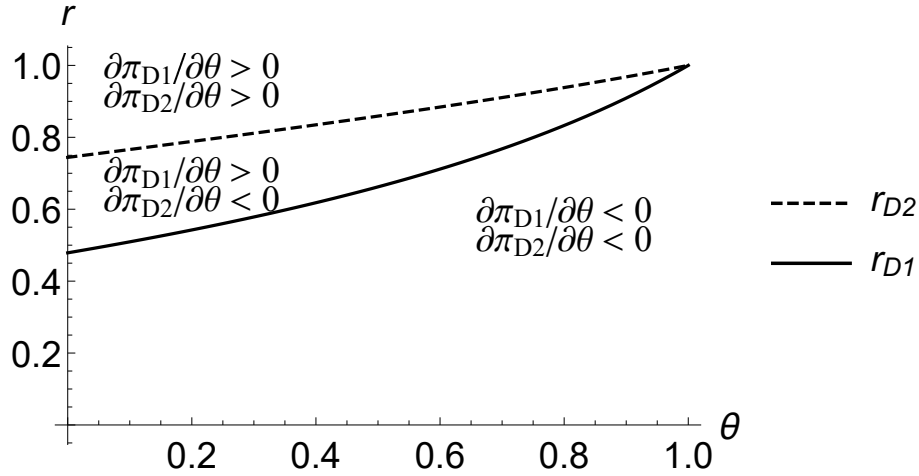


Figure 1: The effects of common ownership on operating profit of downstream firms.

Figure 1 shows that the degree of common ownership must be small if it harms downstream firms. Furthermore, we can confirm  $0.479 < r_{D1} < 1$  and  $0.744 < r_{D2} < 1$ . Thus, if  $r$  has an intermediate value for each downstream firm, then  $\theta$  maximizes its operating profits. We implicitly define  $\theta_{Di}$  such that  $r = r_{Di}$  at  $\theta = \theta_{Di}$ . Then, we obtain the following result:

**Corollary 1** For  $0.479 < r < 1$  and  $0.744 < r < 1$ , the operating profits of downstream firms  $D1$  and  $D2$  are maximized at  $\theta_{D1}$  and  $\theta_{D2}$ , respectively.

An intuition behind this result is as follows. As  $\theta$  approaches 1, the downstream firm's objective function approaches that of the merging firms. When input prices are fixed, the competition-reducing effect of common ownership disappears at  $\theta = 1$  because the downstream firm's operating profit is maximized in the monopoly. However, because the effect of changing input prices persists, the effect of common ownership on input prices dominates the competition-reducing effect when  $\theta$  is large. Thus, at some intermediate  $\theta$ , the operating profits of the downstream firms are maximized.

Finally, we provide the results of the comparative statics for consumer, producer, and total surpluses.

$$\begin{aligned}\frac{\partial CS^*}{\partial \theta} &= \frac{\beta^2 \left[ -4r^2(5\theta^3 + 69\theta^2 + 343\theta + 607) + r(\theta^4 + 40\theta^3 + 354\theta^2 + 1216\theta + 1461) - (\theta + 15)(\theta + 3)^3 \right]}{4(3 + \theta)^3(7 + \theta)^3} < 0, \\ \frac{\partial PS^*}{\partial \theta} &= \frac{\beta^2 \left[ -4r^2(\theta^4 + 15\theta^3 + 81\theta^2 + 157\theta + 2) - r(705 - 3\theta^4 - 24\theta^3 - 6\theta^2 + 352\theta) + (1 - \theta)(\theta + 3)^3 \right]}{2(3 + \theta)^3(7 + \theta)^3} < 0, \\ \frac{\partial TS^*}{\partial \theta} &= \frac{\beta^2 \left[ -4r^2(2\theta^4 + 35\theta^3 + 231\theta^2 + 657\theta + 611) + r(7\theta^4 + 88\theta^3 + 366\theta^2 + 512\theta + 51) - (3\theta + 13)(\theta + 3)^3 \right]}{4(3 + \theta)^3(7 + \theta)^3} < 0,\end{aligned}$$

where the signs of the inequalities above are satisfied because we assume  $r_{min} < r < r_{max}$ . Then, we obtain the following proposition:

**Proposition 3** Consumer, producer, and total surpluses decrease with the degree of common ownership.

**Proof** See Appendix.

An intuition behind this result is straightforward. Since an increase in the degree of common ownership relaxes competition in market  $A$ , the double marginalization prob-

lem worsens, leading to small total output. Therefore, consumer, producer, and total surpluses decrease with the degree of common ownership.

## 4 Robustness

### 4.1 A large class of inverse demand functions

In the basic model, we use linear inverse demand functions. Here, we demonstrate that the main results hold under more general inverse demand functions. We denote the inverse demand function in market  $k \in A, B$  by  $p_k(Q_k)$ , where  $Q_k$  is the total output in market  $k$ . We assume that the inverse demand function in market  $k$  has a constant curvature  $z_k$ ; more precisely, we define  $z_k \equiv -p_k''(Q_k)Q_k/p_k'(Q_k)$ , where for any  $Q_k > 0$ ,  $z_k$  is constant. This assumption leads to the following parametric form:  $p_k(Q_k) = a - bQ_k^{1-z_k}/(1-z_k)$  if  $Q_k < [a(1-z_k)/b]^{1/(1-z_k)}$ ;  $p_k(Q_k) = 0$  if  $Q_k \geq [a(1-z_k)/b]^{1/(1-z_k)}$ , where  $a, b > 0$ . This type of inverse demand function is used in the literature on industrial organization (Hu et al. [9]; López and Vives [16]). For simplicity, we assume  $z_k < 0$ . Thus, each inverse demand function is concave. The other settings are identical to those used for the basic model.

The main results of the basic model, Propositions 1 and 2, are obtained under a small  $r$ . In other words, the main results are obtained when market  $A$  is sufficiently smaller than market  $B$ . Therefore, we restrict our analysis to a situation in which the equilibrium total output of market  $A$  is positive but sufficiently close to zero.

**Assumption 1** *The equilibrium total output in market  $A$  is positive but sufficiently close to zero.*

**Decision on output of downstream firms** Since the downstream firms are symmetric in market  $A$ , both downstream firms choose the same output in market  $A$ ; we denote the output chosen by each downstream firm in market  $A$  by  $q_A$  and the output

chosen by downstream firm  $D2$  in market  $B$  by  $q_B$ . Substituting these outputs into the first-order conditions of downstream firms, we obtain the following:

$$\frac{\partial V_{Di}}{\partial q_{Ai}} = p_A(2q_A) - w + (1 + \theta)q_A p'_A(2q_A) = 0, \quad \frac{\partial V_{D2}}{\partial q_{B2}} = p_B(q_B) - w + q_B p'_B(q_B) = 0. \quad (2)$$

Differentiating the first-order conditions with respect to  $w$  or  $\theta$ , we obtain the following lemma:

**Lemma 1** *The results of the comparative statistics are as follows:*

$$\begin{aligned} \frac{\partial q_A}{\partial w} &= \frac{1}{[3 + \theta(1 - z_A) - z_A]p'_A(2q_A)} < 0, & \frac{\partial q_A}{\partial \theta} &= -\frac{q_A}{3 + \theta(1 - z_A) - z_A} < 0, \\ \frac{\partial q_B}{\partial w} &= \frac{1}{(2 - z_B)p'_B(q_B)} < 0. \end{aligned}$$

**Proof** See Appendix.

An intuition behind Lemma 1 is simple. An increase in the input price raises the marginal cost of downstream firms. Thus, a large  $w$  results in small outputs:  $\partial q_k / \partial w < 0$ . As the degree of common ownership increases, competition in the downstream market  $A$  becomes more moderate. Therefore, output in market  $A$  decreases with  $\theta$ .

**Decision on input price** By substituting the outputs in the second stage,  $q_A$  and  $q_B$ , into the profit of the upstream firm, differentiating it with respect to  $w$ , and using Lemma 1, we obtain the first-order condition in the first stage.

$$\frac{\partial \pi_U}{\partial w} = 2q_A + q_B + \frac{2w}{[3 + \theta(1 - z_A) - z_A]p'_A(2q_A)} + \frac{w}{(2 - z_B)p'_B(q_B)} = 0. \quad (3)$$

By solving the first-order condition for  $w$ , we obtain the equilibrium input price.

$$w(\theta) = \frac{(2q_A + q_B)[3 + \theta(1 - z_A) - z_A](2 - z_B)p'_A(2q_A)p'_B(q_B)}{-[3 + \theta(1 - z_A) - z_A]p'_A(2q_A) - 2(2 - z_B)p'_B(q_B)}. \quad (4)$$



**Comparative statics** By applying the implicit function theorem to (3) and using Lemma 1 and the definition of  $z_k$ , we obtain a comparative static result in the first stage.

$$w'(\theta) = \frac{2q_Aq_B(2-z_B)^2p'_A(2q_A)p'_B(q_B)^2[w+q_A(3+\theta-z_A-\theta z_A)p'_A(2q_A)]}{2q_Bwz_A(2-z_B)^2p'_B(q_B)^2-q_A(3+\theta-z_A-\theta z_A)p'_A(2q_A)\Phi},$$

where  $\Phi \equiv -4q_B(2-z_B)^2p'_B(q_B)^2 - (3+\theta-z_A-\theta z_A)p'_A(2q_A)[wz_B+2q_B(2-z_B)p'_B(q_B)]$ .

From Assumption 1, we obtain the following result:

**Proposition 4** *If the equilibrium total output in market A is positive but sufficiently close to zero, then the input price increases with the degree of common ownership:  $w'(\theta) > 0$ .*

**Proof** See Appendix.

We find that our main result is robust even if we allow the inverse demand function to be nonlinear. The intuition behind this proposition is the same as that for Proposition 1.

Regarding the effect of common ownership on the operating profits of downstream firms, differentiating the equilibrium operating profit of downstream firm  $D1$ , denoted by  $\pi_{D1}(\theta)$ , with respect to  $\theta$  and using Lemma 1, we obtain the following:

$$\frac{\partial \pi_{D1}(\theta)}{\partial \theta} = \frac{[p_A(2q_A) - w][w'(\theta) - q_A p'_A(2q_A)]}{[3 + \theta(1 - z_A) - z_A]p'_A(2q_A)} - q_A w'(\theta).$$

From Proposition 4, we have  $w'(\theta) > 0$  if  $q_A$  is positive but sufficiently close to zero. Therefore, under Assumption 1, the first and second terms of  $\partial \pi_{D1}(\theta)/\partial \theta$  are negative, which means that the operating profit of the downstream firm  $D1$  decreases with the degree of common ownership:  $\partial \pi_{D1}(\theta)/\partial \theta < 0$ .

Next, we consider the effect of common ownership on the operating profit of the downstream firm  $D2$ . Differentiating the equilibrium operating profit of the downstream

firm  $D2$ , denoted by  $\pi_{D2}(\theta)$ , with respect to  $\theta$  and applying Lemma 1, we obtain the following:

$$\frac{\partial \pi_{D2}(\theta)}{\partial \theta} = \frac{[p_A(2q_A) - w][w'(\theta) - q_A p'_A(2q_A)]}{[3 + \theta(1 - z_A) - z_A]p'_A(2q_A)} - q_A w'(\theta) - \frac{q_B(1 - z_B)w'(\theta)}{2 - z_B} + \frac{[p_B(q_B) - w]w'(\theta)}{(2 - z_B)p'_B(q_B)}.$$

Thus, under Assumption 1, the operating profit of downstream firm  $D2$  also decreases with the degree of common ownership:  $\partial \pi_{D2}(\theta)/\partial \theta < 0$ .

Finally, we consider the effect of common ownership on the upstream firm's profit. Differentiating the equilibrium profit of upstream firm  $U$ , denoted by  $\pi_U(\theta)$ , and using Lemma 1 and the equilibrium input price (4), we obtain the following:

$$\frac{\partial \pi_U(\theta)}{\partial \theta} = -\frac{2q_A(2q_A + q_B)(1 - z_B)p'_A(2q_A)p'_B(q_B)}{-[3 + \theta(1 - z_A) - z_A]p'_A(2q_A) - 2(2 - z_B)p'_B(q_B)}.$$

Thus,  $\partial \pi_U(\theta)/\partial \theta < 0$ . Summarizing the above results, we obtain the following proposition:

**Proposition 5** *The profit of upstream firm  $U$  always decreases with an increase in the degree of common ownership. If the equilibrium total output in market  $A$  is positive but sufficiently close to zero, then the operating profits of downstream firms  $D1$  and  $D2$  decrease with the degree of common ownership:  $\partial \pi_{D_i}(\theta)/\partial \theta < 0$ .*

From this result, we find that Proposition 2 is robust, even when we consider a case with a nonlinear inverse demand function. The intuition behind Proposition 5 is the same as that for Proposition 2.

## 4.2 Symmetric supply of downstream firms

In the basic model, we assume that downstream firm  $D1$  supplies its product only to market  $A$  and downstream firm  $D2$  supplies its product to both markets  $A$  and  $B$ . To emphasize the importance of supply asymmetry, we show that our results are not robust

in symmetric situations. There are two types of symmetrical cases: (i) downstream firms  $D1$  and  $D2$  exclusively supply to markets  $A$  and  $B$ , respectively; and (ii) downstream firms  $D1$  and  $D2$  supply to both markets  $A$  and  $B$ .

First, we consider case (i). Since we assume that markets  $A$  and  $B$  are independent and that the downstream firms do not compete with each other, the equilibrium output is independent of the degree of common ownership. Therefore, the input price and profits of the downstream firms  $D1$  and  $D2$  are also independent of the degree of common ownership. Therefore, our results are not robust in this case.

Next, we consider case (ii). Since the downstream firms supply both markets, we assume that the inverse demand functions in markets  $A$  and  $B$  are  $p_A = \alpha - q_{A1} - q_{A2}$  and  $p_B = \beta - q_{B1} - q_{B2}$ , respectively. To guarantee positive outputs in equilibrium, we assume  $1/3 < r < 3$ , where  $r = \alpha/\beta$ . The other settings are the same as that for the basic model.

In the second stage, the first-order condition yields the following outputs:

$$q_{Ai} = \frac{\alpha - w}{3 + \theta}, \quad q_{Bi} = \frac{\beta - w}{3 + \theta}.$$

In the first stage, the upstream firm  $U$  sets the following input price:

$$w = \frac{\alpha + \beta}{4}.$$

Thus, the equilibrium input price is independent of the degree of common ownership, indicating that Proposition 1 is not robust under the symmetric supply of downstream firms.

From these outcomes, the equilibrium profits of the upstream and downstream firms are as follows:

$$\pi_U = \frac{\beta^2(1+r)^2}{8(3+\theta)}, \quad \pi_{Di} = \frac{(5-6r+5r^2)\beta^2(1+\theta)}{8(3+\theta)^2}.$$

The equilibrium profit of the upstream firm  $\pi_U(\theta)$  decreases with the degree of common ownership:  $\partial\pi_U(\theta)/\partial\theta < 0$ . By differentiating  $\pi_{Di}$  with respect to  $\theta$ , we show that

the downstream firm's' equilibrium profits increase with the degree of common ownership.

$$\frac{\partial \pi_{Di}}{\partial \theta} = \frac{(5 - 6r + 5r^2)\beta^2(1 - \theta)}{8(3 + \theta)^3} > 0,$$

where because of  $1/3 < r < 3$ , the inequality is satisfied. Therefore, our results are robust for upstream firm profits but not for downstream firm profits.

### 4.3 Differentiated products in market A

In the basic model, we assume that each downstream firm produces a homogenous product. Here, we consider the case of differentiated products in market A. For simplicity, we use linear inverse demand functions:  $p_{A1} = \alpha - q_{A1} - \gamma q_{A2}$ ,  $p_{A2} = \alpha - q_{A2} - \gamma q_{A1}$ , and  $p_B = \beta - q_{B2}$ , where  $p_{Ai}$  is the price of downstream firm  $Di$  in market A and  $\gamma \in (0, 1)$  is the degree of product substitutability. To guarantee positive outputs in equilibrium, we assume that  $\hat{r}_{min} \equiv (2 + \gamma + \gamma\theta)/(8 + 2\gamma + 2\gamma\theta) < r < (10 + \gamma + \gamma\theta)/4 \equiv \hat{r}_{max}$ . The remaining settings are identical to that for the basic model.

In the second stage, from the first-order conditions, the following outputs are obtained:

$$q_{A1} = q_{A2} = \frac{\alpha - w}{2 + \gamma + \gamma\theta}, \quad q_{B2} = \frac{\beta - w}{2}.$$

In the first stage, the upstream firm chooses the following input price:

$$w(\theta) = \frac{4\alpha + \beta(2 + \gamma + \gamma\theta)}{2(6 + \gamma + \gamma\theta)}.$$

By substituting these outcomes, we obtain the equilibrium profits of the upstream and downstream firms.

$$\begin{aligned} \pi_U(\theta) &= \frac{\beta^2(2 + 4r + \gamma + \gamma\theta)^2}{8(2 + \gamma\theta)(6 + \gamma + \gamma\theta)}, & \pi_{D1}(\theta) &= \frac{\beta^2(1 + \gamma\theta)[2 + \gamma + \gamma\theta - 2r(4 + \gamma + \gamma\theta)]^2}{4(2 + \gamma + \gamma\theta)^2(6 + \gamma + \gamma\theta)^2}, \\ \pi_{D2}(\theta) &= \frac{\beta^2(1 + \gamma\theta)[2 + \gamma + \gamma\theta - 2r(4 + \gamma + \gamma\theta)]^2}{4(2 + \gamma + \gamma\theta)^2(6 + \gamma + \gamma\theta)^2} + \frac{\beta^2(10 - 4r\gamma + \gamma\theta)^2}{16(6 + \gamma + \gamma\theta)^2}. \end{aligned}$$

By differentiating the equilibrium outcomes with respect to  $\theta$ , we obtain the following proposition:

**Proposition 6** *When the degree of common ownership increases, (i) input price increases if  $r < 1$ , (ii) the profit of the upstream firm decreases, (iii) the operating profit of downstream firm D1 decreases if  $r < \hat{r}_{D1}$ , and (iv) the operating profit of downstream firm D2 decreases if  $r < \hat{r}_{D2}$  where*

$$\hat{r}_{D1} = \frac{[4 + \gamma(1 - \theta)](\gamma\theta + \gamma + 2)^2}{2[\gamma^3(1 - \theta)(\theta + 1)^2 + 2\gamma^2(5 + 2\theta - 3\theta^2) + 4\gamma(7 - 3\theta) + 8]},$$

$$\hat{r}_{D2} = \frac{-\hat{\Phi}_1 + \sqrt{\hat{\Phi}_1^2 - 4\hat{\Phi}_2\hat{\Phi}_0}}{2\hat{\Phi}_2},$$

$$\hat{\Phi}_2 \equiv 64 + 4\gamma(\gamma\theta + \gamma + 4)[\gamma(1 - \theta^2)(\gamma\theta + \gamma + 8) + 8(3 - 2\theta)],$$

$$\hat{\Phi}_1 \equiv 2\gamma^4(\theta + 1)^3(3\theta - 1) + 8\gamma^3(\theta + 1)^2(8\theta - 1) + 8\gamma^2(27\theta^2 + 26\theta - 1) + 32\gamma(8\theta + 1) + 64,$$

$$\hat{\Phi}_0 \equiv -(\gamma\theta + \gamma + 2)^3(3\gamma\theta + \gamma + 16).$$

**Proof** See Appendix.

From Proposition 6, we find that our main results are robust, even when we introduce product differentiation in market  $A$ . Thus, the intuition behind this proposition is the same as that for the basic model.

## 5 Conclusions

We consider a market with one upstream and two downstream firms whose shares are held by an institutional investor; that is, downstream common ownership. One downstream firm can supply its product to all markets, whereas the other can supply its product to one of the markets. This channel asymmetry may reverse the effect of common ownership on input price. That is, an increase in the degree of common ownership may increase the input price and decrease downstream firms' profits. In this case, common ownership worsens the double marginalization problem, resulting in small consumer, producer, and total surpluses.

We consider only quantity competition in a downstream market. However, it is

worth considering price competition between downstream firms. We also assume that downstream markets are independent. However, if each market is expressed as a product distinction rather than a spatial distinction, substitutability arises for the products sold in each market. Therefore, it is worthwhile to explore this possibility. These extensions should be the subject of future research.

# Appendix

## A1. Proof of Proposition 2

First, we consider the effect of  $\theta$  on an upstream firm's operating profit. By solving  $\partial\pi_U^*/\partial\theta < 0$  for  $r$ , we have  $r > (3 + \theta)/(10 + 2\theta)$ . Assuming that  $r_{min} < r < r_{max}$  and  $r_{min} = (3 + \theta)/(10 + 2\theta)$ , we obtain  $\partial\pi_U^*/\partial\theta < 0$ .

Next, we consider the sign of  $\partial\pi_{D1}^*/\partial\theta$ . By solving  $\partial\pi_{D1}^*/\partial\theta < 0$  for  $r$ , we obtain the following condition:

$$r < \frac{(5 - \theta)(3 + \theta)^2}{2(47 - 7\theta - 7\theta^2 - \theta^3)} \equiv r_{D1}.$$

A comparison of  $r_{D1}$  with  $r_{min}$  and  $r_{max}$  yields that  $r_{min} < r_{D1} < r_{max}$ . Thus, we obtain  $\partial\pi_{D1}^*/\partial\theta < 0$  if  $r < r_{D1}$ .

Finally, we show the condition under which  $\partial\pi_{D2}^*/\partial\theta < 0$ . By solving  $\partial\pi_{D2}^*/\partial\theta < 0$  for  $r$ , we have  $r_{D2'} < r < r_{D1}$ , where

$$r_{D2} \equiv \frac{(3 + \theta) \left[ (7 + \theta)\sqrt{757 - 3\theta^4 - 44\theta^3 - 170\theta^2 + 36\theta} - 3\theta^3 - 31\theta^2 - 81\theta - 13 \right]}{4(181 - \theta^4 - 14\theta^3 - 60\theta^2 - 42\theta)},$$

$$r_{D2'} \equiv \frac{(3 + \theta) \left[ -(7 + \theta)\sqrt{757 - 3\theta^4 - 44\theta^3 - 170\theta^2 + 36\theta} - 3\theta^3 - 31\theta^2 - 81\theta - 13 \right]}{4(181 - \theta^4 - 14\theta^3 - 60\theta^2 - 42\theta)}.$$

Furthermore, for any  $\theta \in (0, 1)$ , we can numerically confirm  $r_{D2'} < r_{min} < r_{D2} < r_{max}$ . Hence, we complete the proof of Proposition 2.  $\square$

## A2. Proof of Proposition 3

First, the sign of  $\partial CS^*/\partial\theta$  depends on the terms in square brackets of the numerator. This is a quadratic function of  $r$  and the discriminant is  $-(21 + 10\theta + \theta^2)^2(4079 + 2612\theta + 458\theta^2 + 20\theta^3 - \theta^4) < 0$ . Thus, we obtain  $\partial CS^*/\partial\theta < 0$ .

Next, we demonstrate that  $\partial PS^*/\partial\theta < 0$ . By solving  $\partial PS^*/\partial\theta < 0$  for  $r$ , we obtain  $r < r_{PS'}$  or  $r > r_{PS}$ , where  $r_{PS'}$  and  $r_{PS}$  are the roots of  $\partial PS^*/\partial\theta = 0$  and  $r_{PS'} < r_{PS}$ . We can numerically show that  $r_{PS} < r_{min}$ . Thus, for any  $r \in (r_{min}, r_{max})$ , we have  $\partial PS^*/\partial\theta < 0$ .

Finally, we consider the sign of  $\partial TS^*/\partial\theta$ . The sign of the derivative depends on the terms in the square brackets of the numerator. This is a quadratic function of  $r$  and the discriminant is  $-(21 + 10\theta + \theta^2)^2(7775 + 10420\theta + 4522\theta^2 + 788\theta^3 + 47\theta^4) < 0$ . Therefore, we obtain  $\partial TS^*/\partial\theta < 0$ .  $\square$

### A3. Proof of Lemma 1

By differentiating the first-order conditions (2) with respect to  $w$  or  $\theta$  and substituting  $p'_k(2q_k) = -z_k p'_k(2q_k)/(2q_k)$ , we obtain the following equations:

$$\begin{aligned}\frac{\partial}{\partial w} \left[ \frac{\partial V_{Di}}{\partial q_{Ai}} \right] &= [3 + \theta(1 - z_A) - z_A] p'_A(2q_A) \frac{\partial q_A}{\partial w} - 1 = 0, \\ \frac{\partial}{\partial \theta} \left[ \frac{\partial V_{Di}}{\partial q_{Ai}} \right] &= p'_A(2q_A) \left[ q_A + (3 + \theta - \theta z_A - z_A) \frac{\partial q_A}{\partial \theta} \right] = 0, \\ \frac{\partial}{\partial w} \left[ \frac{\partial V_{D2}}{\partial q_{B2}} \right] &= 1 - (2 - z_B) p'_B(q_B) \frac{\partial q_B}{\partial w} = 0.\end{aligned}$$

By solving the equations for  $\partial q_A/\partial w$ ,  $\partial q_A/\partial \theta$ , and  $\partial q_B/\partial w$ , we complete the proof.  $\square$

### A4. Proof of Proposition 4

First, we consider the sign of the denominator in  $w'(\theta)$ . The first term  $2q_B w z_A (2 - z_B)^2 p'_B(q_B)^2$  is negative. When  $q_A$  is positive but sufficiently small, the second term  $-q_A (3 + \theta - z_A - \theta z_A) p'_A(2q_A) \Phi$  is sufficiently close to zero. This is because, for any  $q_k \geq 0$ ,  $p_k(2q_k)$  and  $p'_k(2q_k)$  are bounded; note that we use the inverse demand function with a constant curvature. Therefore, under Assumption 1, the denominator is negative.

Next, we consider the numerator. The term outside the square brackets  $2q_A q_B (2 - z_B)^2 p'_A(2q_A) p'_B(q_B)^2$  is negative. Owing to Assumption 1, the terms in square brackets are positive. Thus, the numerator is also negative. Therefore, when the total output in the downstream market  $A$  is sufficiently small, we obtain  $w'(\theta) > 0$ .  $\square$



## A4. Proof of Proposition 6

First, we consider result (i). The first derivative of  $w(\theta)$  leads to  $w'(\theta) = 2(1-r)\beta\gamma/(6+\gamma+\gamma\theta)^2$ . Therefore, we obtain (i) from Proposition 6.

Second, we demonstrate that  $\partial\pi_U(\theta)/\partial\theta < 0$ . By differentiating  $\pi_U(\theta)$  with respect to  $\theta$ , we obtain the following:

$$\frac{\partial\pi_U(\theta)}{\partial\theta} = -\frac{\beta^2\gamma(2+4r+\gamma+\gamma\theta)[r(8+2\gamma+2\gamma\theta)-2-\gamma-\gamma\theta]}{2(2+\gamma+\gamma\theta)^2(6+\gamma+\gamma\theta)^2} < 0,$$

where the terms in the square brackets of the numerator are positive because we assume  $\hat{r}_{min} < r < \hat{r}_{max}$ .

Third, we consider the effect of  $\theta$  on  $\pi_{D1}(\theta)$ . By solving the first derivative  $\partial\pi_{D1}(\theta)/\partial\theta < 0$  for  $r$ , we obtain the following condition:

$$r < \frac{[4+\gamma(1-\theta)](\gamma\theta+\gamma+2)^2}{2[\gamma^3(1-\theta)(\theta+1)^2+2\gamma^2(5+2\theta-3\theta^2)+4\gamma(7-3\theta)+8]} \equiv \hat{r}_{D1}.$$

Furthermore, we show that  $\hat{r}_{min} < \hat{r}_{D1} < \hat{r}_{max}$ . Thus, we obtain (iii) from Proposition 6.

Finally, we consider the sign of  $\partial\pi_{D2}(\theta)/\partial\theta$ .

$$\frac{\partial\pi_{D2}(\theta)}{\partial\theta} = \frac{\beta^2\gamma(\hat{\Phi}_2r^2+\hat{\Phi}_1r+\hat{\Phi}_0)}{4(2+\gamma+\gamma\theta)^3(6+\gamma+\gamma\theta)^3},$$

where  $\hat{\Phi}_2 \equiv 64+4\gamma(\gamma\theta+\gamma+4)[\gamma(1-\theta^2)(\gamma\theta+\gamma+8)+8(3-2\theta)]$ ,  $\hat{\Phi}_1 \equiv 2\gamma^4(\theta+1)^3(3\theta-1)+8\gamma^3(\theta+1)^2(8\theta-1)+8\gamma^2(27\theta^2+26\theta-1)+32\gamma(8\theta+1)+64$ , and  $\hat{\Phi}_0 \equiv -(\gamma\theta+\gamma+2)^3(3\gamma\theta+\gamma+16)$ .

Owing to  $\hat{\Phi}_2 > 0$ , solving  $\partial\pi_{D2}(\theta)/\partial\theta < 0$  for  $r$ , we obtain  $\hat{r}_{D2'} < r < \hat{r}_{D2}$ , where  $\hat{r}_{D2'}$  and  $\hat{r}_{D2}$  are the roots for the equation  $\partial\pi_{D2}(\theta)/\partial\theta = 0$ . Additionally, we numerically show that  $\hat{r}_{D2'} < \hat{r}_{min} < \hat{r}_{D2} < \hat{r}_{max}$ . This result leads to (iv) in Proposition 6.  $\square$

## References

- [1] Avinadav, T. and Shamir, N. (2023) Partial vertical ownership, capacity investment, and information exchange in a supply chain. *Management Science*.
- [2] Aviv, Y. and Shamir, N. (2021) Financial cross-ownership and information dissemination in a supply chain. *Manufacturing & Service Operations Management*, 23(6), 1524–1538.
- [3] Azar, J., Schmalz, M.C., and Tecu, I. (2018) Anticompetitive effects of common ownership. *Journal of Finance*, 73(4), 1513–1565.
- [4] Backus, M., Conlon, C., and Sinkinson, M. (2019) The common ownership hypothesis: Theory and evidence. Brookings papers.  
Available at: [https://chrisconlon.github.io/site/co\\_brookings.pdf](https://chrisconlon.github.io/site/co_brookings.pdf).
- [5] Chen, L., Matsumura, T., and Zeng, C. (2021) Welfare consequence of common ownership in a vertically related market. *Journal of Industrial Economics*, forthcoming.
- [6] Cho, S.H. (2014) Horizontal mergers in multitier decentralized supply chains. *Management Science*, 60(2), 356–379.
- [7] Dennis, P., Gerardi, K., and Schenone, C. (2022) Common ownership does not have anticompetitive effects in the airline industry. *Journal of Finance*, 77(5), 2765–2798.
- [8] Greenhut, M.L. and Ohta, H. (1979) Vertical integration of successive oligopolists. *American Economic Review*, 69(1), 137–141.
- [9] Hu, Q., Monden, A., and Mizuno, T. (2022) Downstream cross-holdings and upstream R&D. *Journal of Industrial Economics*, 70(3), 775–789.
- [10] Kini, O., Lee, S., and Shen, M. (2023) Common institutional ownership and product market threats. *Management Science*.

- [11] Koch, A., Panayides, M., and Thomas, S. (2021) Common ownership and competition in product markets. *Journal of Financial Economics*, 139(1), 109–137.
- [12] Koulamas, C. and Kyparisis, G.J. (2010) A note on the effects of downstream efficiency on upstream pricing. *European Journal of Operational Research*, 200(3), 926–928.
- [13] Lewellen, K. and Lowry, M. (2021) Does common ownership really increase firm coordination? *Journal of Financial Economics*, 141(1), 322–344.
- [14] Li, Y. and Shuai, J. (2022) Input price discrimination and horizontal shareholding. *Journal of Regulatory Economics*, 61(1), 48–66.
- [15] Lømo, T.L. (2022) Input prices and downstream overlapping ownership.  
Available at SSRN: <https://ssrn.com/abstract=4407897>.
- [16] López, Á.L. and Vives, X. (2019) Overlapping ownership, R&D spillovers, and antitrust policy. *Journal of Political Economy*, 127(5), 2394–2437.
- [17] Mukherjee, A. (2019) Profit raising entry in a vertical structure. *Economics Letters*, 183, 108543.
- [18] Pinopoulos, I.N. (2011) Input pricing by an upstream monopolist into imperfectly competitive downstream markets. *Research in Economics*, 65(3), 144–151.
- [19] Rotemberg, J. (1984) Financial transaction costs and industrial performance. MIT Sloan Working Paper.
- [20] Salinger, M.A. (1988) Vertical mergers and market foreclosure. *Quarterly Journal of Economics*, 103(2), 345–356.
- [21] Shuai, J., Xia, M., and Zeng, C. (2023) Upstream market structure and downstream partial ownership. *Journal of Economics & Management Strategy*, 32(1), 22–47.

- [22] Torshizi, M. and Clapp, J. (2021) Price effects of common ownership in the seed sector. *Antitrust Bulletin*, 66(1), 39–67.
- [23] Tyagi, R.K. (1999) On the effects of downstream entry. *Management Science*, 45(1), 59–73.
- [24] Vives, X. (2020) Common ownership, market power, and innovation. *International Journal of Industrial Organization*, 70, 102528.
- [25] Yenipazarli, A. (2021) Downstream entry revisited: Economic effects of entry in vertically-related markets. *Omega*, 103, 102428.