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Bertrand competition in vertically related markets

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Abstract

We build a successive Bertrand model with a homogenous good. We show that increasing the production efficiency of an industry can reduce firms' profits. We also show that this result holds in the successive Cournot model. Hence, an industrial policy aimed at improving production efficiency may be undesirable for firms.

Key words: Successive Bertrand; Production efficiency; Homogenous good; Vertical market

JEL classification: L13, L11

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1 Introduction

Salinger's (1988) model is famous in the analysis of vertically related markets and it assumes quantity competition in both the upstream and downstream markets.¹ Although Salinger's (1988)-type model is frequently used because of tractability, price competition tends to be overlooked.

In practice, it is sometimes observed an industry with price competition. Winston et al. (2011) stated that the railroad industry (coal transportation) in the US tends to engage in price competition. Furthermore, Roy et al. (2006) empirically reported that in the microprocessor, facial tissue, and automobile industries, the strategic variable of firms is price. Therefore, we believe that identifying the characteristics of price competition remains valuable.

We aim to consider the implications of price competition in vertically related markets. To this end, we incorporate Dastidar (1995)-type Bertrand competition² into a Salinger-type (1988) vertical structure model.³ We show that when upstream and downstream production is efficient, increasing upstream and downstream production efficiency reduces firms' upstream and downstream profits, respectively. We also consider the case of successive Cournot competitions and show that a similar result of Bertrand case holds. As improving efficiency in a vertical structure does not necessarily increase firms' profits, our results imply that aiming to improve efficiency is a problematic issue in industrial policy.

2 Model

We consider a market with m upstream and n downstream firms. Each upstream and downstream firm produces a homogenous input and final product, respectively. Upstream firms have a convex cost function γx_k^2 where x_k is the output of the upstream firm k ($= 1, \dots, m$). Convex

¹See, for example, Ghosh and Morita (2007) and Matsushima (2006).

²Dastidar (1995) examined homogenous Bertrand competition with a convex cost.

³Dastidar's model is applied in various scenarios; see, for example, Cabon-Dhersin and Drouhin (2014) and Mizuno and Takauchi (2020).

cost is a popular setting in oligopoly (e.g., Von Weizsacker, 1980). Upstream firm k 's input price is denoted by w_k . Upstream firm k 's profit is $\pi_k^u \equiv w_k x_k - \gamma x_k^2$. Under upstream price competition, each upstream firm chooses its price w_k , and under upstream quantity competition, each upstream firm faces a derived inverse demand $w(\sum_{l=1}^m x_l)$ and chooses its output x_k .

Each downstream firm purchases inputs from upstream firms at price w . Note that, under upstream price competition, the input price is determined as $w \equiv \min\{w_1, \dots, w_m\}$. If multiple upstream firms choose w , the downstream firms purchase an equal quantity of inputs from the upstream firms. We assume that each downstream firm uses one unit of input to produce one unit of output. When the downstream firm i ($= 1, \dots, n$) produces q_i output, its production cost is λq_i^2 .

The downstream market demand function is $Q \equiv (a - p)/b$. Under downstream price competition, each downstream firm chooses its price p_i . Hence, the total demand is determined as $p \equiv \min\{p_1, \dots, p_n\}$. Each downstream firm's demand is determined by $q_i = Q$ if for any j ($= 1, \dots, n$) and $i \neq j$, $p_i < p_j$; $q_i = Q/s$ if p_i is the lowest price and s downstream firms choose it; and $q_i = 0$ if p_i is not the lowest price. Under downstream quantity competition, each downstream firm faces the same inverse demand $p_i = a - b \sum_{j=1}^n q_j$ and chooses its output q_i . The downstream firm i 's profit is $\Pi_i^d \equiv (p_i - w)q_i - \lambda q_i^2$. The consumer surplus is $CS = bQ^2/2$ and the total surplus is $TS = CS + \sum_{k=1}^m \pi_k^u + \sum_{j=1}^n \Pi_j^d$.

We consider two cases: all firms compete in price; all firms compete in quantity. The timing of the game is as follows. In the first stage, each upstream firm chooses its strategic variable (w_k or x_k). In the second stage, each downstream firm chooses its strategic variable (p_i or q_i).

As we consider homogenous price competition with a convex cost, the Nash equilibria of each stage game are given by a closed interval. To choose a unique equilibrium, we use the payoff-dominance criterion as the equilibrium refinement. Note that an equilibrium point is payoff dominant if there is no other equilibrium point with a higher payoff for all players. Because

the payoff-dominance equilibrium is supported by economic experiments (e.g., Rankin et al., 2000) and is often used (e.g., Cabon-Dhersin and Drouhin, 2014; Mizuno and Takauchi, 2020), we employ this criterion. Hence, the equilibrium concept is a subgame-perfect equilibrium with the payoff-dominance criterion. For simplicity, we assume $\lambda < bn/(n-1) \equiv \hat{\lambda}$. Under this assumption, the equilibrium downstream price does not coincide with the price that maximizes the downstream firms' profits.

3 Equilibrium calculation

3.1 Successive Bertrand competition

We consider the cases of upstream and downstream price competition. Because the downstream firms are symmetric, they choose a symmetric price $p(w)$. We define the aggregate and each downstream firm's outputs at $p(w)$ as $Q(w) = [a - p(w)]/b$ and $q(w) = Q(w)/n$, respectively. As downstream firms never deviate from $p(w)$, the following inequalities must be satisfied:

$$[p(w) - w]q(w) - \lambda q(w)^2 \geq [p(w) - w]Q(w) - \lambda Q(w)^2, \quad [p(w) - w]q(w) - \lambda q(w)^2 \geq 0.$$

The first inequality represents the condition under which each downstream firm does not undercut its price, and the second inequality indicates that each downstream firm has no incentive to increase its price.

Solving the above inequalities for $p(w)$, we obtain the Nash equilibria as the interval $p(w) \in [\underline{p}(w), \bar{p}(w)]$, where:

$$\underline{p}(w) = \frac{bnw + a\lambda}{bn + \lambda}, \quad \bar{p}(w) = \frac{bnw + a(1+n)\lambda}{bn + (1+n)\lambda}. \quad (1)$$

As we use the payoff-dominance criterion, we derive the price at which the downstream firms earn the highest profit. Hence, we define the symmetric collusive price $p_{col}(w)$ that maximizes

the downstream firm's profit as follows:

$$p_{col}(w) = \frac{a(bn + 2\lambda) + bnw}{2(bn + \lambda)}. \quad (2)$$

When $\lambda < \hat{\lambda}$, we obtain $\bar{p}(w) < p_{col}(w)$. Hence, for $p(w) \in [\underline{p}(w), \bar{p}(w)]$, the downstream profit increases with p . Therefore, $\bar{p}(w)$ is a unique price that satisfies the payoff-dominance criterion.

We consider the first stage. Because the upstream firms are symmetric, they choose a symmetric input price w^* and obtain symmetric demand $Q(w^*)/m$. In the second stage, the conditions under which upstream firms do not undercut or raise prices are expressed by the following inequalities:

$$w^* \frac{Q(w^*)}{m} - \gamma \left[\frac{Q(w^*)}{m} \right]^2 \geq w^* Q(w^*) - \gamma Q(w^*)^2, \quad w^* \frac{Q(w^*)}{m} - \gamma \left[\frac{Q(w^*)}{m} \right]^2 \geq 0.$$

Then, the input prices in the Nash equilibria are given by the interval $w^* \in [\underline{w}, \bar{w}]$, where:

$$\underline{w} = \frac{a\gamma n}{bmn + \lambda m + \lambda mn + \gamma n}, \quad \bar{w} = \frac{a\gamma(m+1)n}{bmn + \lambda m + n(\gamma + \gamma m + \lambda m)}.$$

To find the equilibrium input price that satisfies the payoff-dominance criterion, we calculate the collusive input price that maximizes each upstream firm's profit.

$$w_{col} = \frac{a(bmn + \lambda m + \lambda mn + 2\gamma n)}{2[bmn + \lambda m + n(\gamma + \lambda m)]}.$$

The input prices above yield Lemma 1:

Lemma 1. (i) $\bar{w} > \underline{w}$. (ii) $w_{col} > \bar{w}$ iff $\gamma < \frac{m[bn+(1+n)\lambda]}{n(m-1)} \equiv \hat{\gamma}$.

From this lemma, for $w^* \in [\underline{w}, \bar{w}]$, the upstream firm's profit is maximized at \bar{w} and w_{col} if $\gamma < \hat{\gamma}$ and $\gamma \geq \hat{\gamma}$, respectively. Therefore, the equilibrium input price is \bar{w} if $\gamma < \hat{\gamma}$, then it is w_{col} if $\gamma \geq \hat{\gamma}$.

For $\gamma < \hat{\gamma}$, we obtain the following equilibrium outcomes:

$$\bar{\pi}^u = \frac{a^2 \gamma m n^2}{[bmn + \lambda m + n(\gamma + \gamma m + \lambda m)]^2}, \quad \bar{\Pi}^d = \frac{a^2 \lambda m^2 n}{[bmn + \lambda m + n(\gamma + \gamma m + \lambda m)]^2}. \quad (3)$$

$$\overline{CS} = \frac{a^2 b m^2 n^2}{2[bmn + \lambda m + n(\gamma + \gamma m + \lambda m)]^2}, \quad \overline{TS} = \frac{a^2 m^2 n^2 [b + 2(\gamma + \lambda)]}{2[bmn + \lambda m + n(\gamma + \gamma m + \lambda m)]^2}. \quad (4)$$

For $\gamma \geq \hat{\gamma}$, equilibrium outcomes are as follows.

$$\pi_{col}^u = \frac{a^2 n}{4[bmn + \lambda m + n(\gamma + \lambda m)]}, \quad \Pi_{col}^d = \frac{a^2 \lambda m^2 n}{4[bmn + \lambda m + n(\gamma + \lambda m)]^2}. \quad (5)$$

$$CS_{col} = \frac{a^2 b m^2 n^2}{8[bmn + \lambda m + n(\gamma + \lambda m)]^2}, \quad TS_{col} = \frac{a^2 mn(3bmn + 2n\gamma + 2m\gamma 4mn\gamma)}{8[bmn + \lambda m + n(\gamma + \lambda m)]^2}. \quad (6)$$

3.2 Successive Cournot competition

In the second stage, the profit of downstream firm i is $\Pi_i^d \equiv (a - b \sum_{j=1}^n q_j - w)q_i - \lambda q_i^2$. The first-order condition yields each firm's output: $q_i(w) = (a - w)/(b + bn + 2\lambda)$.

Substituting $q_i(w)$ into the input market-clearing condition $\sum_{j=1}^n q_j(w) = \sum_{l=1}^m x_l$, we obtain the derived inverse demand for input.

$$w \left(\sum_{l=1}^m x_l \right) = a - \frac{(b + bn + 2\lambda)}{n} \sum_{l=1}^m x_l.$$

Substituting the derived inverse demand into the upstream profit, we obtain $\pi_k^u = w(\sum_{l=1}^m x_l)x_k - \gamma x_k^2$. The first-order condition yields the following output:

$$x_k^C = \frac{an}{b(1+m)(1+n) + 2(n\gamma + \lambda + m\lambda)},$$

where the variable 'C' denotes Cournot competition.

Using the above outcomes, we obtain the upstream and downstream profits, consumer surplus, and total surplus:

$$\pi_C^u = \frac{a^2 n(b + bn + n\gamma + 2\lambda)}{[b(1+m)(1+n) + 2(n\gamma + \lambda + m\lambda)]^2}, \quad \Pi_C^d = \frac{a^2 m^2 (b + \lambda)}{[b(1+m)(1+n) + 2(n\gamma + \lambda + m\lambda)]^2}, \quad (7)$$

$$CS_C = \frac{a^2 b m^2 n^2}{2[b(1+m)(1+n) + 2(n\gamma + \lambda + m\lambda)]^2}, \quad TS_C = \frac{a^2 mn[2b(1+n) + bm(2+n) + 2n\gamma + 2(2+m)\lambda]}{2[b(1+m)(1+n) + 2(n\gamma + \lambda + m\lambda)]^2}. \quad (8)$$

4 Comparative statics

4.1 Successive Bertrand competition with upper bound pricing

We consider the case where $\gamma < \hat{\gamma}$ under successive Bertrand competition. Differentiating (3) and (4) with respect to γ or λ , Proposition 1 is established.

Proposition 1. We assume that $\gamma < \hat{\gamma}$. (i) Each upstream firm's profit is a single-peaked function of γ and takes its maximum value at $\gamma = m[bn + (1 + n)\lambda]/[n(m + 1)] \equiv \bar{\gamma}$; the profit of each upstream firm decreases with λ . (ii) Each downstream firm's profit is a single-peaked function of λ and takes its maximum value at $\lambda = n[bm + (1 + m)\gamma]/[m(n + 1)] \equiv \bar{\lambda}$; the profit of each downstream firm decreases with γ . (iii) The consumer and total surpluses decrease with γ and λ .

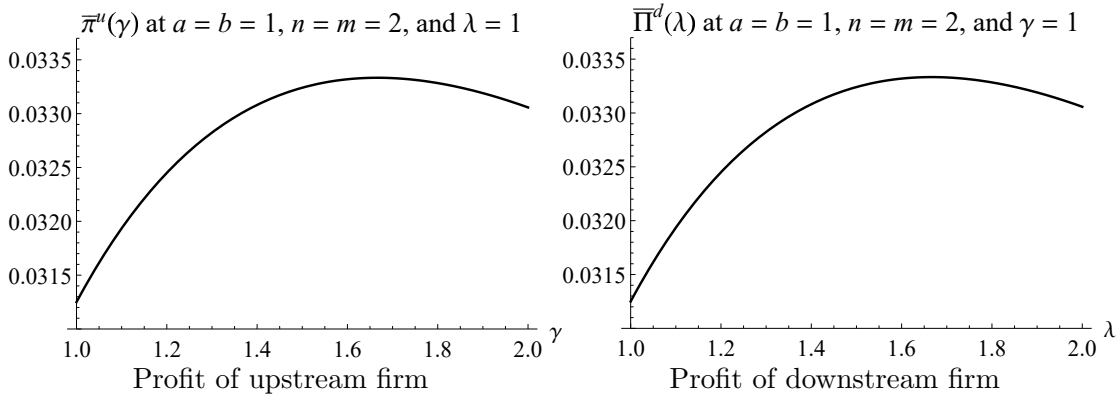


Figure 1: The inverse U-shape curves of profits

The main results of Proposition 1 are shown in Figure 1. The intuition behind this result is as follows. First, we consider the effects of production efficiency in the upstream and downstream markets on upstream firms' profits. Inefficient upstream technology has two effects: production inefficiency and competition mitigation effects. A large γ leads to inefficient upstream production. In addition, upstream firms are less likely to undercut the input price because the undercutting firm must supply the entire market. Thus, an increase in γ relaxes competition.

Next, we consider a scenario in which each effect dominates. As γ converges to zero, upstream competition approaches Bertrand competition with a constant marginal cost. Then, upstream firms' profits converge to zero. Additionally, when γ diverges infinitely, the upstream firms' profits converge to zero. Thus, when γ is small, the competition-mitigation effect dominates, and when γ is large, the production-inefficiency effect dominates. Therefore, the profit of each upstream firm has an inverted-U shape for γ .

When downstream production becomes less efficient, upstream firms have less efficient trading partners. Hence, an increase in λ decreases the upstream firms' profits.

Second, we consider the effects of production efficiency in the upstream and downstream markets on downstream firms' profits. These effects are similar to those of the upstream firms' profits. An increase in λ results in production inefficiency and competition-mitigation effects. Hence, the profit of each downstream firm has an inverted-U shape for λ . Additionally, for large γ , downstream firms face inefficient input suppliers. Hence, the downstream firms' profits decrease with γ .

The effects of γ and λ on consumers and total surpluses are simple. An increase in γ or λ reduces the efficiency of the upstream and downstream firms. Hence, the total output decreases with γ or λ , which reduces consumer and total surpluses.

Proposition 1 leads to the following:

Corollary 1. *The upstream and downstream efficiency levels that maximize upstream and downstream profits increase with λ and γ , respectively.*

As explained in the intuition behind Proposition 1, an increase in γ has a production inefficiency effect and a competition mitigation effect in the upstream market. These effects are balanced at $\gamma = \bar{\gamma}$, where upstream firms' profits are maximized. When the downstream market becomes inefficient, upstream firms can increase their profits by choosing a higher price that achieves less output; that is, the competition-mitigation effect is strengthened. Thus, at a larger γ , upstream firms' profits are maximized, which means that $\bar{\gamma}$ increases with λ .

The reason why $\bar{\lambda}$ increases with γ can be explained in the same manner. Consider the case in which λ increases at $\gamma = \bar{\gamma}$, where downstream firms' profits are maximized. Then, γ that maximizes downstream firms' profits increases because the effect of increasing γ on mitigating competition is strengthened.

4.2 Successive Bertrand competition with upstream collusive pricing

We consider the case with $\gamma \geq \hat{\gamma}$ under successive Bertrand competition. Differentiating (5) and (6) with respect to γ and λ yields the following proposition:

Proposition 2. *We assume that $\gamma \geq \hat{\gamma}$. (i) Each upstream firm's profit decreases with γ and λ . (ii) The profit of each downstream firm is a single-peaked function of λ , and decreases with γ . (iii) The consumer and total surplus decrease with γ and λ .*

In this case, only the conditions that determine the equilibrium input price differ from upper-bound pricing. When the equilibrium input price is collusive, the competition-mitigation effect of increased production inefficiency in the upstream market disappears. Hence, upstream firms' profits decrease with γ . The other results are similar to Proposition 1, as are their intuitions.

4.3 Successive Cournot competition

We consider the case with successive Cournot competition. Differentiating (7) and (8), we obtain the following proposition:

Proposition 3. *(i) Each upstream firm's profit is a single-peaked function of γ , takes its maximum value at $\gamma = (m - 3)(b + bn + 2\lambda)/(2n)$, and decreases with λ . (ii) Each downstream firm's profit is a single-peaked function of λ , takes its maximum value at $\lambda = [2n\gamma + b(1 + m)(n - 3)]/[2(1 + m)]$, and decreases with γ . (iii) The consumer and total surpluses decrease with γ and λ .*

The intuition behind this result is as follows. An increase in γ or λ causes an inefficient effect of the upstream or downstream technology. This effect has a negative impact on both upstream and downstream profits, similar to Proposition 1. In addition, an increase in γ or λ also has an input price effect. However, an increase in γ reduces total upstream output. Because

of the input market-clearing condition, the input price rises, which is profitable for upstream firms. On the contrary, an increase in λ reduces the total downstream output, which lowers the input price and increases downstream profit. In our model, the input price effect dominates the inefficiency effect if γ or λ are small. Hence, the first and second results hold true. Finally, the effect of changing γ or λ on the consumer and total surplus is the same as in Propositions 1 and 2.

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Declaration of interest statement

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References

- [1] Cabon-Dhersin, M. L., Drouhin, N., 2014. Tacit collusion in a one-shot game of price competition with soft capacity constraints. *J. Econ. Manage. Strat.* 23, 427-442.
- [2] Dastidar, K. G., 1995. On the existence of pure strategy Bertrand equilibrium. *Econ. Theor.* 5, 19-32.
- [3] Ghosh, A., Morita, H., 2007. Free entry and social efficiency under vertical oligopoly. *RAND J. Econ.* 38, 541-554.

- [4] Matsushima, N., 2006. Industry profits and free entry in input markets. *Econom. Lett.* 93, 329-336.
- [5] Mizuno, T., Takauchi, K., 2020. Optimal export policy with upstream price competition. *Manch. Sch.* 88, 324-348.
- [6] Rankin, F. W., Van Huyck, J. B., Battalio, R., 2000. Strategic similarity and emergent conventions: Evidence from similar stag hunt games. *Game. Econ. Behav.* 32, 315-337.
- [7] Roy, A., Kim, N., Raju, J. S., 2006. Assessing new empirical industrial organization (NEIO) methods: The cases of five industries. *Int. J. Res. Mark.* 23, 369-383.
- [8] Salinger, M. A., 1988. Vertical mergers and market foreclosure. *Q. J. Econ.* 103, 345-356.
- [9] Winston, C., Dennis, S. M., Maheshri, V., 2011. Duopoly equilibrium over time in the railroad industry. Working Paper, Brookings Institution.
- [10] Von Weizsacker, C. C., 1980. A welfare analysis of barriers to entry. *RAND J. Econ.* 11, 399-420.