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A Common Fixed Points of Mappings

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In this note, we shall generalize K. Iséki's result [2] on some common fixed point theorems about contraction mappings, by considering metric space over a topological semifield [1].

Theorem 1. *Let X be a sequentially complete metric space over a topological semifield E, and f_k $(k = 1, 2, \cdots, n)$ a sequence of mappings of X into itself. Let f_k $(k = 1, 2, \cdots, n)$ satisfy*

(i) $f_k f_\ell = f_\ell f_k$ $(k, \ell = 1, 2, \cdots, n)$, *and*

(ii) *for some positive integers* $m_1, m_2, \cdots, m_n$,

$$(1)\quad d(f_1^{m_1}f_2^{m_2} \cdots f_n^{m_n}x,\ f_1^{m_1}f_2^{m_2} \cdots f_n^{m_n}y)$$
$$\ll \alpha\{d(x,\ f_1^{m_1}f_2^{m_2} \cdots f_n^{m_n}x) + d(y,\ f_1^{m_1}f_2^{m_2} \cdots f_n^{m_n}y)\}$$
$$+ \beta\{d(x,\ f_1^{m_1}f_2^{m_2} \cdots f_n^{m_n}x) + d(y,\ f_1^{m_1}f_2^{m_2} \cdots f_n^{m_n}x)\} + \gamma d(x,\ y).$$

for $x, y \in X$, where $0 \le \alpha, \beta, \gamma$ and $2\alpha + 2\beta + \gamma < 1$. Then f_k $(k = 1, 2, \cdots, n)$ have a unique common fixed point.

Proof. Put $V = f_1^{m_1}f_2^{m_2} \cdots f_n^{m_n}$; from (1), we obtain

$$d(Vx, Vy) \ll \alpha\{d(x, Vx) + d(y, Vy)\}$$
$$\beta\{d(x, Vy) + d(y, Vx)\} + \gamma d(x.\ y)$$

for every $x, y \in X$. Then by a semifield formulation of I. Rus theorem [3] V has a unique fixed point x' in X. Thus $Vx' = x'$, we get

(2) $$f_i(Vx') = f_i x' \quad (i = 1, 2, \cdots, n).$$

Since f_k is commutative, it follows that

$$V(f_i x') = f_i x'.$$

Since V has a unique fixed point x', we have $f_i x' = x'$ $(i = 1, 2, \cdots, n)$. It follows x' is a common fixed point of the sequence $\{f_k\}$.

If x', y' are common fixed points of $\{f_k\}$, from (1), it follows that

$$d(x', y') = d(Vx', Vy') \ll \alpha\{d(x', Vx') + d(y', Vy')\} + \beta\{d(x', Vx') + d(y', Vx')\} + \gamma d(x', y')$$

and this implies

$$d(x', y') \ll 2\beta d(x', y') + \gamma d(x', y')$$

from $\alpha + 2\beta < 1$. We get $d(x', y') = 0$, which implies $x' = y'$.

Theorem 2. *Let* $\{f_k\}$ *satisfy the conditions*:

(i) $f_1 f_2 \cdots f_n$ *commutes with every* f_i,

(ii) $d(f_1 f_2 \cdots f_n x, f_n f_{n-1} \cdots f_1 y)$

$\ll \alpha\{d(x, f_1 f_2 \cdots f_n x) + d(y, f_n f_{n-1} \cdots f_1 y)\}$

$+ \beta\{d(x, f_n f_{n-1} \cdots f_1 y) + d(y, f_1 f_2 \cdots f_n x)\} + \gamma d(x, y)$,

where $0 \le \alpha, \beta, \gamma$ *and* $2\alpha + 2\beta + \gamma < 1$, *and for all* $x, y \in X$ *then* f_k $(k = 1, 2, \cdots, n)$ *have the unique common fixed point.*

Proof. Suppose $V = f_1 f_2 \cdots f_n$, $W = f_n f_{n-1} \cdots f_1$, then from (2), we get

$$d(Vx, Wy) \ll \alpha\{d(x, Vx) + d(y, Wy)\} + \beta\{d(x, Wy) + d(y, Vx)\} + \gamma d(x, y)$$

for every $x, y \in X$. From a semifield formulation of I. Rus theorem [3], V and W have a unique common fixed point x'. Hence $Vx' = Wx' = x'$. For each f_i $(k = 1, 2, \cdots, n)$, $f_i(Vx') = f_i x'$. From the hypothesis

$V(f_i x') = f_i x'$. Similarly $W(f_i x') = f_i x'$. Therefore $f_i x' = x'$, that is x' is a common fixed point of $\{f_k\}$. It is easy to show that x' is unique common fixed point of $\{f_k\}$.

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