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On Implicative BCK-Algebras

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In this Note, the structure of an implicative BCK-algebra is determined. This is stated as an unsolved problem in my Note [2]. The answer is given in the following

Theorem 1. *A* BCK-*algebra is implicative, if and only if it is a positive implicative algebra.*

Proof. Let M be a positive implicative algebra. As well-known, M is a BCK-algebra. Let I be an ideal, and $(x * y) * z \in I, y * z \in I$. Since M is a positive implicative algebra, we have

$$(x * z) * (y * z) \leq (x * y) * z$$

for all $x, y, z \in M$. Hence by $(x * y) * z \in I$, we have

$$(x * z) * (y * z) \in I.$$

Since I is an ideal and $(y * z) \in I$, we have

$$x * z \in I,$$

which means that I is implicative.

We shall show the converse.

Let M be an implicative BCK-algebra. We shall prove the identity relation: $(x * y) * y = x * y$. Then M is a positive implicative algebra.

$(x * y) * y \leq x * y$ holds in any BCK-algebra. In an implicative BCK-algebra, $\{0\}$ is an implicative algebra. Hence the section $A(a)$

of an element a, i.e., $\{x|x \leq a\}$ is an ideal (see [1]). Consider $A((x * y) * y)$. Then $I = A((x * y) * y)$ is an ideal. Hence $(x * y) * y \in I$, and $y * y = 0 \in I$ imply $x * y \in I$, since I is implicative. Therefore $x * y \leq (x * y) * y$. Hence we have $(x * y) * y = x * y$. This means that M is a positive implicative algebra.

From the above proof, we have the following

Theorem 2. *If any section of a* BCK-*algebra* M *is an implicative ideal, then* M *is a positive implicative algebra.*

References

[1] K. Iséki, On ideals in BCK-algebras, this Notes 3(1975), I, Kobe University.

[2] K. Iséki, Remarks on BCK-algbera, this Notes 3(1975), VI, Kobe University.