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Nakagiri, Shin-ichi

Kuroe, Yasuaki

Kitamura, Shinzo

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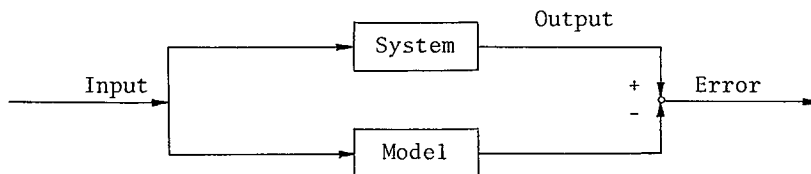
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Identifiability of Spatially Varying Parameters for
Distributed Parameter Systems of Parabolic Type

By Shin-ichi NAKAGIRI^{*}, Yasuaki KUROE^{**} and Shinzo KITAMURA^{**}

To identify unknown parameters in a system, we often use the following model reference method:



In this case, the problem is whether the model parameters agree with those of the system or not when the error between the system output and model output becomes zero. This is, so called the parameter identifiability problem. For this problem, we have obtained in [1] and [2] a number of results for the distributed parameter system of parabolic type. In this short note we report some results about the identifiability of spatially varying parameters in the case of continuous measurement.

* Department of Applied Mathematics, Kobe University, Nada, Kobe, 657, Japan.

** Department of Instrumentation Engineering, Kobe University, Nada, Kobe, 657, Japan.

Now let the system be described by

$$(1) \quad \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left(a(x) \frac{\partial u}{\partial x} \right) + b(x)u + f(x, t), \quad x \in (0, 1), \quad t > 0,$$

and the initial condition is given by

$$(2) \quad u(x, 0) = u_0(x).$$

The boundary condition is given by

$$(3) \quad \begin{cases} \alpha_0 u(0, t) + (1 - \alpha_0) \frac{\partial u}{\partial n}(0, t) = g_0(t) \\ \alpha_1 u(1, t) + (1 - \alpha_1) \frac{\partial u}{\partial n}(1, t) = g_1(t) \end{cases}$$

where $(\alpha_0, \alpha_1) \in [0, 1] \times [0, 1]$.

We assume that the measurement output is given by the following continuous measurement:

$$(4) \quad y(x, t) = u(x, t), \quad x \in [0, 1], \quad t \geq 0.$$

Moreover, suppose the following:

- 1° The form of the equation (1) is *a priori* known, and the parameters $a(x)$ and $b(x)$ are unknown.
- 2° Constant parameters α_0 and α_1 in the boundary condition are known.
- 3° Input functions f , u_0 , g_0 and g_1 are known and are sufficiently smooth functions.

Under these assumptions, the model equation is given by replacing $a(x)$ and $b(x)$ in (1) by $a_m(x)$ and $b_m(x)$ and also by replacing $u(x, t)$ in (1) by $u_m(x, t)$.

The boundary condition, the initial condition and the output equation for the model have the same form as in (2), (3) and (4), respectively.

Now we define the identifiability of the parameters $a(x)$ and $b(x)$. We say that unknown parameter, e.g., $a(x)$ *identifiable* if $a(x) = a_m(x)$ for all $x \in [0, 1]$ follows from the relation $\varepsilon(x, t) = y(x, t) - y_m(x, t)$

$$= u(x, t) - u_m(x, t) = 0 \quad \text{for all } x \in [0, 1] \quad \text{and } t \geq 0.$$

As is well-known (for example, see [3]), there exists a sequence of eigenvalues λ_n^m and eigenfunctions ψ_n^m , $n = 1, 2, \dots$ of the elliptic eigenvalue problem corresponding to the model system of parabolic type. We note that these λ_n^m and ψ_n^m are considered to be known quantities. Now we can state the main result.

Theorem. $a(x)$ and $b(x)$ are simultaneously identifiable if the following condition (C_i) in each case is satisfied ($i = 1, 2, 3, 4$):

(Case 1) $f(x, t) = 0$, $g_0(t) = g_1(t) = 0$ and $u_0(x) \neq 0$,

(C_1) $(\psi_n^m, u_0) \neq 0$ for all $n = 1, 2, \dots$.

(Case 2) $f(x, t) = 0$, $u_0(x) = 0$, $g_1(t) = 0$ and $g_0(t) \neq 0$,

(C_2) $g_0(t)$ is Laplace transformable.

(Case 3) $f(x, t) = 0$, $u_0(x) = 0$, $g_0(t) = 0$ and $g_1(t) \neq 0$,

(C_3) $g_1(t)$ is Laplace transformable.

(Case 4) $f(x, t) = f_1(x)f_2(t) \neq 0$, $g_0(t) = g_1(t) = 0$ and $u_0(x) = 0$,

(C_4) $(\psi_n^m, f_1) \neq 0$ for all $n = 1, 2, \dots$ and f_2 is Laplace transformable.

Here (ψ_n^m, u) denotes the inner product in $L_2(0, 1)$.

The conditions $(\psi_n^m, u_0) \neq 0$ for all $n = 1, 2, \dots$ in the case 1 and $(\psi_n^m, f_1) \neq 0$ for all $n = 1, 2, \dots$ in the case 4 imply that the initial function u_0 and input function f_1 should excite all the modes of the model state, respectively. These conditions for the parameter identifiability obtained here are very weak since we have assumed the continuous measurement as in the equation (4). For the case of the pointwise measurement, refer to [1].

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