# Tool path generation in pocket machining considering workpiece deformation using Finite Element Method (FEM)

Kuroda, Koki

Nakatsuji, Hidenori

Nishida, Isamu

(Citation)
Journal of Advanced Mechanical Design, Systems, and Manufacturing,18(7):JAMDSM0089

(Issue Date)
2024

(Resource Type)
journal article

(Version)
Version of Record

Bulletin of the JSME
Vol.18, No.7, 2024
Journal of Advanced Mechanical Design, Systems, and Manufacturing

# Tool path generation in pocket machining considering workpiece deformation using Finite Element Method (FEM)

Koki KURODA*, Hidenori NAKATSUJI* and Isamu NISHIDA*
*Graduate School of Engineering, Kobe University
1-1 Rokko-dai, Nada-ku, Kobe, Hyogo, 657-8501, Japan
E-mail: nishida@mech.kobe-u.ac.jp

**Received: 22 March 2024; Revised: 6 May 2024; Accepted: 12 June 2024**

**Abstract**
The metalworking industry is currently facing a shortage of human resources. Furthermore, there is a need for a shift to mass customization, which enables low-volume production at a cost equivalent to that of mass production. To satisfy the demand, it is necessary to increase labor productivity per worker by shortening production lead times. Automated tool path generation is one of the means to reduce production lead time. However, the generation of NC programs that enable high-precision machining requires enormous amounts of time and labor, since it is necessary to modify the NC programs according to the results of test cutting and reviewing the machining conditions. One of the factors causing machining errors in cutting is considered the deformation of the workpiece due to the clamping in a vise. Hence, even if dimensional tolerance measured on the machine is satisfied, dimensional errors may occur when the workpiece is removed from the vise. The purpose of this study is to realize the automated NC program generation for high-precision pocket machining. This study developed a system that predicts the elastic deformation of the workpiece due to the clamping force using Finite Element Method (FEM) and automatically generates a tool path for machining that satisfies the dimensional tolerance when the workpiece is removed from the vise. As a result of case study, it was confirmed that the proposed system can automatically generate tool paths that improves machining accuracy.

***Keywords*** : Tool path generation, Compensation, Vise, High-precision machining, Finite Element Method (FEM), Computer Aided Manufacturing (CAM), Computer Aided Process Planning (CAPP)

## 1. Introduction

Recently, manufacturing sites are facing a manpower shortage due to a decrease in the number of skilled workers caused by the retirement of older workers and a decrease in the number of new workers due to the declining birthrate. On the other hand, as needs diversify and product life cycles shorten, production is shifting from traditional mass production to high-mix low-volume production and even variable-mix variable-volume production. In addition, there is a demand for the realization of "mass customization," the production of single items with the same efficiency and cost as mass production (Ueno, 2017). In order to achieve mass customization while compensating for the lack of human resources, it is necessary to increase labor productivity per worker by shortening production lead times through the use of computer technology.

Currently, NC machine tools are widely used for machining machine parts, molds, and other components in the manufacturing field, and are capable of high-precision machining under numerical control by a computer. Machining with NC machine tools requires the generation of NC programs for each machining object using a CAD/CAM system in advance, which requires time for setup task. Particularly in the case of low-volume production, the ratio of setup time per product also increases. Therefore, it is effective to reduce the time required for preparation work such as NC program generation as a means to shorten the production lead time. Various studies have been conducted to reduce the time required to generate NC programs (Shinoki et al., 2015; Nishida and Shirase, 2018a).

However, for high-precision machining, in addition to NC program generation, programs must be corrected to

Paper No.24-00122
[DOI: 10.1299/jamdsm.2024jamdsm0089]

minimize machining errors by repeating test cuts and revisions of machining conditions. This process is one of the factors that make it difficult to shorten production lead time in high-precision machining. Therefore, it should be possible to shorten the production lead time in high-precision machining when NC programs that enable machining with small machining errors can be easily generated.

There are various machining error factors that must be considered in order to achieve high-precision machining. There are some previous researches on error prediction and correction methods based on the prediction results. To reduce errors in machining, methods are proposed to correct errors due to thermal displacement of the machine tool (Kaneko et al., 2007), to correct errors due to elastic deformation of the tool system (Nishida and Shirase, 2018b), and to minimize deformation of the workpiece due to cutting force (Wang et al., 2017). Another error factor is the deformation of the work material caused by the fixture (Ramesh et al., 2000). During machining, the workpiece must be clamped in a vise, but the clamping force causes deformation of the workpiece. Therefore, as shown in Fig. 1, even if the workpiece is machined to satisfy dimensional tolerances on the machine, dimensional errors may occur when the workpiece is removed from the vise.

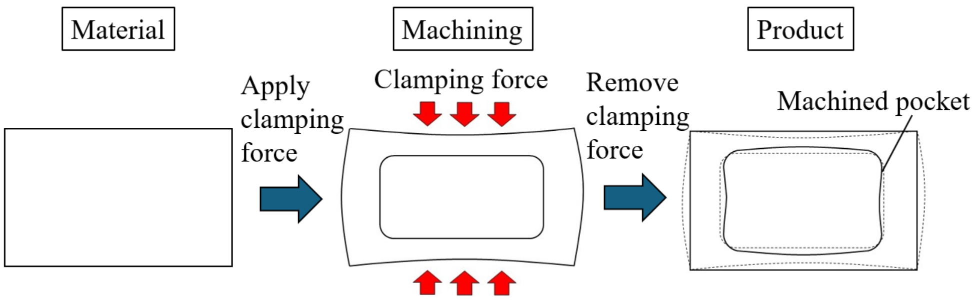



Fig. 1 Machining error due to deformation of workpiece caused by clamping in the vice.

Studies have been conducted to estimate the deformation caused by fixturing forces (Teramoto, 2017; Zeng et al., 2021). However, to improve the efficiency of machining pocket shapes used for fitting, it is necessary to predict deformation with high accuracy, even when strong clamping forces are applied, and to apply these predictions quickly to the machining process. The purpose of this study is the automatic generation of tool paths that enable high-precision pocket milling. This study proposes a method of generating tool paths considering the deformation of the workpiece due to the clamping of the vise. To improve machining accuracy when the workpiece is removed from the vise, this study develops a system to generate an NC program that can perform machining with a tool path modified for deformation of the workpiece due to the clamping force of the vise, as shown in Fig. 2. In this system, the input information is just a STL (Standard Triangulated Language) data exported from 3D CAD and loading conditions from a vise. The advantage of the STL format is its high compatibility, in which the surface shape of the model is represented by a triangular mesh. The system uses the FEM (Finite Element Method) to predict the deformation of the workpiece due to the clamping force with the vise and modifies the tool path based on the analysis.

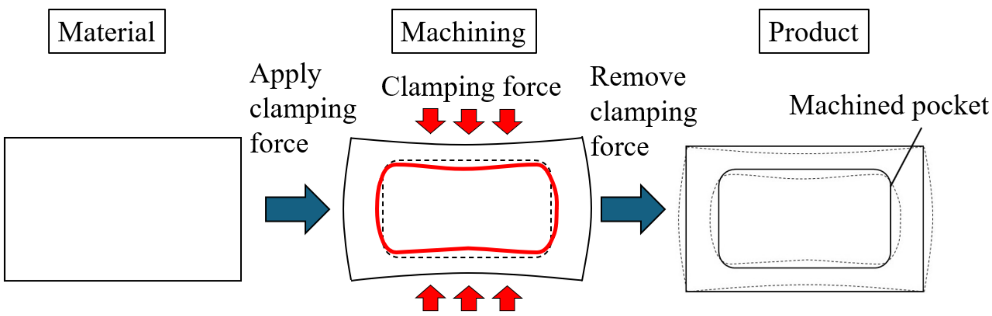



Fig. 2 Machining with tool path modified considering workpiece deformation due to vice clamping.

## 2. Tool path generation that considers deformation of work material due to clamping in the vice

### 2.1 Overview of the proposed system

This study proposes a system that automatically generates tool paths considering the deformation of the workpiece due to clamping in a vise in order to improve the efficiency of NC program generation for high-precision machining. An overview of the proposed system is shown in Fig. 3. First, the situation in which the product is clamped by the vise is represented using elements for the FEM based on the input STL data of the product and the clamping condition of the workpiece. Based on the set of elements, a finite element equation is established and solved to predict the deformation of the workpiece due to the vice's clamping. If there is an overlapping area between the product and the vise, the position where the clamping force is transmitted from the vise to the product is adjusted and the deformation is re-calculated. According to the predicted deformation results, the shape of the pocket to be machined is modified, and the tool path is calculated for the modified pocket shape. Finally, an NC program is generated based on the obtained tool path.

The proposed system automatically compensates for machining errors caused by deformation of the workpiece due to clamping in a vise. This can reduce test cutting and tool path modification, which were conventionally required. As the result, this also contributes to the reduction of production lead time and labor cost. The proposed system uses a STL data exported from 3D CAD as input information, which is a product model data that represents the surface shape of the product using only a combination of triangular meshes, and is independent of the CAD software. This enables a system with high compatibility, which overcomes issues related to data format compatibility of 3D CAD models.

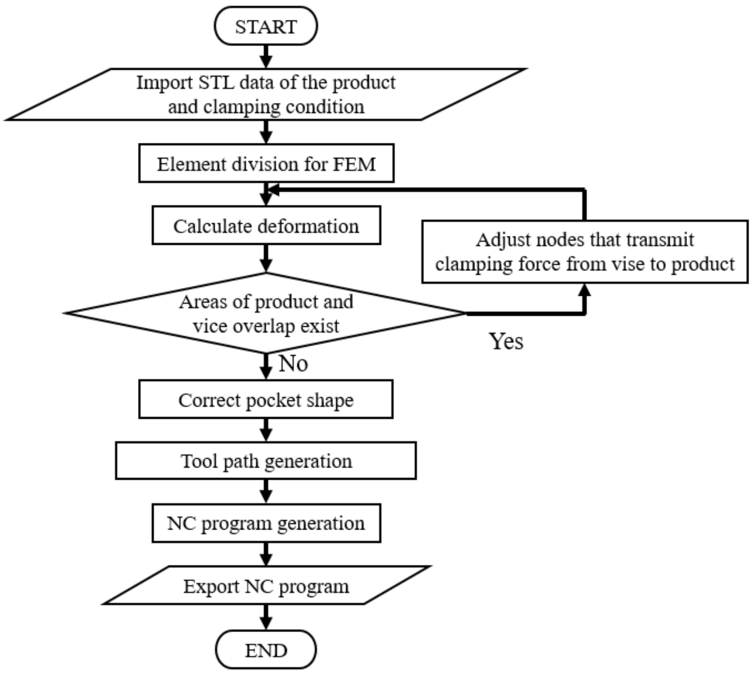



Fig. 3 Process of the system proposed in this study.

### 2.2 Element division for FEM

In this study, the STL data exported from 3D CAD is used as input information. The deformation of workpiece is predicted using a two-dimensional FEM. Since the surface profile in the STL format is divided into a triangular mesh, it is necessary to re-divide the mesh into triangles suitable for FEM because the non-uniformity of the size and shape of the triangles may reduce the accuracy of the analysis. A contour line (Nishida and Shirase, 2021) representing the product shape with one or more closed polyline is obtained from the STL data as shown in Fig. 4, and the elements for FEM are created based on the contour line. The algorithm for the contour line is described as follows. For the XY plane, the intersection between the plane and all triangular meshes is geometrically extracted. The line segment, where the triangular mesh and the target plane intersect, is calculated. The contour lines of the plane are calculated by searching for a line with the same start of the end point and connecting the same points, as shown in Fig. 5. The obtained line segments composing the contour line are then of non-uniform lengths, as shown in Fig. 6. By using linear completion to insert points into the obtained line segment, a closed polyline with intervals less than or equal to the specified resolution is created, as shown in Fig. 7. The accuracy of the contour shape representation depends on the resolution at which the STL data is created.

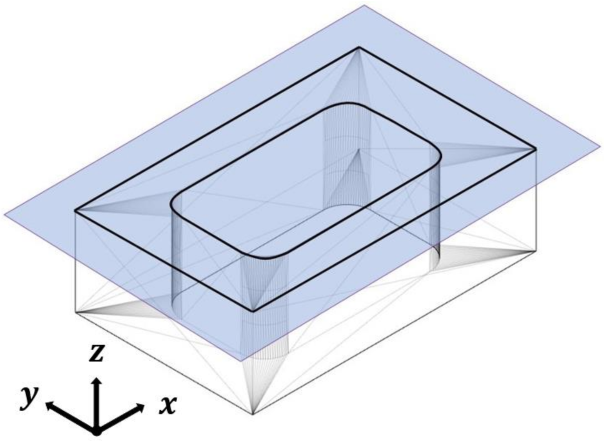



Fig. 4 Contour line representing the two-dimensional shape of the product.

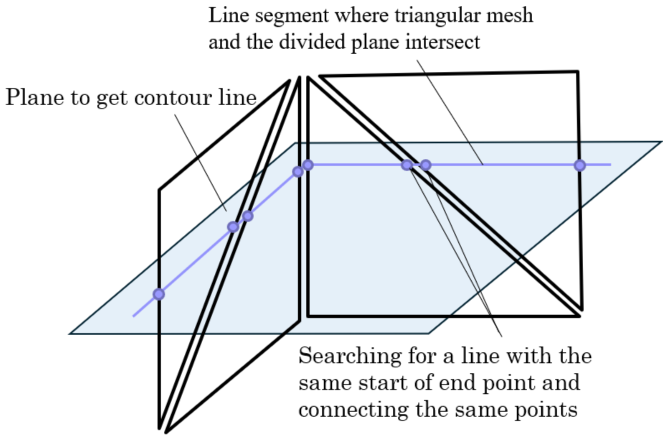



Fig. 5 Geometrical relationship between triangular mesh and the divided plane.

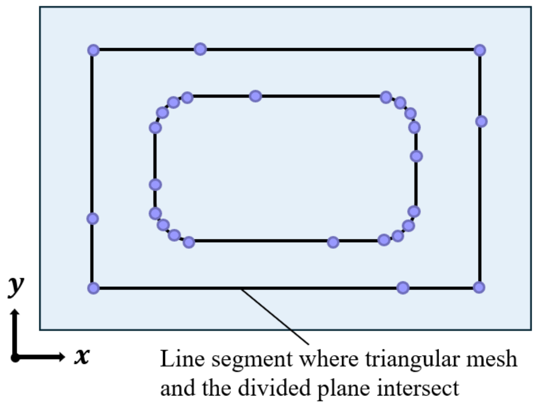



Fig. 6 Contour line obtained from the line segment where the triangular mesh and divided plane intersect.

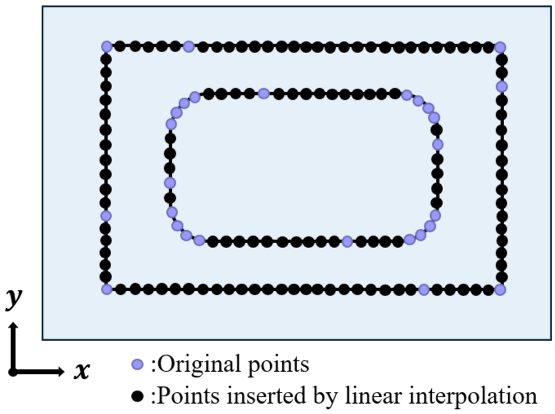
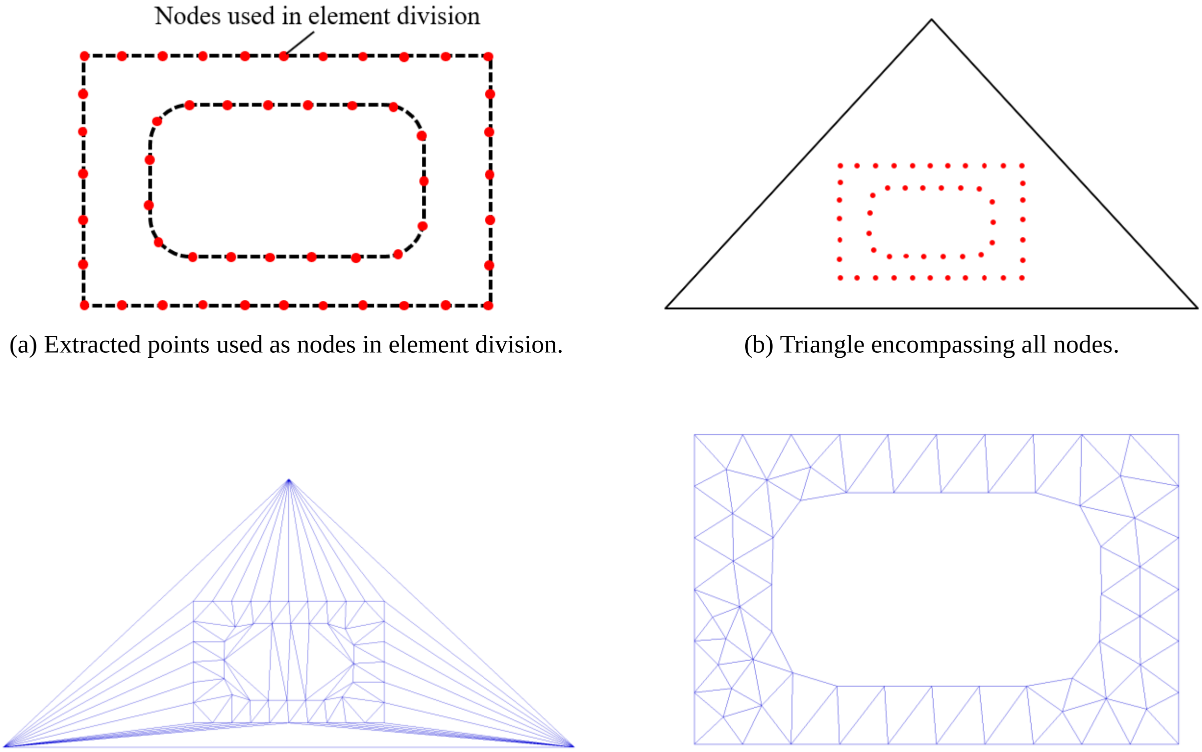



Fig. 7 Contour line expressed using points at intervals of the resolution.

Delaunay triangulation (Sloan, 1987) has been proposed as a method for dividing triangles. This method connects arbitrary collections of nodes and divides them into triangles, and the triangles obtained by this method have the property that their circumscribed circles do not contain the vertices of other triangles. This geometric property is desirable in terms of accuracy in the finite element method. The problems with this method are that it can only be applied to convex shapes and that all nodes must be defined in advance. To solve these problems, a modified Delaunay triangulation method (Taniguchi, 1992) has been proposed. In this study, a modified Delaunay triangulation method is used to divide the elements and represent the product shape as a set of elements. The procedure is shown in Fig. 8(a)~(d). First, as shown in Fig. 8(a), a set of nodes constituting the boundary used for element division is created by extracting points to be used as nodes at intervals specified as the size of the element from the set of points constituting the contour lines in Fig. 5. In the Delaunay triangulation method, an element division result is created by repeating the process of adding points to the interior of a triangle and modifying the triangles to incorporate the new points, as many times as the number of points to be added. Therefore, at the beginning of the division, a triangle containing all the nodes is set up as shown in Fig. 8(b). By applying Delaunay triangulation to this triangle and nodes, this study obtains

[DOI: 10.1299/jamdsm.2024jamdsm0089]

the result of element division using only the nodes that constitute the boundary, as shown in Fig. 8(c). After adding nodes inside the boundary using the distortion rate method proposed in the modified Delaunay triangulation method, unnecessary nodes and elements are removed to obtain the result of element division using triangle elements that represent the product shape, as shown in Fig. 8(d). As shown in Fig. 9, the distortion rate method generates nodes at the midpoints of the opposite sides of the largest angle of each element in order of the generated triangular elements with the largest angles.




(a) Extracted points used as nodes in element division.

(b) Triangle encompassing all nodes.

(c) Result of division with nodes on the boundary.

(d) Result of dividing the product into elements.

Fig. 8 Procedure for dividing a product model into elements.

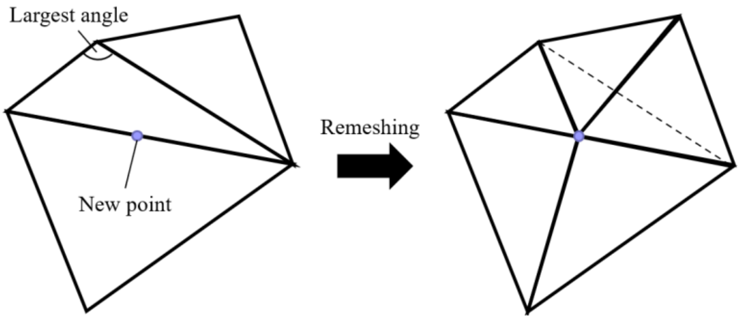
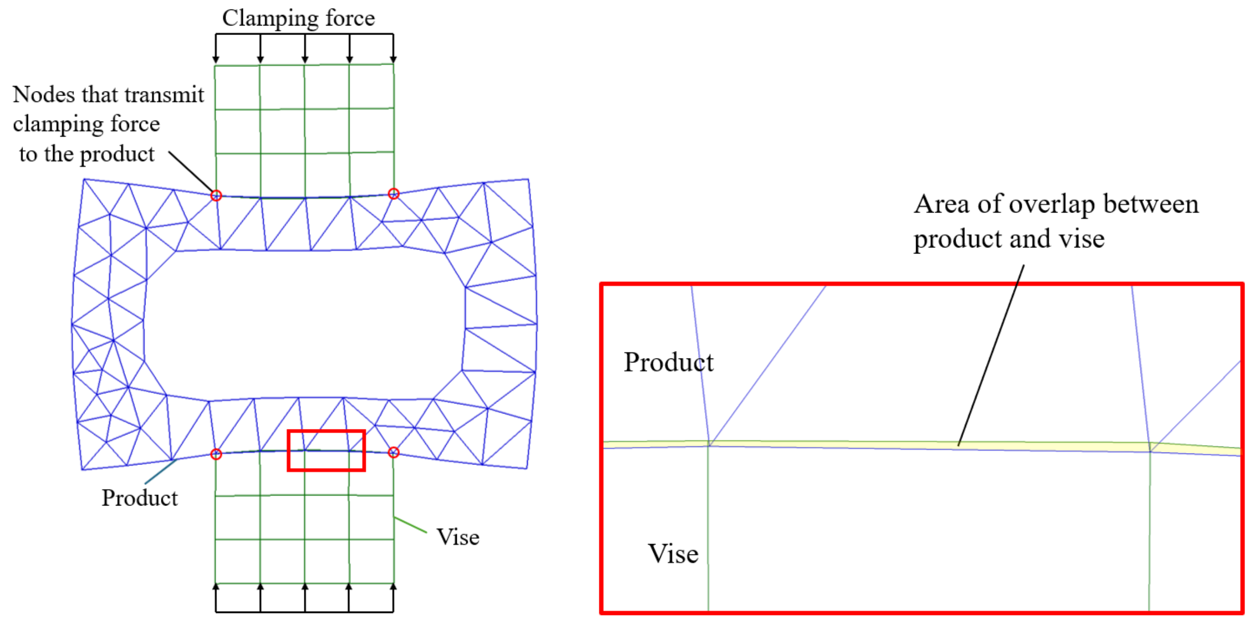



Fig. 9 Remeshing procedure with the distortion rate.

## 2.3 Analysis considering clamping force of vices

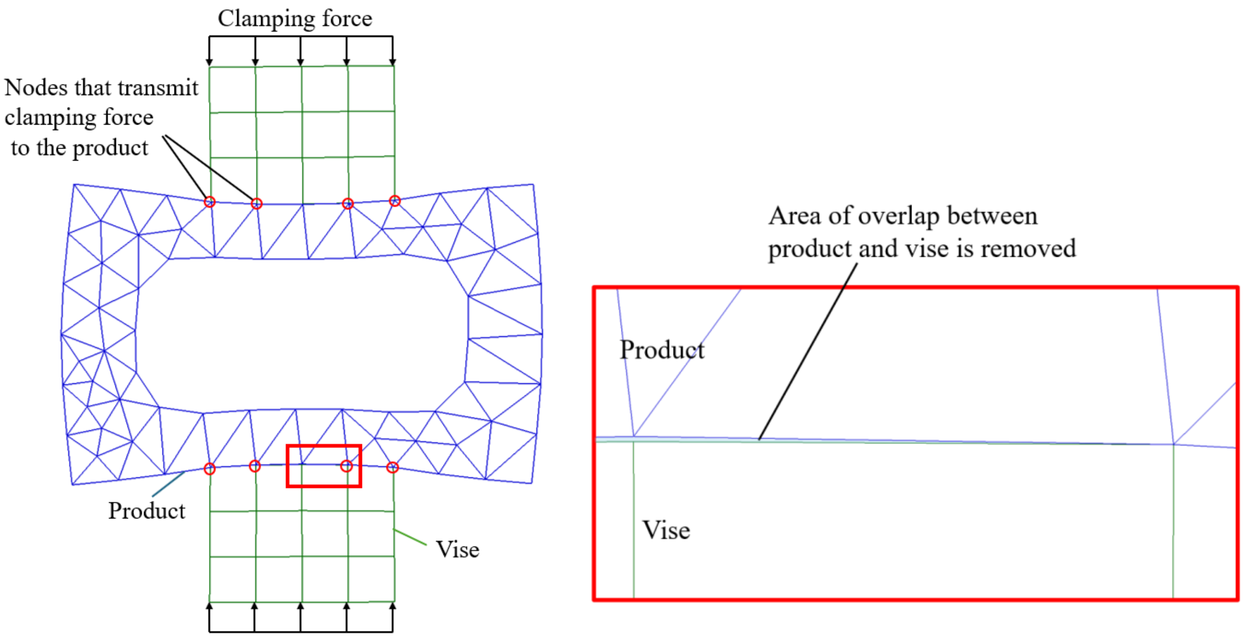
To the result of dividing the product into triangles, a vice that grips the product is added as an element. The vice is represented by a quadrilateral element composed of nodes arranged in a grid as shown in Fig. 10(a). Only the kinematic boundary conditions are given as boundary conditions. The force applied by the vise, given as the kinematic boundary conditions, is distributed loads acting at the nodes shown in Fig. 10(a). In this case, the magnitudes of the upper and lower loads in Fig. 10(a) are equal and balanced. In FEM, the loads are applied to nodes and transmitted through the nodes to the elements. Therefore, the loads applied to the vise are transmitted to the product through nodes that are shared by the vise and the product. Based on the FEM analysis results, deformation of shapes with holes, such as the product shape, reaches tens of times the deformation of shapes without holes, such as a vise, when the same load is applied. Since the product deflects more than the vise and the curvature of the clamping area is larger, this study can assume that the force transmitted from the vise to the product concentrates near the both ends of the clamping area. Therefore, the nodes, at which the force is transmitted from the vise to the product, are initially defined as the nodes circled in red in Fig. 10(a). However, as shown in Fig. 10(b), which is an enlarged view of Fig. 10(a), since the nodes of the vise and product, that are not shared in the clamping area, move independently, the area of the vise and the area of the product may overlap. This is considered to be caused by the force concentrating locally beyond the actual clamping condition, leading to the vise locally covering on the product. Since this overlap should not occur in actual, the nodes at which force is transmitted are adjusted and the analysis is conducted again. As shown in Fig. 11(a), the process of increasing the nodes at which force is transmitted from the vise to the product in the inward direction of the vise and conducting the analysis is repeated. As shown in Fig. 11(b), the analysis result in which the product and vise do not overlap for the first time is adopted as the predicted deformation result. The gap between the vise and the product in Fig. 11(b) occurs because the nodes of the vise and the product, which are not shared in the clamping area, move independently, and their coordinates do not match after deformation.




(a) Nodes that transmit clamping force from the vise to the product at the start of the analysis.

(b) Positional relationship between product and vise for re-analysis.

Fig. 10 Applied clamping force to analysis.




(a) Added nodes to transmit clamping force from the vise to the product.

(b) Positional relationship between product and vise when adopting the analysis results.

Fig. 11 Adjustment of the positional relationship between product and vise by iterating analysis.

### 2.4 Calculation of deformation by FEM

Based on the generated set of elements, the deformations of the product and vice are calculated using FEM. Matrices $[D]$, $[B]$, and $[K_e]$ are obtained to set up the equations for calculating the element displacements. $[D]$ is a matrix representing the relationship between strain $\{\varepsilon\}$ and stress $\{\sigma\}$ as in Eq. (1), and $[B]$ is a matrix representing the relationship between strain $\{\varepsilon\}$ and nodal displacements $\{u\}$ as in Eq. (2). $[K_e]$ is expressed in Eq. (3) using $[D]$ and $[B]$, and $[K_e]$ is used to obtain Eq. (4), which expresses the relationship between nodal displacements $\{u\}$ and external forces $\{f\}$ at each element. Equation (4), which is formulated for each element, can be summed up to obtain Eq. (5), which expresses the relationship between {U}, which is the displacement of all nodes, and {F}, which is the load applied to all nodes. By solving Eq. (5), the displacements of all nodes can be obtained. In this study, Conjugate Gradient Method is used to solve the equation (Eq. 5).

$$\{\sigma\} = [D]\{\varepsilon\} \tag{1}$$

$$\{\varepsilon\} = [B]\{u\} \tag{2}$$

$$[K_e] \equiv \int_V [B]^T [D]^T [B]\, dV \tag{3}$$

$$[K_e]\{u\} - \{f\} = 0 \tag{4}$$

$$[K]\{U\} - \{F\} = 0 \tag{5}$$

### 2.5 Modification of pocket shape

The proposed system calculates the tool path based on the contour lines of the pocket to be machined. Therefore, the shape of the pocket contour lines is modified based on the results of the FEM. The displacement of a nodes can be directly applied as the displacement of the points constituting the pockets extracted as nodes from the contour lines.

[DOI: 10.1299/jamdsm.2024jamdsm0089]  

The points, which were not extracted, exist between the extracted points. The displacements of the unextracted points are obtained using the nodal displacements of the element with the two points at both ends as nodes. The displacement at the point $(x, y)$ inside the triangular element is expressed by Eq. (6). In this case, parameter $\alpha$ corresponds to node numbers 1~3 in Fig. 12. $N_\alpha^e$ is the shape function of node $\alpha$ in a triangular element $e$. It can be obtained from the coordinates of the point for which the displacement is calculated and the coordinates of the nodes constituting the element. $u_\alpha^e$ is the displacement at the nodes, which is obtained by FEM analysis. From the coordinates of the points and the coordinates and displacements of the nodes constituting the element containing the point, the displacements of all the points constituting the contour line can be obtained. By adding the displacements to the original coordinates, the pocket shape after the deformation is obtained as a contour line.

$$u = \sum_{\alpha=1}^{3} N_\alpha^e(x, y) u_\alpha^e \tag{6}$$

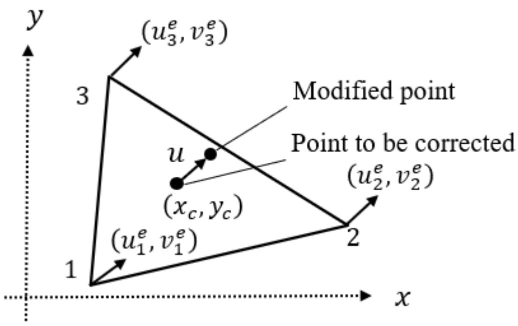



Fig. 12 Modification of the coordinates of the points constituting the pocket based on the predicted deformation.

## 2.6 Tool path generation considering deformation of workpiece

The following shows the method to generate tool paths in pocket machining. As shown in Fig. 13, a tool path is generated to finish the product shape by generating a set of points offset from the target pocket by the tool radius $r$. In this study, the pocket shape is represented as contour lines by points arranged in counterclockwise order. As shown in Fig. 14, a vector is obtained from one point to the next that constitutes the contour line. The vector is rotated 90 degrees counterclockwise, and the direction of the rotated vector is the direction of offset. The position offset by the tool radius $r$ in the offset direction is taken as the command point of the tool center. This process is applied to all points representing the pocket shape to generate the tool path for finish machining. Furthermore, a tool path is generated that is offset from the generated tool path by a radial depth $d_r$. This process is repeated until there are no more areas to be removed. As shown in Fig. 15, the proposed system generates a modified tool path by the proposed method to contour lines that reflect workpiece deformation. The proposed system converts the generated tool path to G-code and outputs it as an NC program.

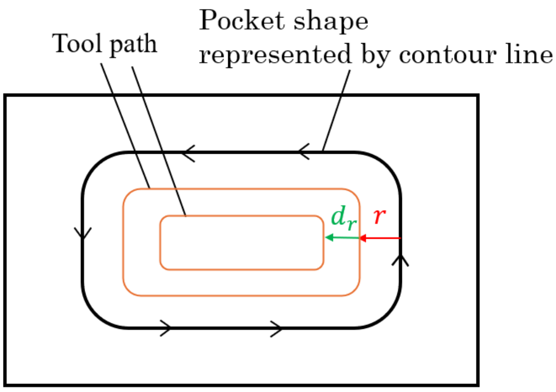



Fig. 13 Tool path generation without considering deformation of workpiece.

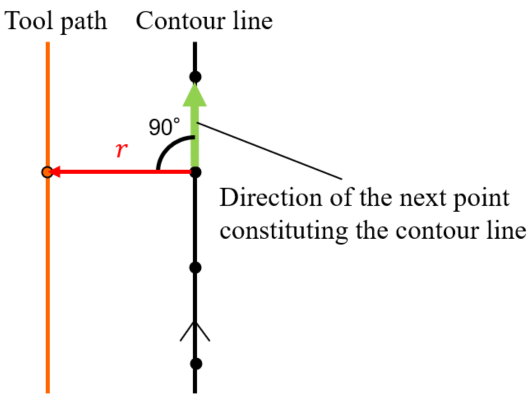



Fig. 14 Tool path generation from the contour lines by points.

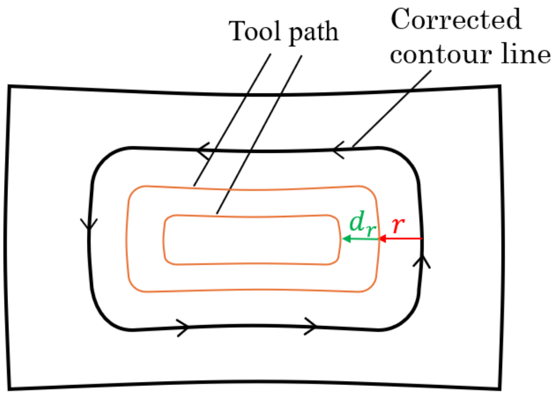



Fig. 15 Tool path generation with considering deformation of workpiece.

## 3. Case study

To verify the validity of the method proposed in this study for generating tool paths considering the deformation of the workpiece due to the clamping in the vise, machining experiment was conducted using the product model shown in Fig. 16. In this case study, two NC programs were generated, one is modified with consideration of the workpiece deformation for the shape of the target pocket and the other is not modified. Comparison of the dimensional errors was conducted by measuring the width of the pocket at position A in the center of the pocket. Aluminum (A5052) was used as the work material.

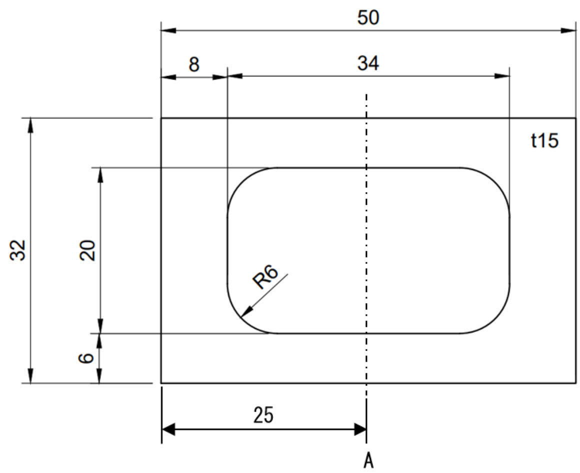



Fig. 16 Product model with a through pocket for case study.

[DOI: 10.1299/jamdsm.2024jamdsm0089]

In the case study, the load position was limited by placing square bars of aluminum (A2017) between the vise and the workpiece, as shown in Fig. 17, which assumes a small vise relative to the workpiece. In Fig. 17, the top surfaces of the vise, the square bar, and the workpiece are the same height, and the height of the square bar is higher than that of the workpiece. So, the workpiece is clamped in the entire height direction. Roughing and finishing tool paths are generated based on the analysis results. The clamping force magnitude was 10 kN, measured by a load cell, to predict deformation. The clamping force was re-adjusted to 10 kN just before the finish machining because the clamping force decreases as the workpiece is ground off during the machining. The predicted deformation results obtained from the STL data exported from 3D CAD shown in Fig. 18 and the clamping situation of the vise are shown in Fig. 19(a). The predicted deformation results are shown with the amount of deformation multiplied by a factor of 10. The number of triangular elements used in the analysis was 11297 and the number of quadrilateral elements was 2400. The deformation analysis was repeated five times to adjust the nodes that transmit the load from the vise to the product. Some of the nodes used as the nodes finally transmitting the load are circled in red in Fig. 19(b), which is a close-up of the area enclosed by the red rectangle in Fig. 19(a). The calculation is carried out using a computer with a 3.50 GHz Intel(R) Core (TM) i9-11900K CPU. The algorithm and FEM code are written in C# and are independently developed. The calculation took 49 minutes in total. FANUC ROBODRILL α-D14MiB5 was used as the machine tools, and the tools and machining conditions used are shown in Table 1. The deformation of the pocket width at position A shown in Fig. 16, where the maximum displacement occurs, is predicted to be 0.246 mm. Fig. 20(a) and Fig. 20(b) show the tool paths for roughing and finishing without modification. Figures 20(c) and 20(d) show the roughing and finishing tool paths with modification. Fig. 21 shows the tool path of the finish machining without modification superimposed on the tool path of the finishing machining with modification. The vertical light blue and purple lines in the figures are the x- and y-axes of the machine coordinate, respectively. In roughing, 0.3 mm machining allowance remains for the target pocket shape. To remain a uniform allowance, the modification is also applied to the roughing tool path. Then, the allowance is set to be large enough to modify the deformations. In the finishing operation, the machining process is repeated twice to remove the machining allowance remaining in the roughing process. Finishing is repeated twice to eliminate the effect of tool deflection.

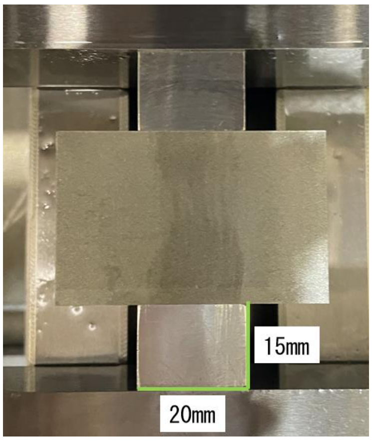



Fig. 17 Device for clamping workpiece.

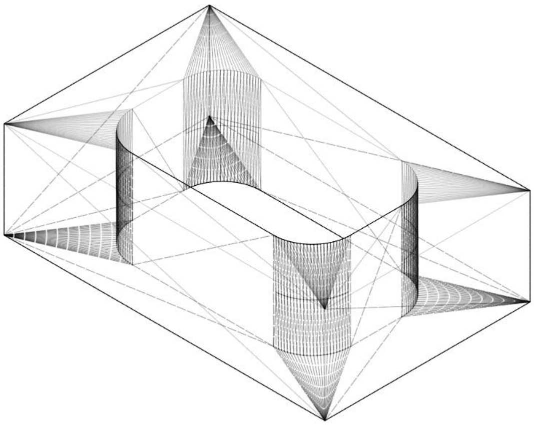
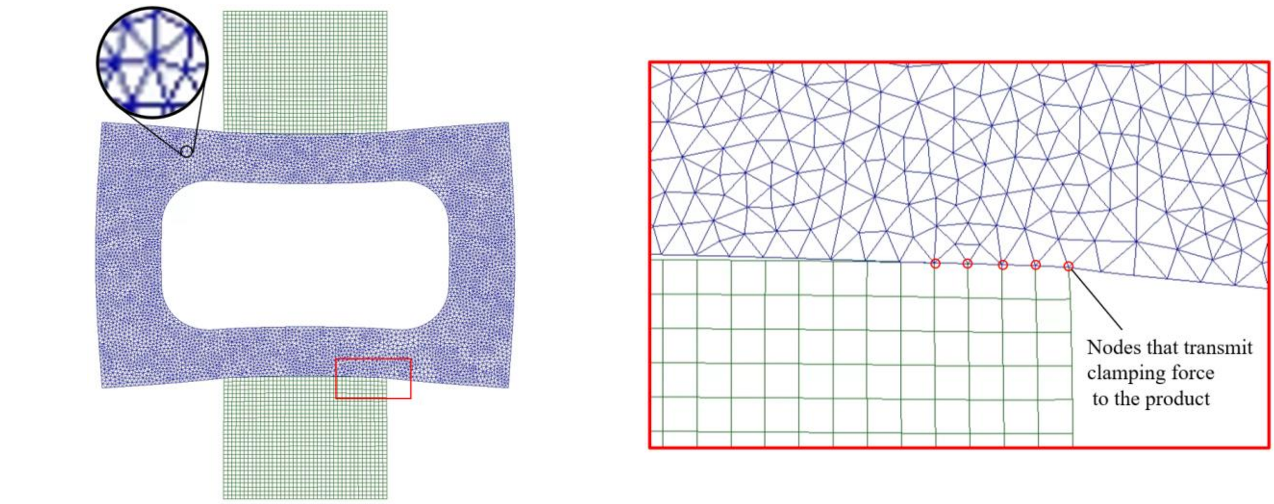
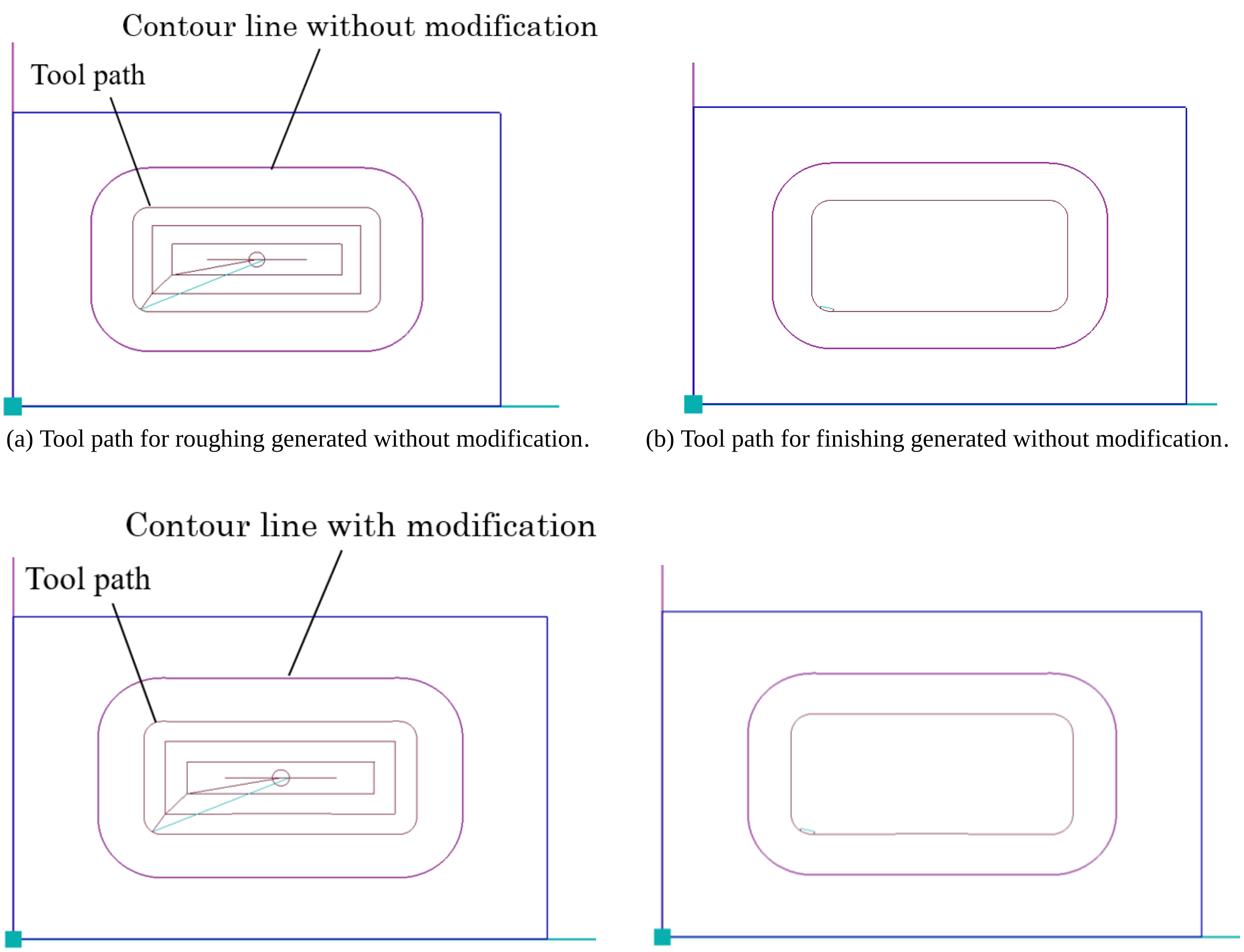


Fig. 18 Product model in STL format.




(a) Deformed product shape represented by the elements. (b) Position of the nodes transmitting the load.

Fig. 19 Prediction results of workpiece deformation by FEM.




(a) Tool path for roughing generated without modification. (b) Tool path for finishing generated without modification.

(c) Tool path for roughing generated with modification. (d) Tool path for finishing generated with modification.

Fig. 20 Tool paths used in machining.

[DOI: 10.1299/jamdsm.2024jamdsm0089]

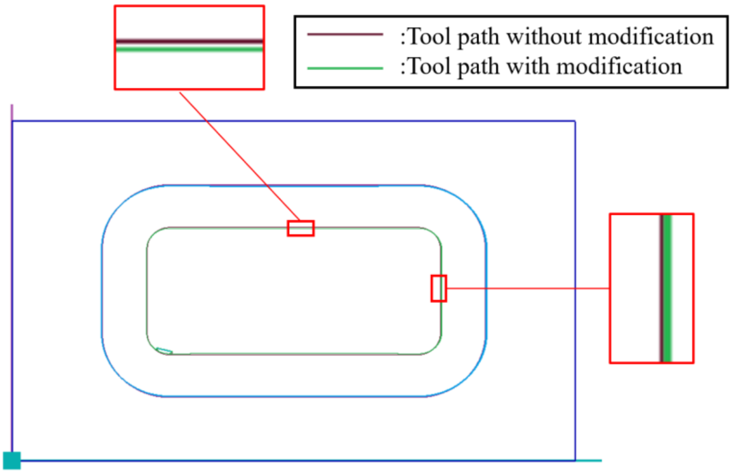



Fig. 21 Difference in tool paths due to modification.

Table 1 Machining conditions used in the case study.

| Process | Tool type | Tool diameter | Axial depth | Radial depth | Spindle speed | Feed Rate |
|---|---|---|---|---|---|---|
| Roughing | Square endmill | 8[mm] | 1[mm] | 2[mm] | 10000[rpm] | 50[mm/min] |
| Finishing | Square endmill | 8[mm] | 1[mm] | | 10000[rpm] | 50[mm/min] |

Figure 22 shows the results of machining using a tool path that does not consider the deformation of the workpiece due to the clamping in the vice. Figure 23 shows the results of machining using a tool path that considers the deformation of the workpiece due to the clamping in the vice. A marking off line was drawn on center of the workpiece and pocket (position A in Fig. 16), and the width of the pocket was measured on the line. A digital caliper (CD-P15S, Mitutoyo) was used for measurement. The internal jaws were inserted as deeply as possible to reduce measurement error. Measurements were taken five times, and the average value was used as the measurement result. The width machined without pocket geometry modification was 20.24 mm, as shown in Fig. 22, resulting in a machining error of 0.24 mm compared to the target width of 20 mm. The width machined with pocket geometry modification was 20.00 mm, as shown in Fig. 23, confirming that the error was reduced. According to the results considering the deformation of the workpiece due to clamping in the vice, the resulting error satisfies the fit tolerance JS7 (±0.0105 mm for a size of 20 mm in the case study model). This confirms the validity of the tool path generation method proposed in this study.

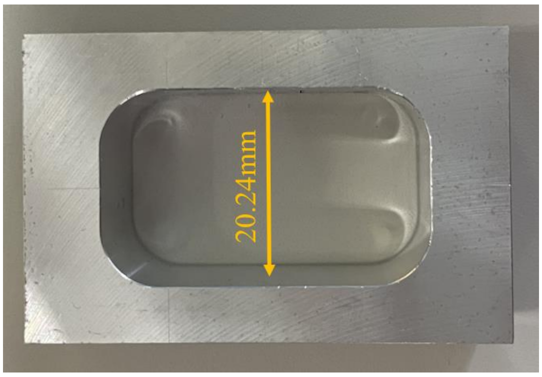



Fig. 22 Through pocket machined without modification and width of center of the pocket.

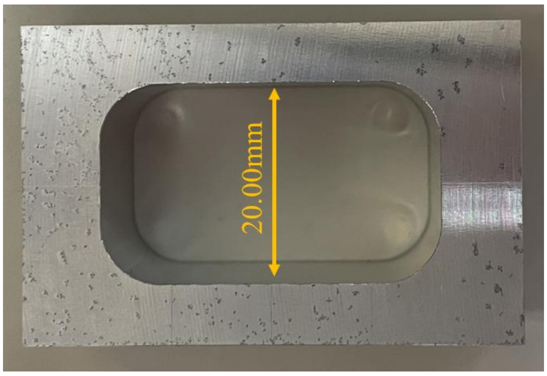



Fig. 23 Through pocket machined with modification and width of center of the pocket.

## 4. Conclusion

The purpose of this study is to realize the high-precision pocket machining. This study proposed a method for generating a tool path considering the deformation of the workpiece due to clamping in the vice. The conclusions of this study are as follows.

[DOI: 10.1299/jamdsm.2024jamdsm0089]

1. The analysis model for the Two-Dimensional Finite Element Method could be generated from a STL data exported from 3D CAD, a highly compatible format.
2. The tool path considering the deformation of the workpiece due to the clamping force in the vice could be automatically generated.
3. A system incorporating the method proposed in this study was constructed, and machining experiments were conducted using the NC program generated by the system. As a result, it was confirmed that the tool path generated by the method proposed in this study can reduce machining errors.

## References

Kaneko Y., Tachiya H., Tamura H., Shinjo H., and Isobe M., A Simple Method of Machining Error Compensation by Formulation of Thermal Deformation of Machine Tools: Application under Continuous Operation Condition, Transactions of the Japan Society of Mechanical Engineers, Part C, Vol. 73, No. 726 (2007), pp. 371-378 (in Japanese).

Nishida, I. and Shirase, K., Automatic determination of cutting conditions for NC program generation by reusing machining case data based on geometric properties of removal volume, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 12, No. 4 (2018a), DOI:10.1299/jamdsm.2018jamdsm0093.

Nishida, I. and Shirase, K., Machining Error Correction Method Based on Prediction Result of Elastic Deformation of Tool System in Endmilling, Proceedings of JSPE Semestrial Meeting, Vol. 2018A (2018b), pp. 381-382, DOI: 10.11522/pscjspe.2018A.0_381 (in Japanese).

Nishida, I. and Shirase, K., Automated Process Planning System for End-Milling Operation by CAD Model in STL Format, International Journal of Automation Technology, Vol.15, No.2 (2021), pp. 149-157.

Ramesh, R., Mannan, M. A., and Poo, A. N., Error compensation in machine tools—a review: part I: geometric, cutting-force induced and fixture-dependent errors. International Journal of Machine Tools and Manufacture, Vol. 40, Issue 9 (2000), pp. 1235-1256.

Shinoki, Y., Isnaini, M. M., Sato, R. and Shirase, K., Machining operation planning system which utilize past machining operation data to generate new NC program, Trans. of the Japan Society of Mechanical Engineers (in Japanese), Series C, Vol.81, No. 832 (2015), DOI: 10.1299/transjsme.15-00280.

Sloan, S. W., A fast algorithm for constructing Delaunay triangulations in the plane, Advances in Engineering Software, Vol. 9, No. 1 (1987), pp. 34-55.

Teramoto, K., On-Machine Estimation of Workpiece Deformation for Thin-Structured Parts Machining, Int. J. Automation Technol., Vol.11 No.6 (2017), pp. 978-983.

Taniguchi, T., Automatic Element Segmentation for FEM: Use of Delaunay Triangulation Method (1992), p.34, Morikita Publishing Co., Ltd. (in Japanese).

Ueno, H., Intelligent Technology for Machine Tools, Flexible Automation, General Special Issue, Vol.61, No. 3 (2017), pp. 107-112, DOI: 10.11509/isciesci.61.3_107 (in Japanese).

Wang, J., Ibaraki, S., and Matsubara, S., A cutting sequence optimization algorithm to reduce the workpiece deformation in thin-wall machining, Precision Engineering, Vol. 50 (2017), pp.506-514.

Zeng, J., Teramoto, K., and Matsumoto, H., On-Machine Estimation of Workholding State for Thin-Walled Parts, Int. J. Automation Technol., Vol.15 No.6 (2021), pp. 860-867.

[DOI: 10.1299/jamdsm.2024jamdsm0089]