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First mover or second mover? Endogenous timing with partial vertical ownership

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Abstract

We investigate a vertically related market with a upstream firm offering inputs to two downstream firms. Considering the downstream firms hold partial vertical ownership of the upstream firm without control rights, we analyze the endogenous order for the downstream firms and find they may choose a more competitive competition, i.e., simultaneous pricing under Bertrand competition or sequential producing under Cournot competition. Our results happen when the degree of product substitutability is small and the degree of partial vertical ownership is high.

JEL codes: L00, L13, L22.

Keywords: endogenous timing, partial vertical ownership, vertical relationship.

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1 Introduction

We analyze firms' endogenous order of moves under both Bertrand and Cournot competition. It is commonly known that sequential pricing weakens competition and provides larger profits than simultaneous pricing under Bertrand competition, while the opposite is true under Cournot competition. In a sequential play, the firms can anticipate each other's moves. Hence, under Bertrand competition, sequential pricing allows firms to relatively avoid the intense price undercutting that occurs in simultaneous pricing, leading weaker competition with higher prices and larger profits. By contrast, under Cournot competition, the sequential play encourages the leader to produce more as a commitment anticipating that the follower will adjust the output by observing the leader's output, resulting a more competitive market with larger outputs and lower profits than simultaneous quantity competition. Hamilton and Slutsky (1990) analyzed this firms' endogenous order of moves problem and showed that sequential pricing occurs in equilibrium under Bertrand competition whereas simultaneous play is an equilibrium under Cournot competition. In this study, we challenge this well-known result by introducing partial vertical ownership (PVO).

PVO, referred as a business phenomena where downstream firms acquire partial ownership of their upstream supplier with no control rights, has been commonly observed (Allen & Phillips 2000; Greenlee & Raskovich, 2006; Hunold & Stahl, 2016; Fang et al., 2022). For instance, JD.com acquired an 8.8% stake in the furniture manufacturer Shangpin Home Collection and Walgreens acquired 26% of the drug wholesaler AmerisourceBergen (Fang et al., 2022). Hence, it is natural to introduce PVO into a discuss of firms' endogenous order of moves.

Specifically, we consider a three-stage game played by a monopolistic upstream firm and two downstream firms holding PVO of the upstream firm. In the first (pre-play) stage, the downstream firms play observable delay game (Hamilton and Slutsky, 1990) to choose either act early or late. In the second stage, the upstream firm decides the wholesale price to the downstream firms. In the final stage, the downstream firms set prices under Bertrand competition (quantities under Cournot competition) either simultaneously or sequentially, depending upon their moves in the first stage.

We find that in the subgame perfect equilibrium, if the degree of product substitutability is small or the degree of PVO is high, simultaneous pricing occurs under Bertrand competition and sequential play occurs under Cournot competition. This is in contrast to the well-known results.[1]

The intuition for the results is as follows. PVO makes downstream firms caring about the profit of the upstream firm. Hence, a more competitive downstream market, i.e., simultaneous price competition or sequential quantity competition, leads to more outputs and thus more demand to the upstream firms and then in turn more downstream profits as well. This beneficial effect becomes stronger when the downstream firms care more of the upstream firm, i.e., a higher degree of PVO. In contrast, a higher competition harms the downstream firms' profits. However, this negative effect becomes less important when the degree of product product substitutability is smaller. Therefore, if the degree of product product substitutability is small or the degree of PVO is high, the beneficial effect from the upstream profit gains dominates the negative effect of fiercer downstream competition, leading downstream firms prefer simultaneous price competition or sequential quantity competition.

Additionally, it is commonly believed that the upstream firm prefers a more competitive downstream market (simultaneous price competition or sequential quantity competition) for more demands. However, with PVO, we find that when the degree of product substitutability is sufficiently small and the degree of PVO is high, the upstream firm's profit is larger under less competitive downstream market, i.e., sequential price competition or simultaneous quantity competition, which reverses the well-known result. This is because the wholesale price is always larger under less competitive downstream market. With PVO, when the degree of product substitutability is sufficiently small, the upstream gains effect from downstream market expansion becomes smaller. At this time, a higher wholesale price bringing more profit for the upstream firm appears to become more important. This whole-

[1] It is well-known that being a follower is preferred in price games, while being a leader is preferred in quantity games. In other words, firms take decisions sequentially in price games and simultaneously in quantity games (Gal-Or, 1985; Dowrick 1986; Boyer and Moreaux, 1987a, 1987b)

sale price raise effect under sequential pricing (or simultaneous quantity competition) raises more upstream profit than the effect by the downstream market expansion of more competition under simultaneous pricing (or sequential quantity competition), when the degree of product substitutability is small and the degree of PVO is high.

Our study is related to previous studies that reexamine the well-known first-mover advantage property of Cournot competition (Pal, 1998; Fanti, 2017) and they show that sequential quantity competition may occur in equilibrium. Many studies also tackle the second-mover advantage property of Bertrand market games in various contexts (Pal, 1998; Bárcena-Ruiz, 2007; Naya, 2015; Fanti, 2016; Hu and Mizuno, 2020). These show that the subgame perfect equilibrium could be the simultaneous choice of prices. However, all the above-mentioned studies consider models without PVO.[2] Our work also contributes to the studies about PVO (Flath, 1989; Greenlee and Raskovich, 2006; Lestage, 2021; Fang et al. 2022; Sun et al.2023), which analyze the effects related to PVO in various contexts.

The rest of the paper is organized as follows. Section 2 describes the basic model. Section 3 presents the analysis under Bertrand competition. In section 4, we examine the case under Cournot competition, and section 5 concludes.

2 Model

We consider a vertically related market with a monopolistic upstream firm M and two downstream firms (firm i and firm j, $i, j = 1, 2$ and $i \neq j$). To produce on unit of the final product, the downstream firms purchase one unit of the input from the upstream firm at wholesale price w. We assume that the marginal production costs of the upstream firm and the downstream firms are normalized to zero. The final products sold by the downstream firms are horizontally differentiated (e.g., each downstream firm sells a product with differentiated service). The downstream firms compete on price, and the price chosen by downstream firm i and its demand are denoted by p_i and q_i, respectively. Then, the

[2]Fanti (2016, 2017) also considers endogenous timing game with a vertical structure. However, the profit rankings in our result are different from those in Fanti (2016, 2017).

profits of downstream firm i and the upstream firm are $\pi_i \equiv (p_i - w)q_i$ and $\pi_M \equiv w(q_i + q_j)$, respectively.

In our model, we consider PVO between the upstream firm and downstream firms. For simplicity, we assume that the each shareholder of the downstream firm has a same share $s \in (0, 1/4)$ of the upstream firm in the form of passive investments with no control rights (e.g., nonvoting shares; Gilo et al., 2006)[3]. Hence, the shareholder of the upstream firm owns a share $1 - 2s$ of its property. Therefore, the managers of the downstream firms and the upstream firm M engage in maximize the total values $V_i = \pi_i + s\pi_M$ and $V_M = (1 - 2s)\pi_M$, respectively.

We assume that the utility function of a representative consumer is $u(q_i, q_j, m) \equiv a(q_i + q_j) - b(q_i^2 + 2\gamma q_i q_j + q_j^2)/2 + m$, where q_i and q_j are the consumption levels for products i and j, respectively; m is the quantity of a numeraire good; $\gamma \in (0, 1)$ is the measure of product substitutability; and a and b are positive parameters. This utility function yields the following demand function: $q_i = [a(1 - \gamma) - p_i + \gamma p_j]/[b(1 - \gamma^2)]$.

Given that we focus on endogenous timing for the downstream firms, we employ the observable delay game (Hamilton and Slutsky, 1990). The timing of this game is as follows. In the first (pre-play) stage, the downstream firms can choose to sell early (e) or late (l). In the second stage, the upstream firm decides the wholesale price w. In the third stage, the downstream firms play price competition where the timing of each downstream firm's decision depends on the choices in the first stage. Specifically, in the first stage, if both downstream firms make the same decision, that is either (e, e) or (l, l), the downstream firms play Bertrand competition, i.e., price competition with simultaneous timing, in the final stage of the game. By contrast, if the downstream firms choose different timing, that is either (e, l) or (l, e), they play price competition with sequential timing in the final stage; a downstream firm choosing e and l becomes a leader and a follower, respectively. We solve the game using backward induction.

[3]The assumption $s \in (0, 1/4)$ also guarantees all the equilibrium outcomes are positive.

3 Analysis

In the third stage, the downstream firms decide the prices in two scenarios depending on the choices in the first stage: Bertrand competition (simultaneous price competition) and sequential price competition. Solving the first-order conditions $\partial V_i/\partial p_i = 0$ yield

$$\begin{aligned} p^B(w) &\equiv \arg\max_p \ (p-w)q_i(p, p^B(w)) + sw[q_i(p, p^B(w)) + q_j(p, p^B(w))] = \frac{a(1-\gamma) + w(1-s+s\gamma)}{2-\gamma}, \\ p^F(p^L, w) &\equiv \arg\max_p \ (p-w)q_i(p, p^L) + sw[q_i(p, p^L) + q_j(p, p^L)] = \frac{a + w + sw(\gamma-1) - a\gamma + p^L\gamma}{2}, \\ p^L(w) &\equiv \arg\max_p \ (p-w)q_i(p, p^F(p,w)) + sw[q_i(p, p^F(p,w)) + q_j(p, p^F(p,w))] \\ &= \frac{w[2+2s(\gamma-1)-\gamma](1+\gamma) + (2-\gamma-\gamma^2)}{2(2-\gamma^2)}, \end{aligned}$$

where the superscript B denotes Bertrand competition and the superscripts L and F represent the leader and follower in the sequential price competition, respectively.

In the second stage, the upstream firm chooses the wholesale price. For the simultaneous price competition case, maximizing $V_M^B(w) = (1-2s)w[q_i(p^B(w), p^B(w)) + q_j(p^B(w), p^B(w))]$ for w, while for the sequential price competition case, maximizing $V_M^S(w) = (1-2s)w[q_i(p^L(w), p^F(w)) + q_j(p^L(w), p^F(w))]$ for w, we have

$$w^B = \frac{a}{2-2s(1-\gamma)}, \quad w^S = \frac{a(8+4\gamma-3\gamma^2-\gamma^3)}{16+8\gamma-6\gamma^2-2\gamma^3-4s(4-\gamma-3\gamma^2)},$$

where the superscript S denotes the case with sequential pricing. Note that when PVO does not exist, i.e., $s = 0$, the wholesale price is same in the cases with simultaneous and sequential pricing: $w^S = w^B = a/2$.

In the first stage, the downstream firms choose to sell early (e) or late (l). Using the outcomes in the second and third stages, we obtain the downstream total values $V^B(w^B) = [p^B(w^B) - w^B]q_i^B(w^B) + sw^B[q_i^B(w^B) + q_j^B(w^B)]$ under simultaneous pricing, while $V^L(w^S) = (p^L(w^S) - w^S)q_i^L(w^S) + sw^S[q_i^F(w^S) + q_j^L(w^S)]$ and $V^L(w^S) = (p^F(w^S) - w^S)q_i^F(w^S) + sw^S[q_i^F(w^S) + q_j^L(w^S)]$ as the total values of leader and follower under sequential pricing, respectively. Using these total values V^B, V^L and V^F, the payoff matrix in the first stage is presented in Table 1.

		Firm j	
		early	late
Firm i	early	V^B, V^B	V^L, V^F
	late	V^F, V^L	V^B, V^B

Table 1: Payoff matrix in the endogenous timing with PVO

We compare V^B, V^L and V^F and show the results in Figure 1.[4] There are two asymmetric equilibria in pure strategies, that is, (e, l) and (l, e) in region C with $V^F > V^L > V^B$, which yield sequential pricing. In region B with $V^F > V^B > V^L$, the dominant strategy is "late pricing (l)." Hence, simultaneous pricing occurs in a unique equilibrium. Finally, for the case in region A with $V^B > V^F > V^L$, we have two pure strategy equilibria (e, e) and (l, l), which means two firms choose simultaneous pricing. Therefore, we can find that simultaneous pricing tends to happen when the degree of product substitutability γ is small or the degree of PVO s is high from Figure 1. All proofs are shown in the Appendix. Then, we have the following proposition.

Proposition 1 *With PVO, simultaneous pricing occurs in equilibrium if the degree of product substitutability is small and the degree of PVO is high.*

The intuition is as follows. PVO makes the downstream firms caring about the profit of the upstream firm. A more competitive downstream market contributes to the profit of the upstream firm by generating more demands. It is well-known that the downstream market is more competitive under simultaneous pricing than under sequential pricing. Hence, comparative to sequential pricing, simultaneous pricing leads to more output and thus more demand to the upstream firm, which benefits the upstream profit and in turn the downstream profits as well. This beneficial effect becomes stronger when the downstream firms care more

[4] We also obtain the similar results as Figure 1 by comparing the profits π^B, π^L and π^F, when the managers of the downstream firms focus on the short-term immediate financial gains from the order of moves, for example. However, in our model, the decision-making of the order of moves is considered in the first stage, hence we compare the long-term total values of the downstream firms.

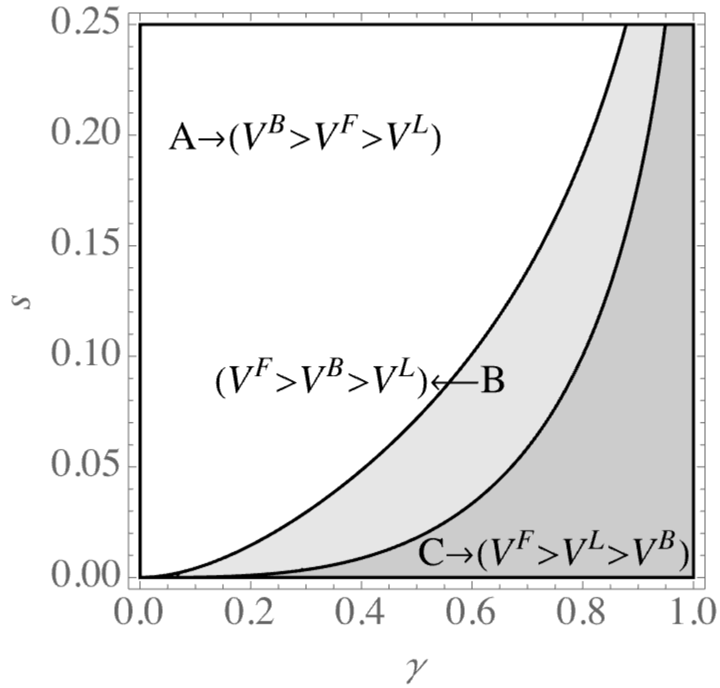

Figure 1: Total value rankings with region A: $V^B > V^F > V^L$; region B: $V^F > V^B > V^L$; and region C: $V^F > V^L > V^B$.

of the upstream firm, i.e., a higher degree of PVO (large s). In contrast, a higher competition harms the downstream firms' profits. However, this negative effect becomes less important when the degree of product substitutability is smaller (small γ). Therefore, if the degree of product substitutability is small or the degree of PVO is high, the beneficial effect from the upstream profit gains dominates the negative effect of fiercer downstream competition, leading downstream firms prefer simultaneous pricing. This result reverses the conventional wisdom that sequential pricing is always preferred when firms compete in prices.

The profit of the upstream firm Now, we also examine the effect of PVO on the upstream profit. First, we analyze the wholesale price. Recall that $w^S = w^B = a/2$ without PVO, i.e., $s = 0$. However, with $s > 0$, in the second stage, it is easy to find that $w^S - w^B = as\gamma(2+\gamma-2\gamma^2-\gamma^3)/[2\Phi(1-s(1-\gamma)] > 0$ where $\Phi = 8+4\gamma-3\gamma^2-\gamma^3-2s(4-\gamma-3\gamma^2) > 0$. In addition, $\partial(w^S - w^B)/\partial s = a\gamma\Theta(2+\gamma-2\gamma^2-\gamma^3)/[2(1-s+s\gamma)^2\Phi^2] > 0$, where $\Theta = 8+4\gamma-3\gamma^2-\gamma^3-2s^2(1-\gamma)^2(4+3\gamma) > 0$. Hence, we obtain the following result.

Lemma 1 *With PVO, (i) the upstream firm always chooses larger wholesale price under sequential pricing than the one under simultaneous pricing, i.e.,* $w^S > w^B$*; (ii) The wholesale price difference becomes larger as the degree of PVO increases, i.e.,* $\partial(w^S - w^B)/\partial s > 0$.

The intuition is as follows. Without PVO ($s = 0$), the choke prices when the downstream firms' outputs are zero are same under both simultaneous and sequential pricing, leading to the same wholesale price $w^S = w^B = a/2$. However, with PVO ($s > 0$), The downstream managers maximize the total value $V_i = \pi_i + s\pi_U$. Hence, the choke price is the price when $V_i = 0$, thus the operating profit π_i could be negative if the part $s\pi_U$ is sufficiently large. With the second mover property under Bertrand competition, the follower gains the most and thus the price is easier to become positive, leading to a higher choke price under sequential pricing than under simultaneous pricing: $w^S > w^B$. As the degree of PVO s increases, the downstream firms care more of the upstream firm, the difference between choke prices under sequential pricing and simultaneous pricing become larger: $\partial(w^S - w^B)/\partial s > 0$.

Now, we analyze the effect of PVO on the upstream firm. Recalling the case without PVO, substituting $s = 0$ into the total values and then we have $V_M^S(w^S) - V_M^B(w^B) = [a^2\gamma^2(2-\gamma-\gamma^2)]/[16b(2-\gamma)(1+\gamma)(-2+\gamma^2)] < 0$, which is a well-known result that the upstream firm's profit is always larger under simultaneous pricing than under sequential pricing, because the output is more under simultaneous pricing (a more competitive market). With PVO, solving $V_M^S(w^S) - V_M^B(w^B) > 0$ for s, we have the following result.

Proposition 2 *With PVO, the upstream firm's profit is larger under sequential pricing if* $\gamma < 0.634$ *and* $s > \gamma(8 + 4\gamma - 3\gamma^2 - \gamma^3)/(16 + 24\gamma - 12\gamma^2 - 15\gamma^3 + 2\gamma^4 + \gamma^5)$.

With PVO, when the degree of product substitutability is sufficiently small, the upstream gains effect from downstream market expansion becomes smaller. Especially when the degree of PVO is relatively high, this market expansion effect appears to be less workable. At this time, a higher wholesale price bringing more profit for the upstream firm appears to become more important. As mentioned in the part (i) of Lemma 1, $w^S > w^B$ always holds with PVO. This effect under sequential pricing by a higher wholesale price raises more upstream profit than the effect by the downstream market expansion of more competition under simultaneous pricing, when the degree of product substitutability is small and the degree of PVO is high (as mentioned in the part (ii) of Lemma 1: $\partial(w^S - w^B)/\partial s > 0$), leading that the upstream firm's profit is larger under sequential pricing than under simultaneous pricing. This result reverses the well-known result that the upstream firm always prefers to downstream simultaneous pricing.

4 Extensions

4.1 Cournot competition between downstream firms

In this subsection, we consider quantity competition in the downstream market. Solving $q_i(p_i, p_j) = [a(1-\gamma) - p_i + \gamma p_j]/[b(1-\gamma)]$ for p_i and p_j, we obtain the inverse demand for product i: $p_i(q_i, q_j) = a - b(q_i + \gamma q_j)$.

Substituting the inverse demand function into the profit of downstream firm i, we have $\pi_i = [a - b(q_i + \gamma q_j) - w]q_i$. In the third stage, solving the first-order conditions, we obtain the quantities that the downstream firms choose as follows. Under simultaneous production, the downstream firms choose q^C; under sequential production, the follower and leader choose $q^{FC}(q^{LC})$ and q^{LC}, respectively. Note that the superscript C denotes simultaneous quantity competition and the superscripts FC and LC indicate the follower and leader in the sequential quantity competition, respectively.

$$q^C(w) = \frac{a-(1-s)w}{b(2+\gamma)},$$
$$q^{FC}(q^{LC}, w) = \frac{a-(1-s)w - bq^{LC}\gamma}{2b}, \quad q^{LC} = \frac{w[2-2s(1-\gamma)-\gamma] + a(\gamma-2)}{2b(\gamma^2-2)},$$

In the second stage, the upstream firm chooses the wholesale price. For the simultaneous quantity competition case, maximizing $V_M(w) = (1-2s)w[q_i(q^C(w), q^C(w)) + q_j(q^C(w), q^C(w)]$ for w, while for the sequential price competition case, maximizing $V_M(w) =$

$(1-2s)w[q_i(q^{LC}(w), q^{FC}(w)) + q_j(q^{LC}(w), q^{FC}(w))]$ for w, we have

$$w^C = \frac{a}{2-2s}, \quad w^{SC} = \frac{a(8-4\gamma-\gamma^2)}{2(8-s(8-6\gamma)-4\gamma-\gamma^2)},$$

where the superscript SC denotes the case with sequential quantity competition.

In the first stage, the downstream firms choose to sell early (e) or late (l). Substituting back the outcomes in the second and third stages, we obtain the downstream profit $V^C(w^C)$ under simultaneous competition, while $V^{LC}(w^{SC})$ and $V^{FC}(w^{SC})$ as the total values of leader and follower under sequential competition, respectively. We compare these profits V^C, V^{LC} and V^{FC} and show the results in Figure 2. There are two asymmetric equilibria in pure strategies, that is, (e,l) and (l,e) in the region with $V^{LC} > V^{FC} > V^C$, which yields sequential competition. In the region with $V^{LC} > V^C > V^{FC}$, the dominant strategy is to act "early (e)." Hence, simultaneous competition occurs in a unique equilibrium. We can find that sequential competition tends to happen when the degree of product substitutability γ is small or the degree of PVO s is high. Then, we have the following proposition.

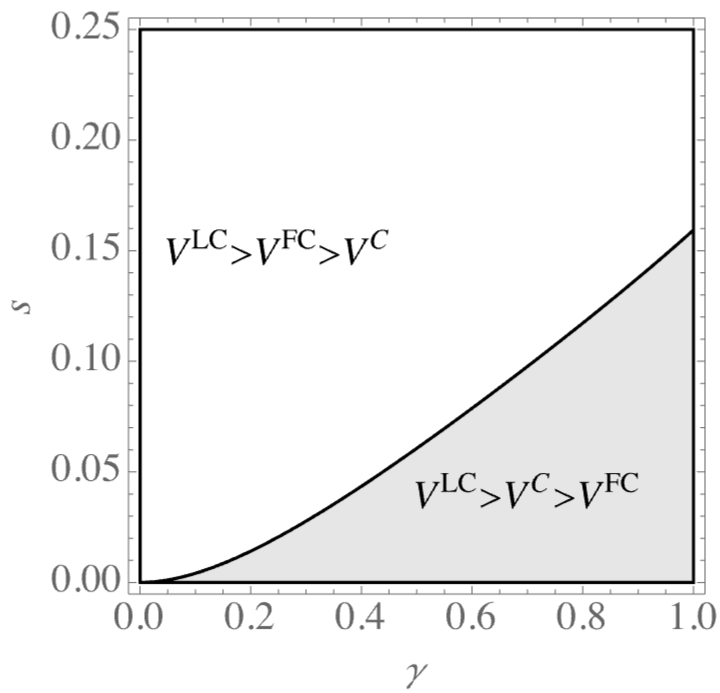

Figure 2: Total value rankings under Cournot competition.

Proposition 3 *With PVO, sequential quantity competition occurs in equilibrium if the degree of product substitutability is small or the degree of PVO is high.*

The conventional wisdom indicates that firms always take decisions simultaneously under Cournot competition (Gal-Or, 1985; Dowrick 1986; Boyer and Moreaux, 1987a, 1987b). However, the standard result reverses by introducing PVO. The intuition behind this result is the same as that under price competition.

The profit of the upstream firm Now, we examine the the effect of PVO on the upstream profit. First, we analyze the wholesale price. Recall that $w^S = w^B = a/2$ without PVO, i.e., $s = 0$. However, with $s > 0$, in the second stage, it is easy to find that $w^C - w^{SC} = as\gamma(2-\gamma)/[2(1-s)(8-4\gamma-\gamma^2-8s+6s\gamma)] > 0$. Additionally, we have $\partial(w^C - w^{SC}) = a\gamma(2-\gamma)(8-8s^2+6s^2++6\gamma s^2-4\gamma-\gamma^2)/[2(1-s)^2\Psi] > 0$ where $\Psi = (8-8s+6\gamma s-4\gamma-\gamma^2)^2 > 0$. Hence, we obtain the following result.

Lemma 2 *With PVO, (i) the upstream firm chooses larger wholesale price under simultaneous competition than the one under sequential competition when downstream firms compete in quantities; (ii) The wholesale price difference becomes larger as the degree of PVO increases.*

The intuition is parallel to Lemma 1.

Now, we examine the upstream firm. Solving $V_M^C(w^C) - V_M^{SC}(w^{SC}) > 0$ for and s, we have the following result.

Proposition 4 *With PVO, the upstream firm's profit is larger under simultaneous competition than under sequential competition if* $\gamma < 0.882$ *and* $s > \gamma(8-4\gamma-\gamma^2)/(16+8\gamma-12\gamma^2+\gamma^3)$.

Recalling that the upstream firm's profit is always larger under sequential competition than simultaneous competition when firms compete in quantities, because the market is more competitive under sequential competition than under simultaneous competition. However, our result reverses this well-known result. The explanations of Proposition 4 are parallel to Proposition 3.

5 Conclusions

We considered a vertical structure with a upstream and two downstream firms considering PVO. A more competitive environment, that is, simultaneous price competition or sequential quantity competition, leads to more outputs and thus more demand to the upstream firms and then in turn more downstream profits as well. This beneficial effect becomes stronger when the downstream firms care more of the upstream firm, i.e., a higher degree of PVO. In contrast, a higher competition harms the downstream firms' profits. However, this negative effect becomes less important when the degree of product substitutability is smaller. Therefore, if the degree of product substitutability is small or the degree of PVO is high, the beneficial effect from the upstream profit gains dominates the negative effect of fiercer downstream competition, leading downstream firms prefer simultaneous price competition or sequential quantity competition.

Appendix

Proof of Proposition 1

We prove this proposition using a numerical method.

First, comparing V^F with V^L yields

$$V^F - V^L = \frac{a^2\gamma\left(4-\gamma-3\gamma^2\right)\left[\gamma\left(\gamma^3+3\gamma^2-4\gamma-8\right)-2\left(7\gamma^3+5\gamma^2-12\gamma-8\right)s\right]^2}{64b(\gamma+1)\left(\gamma^2-2\right)^2\left[\gamma^3+3\gamma^2-4\gamma-2\left(3\gamma^2+\gamma-4\right)s-8\right]^2} > 0.$$

Hence, $V^F > V^L$ always. Next, we compare V^B and V^F. The region where $V^B > V^F > V^L$ is shown in Figure 3.

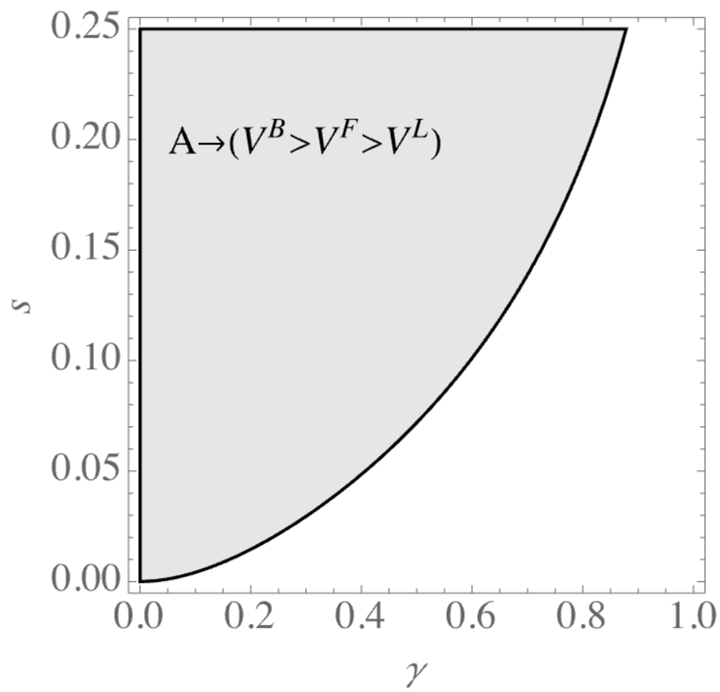

Figure 3: The region where $V^B > V^F > V^L$.

Then, we show the region where $V^F > V^B > V^L$ in Figure 4 and the region where $V^F > V^L > V^B$ in Figure 5.

Combining the results in Figure 3-5, we obtain the profit rankings shown in Figure 1 and complete the proof.□

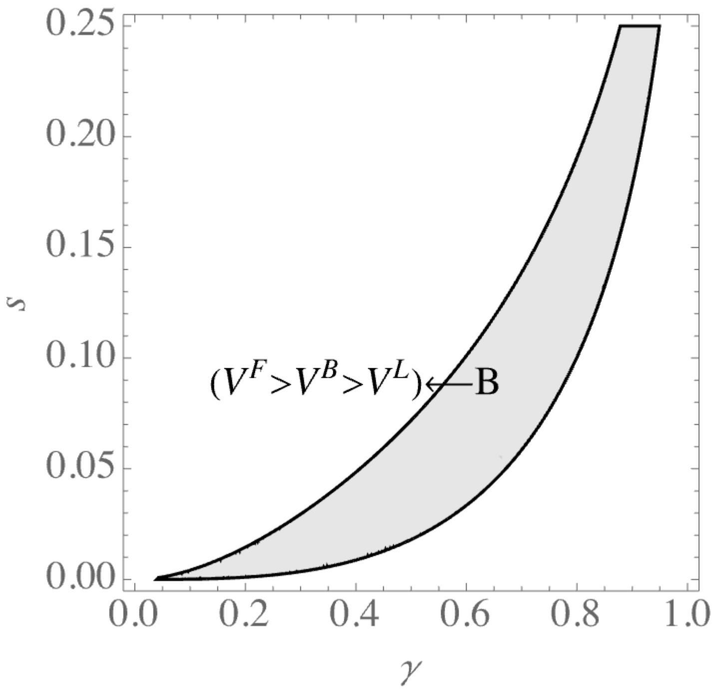

Figure 4: The region where $V^F > V^B > V^L$.

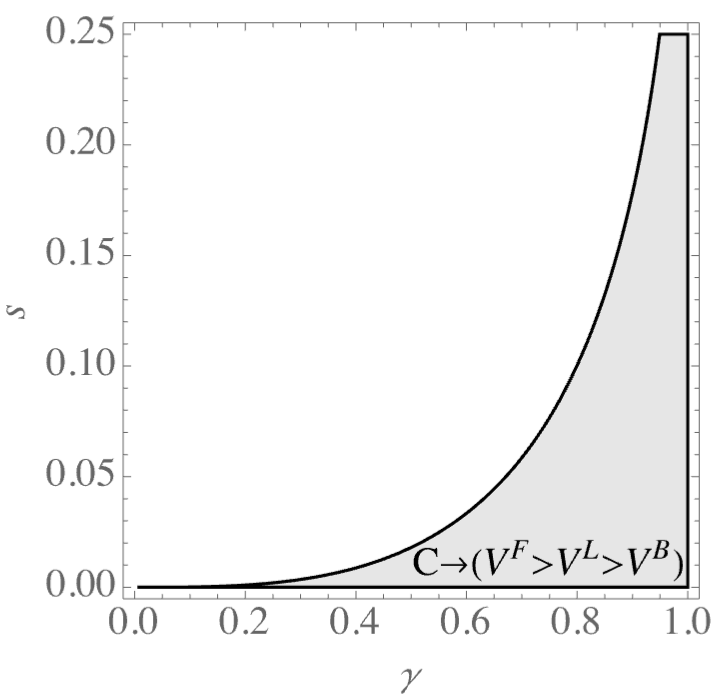

Figure 5: The region where$V^F > V^L > V^B$.

Proof of Proposition 2

We consider the sign of $V_M^S - V_M^B$.

$$V_M^S - V_M^B = \frac{a^2(1-2s)(1-\gamma)\gamma(\gamma+2)\begin{pmatrix} \gamma^4+3\gamma^3-4\gamma^2-8\gamma+\gamma^5 s+2\gamma^4 s \\ -15\gamma^3 s-12\gamma^2 s+24\gamma s+16s \end{pmatrix}}{16b(\gamma-2)(\gamma+1)\left(\gamma^2-2\right)(\gamma s-s+1)\left(-\gamma^3-3\gamma^2+4\gamma+6\gamma^2 s+2\gamma s-8s+8\right)}.$$

It is easy to find the sign depends on the part $(\gamma^4+3\gamma^3-4\gamma^2-8\gamma+\gamma^5 s+2\gamma^4 s-15\gamma^3 s-12\gamma^2 s + 24\gamma s+16s)$, which is a linear function of s and the coefficient is positive. Solving it for s, we have $V_M^S - V_M^B > 0$ if

$$s > -\frac{\gamma\left(\gamma^3+3\gamma^2-4\gamma-8\right)}{\gamma^5+2\gamma^4-15\gamma^3-12\gamma^2+24\gamma+16} \equiv \underline{s}.$$

At the same time, we need to keep $\underline{s} < 1/4$ following our assumption. By numerical solving $\underline{s} - 1/4 < 0$, we have $\gamma < 0.634$.□

Proof of Proposition 3

We can find the region where $V^{LC} > V^{FC} > V^C$ and the area where $V^{LC} > V^C > V^{FC}$ is shown in Figure 6 and Figure 7, respectively.

Combining the results in Figure 6 and 7, we complete the proof.□

Proof of Proposition 4

We consider the sign of $V_M^C - V_M^{SC}$.

$$V_M^C - V_M^{SC} = \frac{a^2(1-2s)(2-\gamma)\gamma\left(\gamma^3+4\gamma^2-8\gamma-\gamma^3 s-12\gamma^2 s+8\gamma s+16s\right)}{16b(\gamma+2)\left(\gamma^2-2\right)(s-1)\left(-\gamma^2-4\gamma+6\gamma s-8s+8\right)}.$$

It is easy to find the sign depends on the part $\gamma^3+4\gamma^2-8\gamma-\gamma^3 s-12\gamma^2 s+8\gamma s+16s$, which is a linear function of s and the coefficient is positive. Solving it for s, we have $V_M^C - V_M^{SC} > 0$

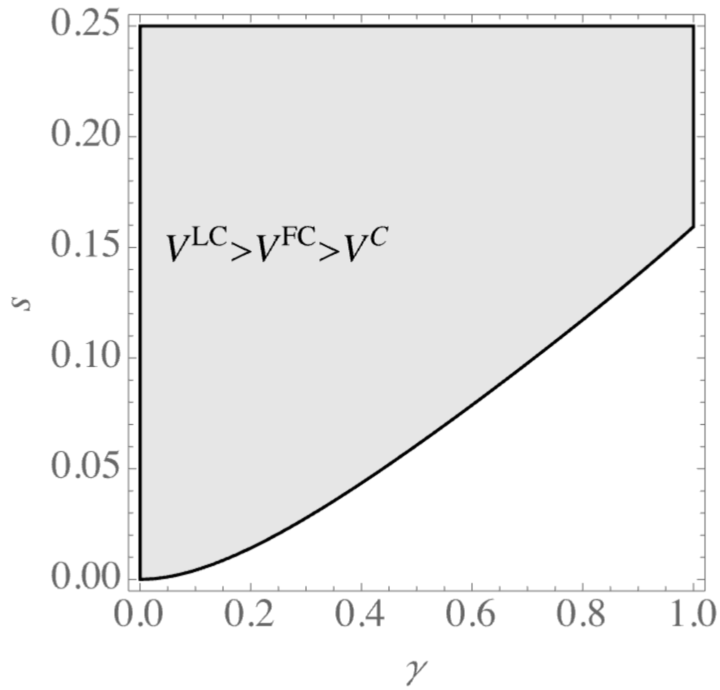

Figure 6: The region where $VLC > V^{FC} > V^C$.

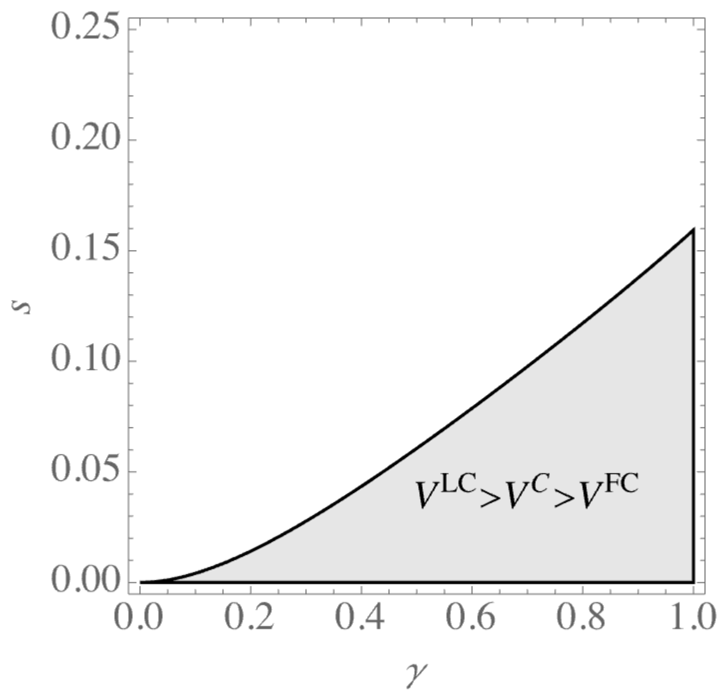

Figure 7: The region where $V^{LC} > V^C > V^{FC}$.

if

$$s > \frac{\gamma\left(\gamma^{2}+4\gamma-8\right)}{\gamma^{3}+12\gamma^{2}-8\gamma-16} \equiv \underline{s}^{C}.$$

At the same time, we need to keep $\underline{s}^{C} < 1/4$ following our assumption. By numerical solving $\underline{s}^{C} - 1/4 < 0$, we have $\gamma < 0.882.\square$

References

Allen, J. W., and Phillips, G. M. (2000). Corporate equity ownership, strategic alliances, and product market relationships. *The Journal of Finance*, 55(6), 2791-2815.

Bárcena-Ruiz, J.C. (2007) Endogenous timing in a mixed duopoly: Price competition. *Journal of Economics*, 91, 263–72.

Boyer, M and Moreaux, M. (1987a) Being a leader or a follower: Reflections on the distribution of roles in duopoly. *International Journal of Industrial Organization*, 5(2), 175–92.

Boyer, M and Moreaux, M. (1987b) On Stackelberg equilibria with differentiated products: The critical role of the strategy space. *Journal of Industrial Economics*, 36(2), 217–30.

Dowrick, S. (1986) von Stackelberg and Cournot duopoly: Choosing roles. *RAND Journal of Economics*, 17(2), 251–60.

Gilo, D., Moshe, Y., and Spiegel, Y. (2006) Partial cross ownership and tacit collusion. *RAND Journal of Economics*, 37(1), 81 – 99.

Greenlee, P., and Raskovich, A. (2006). Partial vertical ownership. *European Economic Review*, 50(4), 1017-1041.

Hunold, M., and Stahl, K. (2016). Passive vertical integration and strategic delegation. *The RAND Journal of Economics*, 47(4), 891-913.

Hu, Q., and Mizuno, T. (2021). Endogenous timing and manufacturer advertising: A note. *Metroeconomica*, 72(2), 309-320.

Fang, F., Jiang, B., and Sun, J. (2023). Partial vertical ownership in the presence of downstream competition. *Production and Operations Management*, 32(6), 1692-1704.

Fanti, L. (2016) Endogenous timing under price competition and unions. *International Journal of Economic Theory*, 12(4), 401–413.

Fanti, L. (2017) An observable delay game with unionised managerial firms. *Scottish Journal of Political Economy* , 64(1), 50–69.

Flath, D. (1989). Vertical integration by means of shareholding interlocks. *International Journal of Industrial Organization*, 7(3), 369-380.

Gal-Or, E. (1985) First mover and second mover advantages. *International Economic Review*, 26(3), 29–46.

Hamilton, J.H and Slutsky, S.M. (1990) Endogenous timing in a duopoly games: Stackelberg or Cournot equilibria. *Games and Economic Behavior*, 2(1), 29–46.

Lestage, R. (2021). Input price discrimination and non-controlling vertical shareholding. *Journal of Regulatory Economics*, 59(3), 226-250.

Naya, J.M. (2015) Endogenous timing in a mixed duopoly model. *Journal of Economics*, 116, 165–174.

Pal, D. (1998) Endogenous timing in a mixed oligopoly. *Economics Letters*, 61(2), 181–185.

Shy, O. (1995). *Industrial Organization: Theory and Applications*. Cambridge, MA: MIT Press.