



# Oscillations of Differential Equations with Deviating Arguments

Sficas, Y. G.  
Staikos, V. A.

---

(Citation)

Funkcialaj Ekvacioj, 19(1):35-43

(Issue Date)

1976-04

(Resource Type)

journal article

(Version)

Version of Record

(JaLCD0I)

<https://doi.org/10.24546/0100498981>

(URL)

<https://hdl.handle.net/20.500.14094/0100498981>



# Oscillations of Differential Equations with Deviating Arguments

By

Y. G. SFICAS and V. A. STAIKOS

(University of Ioannina, Greece)

## 1. Preliminaries.

In this paper we consider differential equations with deviating arguments of the forms

$$x^{(n)}(t) + \delta \left\{ \sum_{i=1}^r p_i(t) f_i(x[\tau_i(t)]) + F(t; x\langle\sigma_0(t)\rangle, \dots, x^{(n-1)}\langle\sigma_{n-1}(t)\rangle) \right\} = 0 \quad (E_1, \delta)$$

and

$$x^{(n)}(t) + \delta \left\{ p(t) \prod_{i=1}^r \varphi_i(x[\tau_i(t)]) \prod_{i=1}^{2\nu+1} \operatorname{sgn} x[\tau_i(t)] + F(t; x\langle\sigma_0(t)\rangle, \dots, x^{(n-1)}\langle\sigma_{n-1}(t)\rangle) \right\} = 0 \quad (E_2, \delta)$$

where  $\delta = \pm 1$ ,  $\nu$  is a nonnegative integer with  $2\nu + 1 \leq r$  and

$$x^{(i)}\langle\sigma_i(t)\rangle = (x^{(i)}[\sigma_{i1}(t)], \dots, x^{(i)}[\sigma_{i, m_i}(t)]).$$

For the real-valued functions  $\tau_i$  ( $j=1, 2, \dots, r$ ) and  $\sigma_{ij}$  ( $j=1, 2, \dots, m_i; i=0, 1, \dots, n-1$ ) we suppose that they are continuous on the half-line  $[t_0, \infty)$  and satisfy the following:

- (i)  $\lim_{t \rightarrow \infty} \sigma_{ij}(t) = \infty$
- (ii)  $\lim_{t \rightarrow \infty} \tau_i(t) = \infty$
- (iii)  $\tau_i$  ( $i=1, 2, \dots, r$ ) are nondecreasing and such that  $\tau_i(t) \leq t$  for every  $t \geq t_0$ .

In what follows, we restrict our attention only to solutions  $x$  of  $(E_1, \delta)$  or  $(E_2, \delta)$  which are defined on a half-line  $[t_x, \infty)$ . The oscillatory character is considered in the usual sense, i.e., a continuous real-valued function  $h$  defined on a half-line  $[t_h, \infty)$  is called *oscillatory* if it has no last zero, otherwise it is called *nonoscillatory*.

We introduce, now, the following conditions:

(C<sub>1</sub>) The functions  $p_i$  ( $i=1, 2, \dots, r$ ) are nonnegative and locally integrable on  $[t_0, \infty)$ .

(C<sub>2</sub>) The real-valued functions  $f_i$  ( $i=1, 2, \dots, r$ ) are continuous and such that

$$y \neq 0 \implies y f_i(y) > 0.$$

(C<sub>3</sub>) The real-valued function  $F$  is continuous on  $[t_0, \infty) \times \mathbf{R}^{m_0} \times \dots \times \mathbf{R}^{m_{n-1}}$  and such that

$$(\forall i=1, 2, \dots, m_0) y_{1i} > 0 \implies F(t, y_1, \dots, y_n) \geq 0$$

and

$$(\forall i=1, 2, \dots, m_0) y_{1i} < 0 \implies F(t, y_1, \dots, y_n) \leq 0.$$

(C<sub>4</sub>) The function  $p$  is nonnegative and locally integrable on  $[t_0, \infty)$ .

(C<sub>5</sub>) The real-valued functions  $\varphi_i$  ( $i=1, 2, \dots, r$ ) are continuous on  $\mathbf{R}$  and such that

$$y \neq 0 \implies \varphi_i(y) > 0.$$

The oscillatory and asymptotic behavior of bounded solutions of differential equations  $(E_1, \delta)$  and  $(E_2, \delta)$  is well described by the following two theorems. The proofs of these theorems are omitted, since they follow as in [7] and [8] by using a recent result [9, Th. 1] and applying the comparison principle introduced by the authors in [7] and [8].

**Theorem 1.** Consider the differential equation  $(E_1, \delta)$  subject to the conditions (i), (ii) and (C<sub>1</sub>), (C<sub>2</sub>), (C<sub>3</sub>). If for some  $i$ ,  $1 \leq i \leq r$

$$(C_6) \quad \int_{t_0}^{\infty} t^{n-1} p_i(t) dt = \infty,$$

then for  $n$  even {resp. odd} all bounded solutions of  $(E_1, 1)$  {resp.  $(E_1, -1)$ } are oscillatory, while for  $n$  odd {resp. even} all bounded solutions of  $(E_1, 1)$  {resp.  $(E_1, -1)$ } are either oscillatory or tending monotonically to zero as  $t \rightarrow \infty$  together with their first  $n-1$  derivatives.

**Theorem 2.** Consider the differential equation  $(E_2, \delta)$  subject to the conditions (i), (ii) and (C<sub>3</sub>), (C<sub>4</sub>), (C<sub>5</sub>). If

$$(C_7) \quad \int_{t_0}^{\infty} t^{n-1} p(t) dt = \infty,$$

then for  $n$  even {resp. odd} all bounded solutions of  $(E_2, 1)$  {resp.  $(E_2, -1)$ } are oscillatory, while for  $n$  odd {resp. even} all bounded solutions of  $(E_2, 1)$  {resp.  $(E_2, -1)$ } are either oscillatory or tending monotonically to zero as  $t \rightarrow \infty$  together with their first  $n-1$  derivatives.

In the particular case, where

$$\begin{aligned}\tau_i(t) &\equiv t & (i=1, 2, \dots, r) \\ \sigma_{ij}(t) &\equiv t & (j=1, 2, \dots, m_i; i=0, 1, \dots, n-1)\end{aligned}$$

equations  $(E_1, \delta)$  and  $(E_2, \delta)$  become ordinary differential equations. That is, to every differential equation with deviating arguments a corresponding ordinary differential equation is associated. Since conditions  $(C_6)$  and  $(C_7)$  do not involve the functions  $\tau_i$  and  $\sigma_{ij}$ , it is obvious that the oscillatory and asymptotic behavior, as it described by the above theorems 1 and 2, is the same for the equations  $(E_1, \delta)$ ,  $(E_2, \delta)$  and their associated ordinary differential equations. However, this is not true for the oscillatory behavior only. Indeed, the retarded differential equation

$$x''(t) - x(t - \pi) = 0$$

admits  $\sin t$  and  $\cos t$  as oscillatory solutions and moreover [2] it has all bounded solutions oscillatory. On the other hand, the associated ordinary equation

$$x'' - x = 0$$

admits the bounded nonoscillatory solution  $x(t) = e^{-t}$  for which, of course,  $\lim_{t \rightarrow \infty} x(t) = 0$ . This change in the oscillatory behavior of a differential equation is obviously generated by the delay and its study is very interesting in applications. For example, oscillations caused by delays should be seriously taken into account in studying the motion of a controlled craft moving with increasing velocities, where it is possible to have a sudden release of oscillations leading to instability (Cf. Minorsky [4, p. 518]).

In view of the above theorems 1 and 2, the problem of establishing conditions under which the bounded and nonoscillatory solutions vanish, makes sense only for the differential equations  $(E_1, -1)$  and  $(E_2, -1)$  for  $n$  even as well as  $(E_1, 1)$  and  $(E_2, 1)$  for  $n$  odd. These equations can be unified in the following two forms:

$$\begin{aligned}x^{(n)}(t) + (-1)^{n+1} \left\{ \sum_{i=1}^r p_i(t) f_i(x[\tau_i(t)]) \right. \\ \left. + F(t; x\langle\sigma_0(t)\rangle, \dots, x^{(n-1)}\langle\sigma_{n-1}(t)\rangle) \right\} = 0\end{aligned}\quad (*)$$

and

$$\begin{aligned}x^{(n)}(t) + (-1)^{n+1} \left\{ p(t) \prod_{i=1}^r \varphi_i(x[\tau_i(t)]) \prod_{i=1}^{2\nu+1} \operatorname{sgn} x[\tau_i(t)] \right. \\ \left. + F(t; x\langle\sigma_0(t)\rangle, \dots, x^{(n-1)}\langle\sigma_{n-1}(t)\rangle) \right\} = 0.\end{aligned}\quad (**)$$

## 2. Oscillations generated by the delays.

In this section we deal with differential equations (\*) and (\*\*) and give conditions involving the functions  $\tau_i$  ( $i=1, 2, \dots, r$ ) under which all bounded solutions are oscillatory. These conditions are not valid for the associated ordinary differential equations. Related results have been obtained in [1], [2], [3] and [5] concerning particular cases of the equations (\*) and (\*\*). Also our results extend and improve, in several directions, a previous one due to Shreve [6, Theorem 1], concerning a first order retarded differential equation.

**Lemma.** *Let  $\psi$  be a bounded  $n$ -times continuously differentiable function on an interval  $[t_\psi, \infty)$  with*

$$\psi(t) > 0 \quad \text{and} \quad (-1)^n \psi^{(n)}(t) \geq 0 \quad \text{for every } t \geq t_\psi.$$

*Then there exists a  $T_\psi \geq t_\psi$  such that for every  $t \geq T_\psi$  and  $k=1, 2, \dots, n$*

$$(-1)^k \psi^{(k)}(t) \geq 0 \tag{1}$$

*and for every  $u, v$  with  $T_\psi \leq u \leq v$*

$$\psi(u) \geq \frac{(-1)^{n-1} \psi^{(n-1)}(v)}{(n-1)!} (v-u)^{n-1}. \tag{2}$$

*Proof.* If for some  $k$ ,  $0 < k < n$ ,

$$\psi^{(k)}(t) \psi^{(k+1)}(t) \geq 0 \quad \text{for all large } t$$

then a simple application of Taylor's formula leads to the contradiction  $\lim_{t \rightarrow \infty} \psi(t) = \pm \infty$ . This proves (1).

Next, by using again the Taylor's formula, we obtain

$$\begin{aligned} \psi(u) &= \psi(v) + \frac{\psi'(v)}{1!} (u-v) + \frac{\psi''(v)}{2!} (u-v)^2 + \dots \\ &\quad + \frac{\psi^{(n-1)}(u^*)}{(n-1)!} (u-v)^{n-1} \\ &= \psi(v) + \frac{(-1)^1 \psi'(v)}{1!} (v-u) + \frac{(-1)^2 \psi''(v)}{2!} (v-u)^2 + \dots \\ &\quad + \frac{(-1)^{n-1} \psi^{(n-1)}(u^*)}{(n-1)!} (v-u)^{n-1} \end{aligned}$$

for every  $u, v$  with  $T_\psi \leq u \leq v$  and some  $u^*$  with  $u \leq u^* \leq v$ . Hence, by (1),

$$\psi(u) \geq \frac{(-1)^{n-1} \psi^{(n-1)}(v)}{(n-1)!} (v-u)^{n-1}$$

for every  $u, v$  with  $T_\psi \leq u \leq v$ .

**Theorem 3.** Consider the differential equation (\*) subject to the conditions (i), (ii), (iii) and  $(C_1), (C_2), (C_3)$ . If

$$(C_8) \quad \limsup_{t \rightarrow \infty} \sum_{i=1}^r \int_{\tau(t)}^t [\tau(t) - \tau_i(s)]^{n-1} p_i(s) ds > (n-1)! \limsup_{y \rightarrow 0} \frac{y}{f(y)}$$

where

$$\tau(t) = \max_{1 \leq i \leq r} \tau_i(t) \quad \text{and} \quad f(y) = \operatorname{sgn} y \min_{1 \leq i \leq r} |f_i(y)|$$

then for  $n \geq 2$  all bounded solutions of (\*) are oscillatory, while for  $n=1$  all solutions of (\*) are oscillatory.

*Proof.* Let  $x$  be a bounded nonoscillatory solution of (\*). Since the substitution  $x = -z$  transforms (\*) into an equation of the same form satisfying the assumptions of the theorem, we can assume that  $x(t) > 0$  for every  $t \geq t_0$ . Moreover, since for every  $t, s$  with  $\tau(t) \leq s \leq t$ ,

$$0 \leq \tau(t) - \tau(s) \leq s$$

from condition  $(C_8)$  we derive that

$$\limsup_{t \rightarrow \infty} \sum_{i=1}^r \int_{\tau(t)}^t s^{n-1} p_i(s) ds > 0$$

which implies  $(C_6)$ . Thus, by Theorem 1, we must have  $\lim_{t \rightarrow \infty} x(t) = 0$ .

By (i) and (ii), we choose a  $t_1 \geq t_0$  such that for every  $t \geq t_1$ ,

$$\sigma_{ij}(t) \geq t_0 \quad (j=1, 2, \dots, m_i; i=0, 1, \dots, n-1)$$

and

$$\tau_i(t) \geq t_0 \quad (i=1, 2, \dots, r).$$

Thus, from the equation (\*) for every  $t \geq t_1$  we obtain

$$(-1)^n x^{(n)}(t) \geq 0$$

where  $x^{(n)}(t)$  is not identically zero for all large  $t$ , since, by  $(C_8)$ , this holds for some of the functions  $p_i$  ( $i=1, 2, \dots, r$ ). So, for some  $t_2 \geq t_1$  and every  $t \geq t_2$

$$x^{(n-1)}(t) \neq 0. \quad (3)$$

By choosing, now, a  $t_3 \geq t_2$  so that for all  $i=1, 2, \dots, r$ ,

$$\tau_i(t) \geq t_2 \quad \text{for every } t \geq t_3$$

and applying the lemma, we derive that for some  $t_4 \geq t_3$  and every  $k=1, 2, \dots, n$

$$(-1)^k x^{(k)}(t) \geq 0 \quad (4)$$

for every  $t \geq t_4$ , as well as

$$x[\tau_i(s)] \geq \frac{(-1)^{n-1} x^{(n-1)}[\tau(t)]}{(n-1)!} [\tau(t) - \tau_i(s)]^{n-1} \quad (5)$$

for every  $i=1, 2, \dots, r$  and  $t, s$  with  $\tau(t) \leq s \leq t$  and  $t \geq t_4$ .

Next, by (\*), integrating from  $\tau(t)$  to  $t$  we obtain

$$(-1)^n \{x^{(n-1)}(t) - x^{(n-1)}[\tau(t)]\} = \int_{\tau(t)}^t \left[ \sum_{i=1}^r p_i(s) f_i(x[\tau_i(s)]) + F(s; \dots) \right] ds$$

for every  $t \geq t_4$ . Hence, by (C<sub>3</sub>) and (4),

$$\begin{aligned} 0 &\geq (-1)^n x^{(n-1)}(t) \geq \sum_{i=1}^r \int_{\tau(t)}^t p_i(s) f_i(x[\tau_i(s)]) ds + (-1)^n x^{(n-1)}[\tau(t)] \\ &= \sum_{i=1}^r \int_{\tau(t)}^t p_i \frac{f_i(x[\tau_i(s)])}{x[\tau_i(s)]} x[\tau_i(s)] ds + (-1)^n x^{(n-1)}[\tau(t)] \\ &\geq \sum_{i=1}^r \inf_{s \geq \tau(t)} \frac{f(x[\tau_i(s)])}{x[\tau_i(s)]} \int_{\tau(t)}^t p_i(s) x[\tau_i(s)] ds + (-1)^n x^{(n-1)}[\tau(t)] \\ &\geq \inf_{0 < y \leq x(\sigma[\tau(t)])} \frac{f(y)}{y} \sum_{i=1}^r \int_{\tau(t)}^t p_i(s) x[\tau_i(s)] ds + (-1)^n x^{(n-1)}[\tau(t)] \end{aligned}$$

where  $\sigma(u) = \min_{1 \leq i \leq r} \tau_i(u)$ . Then using (5), we obtain

$$0 \geq (-1)^{n-1} x^{(n-1)}[\tau(t)] \left[ \frac{1}{(n-1)!} \inf_{0 < y \leq x(\sigma[\tau(t)])} \frac{f(y)}{y} \sum_{i=1}^r \int_{\tau(t)}^t p_i(s) [\tau(t) - \tau_i(s)]^{n-1} ds \right]$$

and consequently, by (3) and (4), after some manipulations,

$$\sum_{i=1}^r \int_{\tau(t)}^t p_i(s) [\tau(t) - \tau_i(s)]^{n-1} ds - (n-1)! \sup_{0 < y \leq x(\sigma[\tau(t)])} \frac{y}{f(y)} \leq 0$$

for every  $t \geq t_4$ . But, since  $\lim_{t \rightarrow \infty} x(t) = 0$ , this inequality contradicts condition (C<sub>8</sub>).

By now we have proved that every solution  $x$  of (\*) is oscillatory or nonoscillatory and unbounded. In the second case we obviously have

$$(-1)^n x(t) x^{(n)}(t) \geq 0 \quad \text{for all large } t$$

and consequently, since  $x$  is unbounded,  $n \geq 2$ . Hence for  $n=1$  all solutions of (\*) are oscillatory.

**Theorem 4.** Consider the differential equation (\*\*) subject to the conditions

(i), (ii), (iii), and  $(C_3)$ ,  $(C_4)$ ,  $(C_5)$ . If there exist nonnegative numbers  $\alpha_i$ ,  $i=1, 2, \dots, r$  with  $\alpha_1 + \alpha_2 + \dots + \alpha_r = 1$  and

$$(C_9) \quad \limsup_{t \rightarrow \infty} \int_{\tau(t)}^t \prod_{i=1}^r [\tau(t) - \tau_i(s)]^{\alpha_i(n-1)} p(s) ds > (n-1)! \limsup_{\substack{y_i \rightarrow 0 \\ 1 \leq i \leq r}} \frac{|y_1|^{\alpha_1} \cdots |y_r|^{\alpha_r}}{\varphi_1(y_1) \cdots \varphi_r(y_r)}$$

where  $\tau(t) = \max_{1 \leq i \leq r} \tau_i(t)$  then for  $n \geq 2$  all bounded solutions of  $(**)$  are oscillatory, while for  $n=1$  all solutions of  $(**)$  are oscillatory.

*Proof.* Let  $x$  be a bounded nonoscillatory solution of  $(**)$ , which as in the proof of Theorem 3, can be supposed positive on  $[t_0, \infty)$ . Also, by the same arguments used in the proof of Theorem 3, it follows that condition  $(C_9)$  implies  $(C_7)$ . Thus, by Theorem 2, we must have  $\lim_{t \rightarrow \infty} x(t) = 0$ . Moreover, as in the proof of Theorem 3, we can choose a  $t_5 \geq t_0$  so that (3), (4) and (5) be satisfied for every  $t \geq t_5$ .

Now, by  $(**)$ , integrating from  $\tau(t)$  to  $t$  and taking into account  $(C_3)$  and (4), for every  $t \geq t_5$  we obtain

$$\begin{aligned} & (-1)^n \{x^{(n-1)}(t) - x^{(n-1)}[\tau(t)]\} \\ &= \int_{\tau(t)}^t \left[ p(s) \prod_{i=1}^r \varphi_i(x[\tau_i(s)]) + F(s; \dots) \right] ds \\ &\geq \int_{\tau(t)}^t p(s) \prod_{i=1}^r \varphi_i(x[\tau_i(s)]) ds \\ &= \int_{\tau(t)}^t p(s) \prod_{i=1}^r \frac{\varphi_i(x[\tau_i(s)])}{x^{\alpha_i}[\tau_i(s)]} \prod_{i=1}^r x^{\alpha_i}[\tau_i(s)] ds \\ &\geq \inf_{s \geq \tau(t)} \prod_{i=1}^r \frac{\varphi_i(x[\tau_i(s)])}{x^{\alpha_i}[\tau_i(s)]} \int_{\tau(t)}^t p(s) \prod_{i=1}^r x^{\alpha_i}[\tau_i(s)] ds. \end{aligned}$$

Then, using (4), (5) and (3), after some manipulations we derive that for every  $t \geq t_5$

$$\int_{\tau(t)}^t p(s) \prod_{i=1}^r [\tau(t) - \tau_i(s)]^{\alpha_i(n-1)} ds - (n-1)! \prod_{i=1}^r \sup_{0 < y_i \leq x(\tau_i[\sigma(t)])} \frac{y_i^{\alpha_i}}{\varphi(y_i)} \leq 0$$

where  $\sigma(u) = \min_{1 \leq i \leq n} \tau_i(u)$ . But, since  $\lim_{t \rightarrow \infty} x(t) = 0$ , this inequality contradicts condition  $(C_9)$ . Thus every solution  $x$  of  $(**)$  is oscillatory or nonoscillatory and unbounded. As in the proof of Theorem 3, it is easy to verify that in the particular case  $n=1$  all solutions of  $(**)$  are oscillatory.

*Example.* By the above Theorems 3 or 4, the retarded differential equation

$$x''' + \frac{6}{t^3} x(\sqrt{t}) = 0, \quad t > 1$$

has no bounded nonoscillatory solutions, since

$$\limsup_{t \rightarrow \infty} \int_{\sqrt{t}}^t \frac{6}{s^3} [\sqrt{t} - \sqrt{s}]^2 ds = 3 > 2!.$$

However, the associated ordinary differential equation

$$x''' + \frac{6}{t^3} x = 0, \quad t > 1$$

admits the bounded and nonoscillatory solution  $x(t) = 1/t$ .

*Remark.* For a retarded differential equation

$$x^{(n)}(t) + (-1)^{n+1} p(t) \varphi(x[\tau(t)]) = 0$$

which is included in both forms (\*) and (\*\*), condition  $(C_8)$  or  $(C_9)$  is impossible in case where

$$\limsup_{y \rightarrow 0} \frac{y}{\varphi(y)} = \infty.$$

This case includes the class of strongly superlinear equations and, in particular, the equation

$$x^{(n)}(t) + (-1)^{n+1} p(t) |x[\tau(t)]|^\alpha \operatorname{sgn} x[\tau(t)] = 0 \quad (\alpha > 1).$$

By the following examples it is shown that, even for such differential equations, the condition

$$\limsup_{t \rightarrow \infty} \int_{\tau(t)}^t p(s) [\tau(t) - \tau(s)]^{n-1} ds = \infty$$

is not enough to maintain the validity of Theorem 3 or 4.

*Example.* The differential equation

$$x''(t) - \frac{2}{t} [x(\sqrt{t})]^4 \operatorname{sgn} x(\sqrt{t}) = 0, \quad t > 1$$

admits the bounded and nonoscillatory solution  $x(t) = 1/t$ , although

$$\limsup_{t \rightarrow \infty} \int_{\sqrt{t}}^t \frac{2}{s} (\sqrt{t} - \sqrt{s}) ds = \infty.$$

*Example 2.* The differential equation

$$x'''(t) + \frac{6}{t} [x(\sqrt{t})]^6 \operatorname{sgn} x(\sqrt{t}) = 0, \quad t > 1$$

admits also the bounded and nonoscillatory solution  $x(t)=1/t$ , although

$$\limsup_{t \rightarrow \infty} \int_{\sqrt{t}}^t \frac{6}{s} [\sqrt{t} - \sqrt{s}]^2 ds = \infty.$$

*Note.* The authors wish to thank the referee for his helpful suggestions.

### References

- [1] Kusano, T. and Onose, H., Oscillatory and asymptotic behavior of sublinear retarded differential equations, *Hiroshima Math. J.*, **4** (1974), 343–355.
- [2] Ladas, G., Ladde, G. and Papadakis, J. S., Oscillations of functional-differential equations general by delays, *J. Differential Equations*, **12** (1972), 385–395.
- [3] Ladas, G., Lakshmikantham, V. and Papadakis, J. S., Oscillations of higher-order retarded differential equations generated by the retarded argument, *Symposium on delay and functional differential equations*, University of Utah, March 7–11, 1972.
- [4] Minorsky, N., *Nonlinear Oscillations*, Van Nostrand, Princeton, NJ, 1962.
- [5] Sficas, Y. and Staikos, V. The effect of retarded actions on nonlinear oscillations, *Proc. Amer. Math. Soc.*, **46** (1974), 259–264.
- [6] Shreve, W., Oscillation in first order nonlinear retarded argument differential equations, *Proc. Amer. Math. Soc.*, **41** (1973), 565–568.
- [7] Staikos, V. and Sficas, Y., Oscillatory and asymptotic properties of differential equations with retarded arguments, *Applicable Anal.*, **5** (1975), 141–148.
- [8] Staikos, V. and Sficas, Y., Some results on oscillatory and asymptotic behavior of differential equations with deviating arguments, *Proceedings of Carathéodory Symposium* (Athens, September 3–7, 1973), pp. 546–553.
- [9] Staikos, V. and Sficas, Y., Oscillatory and asymptotic characterization of the solutions differential equations with deviating arguments, *J. London Math. Soc.*, **10** (1975), 39–47.

nuna adreso:  
Department of Mathematics  
University of Ioannina  
Ioannina, Greece

(Ricevita la 10-an de Decembro, 1974)