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On the Hölder Continuity of the Solutions for Degenerate Elliptic Equations

By

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In the theory of degenerate quasilinear elliptic equations and parabolic ones, regularity properties of solutions are very important. Edmunds and Peletier [2] have established inequalities of Harnack type for weak solutions of the equations of the form

$$\frac{\partial}{\partial x_i}A_i(x, u, u_x)+B(x, u, u_x)=0, \tag{0.1}$$

which admit degeneracy of their ellipticity at fixed point set, that is, in brief, the degeneracy occurs at $\mu(x)=0$ where

$$\mu(x)|p|^2/\mu_0 \leqq p_i A_i(x, u, p) \leqq \mu(x)|p|^2.$$

Moreover Kružkov and Kolodiĭ [4] have established an analogous result for degenerate parabolic equations of somewhat different form.

On the other hand, we can also consider the equations which admit degeneracy at a special value of the solution u, say $u=0$, for example, porous media equations $\Delta u^m=\partial u/\partial t$, whose regularity theorem for non-negative weak solutions has been given by Aronson [1]. For elliptic systems, Uralceva [6] has shown Hölder continuity of the solutions of the systems

$$\frac{\partial}{\partial x_i}\{a_{ij}(x, u)u_{x_j}\}=0,$$

where

$$\nu(|u|)\xi^2 \leqq a_{ij}(x, u)\xi_i\xi_j \leqq \mu\nu(|u|)\xi^2, \qquad \nu(0)=0.$$

Motivated by the above, we will show Hölder continuity of non-negative weak solutions for the form (0.1) that admits degeneracy of ellipticity at $u=0$.

§ 1. Assumption and theorems.

Let Ω be a bounded domain in R^n with boundary $\partial\Omega$. We consider degenerate quasilinear elliptic equations

$$(1.1)\qquad Lu\equiv\frac{\partial}{\partial x_i}\{a_{ij}(x,u)u_{x_j}\}-B_j(x,u)u^{\nu}u_{x_j}-B_0(x,u)=0$$

in Ω, where

$$(1.2)\qquad C_0u^{\tau}|\xi|^2\leqq a_{ij}(x,u)\xi_i\xi_j\leqq C_0^{-1}u^{\tau}|\xi|^2\qquad \text{in } (x,u)\in\bar{\Omega}\times[0,\infty)$$

for some positive constants C_0, τ and for any real vector $\xi=(\xi_1,\cdots,\xi_n)$.

Throughout this note, a summation is carried out over the repeated subscripts and the notations of function spaces and their norms are as usual (cf. [5]).

We assume that the coefficients $a_{ij}(x,u)$ and $B_j(x,u)$ belong to $C(\bar{\Omega}\times[0,\infty))$, and satisfy the following conditions

A_1. $$a_{ij}(x,u)=a_{ji}(x,u)\qquad (i,j=1,\cdots,n)$$

A_2. $$|B_0(x,u)|\leqq C_1u^{\mu}$$

for some positive constant C_1. Here, μ and ν satisfy one of (1) and (2)

(1) $\mu\geqq 0$ and $\nu\geqq 3\tau/4$

(2) $\mu>0$ and $\nu>\tau/2$.

Definition. A non-negative function $u(x)$ is called a weak solution of (1.1), if the following conditions are fulfilled;

(i) $u^{\tau/2+1}\in W_2^1(\Omega)$, $u\in L^2(\Omega)$

(ii) for any function $\phi\in \mathring{W}_2^1(\Omega)$ the following equality is valid;

$$(1.3)\qquad \int_{\Omega}\Big[\frac{2}{\tau+2}a_{ij}(x,u)u^{-\tau/2}(u^{\tau/2+1})_{x_i}\phi_{x_j}$$
$$+\Big\{\frac{2}{\tau+2}B_j(x,u)u^{\nu-\tau/2}(u^{\tau/2+1})_{x_j}+B_0(x,u)\Big\}\phi\Big]dx=0.$$

The main result of this note is the following.

Theorem 1. *Under the conditions of* (1.2), A_1 *and* A_2 *the bounded non-negative weak solution of the equation* (1.1) *is Hölder continuous with some positive exponent* α *in any compact subdomain* Ω' *of* Ω. *And the Hölder norm and exponent depend only on the bound u, the distance from* Ω' *to* $\partial\Omega$, n, C_0, C_1, μ, ν *and* τ.

To state the boundary estimates we assume the following conditions.

A_3. The boundary $\partial\Omega$ satisfies

$$\text{mes}\,(K_\rho-\Omega\cap K_\rho)\geqq\theta_0\,\text{mes}\,K_\rho$$

for any ball K_ρ of radius ρ ($\leqq a_0$) with a center on $\partial\Omega$, where a_0 and θ_0 are some positive constants.

A_4. The boundary value of u belongs to $C^{\beta}(\partial\Omega)$ for some β $(0<\beta<1)$.
Then we have the following.

Theorem 2. *Under the conditions of* (1.2), A_1, A_2, A_3 *and* A_4 *the bounded non-negative weak solution of the equation* (1.1) *is Hölder continuous with some positive exponent* α *on* $\bar{\Omega}$. *And, the Hölder norm and exponent depend only on the bound u, n,* C_0, C_1, μ, ν, τ, β, a_0, θ_0 *and the Hölder norm of the boundary data of u.*

The existence of weak solutions of such type has been given by the authors [3].
If we replace $u^{\tau/2+1}$ by V, the equations (1.1) and (1.3) become as follows;

$$(1.1)' \qquad \tilde{L}V\equiv\frac{\partial}{\partial x_i}\{\tilde{a}_{ij}(x, V)V_{x_j}\}-\tilde{B}_j(x, V)V^{\tilde{\nu}}V_{x_j}-\tilde{B}_0(x, V)=0$$

with

$$C_0V^{\sigma}|\xi|^2\leqq\tilde{a}_{ij}(x, V)\xi_i\xi_j\leqq C_0^{-1}V^{\sigma}|\xi|^2, \ \sigma=\tau/(\tau+2).$$

$$(1.3)' \qquad \int_{\Omega}[\tilde{a}_{ij}(x, V)V_{x_j}\phi_{x_i}+\{\tilde{B}_j(x, V)V^{\tilde{\nu}}V_{x_j}+\tilde{B}_0(x, V)\}\phi]dx=0$$

for any function $\phi\in\mathring{W}_2^1(\Omega)$.
Here the coefficients satisfy the following conditions;

A_1'. $\qquad \tilde{a}_{ij}(x, V)=\tilde{a}_{ji}(x, V) \qquad (i,j=1, \cdots, n)$

A_2'. $\qquad |\tilde{B}_0(x, V)|\leqq C_1V^{\tilde{\mu}}$

for some positive constant C_1. Moreover constants $\tilde{\mu}$ and $\tilde{\nu}$ satisfy one of (3) and (4).

(3) $\tilde{\mu}\geqq 0$ and $\tilde{\nu}\geqq\sigma/2$

(4) $\tilde{\mu}>0$ and $\tilde{\nu}>0$.

The new problem may be more natural than the original one in the definition of weak solution.

§ 2. Auxiliary lemmas.

In this section we present some lemmas to obtain the Hölder estimates of the functions (cf. [2], [3], [5]).

For the function $\phi(x)$, $A_{k,\rho}$ and $B_{k,\rho}$ denote the sets

$$\{x\in K_{\rho}\cap\Omega;\ \phi(x)>k\} \quad \text{and} \quad \{x\in K_{\rho}\cap\Omega;\ \phi(x)<k\}$$

respectively, where K_{ρ} is a ball of radius ρ in R^n, and $\max\phi$ and $\min\phi$ denote essential max ϕ and essential min ϕ, respectively. Furthermore $\operatorname{osc}(\phi, K_{\rho})$ denote $\max\phi-\min\phi$ on K_{ρ}.

Lemma 1. *We consider the function* $\phi(x)$ *in* $W_2^1(\Omega)$ *and assume that*

$$0 \leqq \phi(x) \leqq M, \tag{2.1}$$

$$\int_{A_{k,\rho}-A_{h,\rho}} |\nabla\phi|^2\zeta^2 dx \leqq C\int_{A_{k,\rho}} \{(\phi-k)^2|\nabla\zeta|^2+\zeta^2\}dx \tag{2.2}$$

hold for any ball $K_\rho \subset \Omega$, *for any function* ζ *in* $C_0^\infty(K_\rho)$ *and for any* k, h *such that*

$$\max_{K_\rho}\phi-\delta\operatorname{ocs}(\phi, K_\rho) \leqq k < h \leqq \tfrac{1}{2}(k+\max_{K_\rho}\phi), \tag{2.3}$$

where the constant $\delta \in (0, 1)$.

Moreover assume that

$$\operatorname{mes}\{x \in K_{\rho/2};\ \phi(x) \leqq \max_{K_\rho}\phi-\delta\operatorname{ocs}(\phi, K_\rho)\} \geqq \gamma \operatorname{mes} K_{\rho/2}, \tag{2.4}$$

where a constant $\gamma \in (0, 1)$.

Then for some positive constant s which depend only on M, δ *and* γ *we have the estimates*

$$\operatorname{ocs}(\phi, K_{\rho/2}) \leqq 2^s\rho$$

or

$$\operatorname{osc}(\phi, K_{\rho/4}) \leqq (1-2^{1-s})\operatorname{osc}(\phi, K_\rho),$$

where K_ρ, $K_{\rho/2}$ *and* $K_{\rho/4}$ *are concentric balls.*

Lemma 2. *We consider the function* $\phi(x)$ *in* $W_2^1(\Omega)$ *and assume that*

$$0 \leqq \phi(x) \leqq M, \tag{2.5}$$

$$\int_{B_{k,\rho}-B_{h,\rho}} |\nabla\phi|^2\zeta^2 dx \leqq C\int_{B_{k,\rho}} \{(k-\phi)^2|\nabla\zeta|^2+\zeta^2\}dx \tag{2.6}$$

hold for any ball $K_\rho \subset \Omega$ *and for any function* ζ *in* $C_0^\infty(K_\rho)$ *and for any number* k, h *such that*

$$\tfrac{1}{2}(k+\min_{K_\rho}\phi) \leqq h < k \leqq \min_{K_\rho}\phi+\delta\operatorname{osc}(\phi, K_\rho), \tag{2.7}$$

where the constant $\delta \in (0, 1)$. *Moreover assume that*

$$\operatorname{mes}\{x \in K_{\rho/2};\ \phi(x) \geqq \min_{K_\rho}\phi+\delta\operatorname{osc}(\phi, K_\rho)\} \geqq \gamma \operatorname{mes} K_{\rho/2}, \tag{2.8}$$

where the constant $\gamma \in (0, 1)$.

Then the assertion of Lemma 1 *holds*

Remark. Let us assume that K_ρ is a ball of radius ρ $(\leqq a_0)$ with center on $\partial\Omega$. The above lemmas remain valid for this case if we assume

$$\max\{\phi;\ \partial\Omega\cap K_\rho\}\leqq\max\{\phi;\ \Omega\cap K_\rho\}-\delta\operatorname{osc}(\phi,K_\rho\cap\Omega)$$

and

$$\min\{\phi;\ \partial\Omega\cap K_\rho\}\geqq\min\{\phi;\ \Omega\cap K_\rho\}+\delta\operatorname{osc}(\phi,K_\rho\cap\Omega)$$

instead of conditions (2.4) and (2.8), respectively.

The above lemmas are essentially equivalent to Lemma 6.1, 6.2, 7.1 of Chapter 2 in [3] and those of [6].

§ 3. Proof of the theorems.

First we shall prove Theorem 1.

Proof in the case (1). Let u be a bounded non-negative weak solution of the equation (1.1) and we put $V(x)=u^{\tau/2+1}$ and $M=\max_\Omega V$.

For any positive constant $K(\leqq M)$, for any non-negative function $\zeta\in C_0^\infty(K_\rho)$ and for any ball $K_\rho\subset\Omega$, set

$$\phi(x)=\max\{K-V(x),0\}\zeta^2(x),$$

which belongs to $\overset{\circ}{W}{}_2^1(\Omega)$.

Inserting this function into (1.3) we have

$$\begin{aligned}(3.1)\qquad &\int_{K>V}\Big[\frac{2}{\tau+2}a_{ij}(x,u)u^{-\tau/2}V_{x_i}\{-V_{x_j}\zeta^2+2(K-V)\zeta_{x_j}\zeta\}\\ &\quad+\Big\{\frac{2}{\tau+2}B_j(x,u)u^{\nu-\tau/2}V_{x_j}+B_0(x,u)\Big\}(K-V)\zeta^2\Big]dx=0.\end{aligned}$$

With the aid of the conditions (1.2), A_1, A_2 and $0\leqq u\leqq M^{2/(\tau+2)}$, it follows from (3.1) that

$$\begin{aligned}(3.2)\qquad &\int_{K>V}u^{\tau/2}|\nabla V|^2\zeta^2dx\\ &\leqq C_2\int_{K>V}u^{\tau/2}(K-V)|\nabla V||\nabla\zeta|\zeta+(u^{\tau/4}|\nabla V|+u^{\mu})(K-V)\zeta^2\}dx.\end{aligned}$$

Using Young's inequality we have

$$\begin{aligned}(3.3)\qquad &\int_{K>V}u^{\tau/2}|\nabla V|^2\zeta^2dx\\ &\leqq C_3\int_{K>V}[u^{\tau/2}(K-V)^2|\nabla\zeta|^2+\{(K-V)^2+(K-V)\}\zeta^2]dx\end{aligned}$$

$$\leqq C_4 K^{\tau/(\tau+2)} \int_{K>V} \{(K-V)^2|\nabla\zeta|^2+\zeta^2\}dx.$$

Here and hereafter we denote by C_i $(i\geqq 2)$ the constant which depends only on τ, ν, μ, M, C_0, C_1 and the distance from Ω' to $\partial\Omega$.

For any k, h such that

$$\tfrac{1}{2}(k+\min_{K_\rho} V)\leqq h<k\leqq \min_{K_\rho} V+\delta \operatorname{osc}(V, K_\rho), \tag{3.4}$$

we obtain from (3.3)

$$\begin{aligned}\int_{B_{k,\rho}-B_{h,\rho}} u^{\tau/2}|\nabla V|^2\zeta^2 dx &\leqq \int_{B_{k,\rho}} u^{\tau/2}|\nabla V|^2\zeta^2 dx \\ &\leqq C_5 k^{\tau/(\tau+2)} \int_{B_{k,\rho}} \{(k-V)^2|\nabla\zeta|^2+\zeta^2\}dx,\end{aligned} \tag{3.5}$$

where $B_{k,\rho}=\{x\in K_\rho;\ V(x)<k\}$.

In $B_{k,\rho}-B_{h,\rho}$, $h\leqq V(x)<k$ and $h^{\tau/(\tau+2)}\leqq u^{\tau/2}<k^{\tau/(\tau+2)}$.

Moreover $k\leqq 2h$ from (3.4).

Considering this, from (3.5) we have

$$\int_{B_{k,\rho}-B_{h,\rho}} |\nabla V|^2\zeta^2 dx\leqq C_6 \int_{B_{k,\rho}} \{(k-V)^2|\nabla\zeta|^2+\zeta^2\}dx.$$

This implies that V satisfies the conditions of Lemma 2 except (2.8).

Next for non-negative constant $K(\leqq M)$ set

$$\phi(x)=\max\{V(x)-K, 0\}\zeta^2(x),$$

which belongs to $\mathring{W}_2^1(\Omega)$.

Inserting this function into (1.3), we obtain

$$\begin{aligned}\int_{K<V} \Big[\frac{2}{\tau+2} a_{ij}(x, u)u^{-\tau/2}V_{x_i}\{V_{x_j}\zeta^2+2(V-K)\zeta\zeta_{x_j}\} \\ +\Big\{\frac{2}{\tau+2} B_j(x, u)u^{\nu-\tau/2}V_{x_j}+B_0(x, u)\Big\}(V-K)\zeta^2\Big]dx=0.\end{aligned}$$

By the use of the conditions (1.2), A_1 and A_2, we have

$$\int_{K<V} u^{\tau/2}|\nabla V|^2\zeta^2 dx\leqq C_7 (\max_{K_\rho} V)^{\tau/(\tau+2)} \int_{K<V} \{(V-K)^2|\nabla\zeta|^2+\zeta^2\}dx \tag{3.6}$$

Now, we set $A_{k,\rho}=\{x\in K_\rho;\ V(x)>k\}$. For any k, h such that

$$\max_{K_\rho} V-\delta \operatorname{osc}(V, K_\rho)\leqq k<h\leqq \tfrac{1}{2}(k+\max_{K_\rho} V), \tag{3.7}$$

we obtain

$$\int_{A_{k,\rho}-A_{h,\rho}} |\nabla V|^2\zeta^2 dx \leqq C_8 \int_{A_{k,\rho}} \{(V-k)^2|\nabla\zeta|^2+\zeta^2\}dx. \tag{3.8}$$

This is derived as follows.

It follows from (3.7) and $V\geqq 0$ that

$$k\geqq(1-\delta)\max_{K_\rho} V>0. \tag{3.9}$$

Applying (3.9) to (3.6), we have

$$\begin{aligned}(1-\delta)^{\tau/(\tau+2)}(\max_{K_\rho} V)^{\tau/(\tau+2)} &\int_{k<V} |\nabla V|^2\zeta^2 dx \\ \leqq &\int_{k<V} u^{\tau/2}|\nabla V|^2\zeta^2 dx \\ \leqq C_7(\max_{K_\rho} V)^{\tau/(\tau+2)} &\int_{k<V} \{(V-k)^2|\nabla\zeta|^2+\zeta^2\}dx.\end{aligned} \tag{3.10}$$

From (3.10) we get (3.8).

Consequently, from (3.8) we see that V satisfies the conditions of Lemma 1 except (2.4). Since one of the conditions (2.4), (2.8) always holds, we can apply Theorem 6.1 of Chapter 2 in [5] and Lemmas 1, 2 to obtain desired interior Hölder estimate of V. This completes the proof in the case (1).

Proof in the case (2). Let V, M and $\zeta(x)$ be as above.

Set $\phi(x)=\max\{\min(K-u, K-\varepsilon), 0\}\zeta^2(x)$, which belongs to $\mathring{W}_2^1(\Omega)$, where $0<\varepsilon<K\leqq M^{2/(\tau+2)}$.

Insterting this function into (1.3) we have

$$\begin{aligned}&\int_{K>u>\varepsilon} \frac{2}{\tau+2} a_{ij}u^{-\tau/2}V_{x_i}\{-u_{x_j}\zeta^2+2(K-u)\zeta\zeta_{x_j}\}dx \\ &+\int_{K>u>\varepsilon}\left\{\frac{2}{\tau+2}B_j u^{\nu-\tau/2}V_{x_j}+B_0\right\}(K-u)\zeta^2 dx \\ &+\int_{u\leqq\varepsilon} \frac{2}{\tau+2} a_{ij}u^{-\tau/2}V_{x_i}\cdot 2(K-\varepsilon)\zeta\zeta_{x_j}dx \\ &+\int_{u\leqq\varepsilon}\left\{\frac{2}{\tau+2}B_j u^{\nu-\tau/2}V_{x_j}+B_0\right\}(K-\varepsilon)\zeta^2 dx=0.\end{aligned} \tag{3.11}$$

Applying the conditions (1.2), A_1 and A_2 to (3.11), we obtain

$$\begin{aligned}\int_{K>u>\varepsilon}|\nabla V|^2\zeta^2 dx \leqq C_9\Big[&\int_{K>u>\varepsilon}\{u^{\tau/2}|\nabla V|(K-u)\zeta|\nabla\zeta|+u^{\mu'}(|\nabla V|+1)(K-u)\zeta^2\}dx \\ &+\int_{u\leqq\varepsilon}\{u^{\tau/2}|\nabla V|(K-\varepsilon)\zeta|\nabla\zeta|+u^{\mu'}(|\nabla V|+1)(K-\varepsilon)\zeta^2\}dx\Big],\end{aligned}$$

where $\mu'=\min(\nu-\tau/2, \mu)$.

By the use of Hölder inequality we see easily

$$
\begin{aligned}
&\int_{K>u>\varepsilon} |\nabla V|^2\zeta^2 dx \\
&\leqq C_{10}\Big[\int_{K>u>\varepsilon} \{u^{\tau}(K-u)^2|\nabla\zeta|^2 + u^{2\mu'}(K-u)\zeta^2 + u^{\mu'}(K-u)\zeta^2\} dx \\
(3.12)\qquad &+\Big\{\int_{u\leqq\varepsilon} |\nabla V|^2\zeta^2 dx \cdot \int_{u\leqq\varepsilon} u^{\tau}(K-\varepsilon)^2|\nabla\zeta|^2 dx\Big\}^{1/2} \\
&+\Big\{\int_{u\leqq\varepsilon} (|\nabla V|^2+1)\zeta^2 dx \cdot \int_{u\leqq\varepsilon} u^{2\mu'}(K-\varepsilon)^2\zeta^2 dx\Big\}^{1/2}\Big].
\end{aligned}
$$

On the other hand, for $\alpha>0$ and for $K>u\geqq 0$ the following inequalities are valid;

$$
\begin{aligned}
K^{\alpha+1}-u^{\alpha+1} &= \int_0^1 \frac{d}{dt}\{tK+(1-t)u\}^{\alpha+1} dt \\
&= (\alpha+1)(K-u)\int_0^1 \{tK+(1-t)u\}^{\alpha} dt \\
(3.13)\qquad &\geqq (\alpha+1)(K-u)\int_{1/2}^1 \Big\{\frac{K+u}{2}\Big\}^{\alpha} dt \\
&= \frac{\alpha+1}{2^{\alpha+1}}(K-u)(K+u)^{\alpha} \geqq \frac{\alpha+1}{2^{\alpha+1}}(K-u)u^{\alpha}.
\end{aligned}
$$

Applying (3.13) to (3.12), we have

$$
\begin{aligned}
(3.14)\qquad \int_{K>u>\varepsilon} |\nabla V|^2\zeta^2 dx \leqq C_{11}\int_{K>u>\varepsilon} &\{(K^{\tau/2+1}-u^{\tau/2+1})^2|\nabla\zeta|^2 + u^{\mu'}K\zeta^2\} dx \\
&+ o(\varepsilon^{\tau/2}) + o(\varepsilon^{\mu'})
\end{aligned}
$$

for any k, h such that, $k=K^{\tau/2+1}$,

$$
(3.15)\qquad \tfrac{1}{2}(k+\min_{K_\rho} V) \leqq h < k \leqq \min_{K_\rho} V + \delta \operatorname{osc}(V, K_\rho).
$$

Furthermore $h>0$ from (3.15). Hence for any positive number $\varepsilon^{\tau/2+1}(<h)$, we obtain from (3.14)

$$
\int_{k>V>h} |\nabla V|^2\zeta^2 dx \leqq C_{12}\int_{k>V} \{(k-V)^2|\nabla\zeta|^2+\zeta^2\} dx + o(\varepsilon^{\tau/2}) + o(\varepsilon^{\mu'}).
$$

Let ε tend to zero, then we have

$$
\int_{k>V>h} |\nabla V|^2\zeta^2 dx \leqq C_{12}\int_{k>V} \{(k-V)^2|\nabla\zeta|^2+\zeta^2\} dx
$$

This implies that V satisfies the conditions of Lemma 2.

The rest of the proof can be obtained similarly to the proof in the case (1). Hence we shall omit the proof.

Proof of Theorem 2. By the use of Theorem 7.1 of Chapter 2 in [5] and Remark in § 3, the boundary estimate of V follows in the same way.

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