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Eigenvalues of the Indefinite-Weight p-Laplacian in Weighted Spaces

By

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1. Introduction and results

In this paper we continue to study the following nonlinear eigenvalue problem in $\boldsymbol{R}^N$

$$-\Delta_p u = \lambda g(x)|u|^{p-2}u\,, \tag{1}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian with $p > 1$, $\lambda \in \boldsymbol{R}$, u in some weighted space V to be specified later, and $g \in L^\infty(\boldsymbol{R}^N)$ is an indefinite weight function, i.e. $g^\pm = \max(\pm g, 0) \not\equiv 0$. Here we consider only weak solutions, i.e. (λ, u) is a (nontrivial) solution of (1) in V if $\lambda \in \boldsymbol{R}$, $u \in V\backslash\{0\}$ and

$$\int |\nabla u|^{p-2}\nabla u\nabla\varphi = \lambda \int g(x)|u|^{p-2}u\varphi$$

for all $\varphi \in C_0^\infty(\boldsymbol{R}^N)$. Here and henceforth the integrals are taken over $\boldsymbol{R}^N$ unless otherwise specified.

Eigenvalue problem (1) has been studied by Brown, Cosner and Fleckinger [BCF] and Allegretto [A] for the case $p = 2$ and Huang [H] for $p \neq 2$. Specifically, existence and nonexistence results are obtained for both $p = 2$ ([BCF]) and $p \neq 2$ ([H]), depending on the condition

(H) there exist $K > 0$ and $R' > 0$ such that $g(x) \leq -K$ for $|x| \geq R'$.

In [A], condition (H) is weakened for the case $p = 2$, by using weighted Sobolev spaces and eigencurve arguments. More background material and details can be found in [A], [BCF] and [H]. It is our purpose here to generalize the results in [A] to the case $p \neq 2$. While the form of the results is the same, due to the nonlinear, nonself-adjoint nature of the principal operator Δ_p, the approach of [A] must be modified and we employ the Ljusternik-Schnirelman theory—as in [H].

We write $g = g_1 - g_2$ with $g_1 \geq 0$, $g_2 \geq 0$, and prove the following theorems.

Theorem 1. *Let $1 < p < N$. Assume that $g_1 \in L^\infty(\boldsymbol{R}^N) \cap L^{N/p}(\boldsymbol{R}^N)$, $g_1 \not\equiv 0$. Then* (i) (1) *has a sequence of solutions (λ_k, u_k) with $\int g|u_k|^p = 1$, $0 < \lambda_1 < \lambda_2 \le \cdots \le \lambda_k \to \infty$ as $k \to \infty$.* (ii) *The first eigenfunction u_1 can be taken positive in $\boldsymbol{R}^N$. Moreover, $\lambda_1 > 0$ is simple and unique.*

We note that in Theorem 1, $g_2 \equiv 0$ is allowable.

Corollary 1. *Assume that $g \in L^\infty(\boldsymbol{R}^N) \cap L^{N/p}(\boldsymbol{R}^N)$ and $g^\pm \not\equiv 0$. Then there exist two sequences of solutions $(\lambda_k^\pm, u_k^\pm)$ of* (1) *such that*

$$0 < \pm\lambda_1^\pm < \pm\lambda_2^\pm \le \cdots \le \pm\lambda_k^\pm \to \infty$$

as $k \to \infty$, $\int g|u_k^\pm|^p = \pm 1$ and $u_1^\pm > 0$ in $\boldsymbol{R}^N$.

Following [BM], we introduce, for any weight function $\rho \in L^\infty(\boldsymbol{R}^N)$,

$$M_{\alpha,1}(\rho) = \sup_{x \in \boldsymbol{R}^N} \int_{|y|<1} |\rho(x-y)| \cdot |y|^{\alpha-N},$$

and

$$M_{\alpha,1}(\rho, \Omega) = M_{\alpha,1}(\rho|_\Omega),$$

with $\rho|_\Omega(x) = \rho(x)$ if $x \in \Omega$ and $\rho|_\Omega(x) = 0$ if $x \notin \Omega$. Let $B_R = \{x \in \boldsymbol{R}^N : |x| < R\}$. We have

Theorem 2. *Suppose $1 < p < N$. Assume $g = g_1 - g_2$ satisfies*

(H1) *$g_1 > 0$, $g_2 \ge \varepsilon > 0$ and $M_{\alpha,1}(g_1, \boldsymbol{R}^N \backslash B_R) \to 0$ as $R \to \infty$ for some $\alpha \in (0, p)$.*

Then the conclusion of Theorem 1 holds.

In virtue of the nonexistence results of [BCF] and [H], we must assume $g_2 \ge \varepsilon > 0$ in order to obtain any existence result for the case $p \ge N$.

Theorem 3. *Suppose $p \ge N$. Assume that $g = g_1 - g_2$ satisfies*

(H2) *$g_1 \not\equiv 0$, $g_2 \ge \varepsilon > 0$ and $g_1 \in L^\infty(\boldsymbol{R}^N) \cap L^{N_0/p}(\boldsymbol{R}^N)$, where the integer $N_0 > p$.*

Then the conclusion of Theorem 1 remains valid.

Finally we have

Corollary 2. *Suppose $1 < p < N$. Then the conclusion of Theorem 1 holds if $g_1 \in L^\infty(\boldsymbol{R}^N) \cap L^{N_0/p}(\boldsymbol{R}^N)$, with the integer $N_0 \ge N$.*

2. Proofs

We present proofs of our theorems in this section. We will introduce certain functionals in suitable weighted spaces and verify that the functionals are well defined and satisfy the Palais-Smale condition on some level set. We then proceed by using the Ljusternik-Schnirelman theory to obtain our theorems.

To study the case $p < N$, we introduce the weight function $\omega(x) = (1 + |x|)^{-p}$ for $x \in \boldsymbol{R}^N$ and let V be the completion of $C_0^\infty(\boldsymbol{R}^N)$ with respect to the norm

$$\|u\|_V^p := \int |\nabla u|^p + \int \max(g_2, \omega)|u|^p . \tag{2}$$

Define a functional I on V by

$$I(u) = \frac{1}{p}\int |\nabla u|^p , \tag{3}$$

and denote

$$G = \left\{u \in V: p\Psi(u) := \int g|u|^p = 1\right\} .$$

Obviously the functional I is well defined on V and is bounded below on G. We note that the critical points of I on G are solutions of (1) for certain (Lagrange multiplier) λ. To find critical points of I we employ the Ljusternik-Schnirelman theory.

Lemma 1. *Assume $g_1 \in L^\infty(\boldsymbol{R}^N) \cap L^{N/p}(\boldsymbol{R}^N)$. Then the functional I satisfies the Palais-Smale condition on G, i.e., for $\{u_n\} \subset G$, if $I(u_n)$ is bounded and*

$$I'(u_n) - a_n\Psi'(u_n) \to 0 , \qquad \text{where } a_n = \frac{\langle I'(u_n), u_n\rangle}{\langle \Psi'(u_n), u_n\rangle} , \tag{4}$$

then $\{u_n\}$ has a convergent subsequence in V.

Proof. Let $\{u_n\} \subset G$ be such a sequence. By Hölder's inequality and the Sobolev embedding theorem, we have

$$\int g_1|u_n|^p \le c \cdot \|g_1\|_{N/p} \cdot \|\nabla u_n\|_p^p .$$

Boundedness of $\{I(u_n)\}$ then implies that $\left\{\int g_1|u_n|^p\right\}$ is bounded. Consequently we see $\left\{\int g_2|u_n|^p = \int g_1|u_n|^p - 1\right\}$ is bounded. Noting Hardy's inequality

$$\int \frac{|u|^p}{(1+|x|)^p} \leq \left(\frac{p}{N-p}\right)^p \cdot \int |\nabla u|^p ,$$

we conclude that $\{u_n\}$ is bounded in V. Hence without loss of generality, we may assume, for some $u_0 \in V$, $u_n \to u_0$ weakly in V, and on any bounded domain Ω, $\int_\Omega g|u_0|^p = \lim_{n\to\infty} \int_\Omega g|u_n|^p$. We claim $u_0 \not\equiv 0$. Indeed, if $u_0 \equiv 0$, then on any bounded domain Ω, we have $\int_\Omega g_1 |u_n|^p \to 0$. Let $\tilde{\Omega}$ be a bounded domain so that for all n large,

$$c \cdot \|g_1\|_{N/p,\, \mathbf{R}^N \setminus \tilde{\Omega}} \cdot \|\nabla u_n\|_p^p < \frac{1}{4} ,$$

where c is given in the above inequality. Now we can choose n sufficiently large so that

$$\begin{aligned} \int g_1 |u_n|^p &= \int_{\tilde{\Omega}} g_1 |u_n|^p + \int_{\mathbf{R}^N \setminus \tilde{\Omega}} g_1 |u_n|^p \\ &\leq \int_{\tilde{\Omega}} g_1 |u_n|^p + c \cdot \|g_1\|_{N/p,\, \mathbf{R}^N \setminus \tilde{\Omega}} \cdot \|\nabla u_n\|_p^p < \frac{1}{2} . \end{aligned}$$

We thus derive that $\int g_2 |u_n|^p = \int g_1 |u_n|^p - 1 < -1/2$, a contradiction. Thus $u_0 \not\equiv 0$.

It follows from (4) that for any $\varphi \in C_0^\infty(\mathbf{R}^N)$,

$$(5)_n \qquad \int |\nabla u_n|^{p-2} \nabla u_n \nabla \varphi = a_n \int g |u_n|^{p-2} u_n \varphi + o(1) .$$

Taking $\varphi = u_n - u_m$ in $(5)_n - (5)_m$ (via diagonal arguments if necessary) we obtain

$$\begin{aligned} (6) \qquad & \int (|\nabla u_n|^{p-2} \nabla u_n - |\nabla u_m|^{p-2} \nabla u_m) \nabla (u_n - u_m) \\ & \leq \int g(a_n |u_n|^{p-2} u_n - a_m |u_m|^{p-2} u_m)(u_n - u_m) + o(1) \\ & = \int_\Omega g a_n (|u_n|^{p-2} u_n - |u_m|^{p-2} u_m)(u_n - u_m) \\ & \quad + \int_{\mathbf{R}^N \setminus \Omega} g a_n (|u_n|^{p-2} u_n - |u_m|^{p-2} u_m)(u_n - u_m) \\ & \quad + (a_n - a_m) \int g |u_m|^{p-2} u_m (u_n - u_m) + o(1) , \end{aligned}$$

where Ω is an arbitrary bounded domain. Note here that $a_n = \int |\nabla u_n|^p$, thus is bounded. Observe that, by Hölder's inequality and monotonicity of the function $|t|^{p-2}t$,

$$\begin{aligned}
&\int_{\boldsymbol{R}^N \setminus \Omega} g a_n (|u_n|^{p-2}u_n - |u_m|^{p-2}u_m)(u_n - u_m) \\
&\leq \int_{\boldsymbol{R}^N \setminus \Omega} g_1 a_n (|u_n|^{p-2}u_n - |u_m|^{p-2}u_m)(u_n - u_m) \\
&\leq c \cdot a_n \|g_1\|_{N/p,\, \boldsymbol{R}^N \setminus \Omega} (\|\nabla u_n\|_p^p + \|\nabla u_m\|_p^p)\,,
\end{aligned}$$

which approaches 0 as $\Omega \to \boldsymbol{R}^N$, independent of n and m. While for any given Ω,

$$\int_{\Omega} g a_n (|u_n|^{p-2}u_n - |u_m|^{p-2}u_m)(u_n - u_m) \to 0$$

as (a subsequence of) n, $m \to \infty$, since (a subsequence of) u_n converges to u_0 in $L^p(\Omega)$. Furthermore, Hölder's inequality implies that the integral $\int g |u_m|^{p-2}u_m(u_n - u_m)$ is bounded, and we can choose a subsequence of a_n so that $a_n - a_m \to 0$ as n, $m \to \infty$. Thus we conclude that the right hand side of (6) approaches 0 as n, $m \to \infty$. On the other hand, observe that for any $a, b \in \boldsymbol{R}^N$,

$$|a - b|^p \leq c \cdot \{(|a|^{p-2}a - |b|^{p-2}b)\cdot(a - b)\}^{s/2} \cdot (|a|^p + |b|^p)^{1-s/2}\,, \tag{7}$$

where $s = p$ if $p \in (1, 2)$ and $s = 2$ if $p \geq 2$. We now have

$$\begin{aligned}
|\nabla u_n - \nabla u_m|^p \leq c \cdot \{&(|\nabla u_n|^{p-2}\nabla u_n - |\nabla u_m|^{p-2}\nabla u_m)\nabla(u_n - u_m)\}^{s/2} \\
&\cdot (|\nabla u_n|^p + |\nabla u_m|^p)^{1-s/2}\,.
\end{aligned}$$

By applying Hölder's inequality we obtain

$$\begin{aligned}
\int |\nabla u_n - \nabla u_m|^p \leq c_1 \cdot &\left\{\int (|\nabla u_n|^{p-2}\nabla u_n - |\nabla u_m|^{p-2}\nabla u_m)\nabla(u_n - u_m)\right\}^{s/2} \\
&\cdot \left(\int |\nabla u_n|^p + \int |\nabla u_m|^p\right)^{1-s/2}\,.
\end{aligned}$$

We then derive from the above inequalities that $\nabla u_n \to \nabla u_0$ in $L^p(\boldsymbol{R}^N)$. It then follows that $\int g_i |u_n|^p \to \int g_i |u_0|^p$, $i = 1, 2$, and this together with Hardy's inequality implies that $u_n \to u_0$ in V. The lemma is thus proved. □

Proof of Theorem 1. Proof of Theorem 1 follows directly from application of the Ljusternik-Schnireman theory as in [H]. □

To prove Theorem 2, we define the same functional I on $W^{1,p}(\boldsymbol{R}^N)$ and

$$G = \left\{ u \in W^{1,p}(\boldsymbol{R}^N) : \int g|u|^p = 1 \right\}.$$

In view of proofs of Lemma 1 and Theorem 1, we need only to prove

Lemma 2. *Assume* (H1) *holds. If* $\{u_n\} \subset G$ *and* $\{I(u_n)\}$ *is bounded, then* $\{u_n\}$ *is bounded in* $W^{1,p}(\boldsymbol{R}^N)$.

Proof. By Theorem 2.7 of [BM], for any domain Ω,

$$\int_\Omega g_1|u|^p \le CM_{\alpha,1}(g_1, \Omega)\left(\int_\Omega |\nabla u|^p + |u|^p\right), \qquad u \in W_0^{1,p}(\Omega), \tag{8}$$

where C is a constant independent of Ω. Since $M_{\alpha,1}(g_1, \boldsymbol{R}^N \setminus B_R) \to 0$ as $R \to \infty$, we can find an $\tilde{R}$ so that

$$2^p(2^p + 1)CM_{\alpha,1}(g_1, \boldsymbol{R}^N \setminus B_{\tilde{R}-1}) < \varepsilon/2,$$

where C is given in (8) and ε is given in (H1). We select a function φ as follows:

$$\varphi(x) = \begin{cases} 1, & \text{if } |x| > \tilde{R}; \\ \text{continuous}, & \text{if } \tilde{R} - 1 \le |x| \le \tilde{R}; \\ 0, & \text{if } |x| < \tilde{R} - 1, \end{cases}$$

with $1 \ge \varphi \ge 0$ and $|\nabla\varphi| \le 2$. Then (8) is applicable to φu for $u \in W^{1,p}(\boldsymbol{R}^N)$ and we estimate

$$\begin{aligned} &\int_{\boldsymbol{R}^N \setminus B_{\tilde{R}}} g_1|u|^p \\ &\le \int_{\boldsymbol{R}^N \setminus B_{\tilde{R}-1}} g_1|u|^p \\ &\le CM_{\alpha,1}(g_1, \boldsymbol{R}^N \setminus B_{\tilde{R}-1})\left(\int_{\boldsymbol{R}^N \setminus B_{\tilde{R}-1}} |\nabla(\varphi u)|^p + |\varphi u|^p\right) \\ &\le C2^pM_{\alpha,1}(g_1, \boldsymbol{R}^N \setminus B_{\tilde{R}-1})\left(\int_{\boldsymbol{R}^N \setminus B_{\tilde{R}-1}} \varphi^p|\nabla u|^p + |u|^p|\nabla\varphi|^p + |u|^p\right) \\ &\le 2^p(2^p + 1)CM_{\alpha,1}(g_1, \boldsymbol{R}^N \setminus B_{\tilde{R}-1})\left(\int_{\boldsymbol{R}^N} |\nabla u|^p + |u|^p\right). \end{aligned} \tag{9}$$

This together with (H1) implies that, for $u \in G$,

$$\varepsilon \int_{\boldsymbol{R}^N} |u|^p \le \int_{\boldsymbol{R}^N} g_2 |u|^p \le \int_{\boldsymbol{R}^N} g_1 |u|^p + 1 \tag{10}$$

$$\le 1 + \int_{B_{\tilde{R}}} g_1 |u|^p + \int_{\boldsymbol{R}^N \setminus B_{\tilde{R}}} g_1 |u|^p$$

$$\le 1 + \int_{B_{\tilde{R}}} g_1 |u|^p + \frac{\varepsilon}{2} \int_{\boldsymbol{R}^N} (|\nabla u|^p + |u|^p) .$$

We thus derive that, for $u \in G$,

$$\frac{\varepsilon}{2} \int_{\boldsymbol{R}^N} |u|^p \le 1 + \int_{B_{\tilde{R}}} g_1 |u|^p + \frac{\varepsilon}{2} \int_{\boldsymbol{R}^N} |\nabla u|^p . \tag{11}$$

The lemma then follows. □

Now we turn to the case $p \ge N$. In virtue of the nonexistence results of [BCF] and [H], we assume that $g^{\pm} \not\equiv 0$. We also assume that (H2) holds.

We will study the functional I on $G = \left\{ u \in W^{1,p}(\boldsymbol{R}^N) : \int g |u|^p = 1 \right\}$. We first introduce some notations. Fix $a > 0$ and let m be an integer. Set

$$A_m = \{ y = (y_1, \ldots, y_m) \in \boldsymbol{R}^m : y_i \in (-a, a), i = 1, \ldots, m \} .$$

Let ψ be the positive eigenfunction of the problem

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2} u , & \text{in } A_m \\ u = 0 , & \text{on } \partial A_m \end{cases}$$

with $\int_{A_m} \psi^p = 1$, and let $\lambda(a)$ be the corresponding (positive) eigenvalue. We prove the following embedding theorem, whose $p = 2$ version has been given in [A].

Lemma 3. *Let $f \in L^{\infty}(\boldsymbol{R}^N) \cap L^{N_0/p}(\boldsymbol{R}^N)$ for some integer $N_0 > \max(p, N)$. Then for any $\varphi \in C_0^{\infty}(\boldsymbol{R}^N)$,*

$$\int_{\boldsymbol{R}^N} |f| \cdot |\varphi|^p \le c_1 \cdot (2a)^{(N_0 - N)p/N_0} \cdot \|f\|_{N_0/p} \cdot \left\{ \int |\nabla \varphi|^p + \lambda(a) \int |\varphi|^p \right\} . \tag{12}$$

Proof. Denote $m = N_0 - N$. Set, for $x \in \boldsymbol{R}^N$ and $y \in \boldsymbol{R}^m$,

$$h(x, y) = f(x) \chi_{A_m}(y) , \qquad \tau(x, y) = \varphi(x) \psi(y) ,$$

where χ_{A_m} is the characteristic function of the set A_m and $\varphi \in C_0^{\infty}(\boldsymbol{R}^N)$. The standard Sobolev embedding theorem implies that

$$\int_{R^{N_0}} |h|\cdot|\tau|^p \le \left(\int_{R^{N_0}} |h|^{N_0/p}\right)^{p/N_0}\cdot\left(\int_{R^{N_0}} |\tau|^{N_0 p/(N_0-p)}\right)^{(N_0-p)/N_0}$$
$$\le c_0\left(\int_{R^{N_0}} |h|^{N_0/p}\right)^{p/N_0}\cdot\left(\int_{R^{N_0}} |\nabla\tau|^p\right).$$

A calculation then shows that

$$\left(\int_{R^N} |f|\cdot|\varphi|^p\right)\cdot\left(\int_{A_m}\psi^p\right) \le c_1\cdot\|f\|_{N_0/p}\cdot(2a)^{mp/N_0}$$
$$\cdot\left(\int_{R^N}|\nabla\varphi|^p\cdot\int_{A_m}\psi^p + \int_{R^N}|\varphi|^p\cdot\int_{A_m}|\nabla\psi|^p\right).$$

Hence we obtain

$$\int_{R^N}|f|\cdot|\varphi|^p \le c_1\cdot(2a)^{mp/N_0}\cdot\|f\|_{N_0/p}\cdot\left(\int|\nabla\varphi|^p + \lambda(a)\int|\varphi|^p\right).$$

The lemma is thus proved. □

Proof of Theorem 3. As before, to prove Theorem 3, it suffices to show that, under (H2), any sequence $\{u_n\}\subset G$ with $\{I(u_n)\}$ bounded is indeed bounded in $W^{1,p}(R^N)$. To this end, we need the following lemma, whose proof will follow.

Lemma 4.

$$\lambda(a) \le \frac{(N+p)\cdots(p+1)}{N!a^p}. \tag{13}$$

Now, for $u\in G$, by (H2) and Lemma 3, we have

$$\varepsilon\int_{R^N}|u|^p \le \int_{R^N} g_2|u|^p \le \int_{R^N} g_1|u|^p + 1$$
$$\le 1 + c_1\cdot(2a)^{(N_0-N)p/N_0}\cdot\|g_1\|_{N_0/p}\cdot\left(\int|\nabla u|^p + \lambda(a)\int|u|^p\right).$$

We then derive from Lemma 4 that

$$\varepsilon\int_{R^N}|u|^p \le 1 + c_1\cdot\|g_1\|_{N_0/p}\cdot\Big((2a)^{(N_0-N)p/N_0}\cdot\int|\nabla u|^p$$
$$+ (2a)^{(N_0-N)p/N_0}(N+p)\cdots(p+1)(N!)^{-1}a^{-p}\int|u|^p\Big).$$

One can choose a sufficiently large so that

$$c_1 \cdot \|g_1\|_{N_0/p} \cdot (2a)^{(N_0-N)p/N_0}(N+p)\cdots(p+1)(N!)^{-1}a^{-p} < \frac{\varepsilon}{2}.$$

Then boundedness of $\{u_n\}$ in $W^{1,p}(\boldsymbol{R}^N)$ follows from boundedness of $\{I(u_n)\}$ and the above inequalities. This completes the proof. □

Proof of Lemma 4. Take $\varphi = a - |x|$ if $|x| \le a$, and $\varphi = 0$ if $|x| > a$. Then

$$\lambda(a) \le \frac{\int |\nabla \varphi|^p}{\int |\varphi|^p}.$$

A calculation shows that

$$\int |\nabla \varphi|^p = \frac{a^N}{N} \frac{2\pi^{N/2}}{\Gamma(N/2)},$$

and

$$\int |\varphi|^p = \frac{2\pi^{N/2}}{\Gamma(N/2)} a^{p+N} \int_0^1 (1-s)^p s^{N-1} ds,$$

where $\Gamma(\cdot)$ is the Γ-function. It is known that

$$\int_0^1 (1-s)^p s^{N-1} ds = \frac{\Gamma(N)\Gamma(p+1)}{\Gamma(N+p+1)} = \frac{(N-1)!}{(N+p)\cdots(p+1)},$$

thus we obtain

$$\lambda(a) \le \frac{(N+p)\cdots(p+1)}{N!a^p}$$

The lemma is thus proved. □

Proof of Corollary 2. It is immediate from Lemma 3 and 4. □

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