Kobe University Repository : Kernel

PDF issue: 2026-11-23

Optimal pricing timing for an e-commerce platform selling a private label on its own marketplace

Matsui, Kenji

(Citation)
European Journal of Operational Research,331(3):1005-1021

(Issue Date)
2026-06-16

(Resource Type)
journal article

(Version)
Version of Record

(Rights)
© 2025 The Author(s). Published by Elsevier B.V.
This is an open access article under the Creative Commons Attribution 4.0 International license

(URL)
https://hdl.handle.net/20.500.14094/0100502587

KOBE UNIVERSITY

European Journal of Operational Research 331 (2026) 1005–1021

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Optimal pricing timing for an e-commerce platform selling a private label on its own marketplace

Kenji Matsui

Graduate School of Business Administration, Kobe University, 2-1, Rokkodaicho, Nada-ku, Kobe, Japan

ARTICLE INFO

Keywords:
Supply chain management
Pricing timing
E-commerce platform
Private label
Game theory

ABSTRACT

In this paper, we examine the optimal timing for an e-commerce platform, which produces and sells a private label on its own marketplace, to set the retail price, utilizing the timing game framework of noncooperative game theory. Conventionally, game-theoretic supply chain models describing such a platform assume that simultaneous price competition occurs at the level of the retail market, where the platform and a third-party supplier determine their retail prices at the same point of time. Consistent with this conventional assumption, we first show that under a wholesale contract (i.e., reselling mode), the platform's profit is unaffected by the timing of price setting; thus, the platform may decide and announce the retail price of its private label at any point of time, including making a simultaneous price decision with the supplier. Meanwhile, a more notable result from our model is that under an agency contract (i.e., marketplace mode), such simultaneous price competition between the platform and the supplier never occurs. This suggests that the conventional assumption of simultaneous retail price competition must be reconsidered. Our model yields the useful managerial decision guideline that a platform concluding an agency contract should set the retail price of its private label earlier than or simultaneously with, but not later than, setting the marketplace fee or commission rate charged to a supplier. The result also means that the platform must determine the retail price of the private label prior to the supplier determining the retail price of its product.

1. Introduction

Over the last decade, platform businesses have experienced unprecedented growth, especially in countries with huge markets, such as the United States, China, and India. Recently, several e-commerce platform companies, including Amazon, Walmart.com, JD.Com, and Target, have expanded beyond selling third-party suppliers' products by developing and selling their own private labels directly to consumers on their own marketplaces. For example, Amazon provides well over 20,000 private labels that range from food and beverages and clothing to electronics and automobiles under more than 100 of its own brands, such as Amazon Basics, Amazon Elements, Amazon Essential, Goodthreads, Pinzon, and Presto! (DataWeave, 2020). The private label businesses of platform companies have grown dramatically in recent years. For instance, Amazon increased its product offerings threefold within two years (Davis, 2020), achieving an 81 % increase in the packaged goods category (Numerator, 2019). Moreover, platform private labels yielded $8.1 billion in US sales in 2020, a 50 % increase on 2019 sales (Marker, 2020). Reflecting the prominence of such private label sales, the term *dual-role* platforms has been applied to platforms that not only sell third-party suppliers' products as an intermediary but are also independent sellers of their own products. This dual role of platforms is commanding considerable attention from researchers in the field of operations and supply chain management.

Similar to the physical stores of bricks-and-mortar retailers, the private labels bring higher profit margins to e-commerce platforms overall than the margins they obtain from products sold by third-party suppliers (Waters, 2021). However, the downside of selling private labels is that they are likely to expose the platform to intense competition with suppliers offering similar products in the same category. Therefore, managing conflicts with third-party suppliers is a challenge that requires an urgent response from platform companies playing the dual role in supply chains. One problem typically faced by a platform in competition with a supplier is optimal pricing. As we will review in detail in the next section, a number of studies have constructed game-theoretic models that examine the price level that should be set by an e-commerce platform selling a private label in competition with third-party suppliers' products. Optimal pricing is an important issue because in a situation

E-mail address: kmatsui@b.kobe-u.ac.jp.

https://doi.org/10.1016/j.ejor.2025.10.001
Received 5 November 2024; Accepted 1 October 2025
Available online 8 October 2025
0377-2217/© 2025 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).

involving price competition, a platform's profit margin will fall significantly if it lowers its retail price excessively, whereas if it raises its retail price too high, it will lose considerable market share. Another critical issue for real-world platform companies is determining the timing of price-setting for their private label when they are competing with a supplier. However, there has been little discussion of this topic in the supply chain management literature.

Given the growing importance of sales strategies for dual-role platforms, the objective of this paper is to elucidate the optimal timing for setting the retail price when an e-commerce platform produces and sells a private label on its own marketplace. To this end, we apply the framework of the timing game developed in noncooperative game theory and consider two scenarios in which a platform and a supplier enter into two types of transaction contracts that are used in real-world supply chains, namely, a wholesale contract and an agency contract. The literature refers to the adoption of the wholesale contract as the *reselling mode* of the platform, and the adoption of the agency contract as its *marketplace mode*. The primary distinction between the two contracts or modes is who controls the retail price: the platform establishes the retail price of the supplier's product under the wholesale contract, whereas the supplier establishes the retail price of its product under the agency contract.

Conventionally, game-theoretic supply chain models describing a platform selling on its own marketplace assume that simultaneous price competition occurs at the level of the retail market; that is, the assumption is that a platform and a third-party supplier determine their retail prices at the same point of time. Consistent with this conventional assumption, we first show that under a wholesale contract, the platform's profit is unaffected by the timing of price-setting, such that the platform may decide and announce the retail price of its private label at any point, including making a price decision simultaneously with the supplier. Second, and more notably, our model demonstrates that under an agency contract, such simultaneous price competition between the platform and the supplier never occurs. Because this result is contrary to the conventional assumption of simultaneous retail price competition employed in the literature, our model suggests that this assumption must be reconsidered. As the most important insight of this research, our model yields the following useful guideline for managerial decision-making: to maximize its own profit, a platform concluding an agency contract should set the retail price of its private label earlier than or simultaneously with, but not later than, determining the marketplace fee or commission rate charged to a supplier. The result means that the platform should determine the retail price of the private label prior to the supplier determining the retail price of its product. To use game-theory terminology, the advance determination and announcement of the private label retail price constitute the subgame perfect Nash equilibrium (SPNE) of the noncooperative game between the platform and the supplier. From these perspectives, our study contributes to the literature by providing not only novel academic insights but also useful practical implications.

As mentioned above, a particularly noteworthy point for the motivation of this paper is that many studies describing an e-commerce platform selling its private label and competing with a supplier employ the conventional assumption of simultaneous price competition at the retail level, as we will review in detail in the next section. If the models in the academic literature continue to employ this assumption, their conclusions may provide misleading implications for a real-world platform selling its own private label, suggesting that it determines the retail price of the private label at the same time as its supplier determines the retail price. By challenging this conventional assumption, we reveal that this timing is inappropriate and that a platform should instead determine and announce the retail price of its private label as early as possible. This important managerial implication is why our study is significant. In practice, we expect that a platform will be able to determine an appropriate retail price for its private label as early as possible by referring to historical transaction data. In general, a platform has the advantage of possessing ample sales data from numerous transactions with third-party suppliers selling products on the marketplace.

Next, we consider a real-life case that fits the competition model in this paper. Specifically, we explain actual examples in which a platform has set the price of its private label first, followed by the supplier setting the price of its product. These examples highlight the practical motivation for this paper, and clarify its practical significance. The example that best illustrates the setting of our model concerns a private label smart speaker, known as the Tmall Genie, developed and sold by Alibaba on its platform Tmall. Until Alibaba launched Tmall Genie, smart speakers equipped with an AI assistant function that supports interactive voice control were not widely used in China. Alibaba announced the launch of Tmall Genie on July 5, 2017 at a price of 499 Chinese yuan, which corresponds to USD 71.3 at the rate of 1 USD = 7 yuan (Lee, 2017). The launch of Tmall Genie sparked competition, prompting other manufacturers, including Xiaomi, a prominent high-tech product manufacturing company in China, to enter the smart speaker market. Indeed, just three weeks later, on July 26, 2017, Xiaomi announced the launch of its own smart speaker, the Mi AI speaker, selling at 299 Chinese yuan, or USD 42.7 (Kinsella, 2017). The price range, features, and target market of the Mi AI speaker were nearly identical to those of the Tmall Genie. In fact, Kinsella (2017) reports that "Xiaomi, a top global smartphone manufacturer headquartered in China, earlier today launched a smart speaker rival to take on Alibaba's Tmall Genie X1", suggesting that it was launched in response to Tmall Genie. Furthermore, Xiaomi's Mi AI speaker is sold through Alibaba's platform Tmall, and thus Alibaba and Xiaomi fit the roles of the platform and the supplier described in our model. In other words, this case is a prime real-world example of the situation where the platform (Alibaba) sets and announces the price of its private label (Tmall Genie) first, after which the supplier (Xiaomi) sets the price of its product (Mi AI speaker) next, which supports the conclusion in this paper. In addition, after the entry of Xiaomi, Baidu launched the Xiaodu series in June 2018 and Huawei launched Huawei Sound X, which are their smart speakers.

Another real-world example that illustrates our model is launch by the giant platform Amazon of a range of products, including surge protectors, laptop stands, and packing cubes, under its private label "Amazon Basics." This has caused the entry of multiple suppliers, including Anker and Nulaxy, into the marketplace to manufacture and sell similar products, resulting in increased competition (Pierce, 2017).

Considering the clear price competition from a major supplier in response to the launch of the private label of the platform with a large market size, the real-world example that best fits the products examined in this paper is the private label Tmall Genie launched by platform Alibaba and the Mi AI Speaker sold by the competing supplier Xiaomi. As this demonstrates, there are real-life cases in which a platform has created a new product category by developing and launching its own private label, which has then attracted manufacturers and suppliers to enter the market to supply competing products in the same product category. In addition, the above case shows that there has been an actual situation in the market where a platform sets and announces the price of its product first and the supplier sets its price subsequently, which is consistent with the result and managerial implication shown in this paper. Consequently, in our model, we will assume that the platform and supplier are Alibaba and Xiaomi, the platform's private brand is Tmall Genie, and the supplier's product is the Mi AI speaker.

The following sections of this paper are structured as follows. In Section 2, we examine the studies that have developed game-theoretic models for managing supply chains including e-commerce platforms as members, as well as the literature relating to the timing game. Section 3 delineates the fundamental setting of our operational research (OR) model. In Section 4, we first consider the scenario where a platform and a supplier enter into a wholesale contract, deriving the stable timing of pricing and resulting profits in equilibrium. In Section 5, we consider the scenario in which a platform and a supplier enter into an agency contract that specifies the marketplace fee per unit of product, deriving the stable

timing for pricing and the resulting profits in equilibrium. In Section 6, we consider a further scenario in which an agency contract specifies the commission rate and derive the equilibrium timing for price-setting and the profits. Next, based on the analytical and numerical results derived, we draw managerial implications for each of the scenarios. In Section 7, we summarize our conclusions.

2. Literature review

2.1. Platform competition

Within the OR literature, the present paper is most relevant to game-theoretic research depicting supply chains that involve e-commerce platforms. Many studies in this research stream focus on competition between platform owners and third-party suppliers selling on marketplaces (e.g., Cao et al., 2023; Cheng et al., 2023; Chernonog & Levy, 2023; Gao et al., 2023; Lam & Liu, 2023; Li et al., 2023b, 2024c; Lin et al., 2021; Mandal et al., 2024; Niu et al., 2024; Song et al., 2024; Sui et al., 2023; Xu et al., 2024; Zhang et al., 2023a; Zhu et al., 2023). In particular, a number of recent studies address decision-making problems such as optimal contract selection between platforms and third-party sellers (e.g., Abhishek et al., 2016; Arora & Jain, 2024; Chang et al., 2022; Chen et al., 2021, 2023b; Fang et al., 2024; Gong et al., 2024; Hagiu & Wright, 2015; Hao & Yang, 2023; He et al., 2022; Hu & Fu, 2025; Ji et al., 2023; Kittaka et al., 2022; Li et al., 2021b, 2024b; Liu et al., 2023, 2024, 2025; Matsui, 2024; Pu et al., 2020; Qin et al., 2021; Shen et al., 2019; Siqin et al., 2022; Tan & Carrillo, 2017; Tan et al., 2024; Tian et al., 2018; Wang et al., 2023, 2024; Wei & Dong, 2022; Wei et al., 2020, 2025; Wu et al., 2023; Xi & Zhang, 2023; Xu et al., 2023; Yan et al., 2018; Yenipazarli, 2021; Zhang et al., 2019, 2021, 2023b; Zhang & Zhang, 2020; Zhen & Xu, 2022; Zhen et al., 2022; Zhu et al., 2025). Abhishek et al. (2016) investigate the environments in which an online retailer ought to adopt agency selling over conventional reselling, assuming one upstream supplier and two downstream online retailers. They show that the online retailers favor agency selling (conventional reselling) when electronic channel sales reduce (boost) demand in the traditional channel. In addition, online retailers favor agency selling when competition intensifies or when positive externalities from related product sales exist. Yan et al. (2018) examine the environments in which a marketplace channel should be introduced alongside a resale channel in a dual-channel supply chain, which involves a manufacturer and an online retailer, and consider positive spillovers from online to offline channels. They determine that higher spillovers enhance the manufacturer's willingness to embrace the marketplace channel, whereas they diminish the willingness of the online retailer to do so. They show that when the spillover effect is moderate, the firms can achieve Pareto improvements by introducing the marketplace channel.

Since the early 2020s, a significant number of papers have addressed optimal contract selection between platforms and third-party sellers. Zhang and Zhang (2020) examine the dissemination of demand information from an e-commerce platform to a supplier in agency selling and reselling. They show that when the supplier's cost of entry to an offline channel is very low or very high, the platform keeps its information private in reselling, whereas it shares information in the scenario of agency selling. When the entry cost is moderate, information uncertainty is low, and channel substitutability is high, the platform withholds information in agency selling, but shares information in the case of reselling. Zhang et al. (2021) explore service investments in a supply chain including a supplier and an e-commerce platform that resells products and mediates transactions. They show that investment by the platform in its retail service increases supply chain profits and benefits the supplier, but may not boost the platform's own profit. In addition, the investment can lead to supplier encroachment because it incentivizes the supplier to sell more directly through the platform. Zhen and Xu (2022) analyze the influence of pricing strategies on the configuration of distribution channels when both manufacturer and retailer sell via a third-party platform. They construct a model of a sequential game based on three scenarios in which the manufacturer, the retailer, or both use the platform. Their findings show that pricing strategies, whether uniform or differentiated, significantly affect the choice of the channel structure. Chen et al. (2023b) examine the minimum quantity contract and determine the optimal sales mode for suppliers selling products to loyal customers and general consumers via an online retail platform when the platform provides a marketplace mode, a reselling mode, or both modes. They reveal which mode suppliers should choose under various circumstances, including the market structure, and examine the managerial ramifications for suppliers and platforms when determining sales formats in the context of the minimum quantity contract. Assuming a two-stage supply chain consisting of a manufacturer and a retailer, Fang et al. (2024) investigate which of the parties should invest in blockchain technology when they enter into a wholesale contract or an agency contract. They construct a game-theoretic model to examine the relationship between the retailer's contract selection and the supply chain members' decisions on blockchain investment. Furthermore, they design a cost-sharing contract for the blockchain technology based on a Nash bargaining framework. This is the first line of research to which this paper contributes.

2.2. Platform selling a private label

The second strand of related research consists of studies that explore the optimal decision for a platform playing a dual role to produce and sell its own private label along with third-party sellers' products (e.g., Anderson & Bedre-Defolie, 2022; Bisceglia & Padilla, 2022; Etro, 2021; Hagiu et al., 2022; Hemmati et al., 2024; Huang et al., 2023; Li et al., 2021a, 2023a, 2024a; Long & Amaldoss, 2024; Reza-Gharehbagh et al., 2025; Shopova, 2023; Wu & Zha, 2024; Zennyo, 2022; Zhang & Hou, 2022; Zou & Zhou, 2024). Li et al. (2023a) investigate whether an e-tailer that sells both private-label and national-brand products should manufacture its own product or procure it from a third-party or national-brand manufacturer. They show that procurement from the national-brand manufacturer leads to the greatest agency fee and service quality. In addition, private-label procurement is affected by production costs and consumer service responsiveness, and optimal procurement strategies exist that benefit both the e-tailer and the national-brand manufacturer. Assuming one e-commerce platform and one external seller, Shopova (2023) analyzes how economic outcomes differ when the platform selling its own private label product concludes an agency contract versus a standard wholesale contract with the seller selling its product on the platform's marketplace. She shows that when introducing a private label, the platform using an agency contract has an incentive to reduce the marketplace fee imposed on the third-party supplier and to vertically differentiate the private label to protect the supplier's channel. However, compared with the wholesale contract, the platform offers a lesser quality private label at a higher consumer price, resulting in a reduced improvement in consumer surplus and potentially less detriment to the third-party supplier. Hemmati et al. (2024) examine the impact of selling formats on a supply chain comprising a national-brand manufacturer and an e-tailer providing a private label. Both parties invest in advertising and product quality. The authors compare agency selling and reselling formats and demonstrate that the optimal selling format is influenced by the levels of advertising and product quality. Moreover, when competition between the private label and national brand is intense, agency selling tends to dominate, and can lead to higher prices for both products. In the context of e-commerce platforms beginning to use sponsored advertising to generate additional revenue from private label sales, Long and Amaldoss (2024) investigate the circumstances under which e-commerce platforms may prioritize their private label in sponsored advertising, as well as the impact on third-party sellers and consumers. Their study indicates that self-preferencing decreases consumer surplus in equilibrium despite the platform offering its private label at a price that is lower than the prices

of third-party sellers. In addition, they find that prioritizing its own label in sponsored advertising is not always advantageous for the platform, especially when advertising substantially enhances demand. Zou and Zhou (2024) also focus on the phenomenon of self-preferencing, where a platform prioritizes its first-party products, including private labels, above those of third-party sellers in customer search ranks. They examine the economic impact of search neutrality, which forbids e-commerce platforms' from self-preferencing, and demonstrate that it triggers strategic decisions by the platform and third-party sellers, which may unintentionally harm consumers and third-party sellers.

It is noteworthy that the main models in the abovementioned studies commonly assume simultaneous retail price competition; that is, they assume that the platform and the retailer simultaneously set the retail prices of their respective products. For example, the formulation of the profit function maximization problem and Fig. 1 in Shopova (2023, p. 5) show that the platform and the external seller are assumed to determine their retail prices simultaneously under an agency contract specifying a fee per unit of the product. Fig. 2 in Long and Amaldoss (2024, p. 932) illustrates the timing of the game in their model, showing that the platform and the third-party seller are assumed to set their prices at the same time. Fig. 1 in Zou and Zhou (2024, p. 4091) illustrates the game sequence in their model, in which the platform and the third-party seller are assumed to simultaneously decide their prices only after the commission rate imposed on the supplier is determined. As highlighted in the Introduction, this paper questions the appropriateness of this conventional assumption.

2.3. Decision timing

Finally, this study pertains to research exploring the ideal timing for the various decisions of supply chain participants. Since the seminal work by Hamilton and Slutsky (1990) proposing the timing game concept, a growing number of studies have explored the optimal timing of decision-making by supply chain participants (e.g., Chen et al., 2018, 2021; Hu et al., 2024; Karray & Martín-Herrán, 2022; Liu & Ke, 2020; Liu et al., 2022; Lyu et al., 2025; Matsui, 2017, 2020; Sun et al., 2022; Yan et al., 2020). Matsui (2017) addresses the price-setting problem of a manufacturer selling items directly to end-consumers via an online channel, examining when this manufacturer should set its direct price within a dual-channel supply chain. He shows that the best timing for the manufacturer is to determine the direct selling price in a direct sales channel concurrently with or before setting a wholesale price in an indirect sales channel. Moreover, he demonstrates that this decision constitutes an equilibrium in the timing game. Chen et al. (2018) examine the impact of the order of pricing by two rival retailers on supply chain performance in a situation permitting consumer product returns. They show that the sequence of price decisions in equilibrium is influenced by the handling costs of a retailer relative to the rival retailer and the salvage value of a returned item. Consequently, they demonstrate that the returns policy significantly affects the power structure between the two retailers in equilibrium. Yan et al. (2020) analyze price competition within a dual-channel supply chain with a capital-constrained supplier and an e-retailer offering credit. They determine that financing benefits both parties by increasing market shares and overall profits, even with reduced revenue in the e-retailer's channel. Moreover, they find that the e-retailer gains a first-mover advantage by setting its price before the supplier. Chen et al. (2021) examine the impact of the timing of price-setting in an online promotion channel on the decision of a supplier choosing between wholesale and agency formats in an uncertain environment. They find that the timing of price-setting affects only reselling contracts, leading to lower prices compared with agency selling. Furthermore, they establish that promotional pricing timing can make reselling more attractive in uncertain market environments, contrary to the conventional belief that agency selling is preferable because it eliminates double marginalization. Most recently, Lyu et al. (2025) investigate the optimal pricing timing for a fresh-food retailer considering near-expired food commercialization (NFC). They show that when the size of a fresh-food order increases, the retailer's strategy transitions from not engaging in NFC, to partial NFC, and finally to complete NFC. Moreover, they find that a retailer with strong bargaining power selling near-expired food perceived as high quality will adopt clearance-priority pricing under full NFC, whereas a retailer with weaker power prefers fresh-priority pricing.

Although the timing of price-setting is an urgent and practical issue for e-commerce platform companies, our overview of the literature reveals that no studies apply the timing game framework to determine the optimal timing for pricing in a supply chain that involves an e-commerce platform playing the dual role of both intermediary and independent seller. Hence, we highlight that the novelty of this study is that it is the first to introduce the timing game concept of Hamilton and Slutsky (1990) into the analysis of the management of a supply chain involving an e-commerce platform. Moreover, the models in the papers reviewed above assume the simultaneous determination of retail prices for the platform's private label and the supplier's product. This paper challenges the conventional assumption employed in these models, and thus makes a unique contribution to the supply chain management literature.

3. Model

We first establish the settings that constitute the foundation of our model. Fig. 1 shows the supply chain structure presumed in our model, and Table 1 summarizes the variables employed. As shown in Fig. 1, the platform produces its private label at positive marginal cost c_P, sets the retail price p_P, and sells to end-consumers at that price. The supplier also produces its product at positive marginal cost c_S. Throughout the paper, we examine the two scenarios in which the platform and the supplier conclude the following two alternative contracts in the supply chain: a wholesale contract and an agency contract. In the wholesale contract scenario shown in Panel (i) of Fig. 1, the supplier first sets the wholesale price of its product, w, and sells the product to the platform at that price.

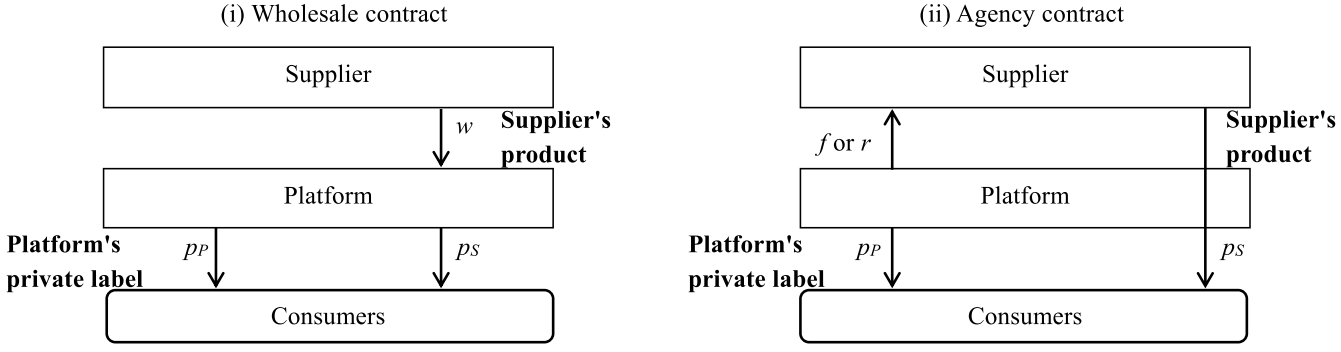


Fig. 1. Supply chain structure.
Note: Each arrow in the figure represents the direction in which a payment is required.

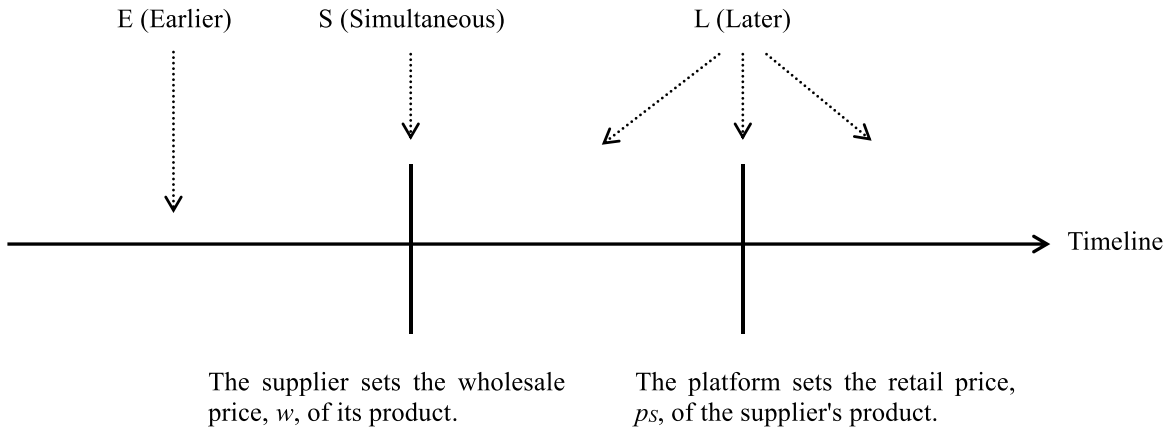


Fig. 2. Timeline under wholesale contract.
Note: E, S, and L, shown by the arrows, represent the time points when the platform sets the retail price of the private label, p_P. Specifically, S (Simultaneous) indicates that the platform sets the private label price simultaneously with the other supply chain member (i.e., the supplier) making a decision (i.e., a decision on w in the case of this wholesale contract). E and L represent the platform setting this price earlier and later, respectively, than the decision by the other supply chain member.

Table 1
Notations.

p_P	private label price; retail price of the platform's private label
p_S	supplier product price; retail price of the supplier's product
w	wholesale price of the supplier's product under the wholesale contract
f	marketplace fee per unit of product specified in the agency contract
r	commission rate specified in the agency contract ($0 < r < 1$)
q_P	quantity of the platform's private label
q_S	quantity of the supplier's product
c_P	platform's marginal cost for its private label
c_S	supplier's marginal cost for its product
c	marginal cost of the platform or the supplier when $c_P = c_S = c$ is assumed
a	intercept of the inverse demand function
b	sensitivity of the retail price to demand
θ	substitutability between the supplier's product and the platform's private label
P	subscript representing the platform's private label
S	subscript representing the supplier's product
Π	profit for the platform
π	profit for the supplier
E	pricing sequence or timing in which the platform sets the retail price of the private label earlier than the supplier makes its pricing decision
S	pricing sequence or timing in which the platform sets the retail price of the private label simultaneously with the supplier making its pricing decision
L	pricing sequence or timing in which the platform sets the retail price of the private label later than the supplier makes its pricing decision

Subsequently, the platform sets the product's retail price p_S and resells to end-consumers at that price. Meanwhile, in the agency contract scenario shown in Panel (ii) of Fig. 1, we consider two cases: one in which the platform determines and charges the supplier a marketplace fee of f per unit of the product, and another in which it determines and charges the supplier a commission rate of r. To clarify, the platform first determines a marketplace fee of f per unit of the product or a commission rate of r and presents it to the supplier. Subsequently, the supplier determines the retail price p_S of its product, and sells directly to end-consumers. Therefore, p_S is set by the platform in the wholesale contract scenario, whereas it is set by the supplier in the agency contract scenario.

Here, we elaborate on why we consider the case in which the platform charges the supplier a marketplace fee of f per unit of product as well as the more commonly examined case in which the platform determines a commission rate of r under the agency contract. The marketplace fee reflects agency contracts being adopted in real-world supply chains and, moreover, it is utilized by agency selling models in the OR literature, which reflect the real-world contracts. Conventionally, platforms charged suppliers relatively simple marketplace fee schedules that increased proportionally with sales revenue at a fixed commission rate. Recently, however, competition between the dominant platforms to attract more third-party sellers, especially in the US and China, has induced platform companies to offer sellers more attractive marketplace fee structures rather than a simple fee increasing in proportion to revenue at a fixed commission rate. For example, dominant platforms such as Amazon and Walmart.com have tended to charge marketplace fees such as order fulfillment fees that increase proportionally with the quantity sold (e.g., Fulfillment by Amazon). As a result, a marketplace fee that increases proportionally with quantity sold represents a good approximation of real-world agency contracts adopted by dominant platforms. Indeed, reflecting this real marketplace fee structure, a number of recent agency selling models describing a platform assume that the platform offers only the structure of a fixed marketplace fee per unit of product (e.g., Chen et al., 2022, 2023a; Fu et al., 2023; Hemmati et al., 2024; Li et al., 2023a). Consequently, we follow the literature in assuming a marketplace fee of f per unit of product charged to the supplier. Meanwhile, we also consider the case wherein the commission rate is determined in the agency contract, as mentioned above.

Next, given these assumptions made on the supply side, we make assumptions on the demand side. We assume the inverse demand functions of Eq. (1):

$$\begin{aligned} p_P &= a - b(q_P + \theta q_S) \\ p_S &= a - b(q_S + \theta q_P) \end{aligned} \tag{1}$$

where q_P and q_S denote the quantity of the platform's private label and the supplier's product, respectively. Correspondingly, p_P and p_S represent the retail prices of one platform's private label and one supplier's product, respectively. Henceforth, we refer to p_P as the *private label price* and p_S as the *supplier product price*. $\theta \in (0,1)$ measures the substitutability between the platform's private label and the supplier's product, such that the products become more differentiated (similar) as θ decreases (increases). a and b are positive constants representing the constant term and slope of the inverse demand function, respectively.

The linear inverse demand functions of Eq. (1) are derived by defining the utility function of a consumer as $U = a(q_P + q_S) - b\left(q_P{}^2 + q_S{}^2 + 2\theta q_P q_S\right)/2$. The consumer surplus based on the utility function is given by $S = U - (p_P q_P + p_S q_S) = a(q_P + q_S) - b\left(q_P{}^2 + q_S{}^2 + 2\theta q_P q_S\right)/2 - p_P q_P - p_S q_S$. The consumer solves $\partial S/\partial q_P = 0$ and $\partial S/\partial q_S = 0$ to maximize S, leading to Eq. (1). Ingene and Parry (2004) provide a detailed description of this derivation and its relevance to various supply chain management issues. Many subsequent dual-channel supply chain models adopt analogous linear inverse demand functions (e.g., Liu et al., 2023; Zhang & Zhang, 2020; Zhen et al., 2022).

To ensure that positive quantities of both the supplier's product and the platform's private label are marketed in equilibrium, we assume that a is greater than c_P and c_S and that the exogenous parameters satisfy the following inequality:

$$\theta^2 < (a-c_S)/(a-c_P) < \left(8-\theta^2-\theta^4\right)/6. \tag{2}$$

If Inequality (2) were not met, the platform would sell only the private label or the supplier product in equilibrium because of the excessively intense competition occurring between the two products. In such a case, only one of the two product types would be sold, and determining the optimal timing of price-setting, which is the aim of this paper, would become irrelevant. If the assumption that $(a-c_S)/(a-c_P) < \left(8-\theta^2-\theta^4\right)/6$ is met, $q_P > 0$ is maintained in equilibrium across all instances in both the wholesale and agency contract scenarios. Similarly, if $\theta^2 < (a-c_S)/(a-c_P)$ is met, $q_S > 0$ is maintained in equilibrium across all instances in the wholesale and agency contract scenarios.

The inverse demand functions of Eq. (1) can be reformulated as the demand functions below:

$$\begin{aligned} q_P &= ((1-\theta)a - p_P + \theta p_S)/\left(b\left(1-\theta^2\right)\right) \\ q_S &= ((1-\theta)a - p_S + \theta p_P)/\left(b\left(1-\theta^2\right)\right). \end{aligned} \tag{3}$$

We consider the framework of the timing game originally developed by Hamilton and Slutsky (1990) based on the aforementioned settings. In brief, the timing game is a model in which not only the decision variables but also the stable order of the decisions made by the players are determined as the equilibrium within a game, such that all players in the game make optimal decisions. Hence, the timing game identifies the timing or sequence of decisions made by the players in equilibrium, as well as the players' decisions. More specifically, in a timing game, the entire game is separated into two stages, the former and latter stages. In the former stage, all players in the game determine the specific point of time at which to determine each of their decision variables. Then, in the latter stage, each player determines the decision variable at the point of time that was already determined in the former stage. By solving this entire game, the equilibrium is obtained. Applying this framework, we examine a game where the platform and the supplier, in the former stage of the game, determine the timing at which they will set their decision variables (i.e., prices) and then, in the latter stage, decide on the prices at the points of time determined in the former stage. As this timing game is obviously a dynamic game with complete information, we adopt the SPNE as the equilibrium concept, following the OR studies that apply the timing game to supply chain management issues, as reviewed in Section 2.

The next three sections identify the equilibrium in the wholesale and agency contract scenarios, respectively. Then, we conclude by discussing the managerial implications of the analytical results.

4. Results: wholesale contract

Based on the setting elaborated in the previous section, we first examine the scenario in which the platform and the supplier enter into a wholesale contract. Under the wholesale contract, the profit functions for the platform and the supplier, denoted by Π_W and π_W, respectively, are:

$$\Pi_W = (p_P - c_P)q_P + (p_S - w)q_S \tag{4}$$

$$\pi_W = (w - c_S)q_S. \tag{5}$$

Inserting Eq. (3) into Eqs. (4) and (5) restates the profits as:

$$\begin{aligned} \Pi_W \quad = \quad & (p_P - c_P)((1-\theta)a - p_P + \theta p_S)/\left(b\left(1-\theta^2\right)\right) \\ & + (p_S - w)((1-\theta)a - p_S + \theta p_P)/\left(b\left(1-\theta^2\right)\right) \end{aligned} \tag{6}$$

$$\pi_W = (w - c_S)((1-\theta)a - p_S + \theta p_P)/\left(b\left(1-\theta^2\right)\right). \tag{7}$$

Next, we make an important and realistic assumption regarding the decision sequence in the wholesale contract scenario; that is, we assume that the platform sets the retail price of the supplier's product, p_S, only after the supplier sets the wholesale price, w. This assumption is commonly used not only in studies constructing e-commerce platform models but also in those constructing general two-echelon game-theoretic supply chain models. This assumption is adopted because, in real-world supply chain environments, it is usual for the reseller (in our model, the platform) to first confirm and accept the transaction terms presented by the supplier, including the wholesale price for purchasing the product, before subsequently setting the retail price of the product. Therefore, we make this assumption following the game-theoretic supply chain models that reflect this real-world supply chain practice. Consequently, the order of the decisions on w and p_S can be fixed on the timeline as described in Fig. 2 because the order of the two variables is not interchangeable.

On the other hand, the other decision variable, p_P, can be determined by the platform at any point on the timeline in Fig. 2. Therefore, we distinguish three possible pricing sequences, denoted as follows. Sequence S denotes the sequence in which p_P is determined by the platform at the same point of time at which the variable w is determined by the supplier. Sequence E and Sequence L denote the sequences in which p_P is determined earlier and later, respectively, than the supplier's decision on w. All possible sequences of price decisions in the wholesale contract scenario are indicated by the arrows shown in Fig. 2.

The profits of the platform and the supplier in the SPNE derived under each sequence are shown in Lemma 1. (All proofs are provided in the Appendix.)

Lemma 1. *The equilibrium profits Π_W and π_W for each sequence of decisions in the wholesale contract scenario are summarized as follows. Superscripts E, S, and L attached to Π_W and π_W denote the equilibrium profits in Sequences E, S, and L, respectively.*

$$\Pi_W^E = \Pi_W^S = \Pi_W^L = \frac{(a-c_P)^2}{4b} + \frac{((1-\theta)a - c_S + \theta c_P)^2}{16b\left(1-\theta^2\right)}$$

$$\pi_W^E = \pi_W^S = \pi_W^L = \frac{((1-\theta)a - c_S + \theta c_P)^2}{8b\left(1-\theta^2\right)}$$

Lemma 1 shows that both the supplier and the platform earn the same profits regardless of the order of their price decisions. Stated differently, Lemma 1 suggests that under the wholesale contract scenario, equilibrium profits do not depend on the sequence of decisions made by the platform and the supplier. Therefore, if we adopt the framework of the timing game, any order of decisions constitutes a stable equilibrium of the timing game in the wholesale contract scenario, as outlined in Proposition 1.

Proposition 1. *In the wholesale contract scenario, all Sequences E, S, and L that specify the time of price decisions by the platform and the supplier constitute the SPNE in the timing game, indicating that to maximize its own profit, the platform may determine the private label price, p_P, at any point of time.*

To reveal the logic behind Proposition 1, let us examine the situation in which the platform determines the private label price, p_P, at one specific point on the timeline in Fig. 2, as a result of which Sequence E, S, or L is realized. Then, the other player, the supplier, has no incentive to deviate from the sequence by changing the time point of the decision regarding w because altering the sequence would not increase its profit according to $\pi_W^E = \pi_W^S = \pi_W^L$ in Lemma 1. In addition, the platform has no incentive to deviate from the sequence by moving p_P to another point in time because of $\Pi_W^E = \Pi_W^S = \Pi_W^L$ in Lemma 1. Therefore, as shown in Proposition 1, all the sequences are stable equilibria from which neither platform nor supplier deviates.

Proposition 1 implies that Sequence S in which the supplier and the platform simultaneously determine the retail prices of their products constitutes a stable equilibrium in the timing game. As explained earlier, models that describe a platform selling a private label assume that this simultaneous retail price setting occurs. Therefore, Proposition 1 justifies the appropriateness of the conventional assumption of existing models that simultaneous retail price determination arises between an upstream supplier and a downstream e-commerce platform selling a private label. In other words, the assumption of simultaneous determination of the private label price and the supplier product price by the platform and the third-party supplier is reasonable in the wholesale contract scenario. However, this conclusion is completely overturned in the agency contract scenario, as we examine in the next section.

5. Results: agency contract specifying a fee per unit of product

Now, we derive the equilibrium outcomes in the agency contract scenario. In this section, we consider the case in which the platform determines a fee per unit of product under the agency contract in line with models in the literature that describe a platform selling a private label. The profit functions for the platform and the supplier concluding the agency contract represented by Π_A and π_A are:

$$\Pi_A = (p_P - c_P)q_P + fq_S \tag{8}$$

$$\pi_A = (p_S - f - c_S)q_S. \tag{9}$$

We substitute the demand function of Eq. (3) into Eqs. (8) and (9), restating the profits as:

$$\Pi_A = (p_P - c_P)((1-\theta)a - p_P + \theta p_S)/\left(b\left(1-\theta^2\right)\right) + f((1-\theta)a - p_S + \theta p_P)/\left(b\left(1-\theta^2\right)\right) \tag{10}$$

$$\pi_A = (p_S - f - c_S)((1-\theta)a - p_S + \theta p_P)/\left(b\left(1-\theta^2\right)\right). \tag{11}$$

Next, we make a realistic assumption regarding the decision sequence to reflect supply chain practice; that is, we assume that the supplier determines the retail price of its product, p_S, only after the platform determines the marketplace fee, f, under the agency contract. In a real-world supply chain environment, a third-party supplier usually enters into an agency contract only after confirming and accepting the terms and conditions, including the marketplace fee presented by the platform. Only after this step is completed does the supplier establish the retail price of its own product sold on the platform's marketplace. Reflecting the timeline of these actual decisions, studies in which e-commerce platform models are constructed to describe agency selling on a marketplace commonly assume this order of pricing. Therefore, this assumption regarding the order of decisions on f and p_S follows the models of agency selling in the literature, which reflect reality. Consequently, we fix the order of the decisions on f and p_S on the timeline, as shown in Fig. 3, and assume that the order of the two variables is not interchangeable.

The remaining decision variable, p_P, can be determined by the platform at any point on the timeline in Fig. 3. Therefore, as in the wholesale contact scenario, we distinguish three possible decision sequences, denoted as Sequences E, S, and L, which signify that the platform establishes the private label price, p_P, respectively, earlier than, simultaneously with, and later than the supplier setting the price of its product, p_S. As a result, all possible decision sequences in the agency contract scenario are indicated by the arrows shown in Fig. 3.

Lemma 2 shows the equilibrium profits of the platform and the supplier by pricing sequences.

Lemma 2. *The equilibrium profits Π_A and π_A for each sequence of decisions in the agency contract scenario are summarized as follows. Superscripts E, S, and L attached to Π_A and π_A denote equilibrium profit in Sequences E, S, and L, respectively.*

Case (I): Sequence E

$$\Pi_A^E = \frac{(a-c_P)^2}{4b} + \frac{((1-\theta)a - c_S + \theta c_P)^2}{8b\left(1-\theta^2\right)}$$

$$\pi_A^E = \frac{((1-\theta)a - c_S + \theta c_P)^2}{16b\left(1-\theta^2\right)}$$

Case (II): Sequence S

$$\Pi_A^S = \frac{(a-c_P)^2}{4b} + \frac{((1-\theta)a - c_S + \theta c_P)^2}{b\left(1-\theta^2\right)\left(8+\theta^2\right)}$$

$$\pi_A^S = \frac{((1-\theta)a - c_S + \theta c_P)^2\left(2+\theta^2\right)^2}{b\left(1-\theta^2\right)\left(8+\theta^2\right)^2}$$

Case (III): Sequence L

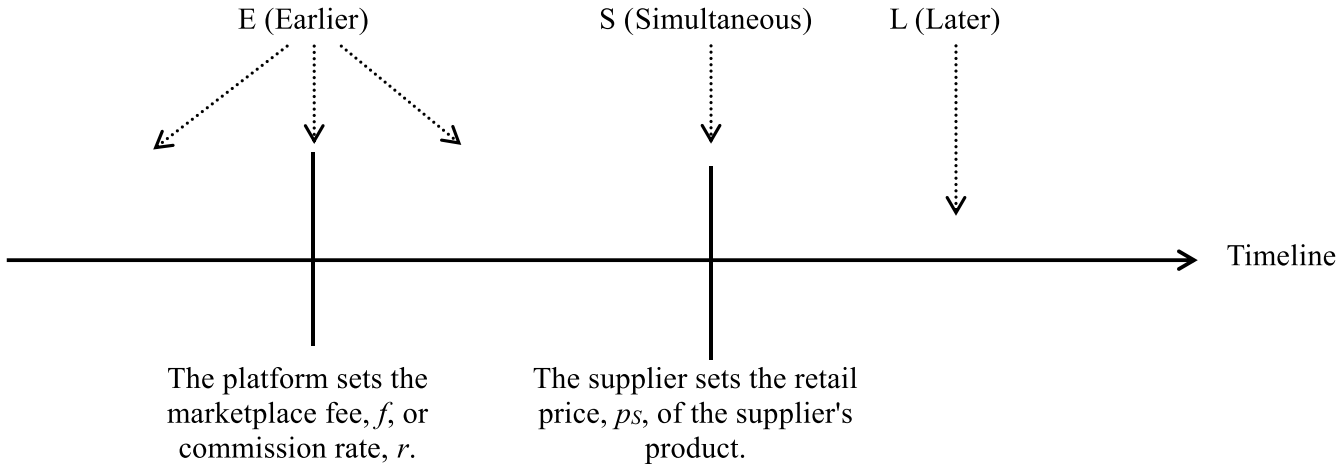


Fig. 3. Timeline under agency contract.
Note: E, S, and L, as shown by the arrows, represent the time points when the platform sets the retail price of the private label, p_P. Specifically, S (Simultaneous) indicates that the platform sets the private label price simultaneously with the other supply chain member (i.e., the supplier) making a decision (i.e., a decision on p_S in the case of this agency contract). E and L represent the platform setting this price earlier and later, respectively, than the decision by the other supply chain member.

$$\Pi_A^L = \frac{(a-c_P)^2}{4b} + \frac{((1-\theta)a - c_S + \theta c_P)^2(2-\theta^2)^2}{4b(1-\theta^2)(8-5\theta^2+\theta^4)}$$

$$\pi_A^L = \frac{2((1-\theta)a - c_S + \theta c_P)^2(2-\theta^2)}{b(1-\theta^2)(8-5\theta^2+\theta^4)^2}$$

Remarkably, Lemma 2 indicates that in the agency contract scenario, the equilibrium profits of both the platform and the supplier vary depending on the sequences of decisions and thus on the timing of pricing decisions. In this respect, the equilibrium result of Lemma 2 differs significantly from Lemma 1 derived in the wholesale contract scenario.

Comparing the equilibrium profits between sequences in Lemma 2 enables us to determine the sequence in which the platform and the supplier earn larger profits than under alternative sequences, as delineated in Lemma 3.

Lemma 3. *In the agency contract scenario, the following inequalities hold concerning the equilibrium profits of the platform and the supplier by sequences of decisions.*

$$\Pi_A^E > \Pi_A^S > \Pi_A^L$$

$$\pi_A^E < \pi_A^S < \pi_A^L$$

$\Pi_A^E > \Pi_A^S > \Pi_A^L$ in Lemma 3 indicates that the platform favors Sequence E, under which it determines the private label price, p_P, at the earliest opportunity before the supplier establishes the retail price of its product, p_S, as shown in Fig. 3. Conversely, $\pi_A^E < \pi_A^S < \pi_A^L$ in the lemma indicates that the supplier prefers Sequence L, in which it determines p_S as early as possible before the platform sets p_P. In these respects, Lemma 3 indicates that the preferences of the platform and the supplier are completely conflicting regarding the sequence of price decisions.

Using the results in Lemma 3, we can determine the optimal timing of pricing for the platform. Proposition 2 identifies the equilibrium of the timing game within the agency contract scenario.

Proposition 2. *The sequence of price decisions constituting the SPNE in the timing game in the agency contract scenario is as follows. First, the platform determines the private label price, p_P, and, either at the same time or subsequently, determines the marketplace fee, f. Finally, the supplier determines the price, p_S, of its product.*

Whereas Propositions 1 and 2 constitute the central findings of this study, Proposition 2 presents a more interesting result than Proposition 1, which confirms the literature. Proposition 2 offers the useful managerial insight that the platform should set the retail price of its private label as early as possible to maximize its profits. That is, according to Fig. 3, the equilibrium sequence suggests that the platform should implement the following decision-making guideline: to maximize its own profit, the platform should set the private label price, p_P, before or at the time of setting the marketplace fee, f, and before the supplier sets its product price, p_S. If, contrary to the implication of Proposition 2, the platform determines the private label price p_P after establishing the marketplace fee f, under that sequence the supplier will be incentivized to preempt the platform by setting the retail price p_S before the platform determines the private label price, p_P. This is because setting p_S before p_P enables the supplier to realize Sequence L or S and to increase its own profit, as $\pi_A^E < \pi_A^S < \pi_A^L$ in Lemma 3 suggests. To prevent this supplier behavior, the platform should always determine the private label price p_P before or at the same time that it determines the marketplace fee f, and never after, as Proposition 2 suggests. This result provides crucial guidance to a real-world platform company regarding when it should determine the retail price of its private label. In this respect, Proposition 2 is the central novel finding of this paper.

Recall that as overviewed in detail in Section 2, game-theoretic models for platforms selling private labels conventionally assume that a platform sets the private label retail price at the same time that the supplier sets the retail price of its product. This means that, in the context of our model, the conventional assumption in the literature is that Sequence S is applied. However, as shown in this section, this simultaneous retail pricing is not a stable equilibrium in the timing game when an agency contract is used. Therefore, our Proposition 2 implies that there is a strong need to reconsider the conventional assumption used in the models describing platforms selling private labels.

Finally, as a supplementary analysis, we explore the environment in which it is especially important for the platform to set the private label price as early as possible to realize Sequence E. The results are presented in Corollary 1.

Corollary 1. *The following inequalities hold with respect to how the difference in the platform's profits between the most profitable Sequence E and Sequence S or L depends on exogenous parameters.*

$$\partial(\Pi_A^E - \Pi_A^S)/\partial c_S < 0, \quad \partial(\Pi_A^E - \Pi_A^L)/\partial c_S < 0$$

$$\partial(\Pi_A^E - \Pi_A^S)/\partial c_P > 0, \quad \partial(\Pi_A^E - \Pi_A^L)/\partial c_P > 0$$

Corollary 1 yields the managerial implication that the difference in the profits between the sequences decreases (increases) as c_S (c_P) increases. This means that because the platform's per-unit profit margin from the private label exceeds that from the product sold by the third-party supplier, the difference in profits between sequences increases as the cost of the private label, c_P, rises. Therefore, Corollary 1 suggests that the larger is the marginal cost of the private label relative to the supplier's product, the more careful the platform must be about the timing of pricing.

6. Results: agency contract specifying a commission rate

In the preceding section, we considered a scenario wherein the platform determines the fee per unit of product. However, in actual business practice, platforms more commonly specify the commission rate imposed on third-party suppliers when concluding agency contracts. Therefore, in this section, we consider the alternative scenario of the commission model, in which the platform determines the commission rate denoted by r instead of the fee per unit, f. Specifically, in this commission model, we assume that the platform receives a commission fee proportional to the revenue of the supplier at the rate of $r \in (0,1)$.

The profit functions for the platform and the supplier concluding the agency contract denoted by Π_A and π_A, respectively, are:

$$\Pi_A = (p_P - c_P)q_P + rp_S q_S \tag{12}$$

$$\pi_A = (1-r)p_S q_S - c_S q_S. \tag{13}$$

We substitute the demand function of Eq. (3) into Eqs. (12) and (13), and restate the profits as:

$$\Pi_A = (p_P - c_P)((1-\theta)a - p_P + \theta p_S)/(b(1-\theta^2)) + rp_S((1-\theta)a - p_S + \theta p_P)/(b(1-\theta^2)) \tag{14}$$

$$\pi_A = ((1-r)p_S - c_S)((1-\theta)a - p_S + \theta p_P)/(b(1-\theta^2)). \tag{15}$$

Next, similar to the fee per unit model in the previous section, we make a realistic assumption regarding the decision sequence to reflect supply chain practice; that is, we assume that the supplier determines the retail price of its product, p_S, only after the platform determines the commission rate, r, under the agency contract. In a real-world supply chain environment, a third-party supplier usually enters into an agency contract only after confirming the terms and conditions, including the commission rate presented by the platform, and then establishes the

retail price of its own product sold on the marketplace. It is common for agency-selling models to assume this pricing sequence given that it reflects the real-world decision-making timeline. Consequently, as shown in Fig. 3, we fix the decision order of r and p_S on the timeline and assume that the order of the decision-making regarding the two variables is not interchangeable.

The remaining decision variable, p_P, can be determined by the platform at any point on the timeline in Fig. 3. Therefore, as in the previous sections, we distinguish three possible decision sequences signifying that the platform establishes the private label price, p_P, earlier than (Sequence E), simultaneously with (Sequence S), and later than (Sequence L) the supplier setting the price of its product, p_S.

Because the profit of the platform becomes a complex function of a polynomial with respect to r in the commission model formulated in this section, it is difficult to obtain analytical solutions of equilibrium prices and profits. Hence, we conduct a numerical analysis, which involves the following steps.

First, we set the exogenous parameters describing the costs of both the platform and the supplier equal to c, that is, $c = c_S = c_P$. We adopt this setting because in actual business practice, it is common for a platform to outsource manufacturing to an external general manufacturer when it launches its private label. In the long term, it is highly likely that technology diffusion will induce the platform and the supplier to use similar technologies and therefore the cost structures of the products are likely to converge to a similar level. Hence, we assume in the numerical analysis that the cost structures of the platform and the supplier are identical. Next, a represents the reservation price for a consumer who most highly values the product, that is, the maximum price that a consumer is willing to pay for the product. In addition, as the intercept of the inverse demand function, a can be interpreted as a measure of the potential of the market. Therefore, the absolute level of a does not significantly influence the equilibrium results. Accordingly, we fix a at a constant value of $a = 10$. Consequently, the key exogenous parameters that significantly influence the equilibrium results are the marginal cost, c, and the degree of substitutability of the products, θ. Specifically, because a is fixed at 10, the level of c significantly affects the equilibrium results and is thus important. Moreover, θ also significantly influences the equilibrium results because it denotes the substitutability of the platform's private label and the supplier's product.

In summary, assuming that $a = 10$ and $c_P = c_S = c$, we conduct the numerical analysis by varying the two parameters, c and θ, to calculate the corresponding equilibrium results. More specifically, because $0 < \theta < 1$ and $0 < c < a$ are assumed as the domains of θ and c, respectively, we consider 25 combinations of θ and c by dividing the two-dimensional exogenous parameter space of $\theta \in (0,1)$ and $c \in (0,10)$ into a $5 \times 5 = 25$ grid at equal intervals to yield a clear picture of the numerical results. More precisely, we set θ as 0.1, 0.3, 0.5, 0.7, or 0.9 and c as 1, 3, 5, 7, or 9, using the values to calculate numerical results. The Appendix provides

Table 2
Results of the numerical analysis.

Parameter value	Profit of the platform in each sequence			Profit of the supplier in each sequence			Profitable sequence		Equilibrium sequence of the timing game
c	Π_A^E	Π_A^S	Π_A^L	π_A^E	π_A^S	π_A^L	Most profitable sequence for the platform	Most profitable sequence for the supplier	
Panel (i): $\theta = 0.1$									
1	33.24	33.23	33.20	2.043	2.050	2.059	E	L	E
3	18.83	18.82	18.80	1.826	1.835	1.839	E	L	E
5	9.265	9.263	9.254	1.079	1.085	1.087	E	L	E
7	3.254	3.253	3.251	0.4233	0.4262	0.4264	E	L	E
9	0.3550	0.3549	0.3546	0.04992	0.05029	0.05030	E	L	E
Panel (ii): $\theta = 0.3$									
1	28.93	28.91	28.72	1.397	1.439	1.502	E	L	E
3	16.61	16.59	16.47	1.249	1.306	1.335	E	L	E
5	8.240	8.226	8.174	0.7298	0.7696	0.7796	E	L	E
7	2.912	2.906	2.889	0.2831	0.3003	0.3026	E	L	E
9	0.3191	0.3184	0.3167	0.03302	0.03520	0.03532	E	L	E
Panel (iii): $\theta = 0.5$									
1	25.71	25.69	25.33	0.8895	0.9555	1.111	E	L	E
3	14.97	14.94	14.72	0.8044	0.9025	0.9736	E	L	E
5	7.486	7.463	7.365	0.4648	0.5342	0.5601	E	L	E
7	2.660	2.650	2.619	0.1781	0.2082	0.2146	E	L	E
9	0.2928	0.2916	0.2885	0.02055	0.02433	0.02478	E	L	E
Panel (iv): $\theta = 0.7$									
1	23.19	23.18	22.73	0.4764	0.5348	0.7881	E	L	E
3	13.70	13.67	13.41	0.4427	0.5431	0.6591	E	L	E
5	6.907	6.884	6.765	0.2533	0.3262	0.3684	E	L	E
7	2.467	2.457	2.420	0.09586	0.1276	0.1384	E	L	E
9	0.2727	0.2714	0.2678	0.01094	0.01490	0.01576	E	L	E
Panel (v): $\theta = 0.9$									
1	21.144	21.138	20.810	0.1406	0.1633	0.3871	E	L	E
3	12.69	12.67	12.50	0.1369	0.1847	0.2751	E	L	E
5	6.447	6.436	6.364	0.07779	0.1139	0.1448	E	L	E
7	2.315	2.310	2.288	0.02908	0.04490	0.05273	E	L	E
9	0.2568	0.2562	0.2541	0.003278	0.005257	0.005893	E	L	E

Note: Each value is calculated by substituting $a = 10$ and $c_P = c_S = c$ as shown in the text. For tractability of the numerical results, $b = 1$ is substituted into the values derived, because b appears only as a factor multiplying the entire denominator of each profit function. For example, in the case of $(\theta, c) = (0.1, 1)$, Π_A^E is calculated as $33.24/b$ if b is not fixed at 1. The equilibrium values are presented with four significant digits, except where five significant digits are required to distinguish the difference in profits between sequences.

further details on the procedure for the numerical analysis.

Table 2 summarizes the results of the numerical analysis. Specifically, Panels (i), (ii), (iii), (iv), and (v) in Table 2 show the values of the equilibrium profits for the platform and the supplier in Sequences E, S, and L given the fixed values of θ at 0.1, 0.3, 0.5, 0.7, and 0.9, respectively. In each of the five panels, c is varied, taking the values of 1, 3, 5, 7, and 9. The third and second columns from the right in each panel show the most profitable sequences for the platform and supplier, respectively, and the rightmost column shows the equilibrium sequence of the timing game. The essential observation obtained from the numerical results can be summarized as follows.

Observation. *The numerical results in Table 2 suggest that Sequence E is always the equilibrium of the timing game, and that the platform's profit* Π_A *is highest in Sequence E, whereas the supplier's profit* π_A *is highest in Sequence L.*

The numerical results in Table 2 show that, remarkably, for all the combinations of exogenous parameters, the profits of the platform and supplier are highest in Sequences E and L, respectively, and that the equilibrium of the timing game is Sequence E. Accordingly, the observation above indicates that the result of Proposition 2, which was analytically demonstrated in the case of the agency contract specifying the marketplace fee per unit, is again valid in the case of the agency contract specifying the commission rate. More specifically, all the inequalities shown in Lemma 3 and thus the result of Proposition 2 continue to hold in this case. Consequently, the numerical results show that Sequence E is the optimal timing for the platform and that it constitutes the equilibrium of the timing game in all the combinations of the parameter values. That is, in the case of the platform determining the commission rate of r, the platform should secure the first-mover advantage when pricing its private label, as in the case of determining the fee per unit, f.

Although the results in the commission model are presented numerically rather than analytically, they are calculated using the values obtained by densely sampling the key exogenous parameters of θ and c at equal intervals on a $5 \times 5 = 25$ grid, proving the validity of the results. Because Sequence E constitutes the equilibrium in all combinations of the parameters, it is highly likely that this result is valid in general environments, as is the case for the result obtained in Proposition 2 under the agency contract specifying the fee per unit.

7. Conclusion

Recently, e-commerce platforms have been selling not only third-party suppliers' products but also their private labels on their own marketplaces, which has enabled them to generate significant sales revenue. Given the profitability of private label sales, this study examines the optimal timing for such an e-commerce platform to set and announce its retail price, utilizing the timing game framework developed in noncooperative game theory. Conventionally, game-theoretic supply chain models describing a platform selling on its own marketplace assume that simultaneous price competition occurs at the level of the retail market, and that the platform and a third-party supplier determine their retail prices at the same point of time. Consistent with this conventional assumption, we first show that under a wholesale contract, the platform's profit remains unaffected by the timing of price-setting, meaning that the platform may decide and announce the retail price of its private label at any point, including making its price decision simultaneously with the supplier. The most interesting result from our model is that under an agency contract, such simultaneous price competition between the platform and the supplier never occurs, suggesting that the conventional assumption of simultaneous retail price competition must be reconsidered. Our model yields the useful managerial decision guideline that to maximize its profit, a platform concluding an agency contract should set the retail price of its private label earlier than or simultaneously with, but not later than, the establishment of the marketplace fee or commission rate charged to a supplier. Moreover, the platform should determine the retail price of the private label before the supplier determines the retail price of its product. Such advance determination and announcement of the private label retail price not only constitute the SPNE of the game between the platform and the supplier, but also maximize the platform's profit.

We conclude by elaborating on the practical implications of the results obtained from our model. Table 3 summarizes specific decision-making guidelines for a platform based on the implications. First, the main result regarding the wholesale contract scenario is that the equilibrium profits generated by the platform and the supplier do not depend on the order of their decision-making. This result is noteworthy not only from the perspective of supply chain management but also from that of game theory because it conflicts with the conventional wisdom of game theory. That is, when price competition transpires between two firms in a horizontal relationship, the literature on game theory presents the following two important insights (e.g., Hamilton & Slutsky, 1990). The first is that both firms can achieve higher profits when they make sequential price decisions than when they make simultaneous price decisions. The second is that the second-moving firm (i.e., the firm that determines the price later) can achieve a higher profit than the first-moving firm when the two firms make sequential price decisions, which is referred to as the second-mover advantage. These insights are the foundation of the conventional wisdom that sequential rather than simultaneous price decisions by players lead to stable equilibria in a timing game involving price competition. Contrary to this conventional wisdom, Lemma 1 of this study shows that in the case of a two-echelon supply chain with an upstream supplier and a downstream platform, equilibrium profits do not depend on the timing of pricing decisions.

In addition, the results from the wholesale contract scenario apply not only to an online platform but also to a more traditional bricks-and-mortar retailer selling its private label as well as a supplier's product in a general two-echelon supply chain. That is, our results hold even if the platform in Panel (i) of Fig. 1 is replaced with a more traditional retailer that sells both its private label and the supplier's product. This is because not only e-commerce platforms but also more traditional bricks-and-mortar retail stores utilize wholesale contracts in reality. Indeed, game-theoretic models exploring desirable sales strategies for general retailers dealing in private labels appear not only in the OR literature but also in the marketing literature. Hence, our model results regarding the wholesale contract scenario provide insights into marketing practice.

As discussed, the results derived in the agency contract scenario are more notable than those derived in the wholesale contract scenario. The equilibrium outcomes obtained from the timing game within the scenario of the agency contract specifying the marketplace fee per unit of product show that in a stable pricing sequence, the platform initially determines the retail price p_P of the private label, and then subsequently or simultaneously determines the marketplace fee f, before, finally, the supplier determines the retail price p_S of its product. Our model proves that this timing strategy is the most profitable strategy for the platform in the case of the agency contract. As well as in the wholesale contract scenario, these results in the scenario of the agency contract specifying

Table 3
Practical price-setting implications for an e-commerce platform.

Contract	Recommendations for a platform
Wholesale contract	• The platform can set the price of its private label at any time
Agency contract	• The platform should set the price of its private label before or at the same time as it determines and presents the marketplace fee or the commission rate to a supplier • To implement the price-setting timing, the platform should utilize past transaction data to establish an appropriate private label retail price before entering into an agency contract

the fee per unit of product are fully analytically proven without relying on numerical analysis. Hence, the above findings are robust and clear-cut in that they hold irrespective of the values of exogenous parameters associated with demand and cost conditions.

Furthermore, in the more realistic scenario of the agency contract specifying the commission rate, the same results are obtained in all considered environments. Although the results in the commission model are presented numerically rather than analytically, they are calculated using the values obtained by densely sampling the key exogenous parameters of θ and c at equal intervals on a $5 \times 5 = 25$ grid, proving the validity of the results. Because Sequence E constitutes the equilibrium of the timing game in all combinations of the parameters considered, it is highly likely that this result is valid in general environments, as in the result obtained in Proposition 2 under the agency contract specifying the fee per unit.

We expect that platforms can feasibly achieve this pricing sequence in practice in the real world. Platforms usually have the advantage of ample past sales data from numerous transactions with third-party suppliers selling similar products within the same category on the marketplace. For example, staff in Amazon's private labels division utilized data from its marketplace to guide their judgments about the introduction of private labels (Mattioli, 2020). The platform can utilize such huge datasets to establish an appropriate private label retail price before entering into an agency contract with a supplier. Indeed, as in our real-life case, Alibaba's decision to launch Tmall Genie into the smart speaker market ahead of its competitors and set a price of 499 yuan is consistent with the findings and implications shown in this paper. Namely, the timing of product pricing was the optimal choice for the platform. Moreover, Xiaomi's decision to release the Mi AI speaker at a price of 299 yuan (well below the 499 yuan for the Tmall Genie) is consistent with insights from the game-theory literature. That is, game-theoretic studies show that it is the best response strategy for the second-mover in a price competition to set a lower price than the first-mover to achieve higher demand, which constitutes the equilibrium (e.g., van Damme & Hurkens, 2004).

Notably, in recent times, platforms have tended to conclude agency contracts with suppliers rather than wholesale contracts. For example, Amazon now strictly selects the suppliers that it permits to enter into wholesale contracts as first-party sellers (Bertherne, 2019). Indeed, if a supplier wishes to conclude a wholesale contract with Amazon, it must first receive an invitation from Amazon (Hensell, 2023). These practices mean that agency contracts have become more prevalent than wholesale contracts in the platform business. Furthermore, a comparison of the equilibrium profits for the platform in Lemmas 1 and 2 shows that $\Pi_{\mathrm{A}}^{\mathrm{E}} - \Pi_{\mathrm{W}}^{\mathrm{E}} = ((1-\theta)a - c_S + \theta c_P)^2/\left(16b\left(1-\theta^2\right)\right) > 0$ holds, meaning that the platform favors the agency contract over the wholesale contract in our model. The fundamental reason for this result is that double marginalization with respect to the sale of the supplier's product arises in the wholesale contract, but not in the agency contract. Given this context, our result that a platform should set and announce its private label retail price as early as possible when entering into the (increasingly adopted) agency contract, is a crucial insight that provides a useful decision guideline for real-world e-commerce platform companies playing the dual role.

CRediT authorship contribution statement

Kenji Matsui: Writing – original draft, Software, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Acknowledgments

The author gratefully acknowledges two anonymous reviewers for their insightful comments. The contents of this research were presented at the 10th Annual International Conference in Hawai'i, held at the University of Hawai'i at Mānoa. The author appreciates the helpful comments from conference participants. Needless to say, the author is solely responsible for any remaining errors. This work was supported by JSPS KAKENHI Grant Numbers JP18K01568, JP20H01551, JP22H00863, and JP24K05153.

Appendix

Proof of Lemma 1. Employing backward induction, we solve the game for each sequence of pricing decision timing.

Case (I): Sequence E

Because the platform sets the retail price of the supplier's product in the last step, we initially maximize Eq. (6) with regard to p_S by using $\partial\Pi_{\mathrm{W}}/\partial p_S = 0$, which results in:

$$p_S = ((1-\theta)a + w + 2\theta p_P - \theta c_P)/2. \tag{A1}$$

Here, we note that all of the objective functions presented in this Appendix are always concave because they are quadratic with regard to prices, which are decision variables. Hence, all the second-order conditions of the maximization problems are met. Accordingly, we will omit stating the second-order conditions hereafter.

Next, we examine the supplier's choice regarding the wholesale price. After Eq. (A1) is substituted for p_P in Eq. (7), we maximize it with respect to w by solving $\partial\pi_{\mathrm{W}}/\partial w = 0$, which yields:

$$w = ((1-\theta)a + c_S + \theta c_P)/2. \tag{A2}$$

Substituting Eqs. (A1) and (A2) into Eq. (6) and solving $\partial\Pi_{\mathrm{W}}/\partial p_P = 0$ to maximize Π_{W} with regard to p_P yield:

$$p_P = (a + c_P)/2. \tag{A3}$$

Finally, substitution of Eqs. (A1)–(A3) into Eqs. (6) and (7) produces the equilibrium profits of the platform and the supplier of $\Pi_{\mathrm{W}} = (a - c_P)^2/(4b) + ((1-\theta)a - c_S + \theta c_P)^2/\left(16b\left(1-\theta^2\right)\right)$ and $\pi_{\mathrm{W}} = ((1-\theta)a - c_S + \theta c_P)^2/\left(8b\left(1-\theta^2\right)\right)$, respectively, which correspond to $\Pi_{\mathrm{W}}^{\mathrm{E}}$ and $\pi_{\mathrm{W}}^{\mathrm{E}}$, respectively, in the lemma.

Case (II): Sequence S

Because the platform sets the retail price of the supplier's product in the last step, we initially maximize Eq. (6) with regard to p_S by using $\partial\Pi_{\mathrm{W}}/\partial p_S$

$= 0$, which yields:

$$p_S = ((1-\theta)a + w + 2\theta p_P - \theta c_P)/2. \tag{A4}$$

Next, we examine the platform's decision on the private label price, p_P, and the supplier's decision on the wholesale price, w. After inserting Eq. (A4) into Eqs. (6) and (7), we maximize Eq. (6) on p_P and Eq. (7) on w, i.e., $\partial\Pi_W/\partial p_P = \partial\pi_W/\partial w = 0$, giving:

$$\begin{aligned} p_P &= (a + c_P)/2 \\ w &= ((1-\theta)a + c_S + \theta c_P)/2. \end{aligned} \tag{A5}$$

Finally, substitution of Eqs. (A4) and (A5) into Eqs. (6) and (7) produces the equilibrium profits for the platform and the supplier as $\Pi_W = (a - c_P)^2/(4b) + ((1-\theta)a - c_S + \theta c_P)^2/\left(16b\left(1-\theta^2\right)\right)$ and $\pi_W = ((1-\theta)a - c_S + \theta c_P)^2/\left(8b\left(1-\theta^2\right)\right)$ respectively, which correspond to Π_W^S and π_W^S, respectively, in this lemma.

Case (III): Sequence L

There are three dotted arrows that indicate Sequence L in Fig. 2. Intuitively, the prices and profits in equilibrium appear to vary depending on the time at which the private label price p_P is determined, as indicated by the three different arrows. However, regardless of which of the three dotted arrows of Sequence L represents the time at which the price p_P is determined, the value of equilibrium profits will be the same due to the envelope theorem. This is essentially because the two variables p_S and p_P, which are determined at the end of any of the three arrows indicated by Sequence L in Fig. 2, are determined by the same player, i.e., the platform. That is, because the platform maximizes its own identical objective function for the two decision variables p_P and p_S irrespective of the timing of pricing, the optimal values of p_P and p_S do not depend on the order of the decisions. To show this, we will derive the SPNE by dividing this case into three distinct subcases, in which the platform determines p_P (III)-(i) before, (III)-(ii) concurrently with, and (III)-(iii) after determining p_S.

Case (III)-(i): The platform determines p_P first and p_S second in Sequence L.

The platform determines the private label price before determining the supplier product price in this instance. Maximization of Eq. (6) concerning p_S by solving $\partial\Pi_W/\partial p_S = 0$ gives:

$$p_S = ((1-\theta)a + w + 2\theta p_P - \theta c_P)/2. \tag{A6}$$

We substitute Eq. (A6) into p_S in Eq. (6), then maximize it with regard to p_P by using $\partial\Pi_W/\partial p_P = 0$, which results in:

$$p_P = (a + c_P)/2. \tag{A7}$$

Subsequently, we insert Eqs. (A6) and (A7) into Eq. (7) and maximize it concerning w by solving $\partial\pi_W/\partial w = 0$, resulting in:

$$w = ((1-\theta)a + c_S + \theta c_P)/2. \tag{A8}$$

Finally, substitution of Eqs. (A6)–(A8) into Eqs. (6) and (7) produces the equilibrium profits for the platform and the supplier of $\Pi_W = (a - c_P)^2/(4b) + ((1-\theta)a - c_S + \theta c_P)^2/\left(16b\left(1-\theta^2\right)\right)$ and $\pi_W = ((1-\theta)a - c_S + \theta c_P)^2/\left(8b\left(1-\theta^2\right)\right)$, respectively, which equate to Π_W^L and π_W^L, respectively, in this lemma.

Case (III)-(ii): The platform determines p_P and p_S simultaneously in Sequence L.

The platform sets both the private label price and the supplier product price simultaneously. Maximization of Eq. (6) concerning p_P and p_S by solving $\partial\Pi_W/\partial p_P = \partial\Pi_W/\partial p_S = 0$ gives:

$$\begin{aligned} p_P &= (a + c_P)/2 \\ p_S &= (a + w)/2. \end{aligned} \tag{A9}$$

We substitute Eq. (A9) into Eq. (7), then maximize it with regard to w by using $\partial\pi_W/\partial w = 0$, which results in:

$$w = ((1-\theta)a + c_S + \theta c_P)/2. \tag{A10}$$

Finally, inserting Eqs. (A9) and (A10) into Eqs. (6) and (7) results in the equilibrium profits for the platform and the supplier of $\Pi_W = (a - c_P)^2/(4b) + ((1-\theta)a - c_S + \theta c_P)^2/\left(16b\left(1-\theta^2\right)\right)$ and $\pi_W = ((1-\theta)a - c_S + \theta c_P)^2/\left(8b\left(1-\theta^2\right)\right)$, respectively, which equate to Π_W^L and π_W^L, respectively, in this lemma.

Case (III)-(iii): The platform determines p_S first and p_P second in Sequence L.

The platform sets the private label price after setting the supplier product price in this case. Solving $\partial\Pi_W/\partial p_P = 0$ to maximize Eq. (6) with respect to p_P yields:

$$p_P = ((1-\theta)a - \theta w + 2\theta p_S + c_P)/2. \tag{A11}$$

After substituting Eq. (A11) into p_P in Eq. (6), we solve $\partial\Pi_W/\partial p_S = 0$ to maximize Π_W with respect to p_S, resulting in:

$$p_S = (a + w)/2. \tag{A12}$$

We insert Eqs. (A11) and (A12) into Eq. (7), then maximize it on w by using $\partial\pi_W/\partial w = 0$, which results in:

$$w = ((1-\theta)a + c_S + \theta c_P)/2. \tag{A13}$$

Finally, substitution of Eqs. (A11)–(A13) into Eqs. (6) and (7) produces the equilibrium profits for the platform and the supplier of $\Pi_W = (a - c_P)^2/(4b) + ((1-\theta)a - c_S + \theta c_P)^2/\left(16b\left(1-\theta^2\right)\right)$ and $\pi_W = ((1-\theta)a - c_S + \theta c_P)^2/\left(8b\left(1-\theta^2\right)\right)$, which correspond to Π_W^L and π_W^L in this lemma.

Note that the identical equilibrium values are derived for Cases (III)-(i), (ii), and (iii) because of the envelope theorem. □

Proof of Proposition 1. Lemma 1 shows that the equilibrium profits for both the platform and the supplier in the wholesale contract scenario are the same regardless of the pricing sequence. Hence, when particular points of time are chosen for price setting by the platform and the supplier and Sequence E, S or L is realized, neither the platform nor the supplier has an incentive to deviate from the realized sequence. This is because even if they deviate, the values of their profits never change. Therefore, Sequences E, S, and L all constitute equilibria in the timing game in the scenario of the wholesale price contract. □

Proof of Lemma 2. Employing backward induction, we solve the game for each sequence of pricing timing, similar to the proof of Lemma 1.

Case (I): Sequence E

In Sequence E, the supplier sets the retail price of its product in the last step. Initially, we maximize Eq. (11) with regard to p_S by using $\partial\pi_A/\partial p_S = 0$, resulting in:

$$p_S = ((1-\theta)a + \theta p_P + f + c_S)/2. \tag{A14}$$

We insert Eq. (A14) into Eq. (10), yielding:

$$\Pi_A = \left(\left(\left(2-\theta-\theta^2\right)a - \left(2-\theta^2\right)p_P + \theta(f+c_S)\right)(p_P - c_P) + ((1-\theta)a - f - c_S + \theta p_P)f\right)/\left(2b\left(1-\theta^2\right)\right). \tag{A15}$$

Next, observe that there are three dotted arrows that indicate Sequence E in Fig. 3. Intuitively, the prices and profits in equilibrium appear to vary depending on the time point at which the private label price p_P is determined, as indicated by the three different arrows. However, regardless of which of the three dotted arrows of Sequence E represents the time at which the price p_P is determined, the value of equilibrium profits will be the same due to the envelope theorem. This is essentially because the two variables f and p_P, determined at the start of any of the three arrows of Sequence E in Fig. 3, are determined by the same player, i.e., the platform. That is, because the platform maximizes its own identical objective function for the two decision variables p_P and f irrespective of the timing at which it sets prices, the optimal values of p_P and f do not depend on the order of the decisions. To show this, we will derive the SPNE by dividing this case into three distinct subcases, in which the platform determines p_P (I)-(i) before, (I)-(ii) concurrently with, and (I)-(iii) after determining f.

Case (I)-(i): The platform determines p_P first and f second in Sequence E.

In this case, the platform determines the private label price before determining the marketplace fee. We maximize Eq. (A15) concerning f by solving $\partial\Pi_A/\partial f = 0$, resulting in:

$$f = ((1-\theta)a + 2\theta p_P - c_S - \theta c_P)/2. \tag{A16}$$

Subsequently, we insert Eq. (A16) into Eq. (A15) and solve $\partial\Pi_A/\partial p_P = 0$ to maximize Π_A with regard to p_P, resulting in:

$$p_P = (a + c_P)/2. \tag{A17}$$

Substituting Eqs. (A16) and (A17) into Eq. (A14) yields:

$$p_S = ((3-\theta)a + c_S + \theta c_P)/4. \tag{A18}$$

Case (I)-(ii): The platform determines p_P and f simultaneously in Sequence E.

The platform sets both the private label price and the marketplace fee simultaneously in this case. Thus, we solve $\partial\Pi_A/\partial f = \partial\Pi_A/\partial p_P = 0$ to maximize Eq. (A15) with respect to both f and p_P, obtaining:

$$\begin{aligned} f &= (a - c_S)/2 \\ p_P &= (a + c_P)/2. \end{aligned} \tag{A19}$$

Substituting Eq. (A19) into Eq. (A14) yields:

$$p_S = ((3-\theta)a + c_S + \theta c_P)/4. \tag{A20}$$

Case (I)-(iii): The platform determines f first and p_P second in Sequence E.

In this case, the platform sets the marketplace fee before setting the private label price. We maximize Eq. (A15) with regard to p_P by using $\partial\Pi_A/\partial p_P = 0$, resulting in:

$$p_P = \left(\left(2-\theta-\theta^2\right)a + \theta(2f + c_S) + \left(2-\theta^2\right)c_P\right)/\left(2\left(2-\theta^2\right)\right). \tag{A21}$$

After substituting Eq. (A21) into Eq. (A15), we maximize the function with regard to f by using $\partial\Pi_A/\partial f = 0$, resulting in:

$$f = (a - c_S)/2. \tag{A22}$$

Replacing f in Eq. (A21) with Eq. (A22), we have:

$$p_P = (a + c_P)/2. \tag{A23}$$

Inserting Eqs. (A22) and (A23) into Eq. (A14) yields:

$$p_S = ((3-\theta)a + c_S + \theta c_P)/4. \tag{A24}$$

Finally, substitution of either Eqs. (A16)–(A18) from Case (I)-(i), Eqs. (A19) and (A20) from Case (I)-(ii), or Eqs. (A22)–(A24) from Case (I)-(iii) into Eqs. (10) and (11) results in the equilibrium profits for the platform and the supplier of $\Pi_A = (a-c_P)^2/(4b) + ((1-\theta)a - c_S + \theta c_P)^2/\left(8b\left(1-\theta^2\right)\right)$ and $\pi_A = ((1-\theta)a - c_S + \theta c_P)^2/\left(16b\left(1-\theta^2\right)\right)$, respectively, which correspond to Π_A^E and π_A^E, respectively, in this lemma. Note that the identical equilibrium values are derived for Cases (I)-(i), (ii), and (iii) because of the envelope theorem.

Case (II): Sequence S

We maximize Eq. (10) with respect to p_P and Eq. (11) with respect to p_S, i.e., $\partial\Pi_A/\partial p_P = \partial\pi_A/\partial p_S = 0$, yielding:

$$\begin{aligned} p_P &= \left(\left(2-\theta-\theta^2\right)a + 2c_P + \theta(c_S+3f)\right)/\left(4-\theta^2\right) \\ p_S &= \left(\left(2-\theta-\theta^2\right)a + \left(2+\theta^2\right)f + 2c_S + \theta c_P\right)/\left(4-\theta^2\right). \end{aligned} \tag{A25}$$

We substitute Eq. (A25) into Eq. (10), then maximize it concerning f by using $\partial\Pi_A/\partial f = 0$, which yields:

$$f = \left(\left(8+\theta^3\right)a - 8c_S - \theta^3 c_P\right)/\left(2\left(8+\theta^2\right)\right). \tag{A26}$$

Finally, substitution of Eqs. (A25) and (A26) into Eqs. (10) and (11) results in the equilibrium profits for the platform and the supplier of $\Pi_A = (a-c_P)^2/(4b) + ((1-\theta)a - c_S + \theta c_P)^2/\left(b\left(1-\theta^2\right)\left(8+\theta^2\right)\right)$ and $\pi_A = ((1-\theta)a - c_S + \theta c_P)^2\left(2+\theta^2\right)^2/\left(b\left(1-\theta^2\right)\left(8+\theta^2\right)^2\right)$, respectively, which correspond to Π_A^S and π_A^S, respectively, in this lemma.

Case (III): Sequence L

We maximize Eq. (10) concerning p_P by solving $\partial\Pi_A/\partial p_P = 0$, yielding:

$$p_P = ((1-\theta)a + \theta(f+p_S) + c_P)/2. \tag{A27}$$

After replacing p_P in Eq. (11) with Eq. (A27), we maximize it regarding p_S by solving $\partial\pi_A/\partial p_S = 0$, resulting in:

$$p_S = \left(\left(2-\theta-\theta^2\right)a + \left(2-\theta^2\right)c_S + \theta c_P + 2f\right)/\left(2\left(2-\theta^2\right)\right). \tag{A28}$$

After substituting Eqs. (A27) and (A28) into Eq. (10), we maximize it with regard to f by solving $\partial\Pi_A/\partial f = 0$, yielding:

$$f = \left(\left(8-6\theta^2+\theta^3+\theta^4\right)a - \left(8-6\theta^2+\theta^4\right)c_S - \theta^3 c_P\right)/\left(2\left(8-5\theta^2+\theta^4\right)\right). \tag{A29}$$

Finally, substitution of Eqs. (A27)–(A29) into Eqs. (10) and (11) produces the equilibrium profits for the platform and the supplier of $\Pi_A = (a-c_P)^2/(4b) + ((1-\theta)a - c_S + \theta c_P)^2\left(2-\theta^2\right)^2/\left(4b\left(1-\theta^2\right)\left(8-5\theta^2+\theta^4\right)\right)$ and $\pi_A = 2((1-\theta)a - c_S + \theta c_P)^2\left(2-\theta^2\right)/\left(b\left(1-\theta^2\right)\left(8-5\theta^2+\theta^4\right)^2\right)$, respectively, which correspond to Π_A^L and π_A^L, respectively, in this lemma. □

Proof of Lemma 3. Using the equilibrium profits in Lemma 2, we obtain the following four inequalities. Note that the assumption of $\theta \in (0,1)$ makes the following four inequalities valid.

$$\begin{aligned} &\Pi_A^E - \Pi_A^S = \theta^2((1-\theta)a + \theta c_P - c_S)^2/\left(8b\left(1-\theta^2\right)\left(8+\theta^2\right)\right) > 0 \\ &\Pi_A^S - \Pi_A^L = \theta^2\left(8-\theta^4\right)((1-\theta)a + \theta c_P - c_S)^2/\left(4b\left(1-\theta^2\right)\left(8+\theta^2\right)\left(8-5\theta^2+\theta^4\right)\right) > 0 \\ &\pi_A^E - \pi_A^S = -3\theta^2\left(16+5\theta^2\right)((1-\theta)a + \theta c_P - c_S)^2/\left(16b\left(1-\theta^2\right)\left(8+\theta^2\right)\right) < 0 \\ &\pi_A^S - \pi_A^L = -\theta^4\left(64-46\theta^2-5\theta^4+6\theta^6-\theta^8\right)((1-\theta)a + \theta c_P - c_S)^2\Big/\left(b\left(1-\theta^2\right)\left(8+\theta^2\right)^2\left(8-5\theta^2+\theta^4\right)^2\right) < 0 \end{aligned}$$ □

Proof of Proposition 2. First, $\Pi_A^E > \Pi_A^S > \Pi_A^L$ in Lemma 3 suggests that if Sequence E is realized, the platform has no incentive to change from Sequence E to Sequence S or L by shifting its decision time point of p_P along the timeline shown in Fig. 3. However, $\pi_A^E < \pi_A^S < \pi_A^L$ in Lemma 3 suggests that the supplier has an incentive to change the timing of setting p_S so that the sequence changes from Sequence E to Sequence L or S. Here, recall the assumption in the agency contract scenario (stated at the beginning of Section 5) that p_S can be determined only after f is determined. Therefore, as long as the platform chooses to set p_P before or at the same time as determining f, the supplier cannot change the sequence from Sequence E to Sequence L or S by setting p_S before p_P. This makes Sequence E a stable sequence from which neither platform nor supplier deviates. Consequently, Sequence E is the equilibrium of the timing game in the agency contract scenario, meaning that the platform should determine p_P before or at the same time as determining f. This also indicates that the platform should determine p_P before the supplier determines p_S according to the timeline shown in Fig. 3. □

Proof of Corollary 1. From the equilibrium profits presented in Lemma 2, the following equations and inequalities hold. Note that all the below inequalities hold due to the assumptions of Inequality (2) and $\theta \in (0,1)$.

$$\begin{aligned} &\partial\left(\Pi_A^E - \Pi_A^S\right)/\partial c_S = -\theta^2((1-\theta)a - c_S + \theta c_P)/\left(4b\left(1-\theta^2\right)\left(8+\theta^2\right)\right) < 0 \\ &\partial\left(\Pi_A^E - \Pi_A^S\right)/\partial c_P = \theta^3((1-\theta)a - c_S + \theta c_P)/\left(4b\left(1-\theta^2\right)\left(8+\theta^2\right)\right) > 0 \\ &\partial\left(\Pi_A^E - \Pi_A^L\right)/\partial c_S = -\theta^2\left(3-\theta^2\right)((1-\theta)a - c_S + \theta c_P)/\left(4b\left(1-\theta^2\right)\left(8-5\theta^2+\theta^4\right)\right) < 0 \\ &\partial\left(\Pi_A^E - \Pi_A^L\right)/\partial c_P = \theta^3\left(3-\theta^2\right)((1-\theta)a - c_S + \theta c_P)/\left(4b\left(1-\theta^2\right)\left(8-5\theta^2+\theta^4\right)\right) > 0 \end{aligned}$$ □

Process of numerical analysis under agency contract specifying commission rate in Section 6. Employing backward induction, we solve the game for each sequence of pricing timing, similar to the proofs of Lemmas 1 and 2.

Case (I): Sequence E

In Sequence E, the supplier sets the retail price of its product in the last step. Initially, we maximize Eq. (15) with regard to p_S by using $\partial\pi_A/\partial p_S = 0$, resulting in:

$$p_S = ((1-r)(1-\theta)a + \theta(1-r)p_P + c_S)/(2(1-r)). \tag{A30}$$

We insert Eq. (A30) into Eq. (14), yielding:

$$\begin{aligned}\Pi_A = {} & (r((1-r)(1-\theta)a + (1-r)\theta p_P - c_S)\\ & ((1-r)(1-\theta)a + (1-r)\theta p_P + c_S)\\ & +2(p_P - c_P)(1-r)\\ & \big((1-r)\big(2-\theta-\theta^2\big)a - (1-r)\big(2-\theta^2\big)p_P + \theta c_S\big)\big)\\ & /\big(4b(1-r)^2\big(1-\theta^2\big)\big).\end{aligned} \tag{A31}$$

Next, in Fig. 3, there are three dotted arrows that indicate Sequence E. Intuitively, the prices and profits in equilibrium appear to vary depending on the time point at which the private label price p_P is determined, as indicated by the three different arrows. However, regardless of which of the dotted arrows of Sequence E represents the time at which the price p_P is determined, the values of equilibrium profits are the same because of the envelope theorem, as in the proof of Lemma 2. That is, because the platform maximizes its own identical objective function for the two decision variables p_P and r irrespective of the timing at which it sets prices in Sequence E, the optimal values of p_P and r do not depend on the order of the decisions. Therefore, in Case (I), we consider only the sequence in which the platform determines r first and p_P second, as indicated by the rightmost arrow in Sequence E in Fig. 3.

We maximize Eq. (A31) with regard to p_P by using $\partial\Pi_A/\partial p_P = 0$, resulting in:

$$p_P = \big((1-r)(1-\theta)(2+\theta+r\theta)a + (1-r)\big(2-\theta^2\big)c_P + \theta c_S\big)/\big((1-r)\big(4-(2+r)\theta^2\big)\big). \tag{A32}$$

Substituting Eq. (A32) into Eq. (A31), we express Π_A as a function of r, θ, c_S, c_P, a, and b. Next, we substitute $c_S = c_P = c$ and $a = 10$ into the profit function, and further substitute θ as 0.1, 0.3, 0.5, 0.7, or 0.9 and c as 1, 3, 5, 7, or 9 into the profit function because $\theta \in (0,1)$ and $c \in (0,a)$ are assumed. Then, the profit function becomes a function of only r and b, which enables us to find the optimal value of r that maximizes the function because b appears only as a factor multiplying the entire denominator of the function. Substituting this optimal value of r derived, the fixed values of θ and c, $c_S = c_P = c$, and $a = 10$ into the profit functions of Π_A and π_A, we have the equilibrium profits of the platform and the supplier.

Case (II): Sequence S

We maximize Eq. (14) with respect to p_P and Eq. (15) with respect to p_S, that is, $\partial\Pi_A/\partial p_P = \partial\pi_A/\partial p_S = 0$, yielding:

$$\begin{aligned}p_P &= ((1-r)(1-\theta)(2+\theta+r\theta)a + 2(1-r)c_P + (1+r)\theta c_S)/\big((1-r)\big(4-(1+r)\theta^2\big)\big)\\ p_S &= \big((1-r)\big(2-\theta-\theta^2\big)a + 2c_S + (1-r)\theta c_P\big)/\big((1-r)\big(4-(1+r)\theta^2\big)\big).\end{aligned} \tag{A33}$$

Substituting Eq. (A33) into Eq. (14), we express Π_A as a function of r, θ, c_S, c_P, a, and b. Next, we substitute $c_S = c_P = c$ and $a = 10$ into the profit function, and further substitute θ as 0.1, 0.3, 0.5, 0.7, or 0.9 and c as 1, 3, 5, 7, or 9 into the profit function because $\theta \in (0,1)$ and $c \in (0,a)$ are assumed. Then, the profit function becomes a function only of r and b, which enables us to determine the optimal value of r that maximizes the function because b appears only as a factor multiplying the entire denominator of the function. Substituting the derived optimal value of r, the fixed values of θ and c, $c_S = c_P = c$, and $a = 10$ into the profit functions of Π_A and π_A, we have the equilibrium profits of the platform and the supplier.

Case (III): Sequence L

We maximize Eq. (14) concerning p_P by solving $\partial\Pi_A/\partial p_P = 0$, yielding:

$$p_P = ((1-\theta)a + \theta(1+r)p_S + c_P)/2. \tag{A34}$$

We insert Eq. (A34) into Eq. (15), yielding:

$$\pi_A = \big((1-r)\big(2-\theta-\theta^2\big)a + \big(2-(1+r)\theta^2\big)c_S - (1-r)\big(2\big(2-(1+r)\theta^2\big)p_S - \theta c_P\big)\big)/\big(2b\big(1-\theta^2\big)\big). \tag{A35}$$

We maximize Eq. (A35) regarding p_S by solving $\partial\pi_A/\partial p_S = 0$, resulting in:

$$p_S = \big((1-r)\big(2-\theta-\theta^2\big)a + \big(2-(1+r)\theta^2\big)c_S + \theta(1-r)c_P\big)/\big(2(1-r)\big(2-(1+r)\theta^2\big)\big). \tag{A36}$$

Substituting Eqs. (A34) and (A36) into Eq. (14), we express Π_A as a function of r, θ, c_S, c_P, a, and b. Next, we substitute $c_S = c_P = c$ and $a = 10$ into the profit function. We also substitute θ as 0.1, 0.3, 0.5, 0.7, or 0.9 and c as 1, 3, 5, 7, or 9 into the profit function because $\theta \in (0,1)$ and $c \in (0,a)$ are assumed. Then, the profit function becomes a function of only r and b, which enables us to find the optimal value of r that maximizes the function because b appears only as a factor multiplying the entire denominator of the function. Substituting this optimal value of r derived, the fixed values of θ and c, $c_S = c_P = c$, and $a = 10$ into the profit functions of Π_A and π_A, we have the equilibrium profits of the platform and the supplier.

References

Abhishek, V., Jerath, K., & Zhang, Z. J. (2016). Agency selling or reselling? Channel structures in electronic retailing. *Management Science, 62*(8), 2259–2280.

Anderson, S., & Bedre-Defolie, Ö. (2022). Online trade platforms: Hosting, selling, or both? *International Journal of Industrial Organization, 84*, Article 102861.

Arora, A., & Jain, T. (2024). Data sharing between platform and seller: An analysis of contracts, privacy, and regulation. *European Journal of Operational Research, 313*(3), 1105–1118.

Berthene, A. (2019). Amazon pushes first-party suppliers toward brand registry and seller central. *Digital Commerce, 360*. March 13 https://www.digitalcommerce360.com/2019/03/13/amazon-pushes-first-party-sellers-toward-brand-registry-and-seller-central.

Bisceglia, M., & Padilla, J. (2022). On sellers' cooperation in hybrid marketplaces. *Journal of Economics & Management Strategy, 32*(1), 207–222.

Cao, K., Xu, Y., Hua, Y., & Choi, T.-M. (2023). Supplier or co-optor: Optimal channel and logistics selection problems on retail platforms. *European Journal of Operational Research, 311*(3), 971–988.

Chang, S., Li, A., Wang, X., & Wang, X. (2022). Joint optimization of e-commerce supply chain financing strategy and channel contract. *European Journal of Operational Research, 303*(2), 908–927.

Chen, J., Chen, B., & Li, W. (2018). Who should be pricing leader in the presence of customer returns? *European Journal of Operational Research, 265*(2), 735–747.

Chen, X., Luo, Z., & Wang, X. (2022). Commission pricing strategy on online retail platforms: Power and dependence in triad. *IEEE Transactions on Engineering Management, 69*(6), 2769–2780.

Chen, X., Wang, X., Zhang, D., & Yang, M. (2023a). Selling models for platforms under service-sensitive demand. *Omega, 115*, Article 102794.

Chen, P., Zhao, R., Yan, Y., & Zhou, C. (2021). Promoting end-of-season product through online channel in an uncertain market. *European Journal of Operational Research, 295*(3), 935–948.

Chen, Y., Zhong, Y., & Cheng, T. C. E. (2023b). Impacts of the minimum quantity contract on an online retail platform. *European Journal of Operational Research, 306*(3), 1236–1247.

Cheng, X., Deng, S., Jiang, X., & Li, Y. (2023). Optimal promotion strategies of online marketplaces. *European Journal of Operational Research, 306*(3), 1264–1278.

Chernonog, T., & Levy, P. (2023). Co-creation of mobile app quality in a two-platform supply chain when platforms are asymmetric. *European Journal of Operational Research, 308*(1), 183–200.

DataWeave, (2020). Amazon triples down on its private label product portfolio. Accessed on October 17, 2024. https://dataweave.com/blog/amazon-triples-down-on-its-private%E2%80%91label-product-portfolio.

Davis, D. (2020). Amazon triples its private label product offerings in 2 years. Accessed on October 17, 2024. https://www.digitalcommerce360.com/2020/05/20/amazon-triples-its-private%E2%80%91label-product-offerings-in-2-years/.

Etro, F. (2021). Product selection in online marketplaces. *Journal of Economics & Management Strategy, 30*(3), 614–637.

Fang, C., Chi, M., Fan, S., & Choi, T.-M. (2024). Who should invest in blockchain technology under different pricing models in supply chains? *European Journal of Operational Research, 319*(3), 777–792.

Fu, F., Chen, S., & Yan, W. (2023). Implications of e-tailers' transition from reselling to the combined reselling and agency selling. *Electronic Commerce Research, 23*, 1885–1920.

Gao, S. Y., Lim, W. S., & Ye, Z. (2023). Optimal channel strategy of luxury brands in the presence of online marketplace and copycats. *European Journal of Operational Research, 308*(2), 709–721.

Gong, C., Ignatius, J., Song, H., Chai, J., & Day, S. J. (2024). The impact of platform's information sharing on manufacturer encroachment and selling format decision. *European Journal of Operational Research, 317*(1), 141–155.

Hagiu, A., Teh, T.-H., & Wright, J. (2022). Should platforms be allowed to sell on their own marketplaces? *RAND Journal of Economics, 53*(2), 297–327.

Hagiu, A., & Wright, J. (2015). Marketplace or reseller? *Management Science, 61*(1), 184–203.

Hamilton, J. H., & Slutsky, S. M. (1990). Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. *Games and Economic Behavior, 2*(1), 29–46.

Hao, C., & Yang, L. (2023). Resale or agency sale? Equilibrium analysis on the role of live streaming selling. *European Journal of Operational Research, 307*(3), 1117–1134.

He, P., He, Y., Tang, X., Ma, S., & Xu, H. (2022). Channel encroachment and logistics integration strategies in an e-commerce platform service supply chain. *International Journal of Production Economics, 244*, Article 108368.

Hemmati, M., Ghomi, S. M. T. F., & Sajadieh, M. S. (2024). The impact of advertising and product quality on sales mode choice in the presence of private label. *International Journal of Production Economics, 269*, Article 109164.

Hensell, L. (2023). Understanding the differences between being an Amazon vendor vs. a seller. *Forbes*. January 27 https://www.forbes.com/sites/forbesbusinesscouncil/2023/01/27/understanding-the-differences-between-being-an-amazon-vendor-vs-a-seller/.

Hu, X., Cai, J., & Yue, X. (2024). Power structure preferences in a dual-channel supply chain: Demand information symmetry vs. asymmetry. *European Journal of Operational Research, 314*(3), 113–126.

Hu, S., & Fu, K. (2025). Equilibrium analysis of the seller's fulfillment channels and sales channels. *European Journal of Operational Research, 320*(3), 544–558.

Huang, Z., Zhang, W., & Liu, B. (2023). Strategic introduction of private brand in the online retail platform. *RAIRO-Operations Research, 57*(3), 1377–1396.

Ingene, C. A., & Parry, M. E. (2004). *Mathematical models of distribution channels*. Springer.

Ji, G., Fu, T., & Li, S. (2023). Optimal selling format considering price discount strategy in live-streaming commerce. *European Journal of Operational Research, 309*(2), 529–544.

Karray, S., & Martín-Herrán, G. (2022). The impact of a store brand introduction in a supply chain with competing manufacturers: The strategic role of pricing and advertising decision timing. *International Journal of Production Economics, 244*, Article 108378.

Kinsella, B. (2017). Xiaomi MI AI smart speaker takes on Alibaba Tmall Genie and Linglong Dingdong. *Voicebot.ai*. July 26 https://voicebot.ai/2017/07/26/xiaomi-mi-ai-smart-speaker-takes-alibaba-tmall-genie-linglong-dingdong/.

Kittaka, Y., Matsushima, N., & Saruta, F. (2022). Negative effect of price-matching policy on traditional retailers in a dual-channel supply chain with different content formats. *Transportation Research Part E: Logistics and Transportation Review, 161*, Article 102682.

Lam, W. M. W., & Liu, X. (2023). Dancing with rivals: How does platform's information usage benefit independent sellers? *European Journal of Operational Research, 309*(1), 421–431.

Lee, M. (2017). Alibaba launches voice assistant Tmall Genie. *Technode*. July 5 https://technode.com/2017/07/05/alibaba-launches-voice-assistant-tmall-genie/.

Li, H., Chen, H., Chai, J., & Shi, V. (2023a). Private label sourcing for an e-tailer with agency selling and service provision. *European Journal of Operational Research, 305*(1), 114–127.

Li, Y., Chu, M., & Bai, X. (2024a). To fight or not? Product introduction and channel selection in the presence of a platform's private label. *Transportation Research Part E: Logistics and Transportation Review, 181*, Article 103373.

Li, D., Liu, Y., Hu, J., & Chen, X. (2021a). Private-brand introduction and investment effect on online platform-based supply chains. *Transportation Research Part E: Logistics and Transportation Review, 155*, Article 102494.

Li, J., Shen, W., Liao, Y., Cai, G., & Chen, X. (2024c). The fulfillment service in online marketplaces. *European Journal of Operational Research, 315*(3), 1139–1152.

Li, P., Tan, D., Wang, G., Wei, H., & Wu, J. (2021b). Retailer's vertical integration strategies under different business modes. *European Journal of Operational Research, 294*(3), 965–975.

Li, J., Wang, H., Shi, V., & Sun, Q. (2024b). Manufacturer's choice of online selling format in a dual-channel supply chain with green products. *European Journal of Operational Research, 318*(1), 131–142.

Li, Q., Zha, Y., & Dong, Y. (2023b). Subsidize or not: The competition of credit card and online credit in platform-based supply chain system. *European Journal of Operational Research, 305*(2), 644–658.

Lin, J., Ma, X., Talluri, S., & Yan, C.-H. (2021). Retail channel management decisions under collusion. *European Journal of Operational Research, 294*(2), 700–710.

Liu, J., & Ke, H. (2020). Firms' pricing strategies under different decision sequences in dual-format online retailing. *Soft Computing, 24*, 7811–7826.

Liu, M., Liang, K., Perera, S., Huang, R., & Ghose, S. (2022). Game theoretical analysis of service effort timing scheme strategies in dual-channel supply chains. *Transportation Research Part E: Logistics and Transportation Review, 158*, Article 102620.

Liu, Y., Lu, J., & Yan, N. (2024). Agency or reselling? Supplier's online channel strategies with platform financing. *European Journal of Operational Research, 318*(3), 1014–1027.

Liu, X., Shao, L., & Qin, X. (2025). Platform acquisition in a triple-channel supply chain. *European Journal of Operational Research, 324*(3), 1035–1046.

Liu, H., Xu, T., Jing, S., Liu, Z., & Wang, S. (2023). The interplay between logistics strategy and platform's channel structure design in B2C platform market. *European Journal of Operational Research, 310*(2), 812–833.

Long, F., & Amaldoss, W. (2024). Self-preferencing in e-commerce marketplaces: The role of sponsored advertising and private labels. *Marketing Science, 43*(5), 925–952.

Lyu, Z., Lan, H., Liu, S. S., Hua, G., & Cheng, T. C. E. (2025). When should the fresh-food retailer embrace near-expired food commercialization? *Journal of Retailing and Consumer Services, 82*, Article 104081.

Mandal, P., Basu, P., Choi, T.-M., & Rath, S. B. (2024). Platform financing vs. bank financing: Strategic choice of financing mode under seller competition. *European Journal of Operational Research, 315*(1), 130–146.

Marker, (2020). The stealthy rise of amazon's private label empire, by the numbers. Accessed on October 17, 2024. https://marker.medium.com/the-stealthy-rise-of-amazons-private-label-empire-by-the-numbers-39875e079ed.

Matsui, K. (2017). When should a manufacturer set its direct price and wholesale price in dual-channel supply chains? *European Journal of Operational Research, 258*(2), 501–511.

Matsui, K. (2020). Optimal bargaining timing of a wholesale price for a manufacturer with a retailer in a dual-channel supply chain. *European Journal of Operational Research, 287*(1), 225–236.

Matsui, K. (2024). Should competing suppliers with dual-channel supply chains adopt agency selling in an e-commerce platform? *European Journal of Operational Research, 312*(2), 587–604.

Mattioli, D. (2020). Amazon scooped up data from its own sellers to launch competing products. *Wall Street Journal*. April 23 https://www.wsj.com/articles/amazon-scooped-up-data-from-its-own-sellers-to-launch-competing-products-11587650015.

Niu, B., Ruan, Y., & Xu, H. (2024). Channel separation to mitigate triple competition and combat copycats in agency e-commerce. *European Journal of Operational Research, 315*(1), 242–270.

Numerator, (2019). Amazon's private label proliferation. Accessed on October 17, 2024. https://www.numerator.com/resources/report/amazons-private-label-proliferation.

Pierce, D. (2017). Anker's Genie is just like the Echo Dot, only cheaper. *Wired*. August 9 https://www.wired.com/story/ankers-genie-is-just-like-the-echo-dot-only-cheaper/.

Pu, X., Sun, S., & Shao, J. (2020). Direct selling, reselling, or agency selling? Manufacturer's online distribution strategies and their impact. *International Journal of Electronic Commerce, 24*(2), 232–254.
Qin, X., Liu, Z., & Tian, L. (2021). The optimal combination between selling mode and logistics service strategy in an e-commerce market. *European Journal of Operational Research, 289*(2), 639–651.
Reza-Gharehbagh, R., Khouja, M., & Hammami, R. (2025). Retailer-manufacturer partnerships in e-commerce: Dual product strategy and market share dynamics. *European Journal of Operational Research, 322*(1), 85–104.
Shen, Y., Willems, S. P., & Dai, Y. (2019). Channel selection and contracting in the presence of a retail platform. *Production and Operations Management, 28*(5), 1173–1185.
Shopova, R. (2023). Private labels in marketplaces. *International Journal of Industrial Organization, 89*, Article 102949.
Siqin, T., Choi, T.-M., & Chung, S.-H. (2022). Optimal E-tailing channel structure and service contracting in the platform era. *Transportation Research Part E: Logistics and Transportation Review, 160*, Article 102614.
Song, H., Wang, R., & Tang, Y. (2024). Competition or cooperation: Strategy analysis for a social commerce platform. *European Journal of Operational Research, 318*(3), 560–574.
Sui, R., Zhang, X., Dan, B., Zhang, H., & Liu, Y. (2023). Bilateral value-added service investment in platform competition with cross-side network effects under multihoming. *European Journal of Operational Research, 304*(3), 952–963.
Sun, L., Jiao, X., Guo, X., & Yu, Y. (2022). Pricing policies in dual distribution channels: The reference effect of publicized official prices. *European Journal of Operational Research, 296*(1), 146–157.
Tan, Y., & Carrillo, J. E. (2017). Strategic analysis of the agency model for digital goods. *Production and Operations Management, 26*(4), 724–741.
Tan, Y., Yu, C., Liu, Y., & Zheng, Q. (2024). Agency models in online platforms: A review of recent developments and future prospects. *European Journal of Operational Research, 319*(3), 679–695.
Tian, L., Vakharia, A. J., Tan, Y., & Xu, Y. (2018). Marketplace, reseller, or hybrid: Strategic analysis of an emerging e-commerce model. *Production and Operations Management, 27*(8), 1595–1610.
van Damme, E., & Hurkens, S. (2004). Endogenous price leadership. *Games and Economic Behavior, 47*(2), 404–420.
Wang, K., Lin, J., Zhang, Q., & Li, J. (2023). Store brand introduction under platform financing and competition. *Transportation Research Part E: Logistics and Transportation Review, 176*, Article 103209.
Wang, Y.-Y., Zhao, J., Wang, J.-C., & Lai, F. (2024). Strategic choice of commission formats in a supply chain with a platform and competing manufacturers. *IEEE Transactions on Engineering Management, 71*, 9661–9678.
Waters, M. (2021). Amazon briefing: Private-label products are winning search results. Accessed on October 17, 2024. https://www.modernretail.co/platforms/amazon-briefing-private-label-products-are-winning-search-results/.
Wei, Y., & Dong, Y. (2022). Product distribution strategy in response to the platform retailer's marketplace introduction. *European Journal of Operational Research, 303*(2), 986–996.
Wei, J., Lu, J., & Zhao, J. (2020). Interactions of competing manufacturers' leader-follower relationship and sales format on online platforms. *European Journal of Operational Research, 280*(2), 508–522.
Wei, L., Zheng, S., Du, S., & Zhang, B. (2025). Agency selling or reselling: The role of cause marketing. *European Journal of Operational Research, 320*(2), 343–357.
Wu, J., Lu, W., & Ji, X. (2023). The interactions between upstream quality disclosure and downstream entry. *European Journal of Operational Research, 309*(2), 545–559.
Wu, X., & Zha, Y. (2024). Optimal advertising structure in the presence of a platform owner's entry. *Transportation Research Part E: Logistics and Transportation Review, 191*, Article 103745.
Xi, X., & Zhang, Y. (2023). The interplay between marketplace channel addition and pricing strategy in an e-commerce supply chain. *International Journal of Production Economics, 258*, Article 108807.
Xu, X., Chen, X., Choi, T.-M., & Cheng, T. C. E. (2024). Platform financing versus bank financing: "when to choose which" for green production systems. *European Journal of Operational Research, 317*(2), 515–532.
Xu, X., He, P., Zhou, L., & Cheng, T. C. E. (2023). Coordination of a platform-based supply chain in the marketplace or reselling mode considering cross-channel effect and blockchain technology. *European Journal of Operational Research, 309*(1), 170–187.
Yan, N., Liu, Y., Xu, X., & He, X. (2020). Strategic dual-channel pricing games with e-retailer finance. *European Journal of Operational Research, 283*(1), 138–151.
Yan, Y., Zhao, R., & Liu, Z. (2018). Strategic introduction of the marketplace channel under spillovers from online to offline sales. *European Journal of Operational Research, 267*(1), 65–77.
Yenipazarli, A. (2021). The marketplace dilemma: Selling to the marketplace vs. selling on the marketplace. *Naval Research Logistics, 68*(6), 761–778.
Zennyo, Y. (2022). Platform encroachment and own-content bias. *Journal of Industrial Economics, 70*(3), 684–710.
Zhang, J., Cao, Q., & He, X. (2019). Contract and product quality in platform selling. *European Journal of Operational Research, 272*(3), 928–944.
Zhang, X., & Hou, W. (2022). The impacts of e-tailer's private label on the sales mode selection: From the perspectives of economic and environmental sustainability. *European Journal of Operational Research, 296*(2), 601–614.
Zhang, Y., Huang, M., Tian, L., Cai, G. G., Jin, D., & Fan, Z. (2023a). Manufacturer's product line selling strategy and add-on policy in product sharing. *European Journal of Operational Research, 308*(3), 1332–1343.
Zhang, X., Li, G., Liu, M., & Sethi, S. P. (2021). Online platform service investment: A bane or a boon for supplier encroachment. *International Journal of Production Economics, 235*, Article 108079.
Zhang, Z., Xu, H., Chen, K., Zhao, Y., & Liu, Z. (2023b). Channel mode selection for an e-platform supply chain in the presence of a secondary marketplace. *European Journal of Operational Research, 305*(3), 1215–1235.
Zhang, S., & Zhang, J. (2020). Agency selling or reselling: E-tailer information sharing with supplier offline entry. *European Journal of Operational Research, 280*(1), 134–151.
Zhen, X., & Xu, S. (2022). Who should introduce the third-party platform channel under different pricing strategies? *European Journal of Operational Research, 299*(1), 168–182.
Zhen, X., Xu, S., Li, Y., & Shi, D. (2022). When and how should a retailer use third-party platform channels? The impact of spillover effects. *European Journal of Operational Research, 301*(2), 624–637.
Zhu, H., Ni, Y., & Xiao, Y. (2025). Information sharing across competing platforms with varying information capabilities. *European Journal of Operational Research, 323*(1), 125–138.
Zhu, W., Xie, J., Xia, Y., Wei, L., & Liang, L. (2023). Getting more third-party participants on board: Optimal pricing and investment decisions in competitive platform ecosystems. *European Journal of Operational Research, 307*(1), 177–192.
Zou, T., & Zhou, B. (2024). Self-preferencing and search neutrality in online retail platforms. *Management Science, 71*(5), 4087–4107.