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Characteristic seepage flow through a test specimen of an anisotropic porous medium – Ellipse of coefficient of permeability and induced hydraulic gradient – Revised version

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Abstract: Anisotropic permeability of soil represents a problem in seepage flow mechanics, such as resistance to seepage failure of soil, quantity of discharge, and location of the free surface in an embankment dam. In this study, the theory of one-dimensional confined flow (1DCF) is initially presented, and then the coefficient of permeability with respect to 1DCF, $k_{n\ 1DCF}$, and the induced hydraulic gradient, $i_{s\ IND}$, are given in the $n - s$ plane. Second, seepage flow through a test specimen, the flow direction of which has an angle to the principal direction of the permeability tensor, is analyzed by the finite element method. Measured values of $i_{s\ IND}$ and $k_{n\ 1DCF}$ are then investigated within the specimen for various cases. The results showed that a large value of B/D is required for accurate estimation of $i_{s\ IND}$ and $k_{n\ 1DCF}$, where B and D are the length and height of the test specimen, respectively. Effects of a regulating filter settled at both ends of the test specimen were also investigated. We can conclude that the regulating filter is an effective method for placing a specimen under the one-dimensional confined flow condition. This paper is a revised version of our previous paper, published in the Treatises in Honor of Professor Takashi Hasegawa July 1999, Kyoto University, with the same title by the same authors with some modifications.

Key words: Seepage flow, Anisotropic permeability, One-dimensional permeability test, Permeability tensor, One-dimensional confined flow, Induced hydraulic gradient

1. INTRODUCTION

Microscopic properties such as shapes and/or orientations of sedimentary solid soil particles have marked effects on seepage flow through porous media. From a macroscopic point of view, a body of soil shows anisotropic properties with respect to the mechanics of soil. Anisotropic permeabilities of soil cause characteristic phenomena of flow through the soil. Many researchers investigated anisotropic properties with respect to the permeability of soil, and conducted one-dimensional permeability tests for anisotropic soils. Almost all research, however, considered the case where the direction of flow is coincident with the principal direction of the permeability tensor in permeability tests on anisotropic soil.

Let us consider a one-dimensional permeability test on anisotropic soil in a laboratory to obtain the components of the permeability tensor: k_{xx} , k_{zx} , k_{xz} , and k_{zz} in the $x - z$ plane. Permeability tests with respect to two different directions are required to obtain the permeability tensor for the two-dimensional case. Consider permeability tests on two samples with flow in x and z directions in an orthogonal $x - z$ coordinate system. From a permeability test in the x direction, i_x , v_x , and $i_{z\ IND}$ are measured and $k_{x\ 1DCF}$ ($= v_x/i_x$) is calculated, and from the test in the z direction, i_z , v_z , $i_{x\ IND}$, and $i_{z\ 1DCF}$ ($= v_z/i_z$) are similarly obtained. Using these values, we can derive all components of the permeability tensor with respect to the $x - z$ plane (Tanaka

et al., 1995):

< From permeability tests on anisotropic soil in the $x - z$ plane >

$$k_{xx} = k_{x \text{ 1DCF}} \frac{1}{1 - \left(\frac{i_{z \text{ IND}}}{i_x} \right) \left(\frac{i_{x \text{ IND}}}{i_z} \right)} \quad (1)$$

$$k_{zx} = k_{x \text{ 1DCF}} \frac{\left(\frac{i_{x \text{ IND}}}{i_z} \right)}{\left(\frac{i_{z \text{ IND}}}{i_x} \right) \left(\frac{i_{x \text{ IND}}}{i_z} \right) - 1} \quad (2)$$

$$k_{xz} = k_{z \text{ 1DCF}} \frac{\left(\frac{i_{z \text{ IND}}}{i_x} \right)}{\left(\frac{i_{z \text{ IND}}}{i_x} \right) \left(\frac{i_{x \text{ IND}}}{i_z} \right) - 1} \quad (3)$$

$$k_{zz} = k_{z \text{ 1DCF}} \frac{1}{1 - \left(\frac{i_{z \text{ IND}}}{i_x} \right) \left(\frac{i_{x \text{ IND}}}{i_z} \right)} \quad (4)$$

where $i_{z \text{ IND}}$ is the hydraulic gradient perpendicular to the flow direction x which is induced by flow through an anisotropic porous medium, $k_{x \text{ 1DCF}}$ is a coefficient of permeability with respect to the flow direction x , and $i_{x \text{ IND}}$ and $k_{z \text{ 1DCF}}$ are similarly defined for the one-dimensional permeability test in direction z . These values are discussed in detail in the following section. The Eqs.(1) to (4) are valid for flow along a stream line or through a cylinder having an infinitesimal cross-sectional area. However, if a seepage cylinder has a finite area of cross-section, the question arises:

whether the theoretical values of $i_{z \text{ IND}}$, $k_{x \text{ 1DCF}}$, $i_{x \text{ IND}}$, and $k_{z \text{ 1DCF}}$ are indeed obtained from permeability tests through the seepage cylinder?

In this study, we consider the case where the direction of water flow in a one-dimensional permeability test is at an angle to the principal direction of the permeability tensor. From FEM seepage flow analyses, characteristic flows through a one-dimensional test soil with anisotropic permeability are investigated. The ellipse of permeability and induced hydraulic gradient are then discussed from theoretical and practical points of view.

2. THEORY OF ONE-DIMENSIONAL CONFINED FLOW

< 1DCF tests in the n direction, where the $n - s$ plane is rotated as α counterclockwise to the $X - Z$ axes >

Consider a special case or the $X - Z$ plane where X and Z are principal directions of the permeability tensor. Then, let the $n - s$ plane be rotated as α counterclockwise to the $X - Z$ axes, as shown in Fig.1. More precisely, α is the angle when the principal direction X is rotated counterclockwise to align with the flow direction n . When a one-dimensional test specimen is cut from anisotropic soil in direction n and subjected to a one-dimensional permeability test, the hydraulic gradient $i_{s \text{ IND}}$ is induced in direction s perpendicular to the flow direction (see Fig.2):

$$i_{s \text{ IND}} = -\frac{k_{ns}}{k_{ss}} i_n \quad (5)$$

The occurrence of flow caused by $i_{s \text{ IND}}$ cancels flow in direction s and flow

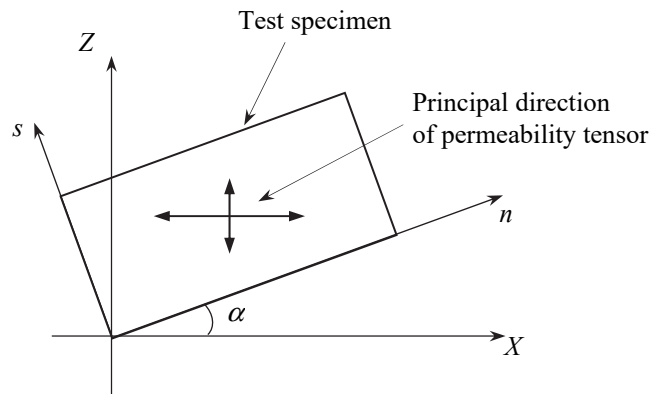


Fig.1 Anisotropic soil in the $X - Z$ plane

perpendicular to n , so v_s does not occur, in which v_s is the velocity of flow in direction s . Thus, flow through a cylinder or tube is eventually confined in the one-dimensional direction n or in the direction of a stream line. The flow of water along a stream line is confined one-dimensionally in the flow direction. We refer to the flow as One-Dimensional Confined Flow (or 1DCF). As a result, the coefficient of permeability with respect to 1DCF, $k_{n\text{ 1DCF}}$, is obtained:

$$k_{n\text{ 1DCF}} = k_{nn} - \frac{k_{ns} k_{sn}}{k_{ss}} \quad (6)$$

We refer to the value $i_{s\text{ IND}}$ as the induced hydraulic gradient, and the value $k_{n\text{ 1DCF}}$ as the coefficient of permeability with respect to 1DCF.

Using components of the permeability tensor k_{XX} and k_{ZZ} , the value $i_{z\text{ IND}}$ is expressed as:

$$i_{s\text{ IND}} = -\frac{(k_{ZZ} - k_{XX}) \sin \alpha \cos \alpha}{k_{XX} \sin^2 \alpha + k_{ZZ} \cos^2 \alpha} \cdot i_n, \quad (7)$$

which shows that a gradient is induced in the direction perpendicular to the direction of flow. Similarly, $k_{n\text{ 1DCF}}$ is expressed as:

$$k_{n\text{ 1DCF}} = \frac{k_{XX} k_{ZZ}}{k_{XX} \sin^2 \alpha + k_{ZZ} \cos^2 \alpha} \quad (8)$$

which is the equation of an ellipse, as shown in Fig.3. $k_{n\text{ 1DCF}}$ (Eq.(8)) is the coefficient of permeability of one-dimensional confined flow along the stream line in direction n . The presence of $k_{n\text{ 1DCF}}$ was supported experimentally by Scheidegger (1974). The theory of 1DCF can be precisely applied to the flow of water along a stream line or tube with an infinitesimal cross-sectional area. When a tube or cylinder for flow has a finite cross-sectional area, such as one-dimensional permeability test specimen, hydraulic heads theoretically have to change at the ends of the test specimen. Conversely, the pattern of flow through the cylinder may deviate from the theory, because the boundary conditions may impose the absence of hydraulic gradients at the two ends of the sample. The state of flow through a tube of an anisotropic porous media is discussed in the following section in detail.

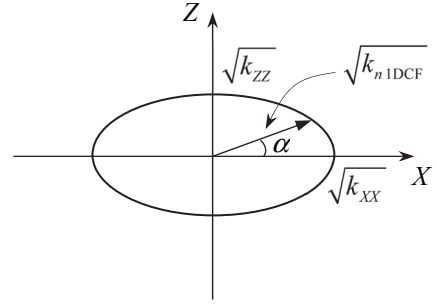


Fig.3 Ellipse of permeability

3. ANALYTICAL RESULTS

Here, we consider the case where X and Z are principal directions and the ratio of two principal values of coefficients of permeability, k_{XX} and k_{ZZ} ($= 0.5 \times 10^{-4}$ m/s); k_{XX}/k_{ZZ} is equal to 1/3, 2, 3, 9, and 25. We analyzed cases where ratios of the length B to height D ($= 2$ m) of a specimen are 1, 2, 3, 5, 7, and 10 cut from anisotropic soil at angles from 0 to 90 degrees, increasing by 15 degrees to the X axis (Fig.1). Let us consider the case where we conduct one-dimensional permeability tests for such specimens.

Tanaka et al. (1995) found that seepage flow through a test specimen can be quite different depending on two flow or boundary conditions: Exerted-pressure type (E-P type) and Uniform-Flow type (U-F type). For the U-F type, seepage flow is precisely in the state of one-dimensional confined flow irrespective of B/D values. However, for the E-P type, the seepage flow differs from the theory. Theoretically speaking, the U-F type test would be better than E-P type test for a permeability test on anisotropic soil, but such a test is impossible to perform. The permeability test is usually conducted for the E-P type because the boundary conditions of such a test are more realistic. Here, the numerical results using FEM analyses for E-P type tests are discussed in detail.

The rate of discharge flowing through a specimen, $q_n^{(m)\text{ 1DCF}}$, and an induced hydraulic gradient in the s direction, $i_{s\text{ IND}}^{(m)}$, are calculated at the center of the specimen. The hydraulic gradient in the n direction, $i_n^{(m)}$, is calculated by the difference between hydraulic heads at both ends of the specimen, h , divided by the

length B . The coefficient of permeability with respect to 1DCF in the n direction $k_n^{(m)}_{1DCF}$ is calculated as :

$$k_n^{(m)}_{1DCF} = q_n^{(m)}_{1DCF} / (i_n^{(m)} \cdot D) \quad (9)$$

3.1 Ellipse of coefficient of permeability (k_n_{1DCF})

Figure 4 shows the ellipses of permeability by theory shown by the solid line and numerical analyses shown in the circle for $B/D = 2, 5$, and 10 , $k_{XX}/k_{ZZ} = 1/3, 2$, and 25 , and $\alpha =$ various values in the E-P type. From Fig.4, when B/D is larger than 5, the numerical results are nearly equal to theoretical ones. So, we can obtain reasonable results using E-P type tests for these cases. The difference between the values by theory and numerical analyses may, however, become large depending on the cut angles of specimens when k_{XX}/k_{ZZ} is large. Additionally, when B/D is smaller than 2, the difference between theoretical and numerical values becomes large, especially in the case when k_{XX}/k_{ZZ} is large. Thus, in order to obtain accurate values of $k_n^{(m)}_{1DCF}$ from E-P type tests, it would be better to make the value B/D of a seepage cylinder large.

3.2 Induced hydraulic gradient ($i_{s\text{ IND}}$)

Figure 5 shows induced hydraulic gradients in theory shown by the solid line and numerical results shown in the circle for $B/D = 2, 5$, and 10 , $k_{XX}/k_{ZZ} = 1/3, 2$, and 25 , and $\alpha =$ various values in the E-P type. Refer to **Appendix A1** and **A2** for the sign (positive or negative values) of $i_{s\text{ IND}}$. From Fig.5, when B/D is larger than 5, the numerical results are nearly equal to theoretical ones. So, we can obtain reasonable values using E-P type tests for this case. The difference between the theoretical and analytical results may, however,

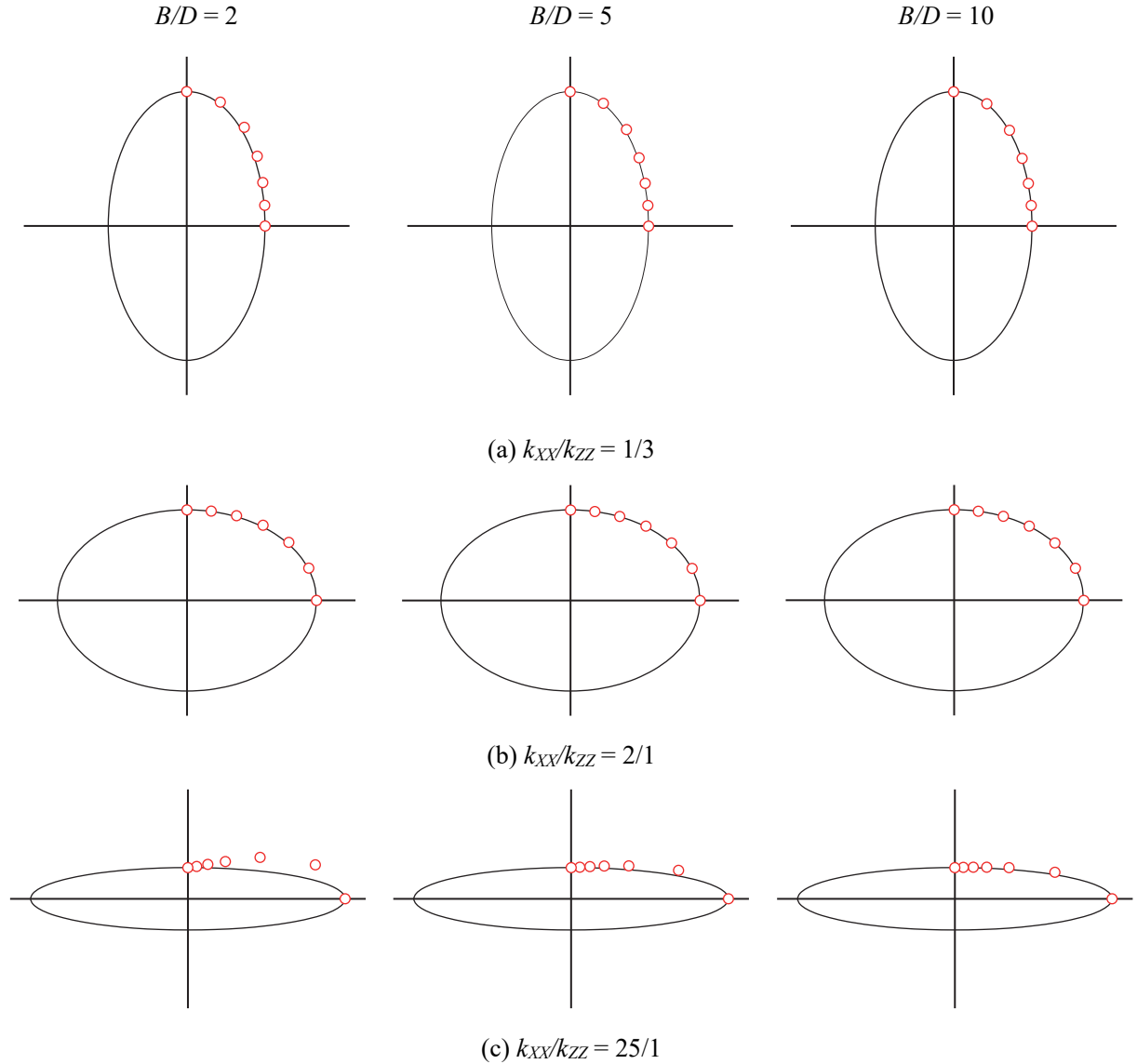


Fig.4 Ellipses of permeability, k_n_{1DCF}

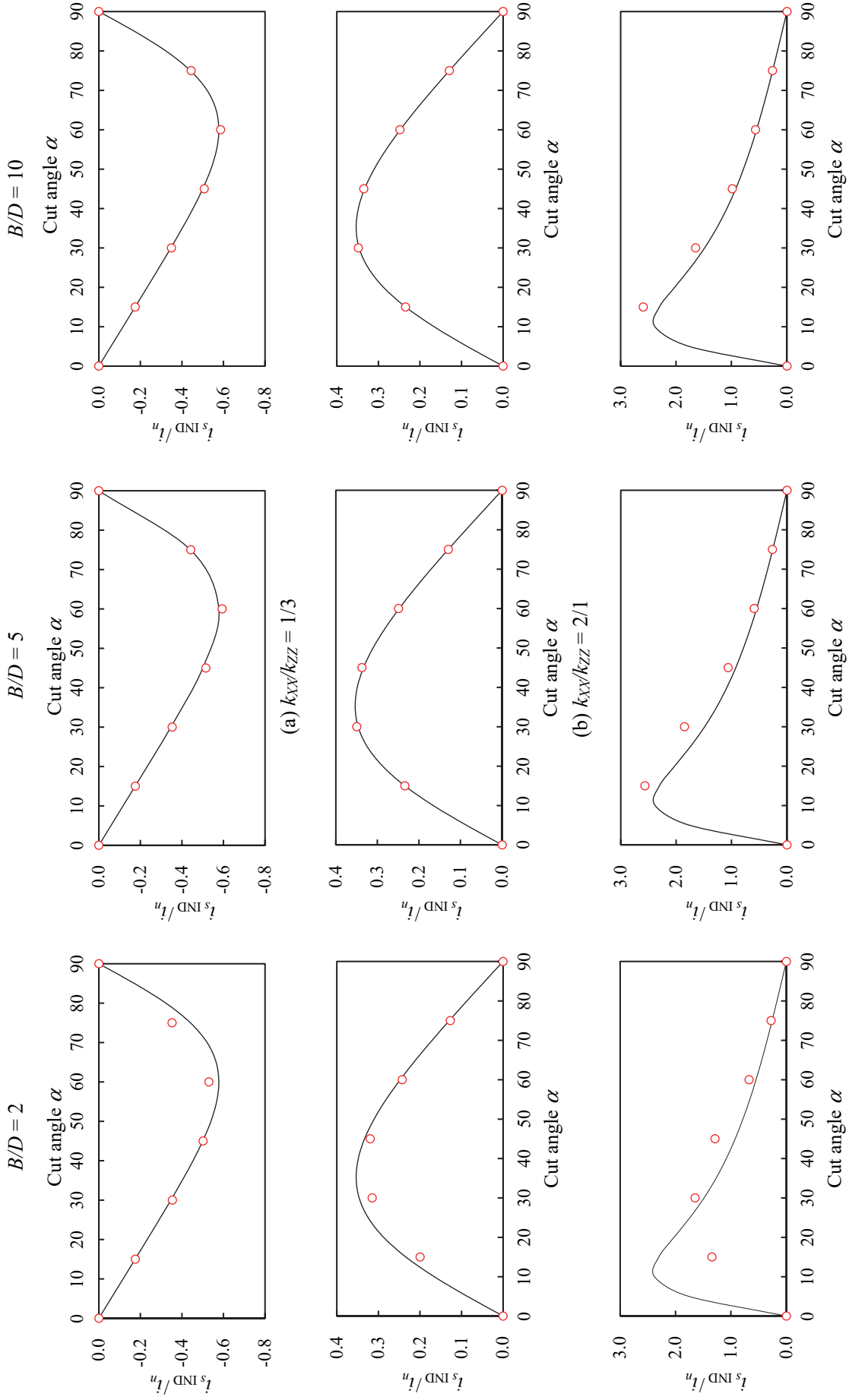


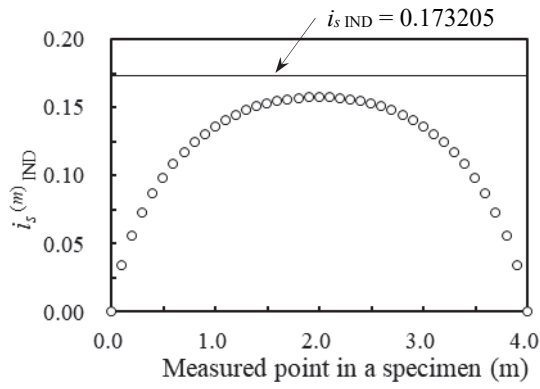
Fig.5 Distribution of induced hydraulic gradient i_s^{IND}/i_n with a cut angle α

become large depending on the cut angles of specimens when k_{XX}/k_{ZZ} is large. Furthermore, when B/D is smaller than 2, the difference between theoretical and numerical values becomes large, especially when k_{XX}/k_{ZZ} is large.

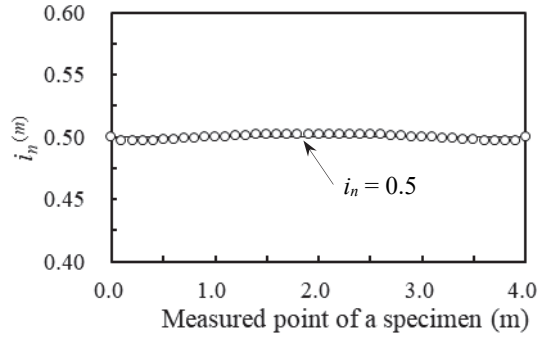
Now, let us closely consider the state of flow through a specimen. Figure 6 shows distributions of $i_s^{(m)}_{IND}$, $i_n^{(m)}$, and $i_s^{(m)}_{IND} / i_n^{(m)}$ for $B/D = 2$, $k_{XX}/k_{ZZ} = 2$, and $\alpha = 30^\circ$, where induced hydraulic gradients, $i_s^{(m)}_{IND}$, are measured through a specimen, and longitudinal hydraulic gradients, $i_n^{(m)}$, are calculated for the part of a specimen within the point from both ends. As shown in Fig.6, it is found that $i_s^{(m)}_{IND}$ becomes a theoretical value at the middle part of the specimen. When B/D is smaller than 2, the value of $i_s^{(m)}_{IND}$ is largely different from the theoretical value $i_{s\ IND}$ even at the middle part of the specimen (see Fig.5).

Thus, in order to obtain precise values of $i_s^{(m)}_{IND}$ and $k_n^{(m)}_{IDCF}$ from E-P type tests, it would be better to make the value B/D of a seepage cylinder large.

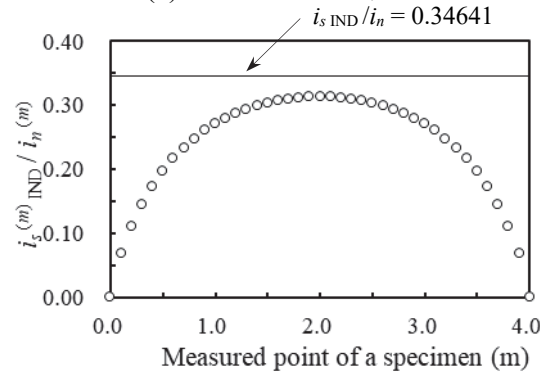
We can now think of another way of getting a flow pattern within a test specimen to approach the theory of IDCF by settling a regulating filter at both ends of the specimen.



(a) Distribution of $i_s^{(m)}_{IND}$

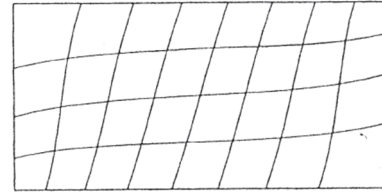


(b) Distribution of $i_n^{(m)}$

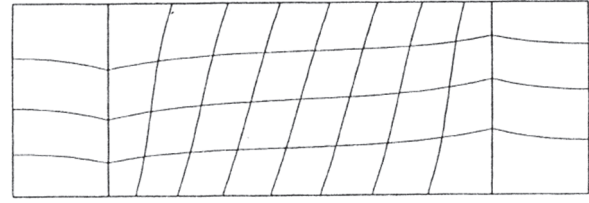


(c) Distribution of $i_s^{(m)}_{IND} / i_n^{(m)}$

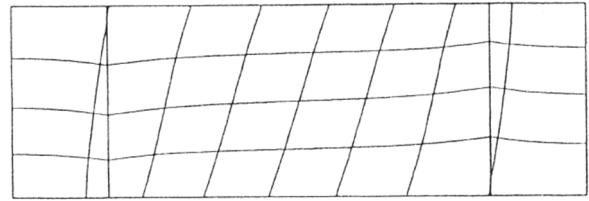
Fig.6 Precise distributions of $i_s^{(m)}_{IND}$, $i_n^{(m)}$ and $i_s^{(m)}_{IND} / i_n^{(m)}$ ($B/D=2$, $k_{XX}/k_{ZZ} = 2$ and $\alpha = 30^\circ$)



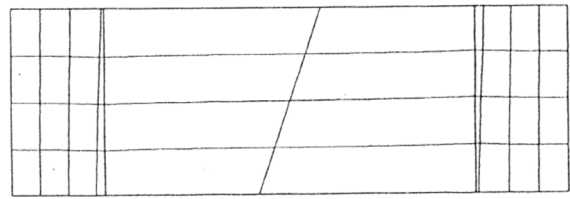
(a) without a filter (The E-P type)



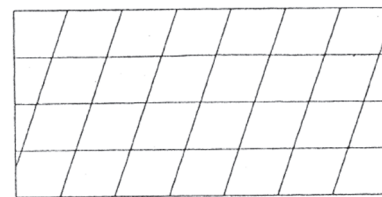
(b) $k_F / k_{ZZ} = 20.0$



(c) $k_F / k_{ZZ} = 2.0$



(d) $k_F / k_{ZZ} = 0.2$



(e) Without a filter (the U-F type)

Fig.7 Flow nets obtained regulating filters

4. EFFECTS OF REGULATING FILTERS

< 1DCF tests in the n direction, where the $n-s$ plane is rotated as θ clockwise to the $X-Z$ plane with a regulating filter at both ends of a test specimen >

Now, let us consider the case where we settle filters with isotropic permeability at both ends of a test specimen to regulate flow through the soil. We analyzed cases of coefficients of isotropic permeability of filter k_F : 0.001, 0.1, 0.5, 1.0, 2.0, 5.0, and 10.0×10^{-4} m/s. Flow nets for cases: $k_F/k_{ZZ} = 20, 2$, and 0.2 are shown in Fig.7, in which the flow net without filters and flow net by the theory of 1DCF are given. It can be noted from Fig.7 that flow through the test specimen approaches the theoretical one-dimensional confined flow as the value k_F/k_{ZZ} becomes smaller, where k_{ZZ} is the minimum principal coefficient of permeability in two dimensions. The flow net of Fig.7(e) without a filter (the U-F type) coincides with one by theory of 1DCF. Figures 8 and 9 show the relationships between k_F/k_{ZZ} and $i_{s\text{ IND}}/i_n$, and k_F/k_{ZZ} and $k_{n\text{ 1DCF}}$, respectively. Refer to **Appendix A3** for the sign (positive or negative values) of $i_{s\text{ IND}}$. From Figs.8 and 9 we can see that as k_F/k_{ZZ} is decreasing, $i_{s\text{ IND}}^{(m)}/i_n^{(m)}$ and $k_{n\text{ 1DCF}}^{(m)}$ approach the theoretical values, respectively. For the value of $k_F/k_{ZZ} < 0.2$, $i_{s\text{ IND}}^{(m)}/i_n^{(m)}$ approaches the theoretical value of $i_{s\text{ IND}}/i_n$ within about 1% error, and for $k_F/k_{ZZ} < 0.4$, $k_{n\text{ 1DCF}}^{(m)}$ approaches the theoretical value of $k_{n\text{ 1DCF}}$ again within about 1% error.

5. CONCLUSION

Seepage flow through a test specimen cut from anisotropic soil at an angle to the principal direction of the permeability tensor was discussed. We can conclude that, in order to obtain exact values of $k_{n\text{ 1DCF}}$ and $i_{s\text{ IND}}$ from Exerted-Pressure type tests, it would be better to make the value of B/D larger, e.g., larger than 5 for our case treated here. It is, however, very difficult for us to make a test sample of a larger value of B/D or cut such a long specimen from block samples. So, the B/D value of a test specimen may have a value of about 2. We could discuss only the observed values at the middle part of a specimen, but just the value of $i_{s\text{ IND}}$ measured at the middle of a specimen markedly deviates from the theory. When a permeability test is conducted with a specimen of $B/D = 2$, it is effective to settle regulating filters with isotropic permeability at both ends of the test specimen. It can be concluded that the condition $k_F/k_{ZZ} < 0.2$ is more effective as a regulating filter, where k_F is the coefficient of isotropic permeability of regulating filter, and k_{ZZ} is the minimum principal coefficient of permeability of test soil.

An abstract in Japanese is given in **Appendix B**.

Appendix A Explanation of the sign of $i_{s\text{ IND}}$

- A1 Test specimen shown in Fig.5(a)
- A2 Test specimen shown in Figs.5(b), (c), and Figs.6(a), (c)
- A3 Test specimen shown in Fig.8

Appendix B Abstract in Japanese

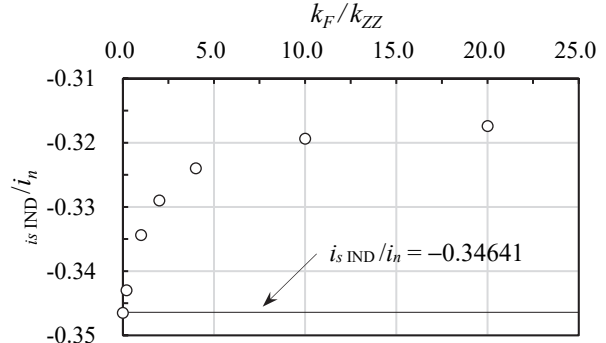


Fig.8 Relationship between k_F/k_{ZZ} and $i_{s\text{ IND}}/i_n$

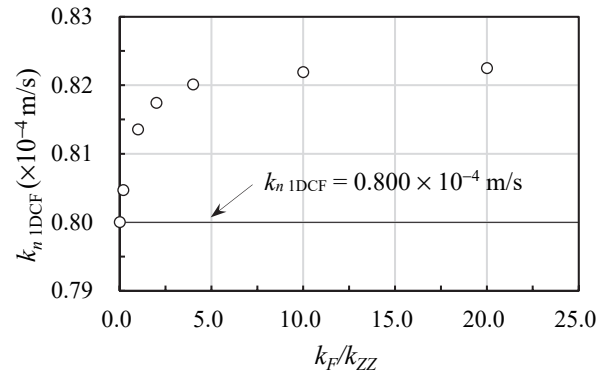
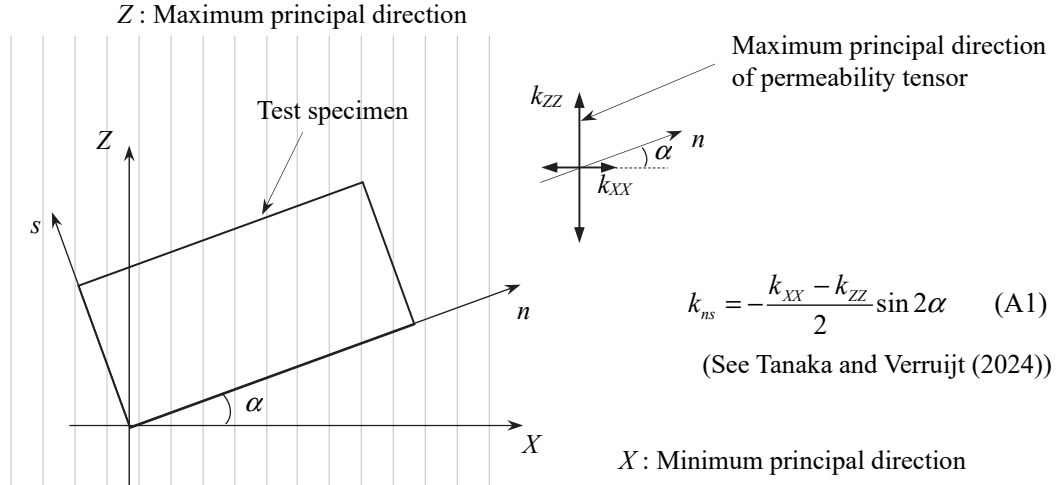


Fig.9 Relationship between k_F/k_{ZZ} and $k_{n\text{ 1DCF}}$

Appendix A Explanation of the sign of $i_{s \text{ IND}}$

A1 The test specimen shown in Fig.5(a)

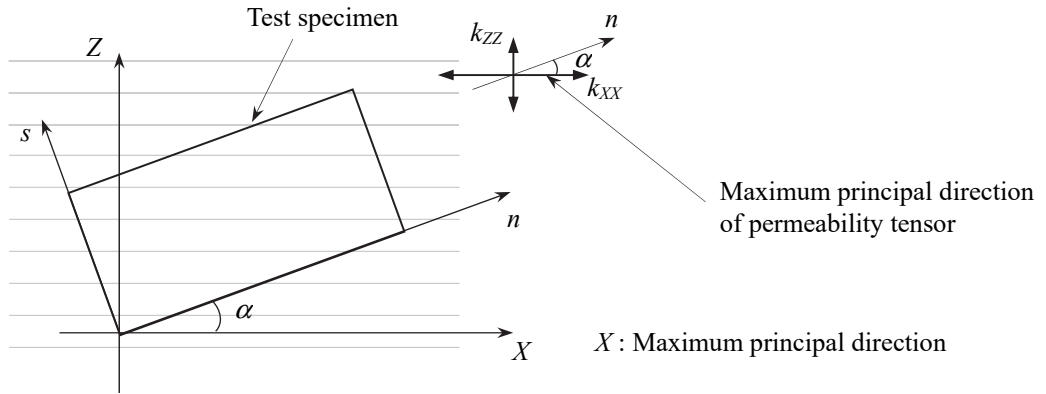


$$0^\circ < \alpha < 90^\circ$$

$k_{ns} > 0$ and $i_{s \text{ IND}} < 0$ from Eqs.(5) and (A1), because $i_n > 0$ and $k_{ss} > 0$

In practice, α is defined as the angle obtained by rotating the principal direction X counterclockwise to the flow direction n .

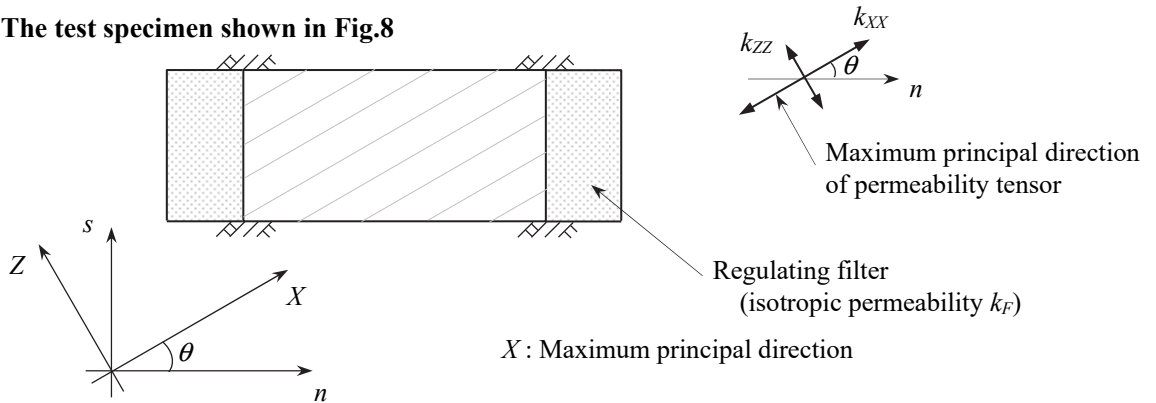
A2 The test specimens shown in Figs.5(b), (c), and Figs.6(a), (c)



$$0^\circ < \alpha < 90^\circ$$

$k_{ns} < 0$ and $i_{s \text{ IND}} > 0$ from Eqs.(5) and (A1), because $i_n > 0$ and $k_{ss} > 0$

A3 The test specimen shown in Fig.8



$$0^\circ < \theta < 90^\circ \quad (\text{or } 0^\circ > \alpha (= -\theta) > -90^\circ)$$

$k_{ns} > 0$ and $i_{s \text{ IND}} < 0$ from Eqs.(5) and (A1), because $i_n > 0$ and $k_{ss} > 0$

θ is defined as the angle obtained by rotating the principal direction X clockwise to the flow direction n , on the other hand α by rotating counterclockwise, so the equation $\alpha = -\theta$ is applied.

異方透水性材料の一次元透水特性 —透水楕円と誘導動水勾配—

Characteristic seepage flow through a test specimen of an anisotropic porous medium –Ellipse of coefficient of permeability and induced hydraulic gradient–

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アーノルド フェライト²⁾

Arnold Verruijt

中村 和弘³⁾

Kazuhiro Nakamura

概要：土の異方透水性は土の浸透破壊に対する安定性や浸透流量、堤体中の自由水面の位置など浸透流の力学的観点から問題となる。ここでは、まず、一次元拘束流れ(1DCF)の理論を提案し、 n 方向の一次元拘束流れの透水係数 $k_{n\text{ 1DCF}}$ と n の直交方向 s に生ずる誘導動水勾配 $i_{s\text{ IND}}$ を定義する。次に、有限要素法を用いて流れの方向が透水係数テンソルの主方向と異なる場合の供試体中の浸透流について解析し、種々の場合における $k_{n\text{ 1DCF}}$ と $i_{s\text{ IND}}$ の観測値について詳細に考察する。そして、 $k_{n\text{ 1DCF}}$ 、 $i_{s\text{ IND}}$ を正確に算定するためには B/D を大きくしなければならないことがわかった。ここに、 B, D はそれぞれ供試体の長さ及び高さである。また、整流フィルターを供試体の両端に設置する場合について考察を行った。整流フィルターは供試体を一次元拘束流れの状態におくのに有効な方法であることがわかった。本論文は、同著者らによる同名の論文について、若干の修正を加え改訂したものである。

キーワード：浸透流，異方透水性，一次元透水試験，透水係数テンソル，一次元拘束流れ，誘導動水勾配

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