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Analysis of vibration characteristics of clamped-clamped shallow arch beam subjected to heat for micromechanical resonant optical sensing

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Abstract— Thermal-type micromechanical resonant sensors are anticipated for highly sensitive optical sensing. The multilayered beams exhibit deflection similar to a shallow arch owing to the residual stress in the thin film, resulting in reduced sensitivity in thermal sensing. This study endeavors to establish a novel analytical method for evaluating the vibration characteristics of a clamped-clamped shallow arch beam under the influence of heat. The thermal deformation and stress of the arch beam are determined using an elastic curve equation, followed by the analysis of the vibration characteristics derived by solving the governing equation of motion. To validate the proposed method, theoretical values of natural frequency and frequency shift are compared with finite element method (FEM) simulations results across varying initial rise, thickness, and span dimensions of the arch beam. The theoretical results are also compared with the experimental results. These comparisons with FEM and experimental outcomes affirm the applicability of the analytical method to the design of thermal-type micromechanical resonant sensors.

Index Terms— Micromechanical resonator, optical sensor, thermal sensor, Galerkin method

I. INTRODUCTION

Near-infrared (NIR) light finds diverse applications, including object detection, security systems, spectroscopic analysis, medical treatment, and optical communications. The NIR sensor market is poised for significant growth. To broaden the applications of NIR light, cost-effective mass production of applicable sensors is crucial. Silicon (Si) photodiode-based sensors have excelled in detecting NIR light with wavelengths below 1100 nm, offering advantages like high responsiveness and sensitivity. However, for wavelengths exceeding 1100 nm, high-cost materials such as indium gallium arsenide (InGaAs) are employed in photodiodes. Additionally, miniaturization of sensors is challenging due to the need for a cooling system. In contrast, thermal-type NIR sensors convert light into heat, facilitating the detection of a broad range of NIR wavelengths without requiring a cooling system. These sensors, produced through

semiconductor and microelectromechanical system (MEMS) fabrication processes of Si, lend themselves to easy miniaturization and are extensively used in NIR applications. One of the candidates is Si micromechanical resonators.

The vibration characteristics of micromechanical resonators play a pivotal role in their sensitivity as various sensing devices [1], [2], [3], [4], [5], [6]. The resonators are the promising highly-sensitive sensing devices for heat, enabling for sensing light by photothermal conversion [7], [8], [9], [10], [11], [12], [13]. The method utilizing thermal stress change owing to heat input enables markedly more sensitive sensing compared to another thermal sensing, where Young's modulus change [13] serves as the sensing principle. The heat induces corresponding compressive thermal stress in the beam, leading to alterations in the flexural vibration characteristics such as natural frequency. Consequently, this highly sensitive optical sensing holds promise for diverse applications, including spectral analysis and biochemical sensors [9], [10], [11]. Integrating wavelength-dependent optical absorbers into resonator beams for spectroscopic analysis has been proposed, with multilayer structures [14], [15], [16], comprising resonator beams and optical absorbing films. The micromechanical resonators may require multilayer structures for amplitude detection. The beams of the multilayer structure are possible to be deflected like a shallow arch due to the residual stress of the thin film, leading lower sensitivity of optical sensing [17], [18], [19]. To design and develop micromechanical resonators for highly sensitive heat sensing, precise prediction of the flexural vibration characteristics of a shallow arch beam under compressive thermal stress is essential.

The vibration characteristics, encompassing natural frequency and mode shape, of the unforced and undamped flexural vibrations of a clamped-clamped straight beam in the presence of an applied axial load were analytically determined [20], [21]. In the straight beam, the conversion from thermal input to axial load is straightforward. Values for the shallow arch beam without an axial load were acquired by solving the corresponding governing equation [22], [23]. In [22], it was noted that the natural frequency significantly depended on the

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arch rise of the clamped-clamped shallow arch, exhibiting an increase with a corresponding rise in the arch. Arched beams have been widely used as bistable actuators (e.g., electrothermal actuators) in the field of MEMS [24], [25]. Studies have been reported on the tuning of natural frequencies and nonlinear vibration characteristics [26] by changing the arch rise with electrothermal and electrostatic actuations.

The vibration characteristics when an axial load is applied to the clamped end of the shallow arch have been investigated extensively for microbeams and microplate [27], [28], [29], [30], [31], [32], [33], [34]. The Galerkin method using the governing equation of motion was frequently used for approximation of dynamic characteristics such as natural frequencies. In these methods, an influence of heat was not taken into account because the axial load was given as the initial condition. Additionally, a mode function for the straight beam was used as a basis function in the Galerkin method to simplify the solution of the governing equation, leading large errors at the large rise in the arch. Precision in assuming an approximate solution closely mirroring the unknown exact solution is crucial. When considering an influence of heat, both the thermal deformation and the thermal axial load of the arch beam owing to heat input must be accurately determined depending on the arch rise, and a mode function for the arch beam must be used instead of a mode function for straight beam to avoid significant errors at the large rise in the arch.

Instead of the Galerkin method, studies have been conducted to obtain exact solutions to the governing equations of motion for shallow arch beams subjected to axial loading [35], [36]. In these studies, the effect of electrothermal heating on the dynamic behavior was investigated instead of setting the axial load as the initial condition. However, the axial loads produced by electrothermal heating were expressed in terms of straight beam equations, which are independent of the arch rise and make it difficult to accurately determine the thermal deflection and thermal axial load of beams after heating.

This study introduces a precise analysis method for evaluating the flexural vibration characteristics of a clamped-clamped shallow arch beam under heat input toward optical sensing. For a meticulous analysis, the arch rise and

compressive axial load after thermal deformation by heat input, contingent on the initial arch rise, were derived by solving the elastic curve equation. Following the acquisition of these solutions, the governing equations, incorporating the compressive load, were solved using the Galerkin method. Although the exact solution of the mode function for a straight beam was utilized as a basis function in the Galerkin method in the conventional studies [27], [37], this study used for the mode function tailored for a shallow arch beam. Previous research has primarily focused on the natural frequency, whereas this study examined the relationship between the input heat and the resonant frequency shift for optical sensing. This study included both calculations and experiments, and the resonant frequency shift were measured when the beam was exposed to light. The natural frequencies, natural frequency shifts, and mode shapes obtained from this analysis were compared with those derived from the finite element method (FEM) and experimental results.

II. THEORETICAL EQUATIONS

A. Thermal deformation and stress

The shapes of the clamped-clamped shallow arch beam before and after heating are denoted by $\hat{\psi}_0$ and $\hat{\psi}$, respectively, as shown in Fig. 1(a). These shapes are assumed to be

$$\begin{aligned}\hat{\psi}_0(\hat{x}) &= \frac{\hat{b}_0}{2} \left(1 - \cos \frac{2\pi\hat{x}}{l} \right), \\ \hat{\psi}(\hat{x}) &= \frac{\hat{b}}{2} \left(1 - \cos \frac{2\pi\hat{x}}{l} \right),\end{aligned}\quad (1)$$

where $\hat{x}$ is the lateral coordinate from the clamped end; $\hat{b}_0$ and $\hat{b}$ are the arch rise of before and after heating, respectively; and l is the span of the arch. The heat q is applied from the cross-section at the beam center. When the temperatures at both beam ends T_e are constant, the temperature at the center T_c is derived using a one-dimensional Fourier's law. The average temperature increase ΔT is given by

$$\Delta T = \frac{1}{2} (T_c - T_e) = \frac{qL_0}{8\lambda A}, \quad (2)$$

where λ denotes the thermal conductivity, L_0 is the length along the curve of $\hat{\psi}_0$, and A is the cross-sectional area of the beam. When the beam is freely expanded by heating, the thermal expansion ΔL is given by

$$\Delta L = \alpha L_0 \Delta T, \quad (3)$$

where α is the coefficient of thermal expansion. $L_0 + \Delta L$ is the length of the arch without any restraints. However, the actual length L along the curve of $\hat{\psi}$, is shorter than $L_0 + \Delta L$ because the thermal expansion is prevented by the immovable clamped ends. A compressive axial load P appears at both ends, as shown in Fig. 1(a), and is defined as follows:

$$P = \frac{L_0 + \Delta L - L}{L_0 + \Delta L} EA = \left(1 - \frac{1}{1 + \alpha \Delta T} \frac{L}{L_0} \right) EA. \quad (4)$$

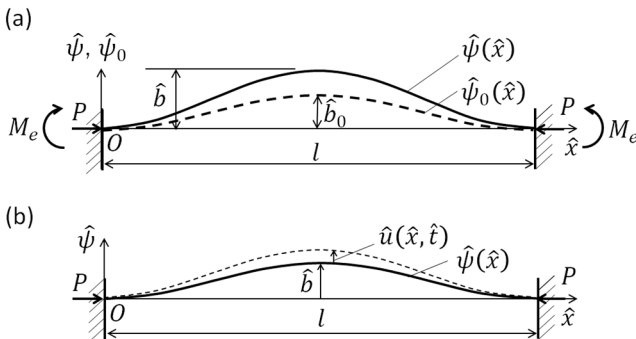


Fig. 1. Schematic of clamped-clamped shallow arch. (a) Shape before and after thermal deformation owing to heat input. (b) Shape of flexural vibration from the thermally deformed shape.

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After Maclaurin expression of L and L_0 , L/L_0 is approximated by substituting Eq. 1 for L and L_0 , as given by Eq. 5.

$$L = \int_0^l \sqrt{1 + \left(\frac{d\hat{\psi}}{d\hat{x}}\right)^2} d\hat{x}, \quad L_0 = \int_0^l \sqrt{1 + \left(\frac{d\hat{\psi}_0}{d\hat{x}}\right)^2} d\hat{x}, \quad (5)$$

$$\frac{L}{L_0} = \frac{\pi^2 \hat{b}^2 + 4l^2}{\pi^2 \hat{b}_0^2 + 4l^2}.$$

The elastic curve equation for the arch beam under a compressive axial load is given by [38].

$$\frac{d^2}{d\hat{x}^2} (\hat{\psi} - \hat{\psi}_0) = -\frac{1}{EI} (P\hat{\psi} - M_e), \quad (6)$$

where M_e denotes the bending moment at the clamped end. From Eqs. 1, 6 and $\partial M_e / \partial \hat{x} = 0$, we have

$$4\pi^2 EI (\hat{b} - \hat{b}_0) - Pl^2 \hat{b} = 0, \quad (7)$$

The cubic equation for $\hat{b}$ was obtained from Eqs. 4, 5, and 7, and the solutions of $\hat{b}$ and P were derived. The thermal deformation at the center and the thermal stress in the beam were calculated using $\hat{b} - \hat{b}_0$ and P/A , respectively.

B. Flexural vibration of clamped-clamped shallow arch

Figure 1(b) shows a schematic of the flexural vibration of the clamped-clamped shallow arch beam after heating. The governing equation for the unforced and undamped flexural vibration is given by [39]:

$$\rho A \frac{\partial^2 \hat{u}}{\partial \hat{t}^2} + EI \frac{\partial^4 \hat{u}}{\partial \hat{x}^4} + P \frac{\partial^2 \hat{u}}{\partial \hat{x}^2} = \left(\frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{\psi}}{\partial \hat{x}^2} \right) \left[\frac{EA}{2l} \int_0^l \left\{ \left(\frac{\partial \hat{u}}{\partial \hat{x}} \right)^2 + 2 \frac{\partial \hat{u}}{\partial \hat{x}} \frac{\partial \hat{\psi}}{\partial \hat{x}} \right\} d\hat{x} \right], \quad (8)$$

where $\hat{t}$ is the time, $\hat{u}(\hat{x}, \hat{t})$ is the displacement, ρ is the density, E is the Young's modulus, and I is the second moment of area. For normalization, we introduce the following nondimensional variables:

$$x = \frac{\hat{x}}{l}, \quad u = \frac{\hat{u}}{r}, \quad \psi = \frac{\hat{\psi}}{r}, \quad t = \frac{\hat{t}}{T}, \quad (9)$$

$$r = \sqrt{\frac{I}{A}}, \quad T = \sqrt{\frac{\rho A l^4}{EI}}, \quad \gamma = \frac{Pl^2}{EI}.$$

Using these variables, Eq. 8 is expressed as

$$\ddot{u} + u'''' + \gamma u'' = \frac{1}{2} (u'' + \psi'') \int_0^1 \{ (u')^2 + 2u'\psi' \} dx, \quad (10)$$

where the dots and primes denote the differential with respect to t and x , respectively. From Eqs. 1 and 9, the normalized shape of the arch beam can be expressed as follows:

$$\psi = \frac{b}{2} (1 - \cos 2\pi x), \quad (11)$$

where b is the non-dimensional rise of the arch. Substituting Eq. 11 into Eq. 10, the governing equation for linear vibration, wherein the nonlinear terms are omitted, is given by

$$\ddot{u} + u'''' + \gamma u'' - 2\pi^3 b^2 \cos 2\pi x \int_0^1 u' \sin 2\pi x dx = 0. \quad (12)$$

The solution of the flexural vibration of the beam is generally written as

$$u(x, t) = \phi_j(x) e^{i\omega_j t}, \quad (13)$$

where $\phi_j(x)$ is the j -th mode shape. Substituting Eq. 13 into Eq. 12, we have

$$-\omega_j^2 \phi_j + \gamma \phi_j'' + \phi_j'''' - 2\pi^3 b^2 \cos 2\pi x \int_0^1 \phi_j' \sin 2\pi x dx = 0. \quad (14)$$

In case that the arch beam is not under the axial load ($\gamma = 0$), the exact solution of $\phi_j(x)$ is given by [22],

$$\phi_j(x) = C_1 \cos \beta_j x + C_2 \sin \beta_j x + C_3 \cosh \beta_j x + C_4 \sinh \beta_j x + C_5 \cos 2\pi x, \quad (15)$$

where $C_1 - C_5$ are the constants and $\beta_j = \sqrt{\omega_j}$. From Eqs. 14 and 15 and the boundary condition of the clamped ends, we have

$$8\pi^4 b^2 \beta_j^3 (\sin \beta_j + \sinh \beta_j - \cos \beta_j \sinh \beta_j - \sin \beta_j \cosh \beta_j) - (\beta_j^4 - 16\pi^4) \{ \beta_j^4 - 2\pi^4 (b^2 + 8) \} (\cos \beta_j \cosh \beta_j - 1) = 0, \quad (16)$$

and the solutions b and β_j are obtained by numerical calculation. When β_j is the value of the even-mode vibration of a straight beam ($b = 0$), Eq. (16) always holds, regardless of the value of b . Then, the mode shape and natural frequency of the arch beam are equivalent to those of a straight beam. We define this vibration mode to be even mode ($j = 2, 4, 6, \dots$) of the arch beam here for convenience. The natural frequency before heating was calculated by replacing b with the initial arch rise $b_0 (= \hat{b}_0/r)$ in Eq. 16.

On the contrary, in case of the arch beam under the axial load ($\gamma \neq 0$), the Galerkin method is applied to approximately solve the differential equation of Eq. 14. In this method, the approximate solution of the mode shape $\phi^*(x)$ is written as

$$\phi^*(x) = \sum_{i=1}^{\infty} a_i v_i, \quad (17)$$

where a_i is a constant and v_i is the basis function. The left side of Eq. 14 is defined as the residual error $\mathcal{L}(\phi^*)$, and the Galerkin equation is

$$\int_0^1 \mathcal{L} \left(\sum_{i=1}^{\infty} a_i v_i \right) v_k dx = 0. \quad (k = 1, 2, 3, \dots, \infty) \quad (18)$$

Although it is preferable that the number of the basis functions i is larger, the calculation becomes more complicated. For simplification, when calculating the j -th natural frequency, the approximate solution $\phi^*(x)$ is redefined by single basis function of $\phi_j(x)$ given by Eq. 15 so that the approximate natural frequency ω_j^* can be expressed in a simple form. Equation 18 is rewritten as

$$\int_0^1 \mathcal{L}(\phi_j) \phi_j dx = 0. \quad (19)$$

Substituting Eq. 15 into Eq. 19, the approximate natural frequency ω_j^* for j -th mode is given by

$$\omega_j^{*2} = \beta_j^4 + \gamma\Gamma_1 + \{(1 + \alpha_{1j})(\beta_j^4 - 16\pi^4) - 2\pi^3 b^2 \Gamma_2\} \Gamma_3 / \Gamma_4 \quad (j = 1, 3, 5, \dots)$$

$$\omega_j^{*2} = \beta_j^4 + \gamma\Gamma_1 - 2\pi^3 b^2 \Gamma_2 \Gamma_3 \quad (j = 2, 4, 6, \dots)$$

$$\Gamma_1 = \int_0^1 \phi_j \phi_j'' dx, \quad (20)$$

$$\Gamma_2 = \int_0^1 \phi_j' \sin 2\pi x dx,$$

$$\Gamma_3 = \int_0^1 \phi_j \cos 2\pi x dx, \quad \Gamma_4 = \int_0^1 \phi_j^2 dx.$$

III. COMPARISONS OF THEORETICAL EQUATION WITH FEM ANALYSIS

A. Basic analysis model

To validate the theoretical equations, we compared the theoretical values of thermal deformation, thermal stress, natural frequency, and natural frequency shift with FEM values. FEM thermal-structural coupling analysis was conducted using Femtet, commercialized by Murata Software Co., Ltd. The FEM models of the clamped-clamped shallow arch beam were formulated as a cosine expression according to Eq. 1. The width w , height h and span l are 15 μm , 3 μm and 800 μm , respectively. The initial arch rise $\hat{b}_0$ is varied within the range from 0 to 9 μm . The temperature of the clamped ends was set to 300 K. The material properties of Si used for the models are $\rho = 2330 \text{ kg/m}^3$, $E = 169 \text{ GPa}$, $k = 156 \text{ W/m}\cdot\text{K}$, and $\alpha = 2.63 \times 10^{-6} / \text{K}$. In the FEM analysis, the heat of 10 μW is given from the 15- μm^2 area formed on the middle of the beam. In the theoretical calculation, the heat is substituted into q in Eq. 2.

B. Dependence of natural frequency and its shift on vibration mode

Figure 2 shows the thermal deformation at the center and thermal stress, obtained from both theoretical equations and FEM thermal-structural coupling analysis, as a function of the initial arch rise. In both thermal deformation and stress comparisons, the theoretical results exhibited similar tendencies to the FEM results. Figure 2(b) shows a decrease in

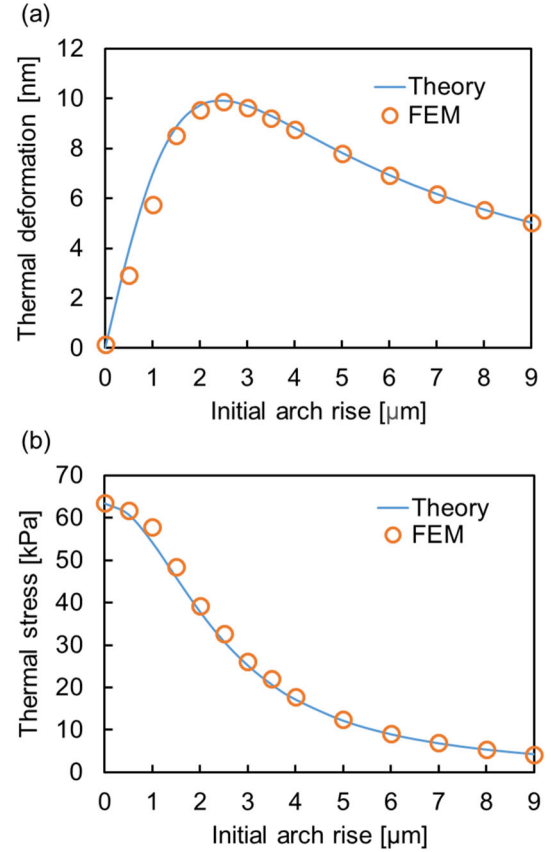


Fig. 2. Dependencies of (a) thermal deformation and (b) thermal stress on the initial arch rise. Solid line and plot indicate calculated results by proposed theory and FEM, respectively.

thermal stress with an increasing initial arch rise due to the reduction in stiffness to axial load. The thermal stresses were highly dependent on the initial arch rise. In this study, the thermal stress and the thermal axial load are derived according to the initial arch rise, and the dynamic characteristics can be calculated accurately. Equation 7 indicates that thermal deformation $\hat{b} - \hat{b}_0$ increases proportionally to the initial arch rise $\hat{b}_0$ and axial load P . When the initial arch rise was small, the deformation increased with the rise because the effect of the rise was relatively larger than that of the stiffness reduction. As the initial arch rise increased, the effect of stiffness reduction became relatively significant, leading to a decrease in deformation. The maximum values of both theoretical and FEM thermal deformation occurred around an initial arch rise of 2.5 μm . Especially for initial arch rises beyond 2.0 μm , the theoretical values of thermal deformation and stress closely matched the FEM results.

The rise $\hat{b}$ after deformation with heat and theoretically calculated compressive axial load P were used to calculate the natural frequencies of the arch. Figure 3(a) depicts natural frequencies from the 1st to the 4th modes as a function of the initial arch rise, with a heat input of 10 μW . Solid lines represent theoretical results, and markers indicate FEM results. The natural frequency depending on the initial arch rise exhibited

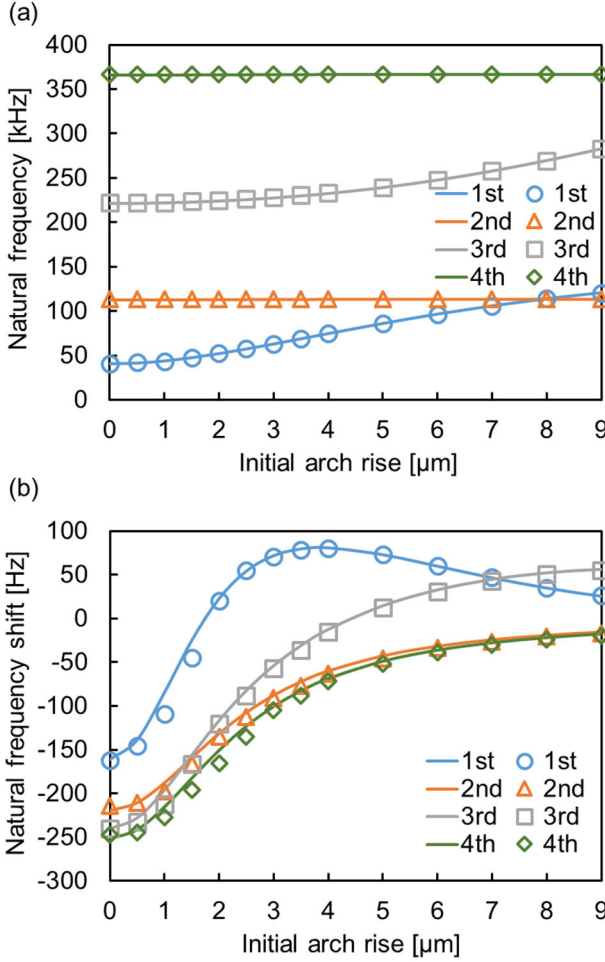


Fig. 3. Theoretical results of (a) natural frequency and (b) its shift as a function of the initial arch rise for each vibration mode.

the same tendency as other reports [22, 23]. The errors in theoretical values were all below 0.1%. In Figure 3(b), natural frequency shifts from the 1st to the 4th vibration modes are shown as a function of the initial arch rise, with a heat input of $10 \mu\text{W}$. According to the theory for the clamped-clamped straight beam, at the low heat input conditions of this study, the compressive axial loads were sufficiently lower than the buckling loads, and consistently lowered the natural frequencies across all vibration modes. In the 1st and 3rd modes, where natural frequencies increased with the initial arch rise, the natural frequency shift changed from negative to positive. Positive values indicated that the effect of the natural frequency increased, surpassing that of the compressive axial load. The largest positive natural frequency shift occurred around an initial arch rise of $4 \mu\text{m}$. As the initial arch rise increased beyond $4 \mu\text{m}$, the natural frequency shift decreased due to reductions in thermal deformation and compressive axial load depicted in Fig. 2. In the 2nd and 4th modes, natural frequency shifts were negative for all initial arch rise. It is because the natural frequencies remained independent of the initial arch rise as shown in Fig. 3(a) and the compressive axial load decreased with the initial arch rise. Despite some errors in natural

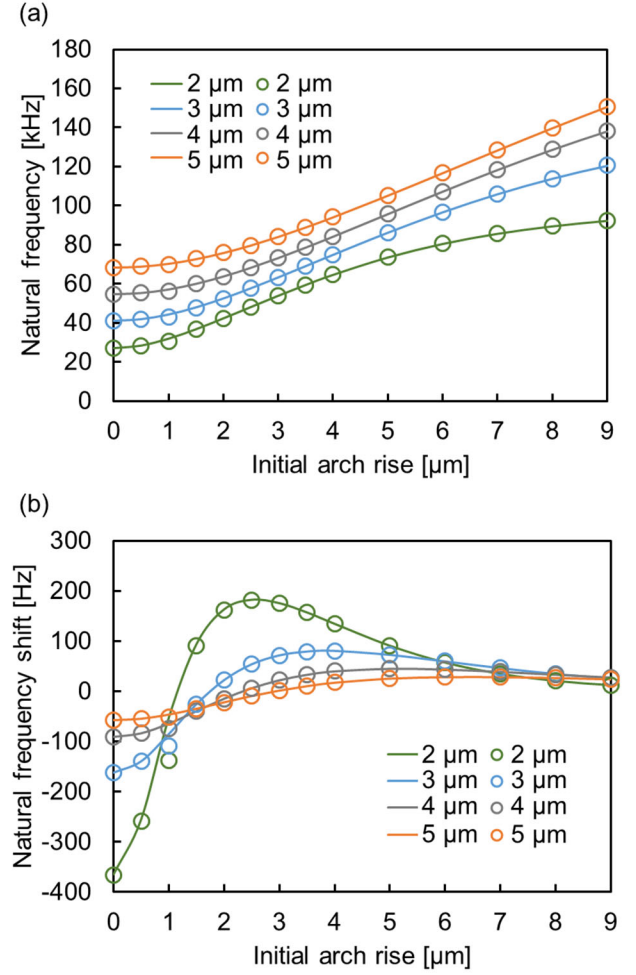


Fig. 4. Theoretical results of (a) natural frequency and (b) its shift as functions of initial arch rise and thickness.

frequency shift for small initial arch rises, the theoretical values closely approximated the FEM results.

C. Dependence of natural frequencies on beam size

In the fundamental analysis model, the beam thickness was systematically varied from 2 to $5 \mu\text{m}$, and the resulting natural frequency along with its shift, calculated using theoretical equations, was compared to values obtained through FEM. Figure 4(a) shows the natural frequencies of the 1st mode as functions of the initial arch rise and beam thickness. As the thickness increases, the nondimensional variables r and T , expressed by Eq. (9) increase and decrease, respectively. The arch rise and natural frequency given by rb and ω/T , respectively, as expressed by Eqs. (1) and (9), increased as the thickness increased as shown in Fig. 4(a). Figure 4(b) shows the natural frequency shifts concerning the initial arch rise and beam thickness. Given that the variable γ is inversely proportional to the cube of thickness, it follows that the natural frequency shift decreases as the thickness increases and the term of $\gamma\Gamma_1$ ($\Gamma_1 < 0$) in Eq. 20 lowers the natural frequency.

Subsequently, the span of the arch l was systematically altered within the range of 400 to $1000 \mu\text{m}$. In Figure 5(a), the natural frequencies are depicted as functions of both the initial

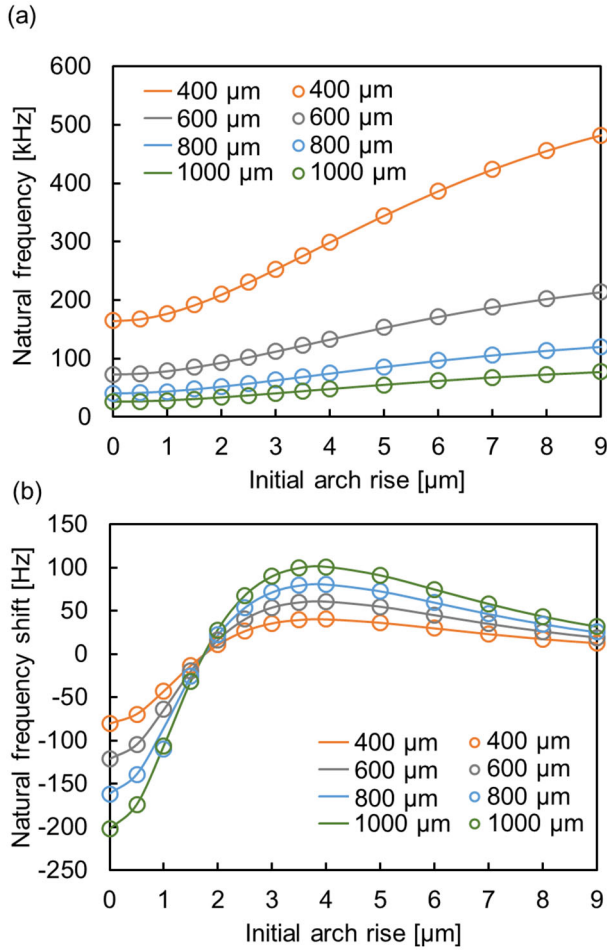


Fig. 5. Theoretical results of (a) natural frequency and (b) its shift as functions of initial arch rise and span.

arch rise and the span. The nondimensional variable r remains independent of the span, while T is directly proportional to the square of the span. Figure 5(b) shows the variations in the natural-frequency shifts concerning the initial arch rise and span. Given that the variable γ is also proportional to the square of the span, the natural frequency shift exhibited an increase with an escalating span. The outcomes suggest that the shift was approximately proportional to the span.

These results confirm that the theoretical values were in good agreement with the FEM values and the validation of the proposed analytical model in the calculated ranges of the beam thickness and span of the arch.

IV. COMPARISONS OF THEORETICAL EQUATION WITH EXPERIMENTS

A. Experimental method

A clamped-clamped shallow-arch Si beam was fabricated from a silicon-on-insulator (SOI) wafer. The arch pattern was defined using photolithography, as illustrated in Fig. 6. The beam underwent in-plane vibrations upon laser exposures for measuring by the Doppler vibrometer and heating to the sidewall of the beam. The SOI wafer comprised a device layer (15 μm), a buried oxide (BOX) layer (1 μm), and a substrate

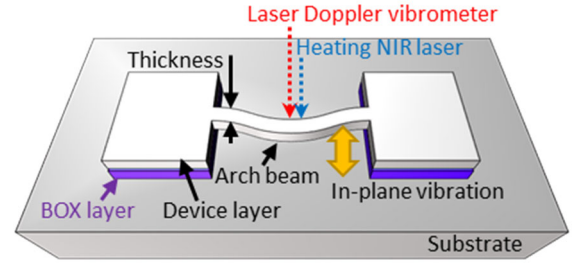


Fig. 6. Schematic of fabricated arch beam from SOI wafer. The arch pattern was defined by photolithography. The resulting beam exhibits in-plane vibrations induced by a piezoelectric ceramic actuator. Lasers for Doppler measurements and heating irradiate the sidewall of the beam.

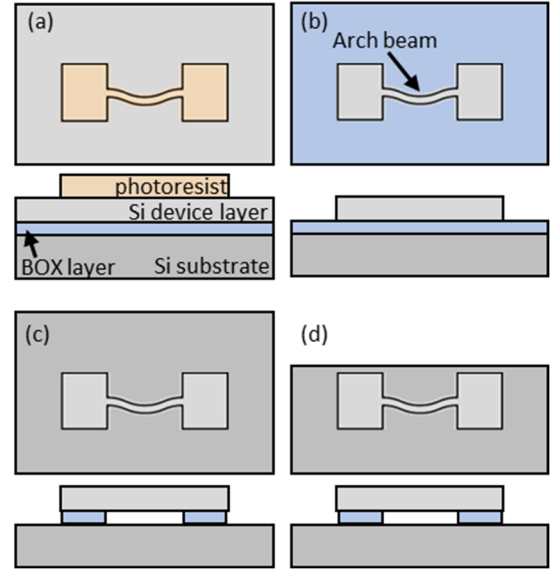


Fig. 7. Fabrication process of the shallow arch beam. (a) Patterning of arch shape on device layer. (b) Si dry etching of the device layer. (c) BOX layer removal by BHF. (d) Dicing of substrate.

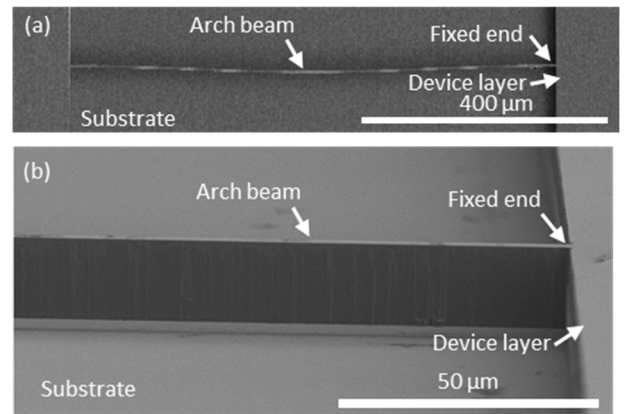


Fig. 8 SEM images of fabricated shallow arch beam. (a) Top view and (b) diagonal view at clamped end.

layer (490 μm). The design dimensions for span, width, and thickness were 800 μm, 15 μm, and 3 μm, respectively, with the initial arch rise varying from 0 to 9 μm. The fabrication process,

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outlined in Fig. 7, involved transferring the arch shape to the device layer through photolithography and subsequent dry etching of Si (Fig. 7a and 7b). The BOX layer was removed via chemical etching with buffered hydrogen fluoride (c), followed by dicing (d). Scanning electron microscopy (SEM) images of the fabricated arch beam are presented in Fig. 8. As the thickness significantly influences the natural frequency and its shift, the thicknesses of the beams were measured using SEM. Figure 9 displays the measured thickness for each beam corresponding to different initial arch rises, ranging from 2.57 to 3.27 μm , while the design dimension was 3 μm . The theoretical natural frequencies were calculated using the measured thicknesses and compared with experimental results.

The experimental setup for natural frequency measurements is depicted in Fig. 10 [7], [8]. The device substrate was mounted on the wall of an L-shaped base for observation and irradiation to turn the sidewall of the beam up. This base, situated on a piezoelectric ceramic actuator, facilitated beam vibration. An AC voltage from a lock-in amplifier (MFLI, Zurich Instruments) was applied to the piezoelectric ceramic actuator, and the resulting vibration amplitude was measured using a laser Doppler vibrometer connected to a lock-in amplifier under a vacuum of less than 10^{-2} Pa. Natural frequencies were obtained from resonance curves in frequency sweep measurements. Amplitudes were measured at 40 μm intervals along the longitudinal direction of the beam to observe the mode shape of vibration. For natural frequency shift measurements due to heating, a near-infrared laser with a wavelength of 1550 nm irradiated the center of the beam. All measurements were conducted at an intensity of 0.2 mW. As the absorption of 1550-nm light by Si is challenging to estimate owing to its low absorption, net input heat was calculated based on the experimental and theoretical results in Section 4.2.

B. Experimental results

Figure 11 presents the measured natural frequencies from the 1st to the 4th vibration modes alongside their corresponding theoretical frequencies before heating. The theoretical values in this figure were calculated using Eq. 16. In the experiments, the natural frequencies of the 1st and 3rd modes noticeably

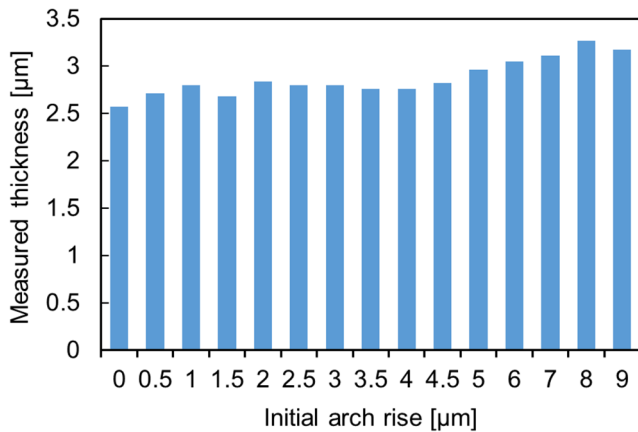


Fig. 9. Measured thickness of the arch beam for each initial arch rise.

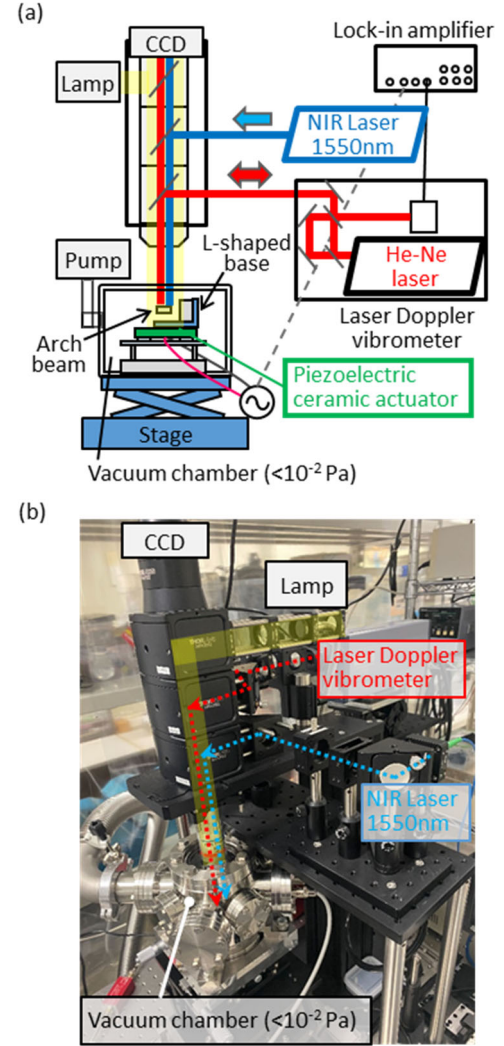


Fig. 10 (a) Schematic and (b) photograph of experimental setup for measuring natural frequency and mode shape.

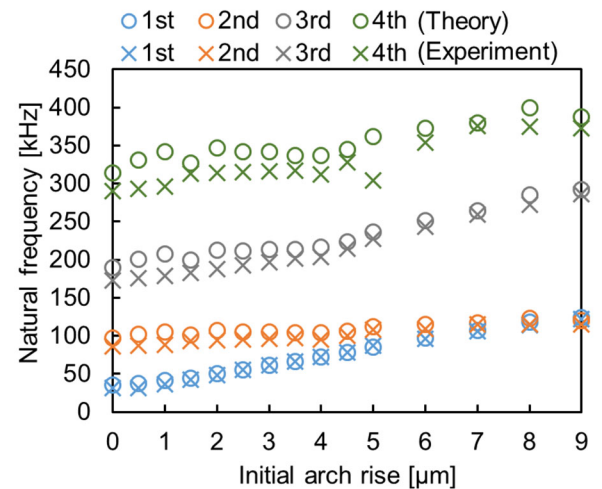


Fig. 11 Calculated and measured natural frequencies as a function of initial arch rise.

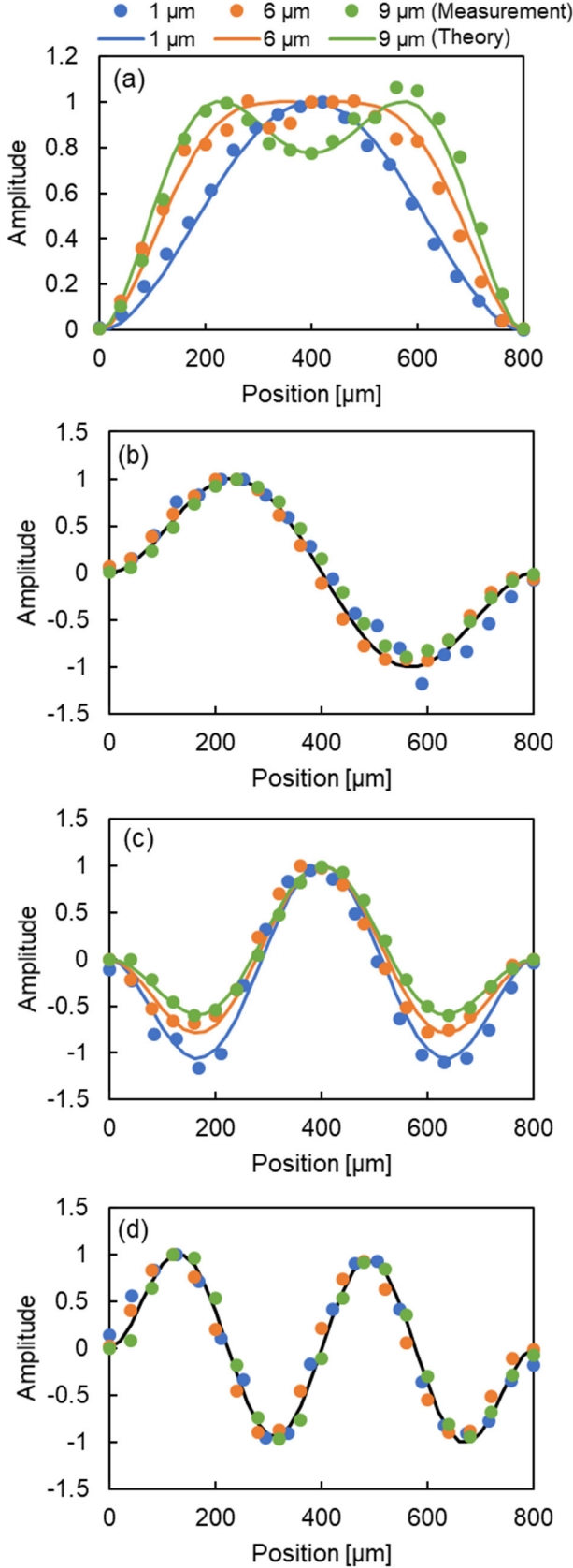


Fig. 12 Calculated and measured mode shape of vibration for initial arch rises of 1, 6, and 9 μm in (a) 1st, (b) 2nd, (c) 3rd, and (d) 4th modes.

increased with an escalating initial arch rise, aligning with theoretical expectations. Conversely, the natural frequencies of the 2nd and 4th modes exhibited a slight elevation when the initial arch rise reached 5 μm or more. This was attributed to the relatively large thicknesses of the fabricated beams, as depicted in Fig. 9. Despite observed differences between theoretical and measured results in this comparison, both demonstrated consistent tendencies in the relationship between the initial arch rise and natural frequency. Possible causes for these differences include fabrication process accuracy, such as nonuniformities in cross-sectional dimensions and shapes at the clamped ends.

Figure 12 shows the calculated and measured mode shapes from the 1st to the 4th modes for initial arch rises of 1, 6, and 9 μm . All mode shapes were normalized, setting the maximum amplitude on the positive side to 1. As shown in Fig. 12, mode shapes were independent of the initial arch rise in even vibration modes (2nd and 4th) and changed with the initial arch rise in odd vibration modes (1st and 3rd). The calculated mode shapes closely matched the measured shapes.

Figure 13 displays the measured natural frequency shifts from the 1st to the 4th vibration modes alongside their

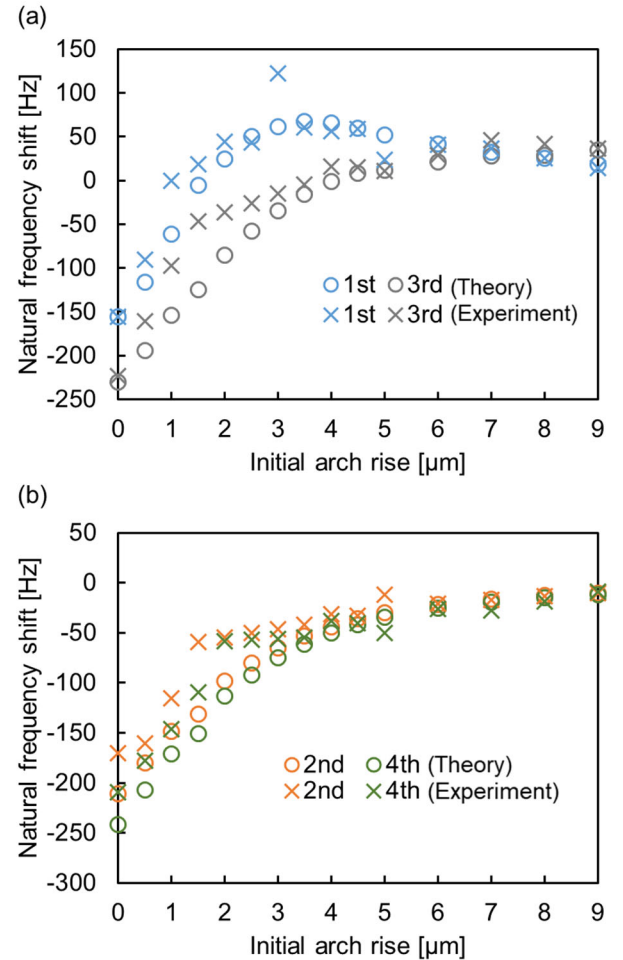


Fig. 13 Calculated and measured natural frequency shifts of (a) odd modes (1st and 3rd modes) and (b) even modes (2nd and 4th modes) as a function of initial arch rise.

theoretical frequencies after heating. Here, the measured natural frequency shift of the straight beam (initial arch rise of 0 μm) is -155 Hz, corresponding to the theoretical value at a heat intensity of $7.1 \mu\text{W}$. The absorption of Si was calculated at 3.5% due to the experimental laser intensity of 0.2 mW . Theoretical values for other initial arch rises were calculated using a heat intensity of $7.1 \mu\text{W}$. The tendencies of the measured shifts for all vibration modes were in good agreement with theoretical values. Differences between theoretical and measured values are attributed to fabrication process accuracy and deviations in light absorption owing to surface roughness, leading to variations in the heat applied to the beam.

V. CONCLUSION

This study introduces a novel analysis method for assessing the vibration characteristics of a clamped-clamped shallow arch beam under the influence of heat. The temperature elevation within the beam is calculated utilizing Fourier's law, while the thermal deformation and stress are determined through the elastic curve equation, accounting for the application of a compressive axial load. The theoretical calculation of the beam vibration characteristics after heating is achieved by solving the governing equation of motion, employing the Galerkin method. The theoretical values of the natural frequency and its shift are compared with the FEM results across varied initial arch rises, thicknesses, and span dimensions. These comparisons reveal a favorable agreement between theoretical and FEM results. Further, a comparative analysis between theoretical and experimental results is conducted. Clamped-clamped shallow arch beams, fabricated without internal stress from an SOI wafer via MEMS process, undergo heating through laser irradiation, and their natural frequencies are experimentally measured. The theoretical predictions align closely with the experimental observations. This consistency across FEM and experimental results affirms the practical utility of the proposed analysis method in the design of thermal-type MEMS resonant sensors for near-infrared light, leading to further applications to various heat sensors.

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