



# Peirce' s Tychism and the Non-Euclidean Nature of Physical Space: 2025 Peirce Essay Prize Winner

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# Peirce's Tychism and the Non-Euclidean Nature of Physical Space<sup>\*</sup>

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## Abstract

This paper offers a new interpretation of a puzzling passage in Charles S. Peirce's 1892 essay "The Doctrine of Necessity Examined" (DNE), where he speaks of the "inevitable consequences" of the hypothesis of tychism that purportedly agree with observed facts. I argue that in this passage, Peirce is alluding to his astronomical work attempting to determine the *curvature of space*, which he was carrying out at the same time as writing DNE. By highlighting a hitherto unnoticed link between tychism and the geometry of space, I show why Peirce should have regarded these investigations as lending empirical support to his anti-deterministic doctrine. This connection will serve to illuminate one specific aspect of the impact that non-Euclidean geometry had on Peirce's philosophy—an influence that, despite its significance, has remained largely unexplored.

## Introduction

Charles S. Peirce's essay "The Doctrine of Necessity Examined" (EP 1:298–311, W 8:111–125; hereinafter abbreviated as "DNE"), published in the April 1892 issue of *The Monist* as the second installment of the *Monist* Metaphysical Series, is a landmark text in his *oeuvre*.<sup>1</sup> It is here that he lays out his main arguments against determinism—which he calls the "doctrine of necessity" or "necessitarianism"—and puts forth his anti-necessitarian doctrine of *tychism*, according to which there is an element of absolute chance in the world. There is a curious passage towards the end of this essay, where Peirce sums up his defense of tychism by giving five reasons why he believes the doctrine to be true (EP 1: 309, W 8:124, 1891). The fifth reason, which Peirce describes as the "chief of my reasons" and the "the strongest reason for my belief," is only hinted at and not explicitly stated at all. The relevant passage is reproduced in its entirety below:

But I must leave undeveloped the chief of my reasons, and can only adumbrate it.

The hypothesis of chance-spontaneity is one whose inevitable consequences are

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capable of being traced out with mathematical precision into considerable detail. Much of this I have done and find the consequences to agree with observed facts to an extent which seems to me remarkable. But the matter and methods of reasoning are novel, and I have no right to promise that other mathematicians shall find my deductions as satisfactory as I myself do, so that the strongest reason for my belief must for the present remain a private reason of my own, and cannot influence others. I mention it to explain my own position; and partly to indicate to future mathematical speculators a veritable goldmine, should time and circumstances and the abridger of all joys prevent my opening it to the world. (EP 1:309, W 8:124, 1891)

What are the “inevitable consequences” of the hypothesis of chance-spontaneity that Peirce is speaking of here, and what are the “observed facts” which he claims agree with these consequences to a remarkable extent? To my knowledge, there have been no specific accounts in the literature so far as to what Peirce may be alluding to in this passage. The editors of *The Writings of Charles S. Peirce* gloss this passage in the annotations as follows:

There are no extant manuscripts in this period whose contents clearly fit the claim, but Peirce may be alluding to work done throughout the 1880s, including his theory of probable inference, his experimental work on small differences of sensation, his logic of relatives, and the elaboration of his grand hypothesis in the various chapters of “A Guess at the Riddle” ... (W 8:389, ann. 124.7)

I suggest that Peirce had something much more specific in mind when he wrote of the observational facts supporting tychism. In this paper, I will argue that Peirce, in the passage quoted above, is alluding to his work attempting to determine the *curvature of space*, which he was carrying out at the same time as writing DNE.

Peirce was immersed in efforts to determine the curvature of space especially in 1891—the same year in which he wrote DNE—and more sporadically in later years, employing an array of arguments based on astronomical observations. These efforts should be understood against the backdrop of his engagement with non-Euclidean geometry, the discovery of which had initiated a revolution in mathematics in the latter half of the 19th century. This was a period in which the overthrow of the axioms of Euclidean geometry, and the advent of various new and exotic geometries, was stirring up debate among scientists and philosophers about the nature of physical space.

As emphasized by Randall Dipert (1978:61), Peirce was an iconoclast in his theory of space. While some scientists and philosophers at the time were open to the possibility that space *might* be non-Euclidean (e.g. Bernhard Riemann, Hermann von Helmholtz), Peirce was unique in maintaining that space *must* be non-Euclidean. His investigations into the curvature of space were motivated by a desire to secure empirical confirmation of this conviction.

The impact that non-Euclidean geometry had on Peirce's philosophy—not only on his theory of space but also his broader evolutionary cosmology—can hardly be overestimated. Yet, despite its importance, this influence is not as well-documented as, say, how Peirce's philosophy was informed by his work in the logic of relations, or the mathematics of the continuum. To be sure, commentators have noted the connection between, on the one hand, Peirce's belief in the non-Euclidean nature of space, and on the other, his evolutionary cosmology and his rejection of determinism in particular. Dipert, for example, notes that the argument Peirce used to establish that space cannot be Euclidean is “essentially the argument he so forcefully used against the ‘doctrine of necessity’” (Dipert 1977:407). Shannon Dea, following the earlier work of Murphey (1993) and Reynolds (2002), highlights the parallels that Peirce himself drew between hyperbolic geometry and his own evolutionary cosmology, and goes further by drawing an explicit link between Peirce's embracement of non-Euclidean geometry and his rejection of determinism. As she puts it:

[F]or Peirce, the non-Euclidean revolution in geometry was part and parcel with a revolution in metaphysics that would mark the demise of strict determinism, and the rise of a metaphysics that embraces chance and possibility ... Indeed, it is no exaggeration to say that, for Peirce, to embrace non-Euclidean geometry was, in a way, to embrace objective chance. (Dea 2008:616)

However, there is a certain aspect of this link which I believe has not received sufficient attention in the literature. The connections that previous commentators have noted between Peirce's embracement of non-Euclidean geometry and his rejection of determinism remain largely *analytical*, whereas, as I will argue, there is a deeper and more immediate relation between the two. Specifically, what I wish to show is that, given Peirce's conception of space as a *law* (just like any other law of nature), the non-Euclidean nature of space can be regarded as an immediate consequence of his doctrine of tychism. This in turn will throw light on the passage from DNE quoted above: If the non-Euclidean nature of space does indeed follow directly from tychism, then Peirce's reference to the “inevitable consequences” of the hypothesis of chance-spontaneity

can be naturally read as an allusion to his work on the curvature of space, which he believed (at the time) supported the hypothesis that space is hyperbolic.

This paper is organized as follows. The first section gives a brief historical sketch of the development of non-Euclidean geometry in the 19th century, insofar as it relates to Peirce's theory of space. In the second section, we turn to Peirce's assimilation of non-Euclidean geometry and the impact it had on his philosophy. Here we will also review his astronomical work attempting to determine the curvature of space. These two sections together provide the background context for my main argument in the third section, which deals with the question of how tychism is related to the geometry of space. After reviewing the existing literature on the topic, I will argue that given Peirce's conception of space as a law, the non-Euclidean nature of space can be regarded as a straightforward consequence of his tychism.

## 1. The Development of Non-Euclidean Geometry

The discovery of non-Euclidean geometry in the early 19th century has its roots in attempts to prove Euclid's fifth postulate, the *parallel postulate*, a task which had occupied mathematicians for centuries since Greek antiquity. In its modern and most well-known form, the postulate states that in a plane, given any line  $l$  and any point  $P$  not on  $l$ , there is exactly one line through  $P$  that is parallel to  $l$  (i.e., does not intersect  $l$ ).<sup>2</sup> Mathematicians across the ages sought to prove this postulate from Euclid's remaining postulates, in part because it seemed to lack the obviousness of those other postulates. Yet, despite centuries of investigation, no such proof seemed to be forthcoming.

While there were some unwitting forays into non-Euclidean geometry in the 18th century (by Girolamo Saccheri and Johann Heinrich Lambert), and while Carl Friedrich Gauss seems to have had at least some faint glimpses of the new geometry, the credit for having undertaken the first systematic study of non-Euclidean geometry must go to Nikolai Lobachevsky and János Bolyai, who worked independently of each other and published their findings in the 1820s and 1830s.<sup>3</sup> The geometry of Lobachevsky and Bolyai is what we today call *hyperbolic geometry*. It is the geometry one obtains by assuming all the postulates of Euclidean geometry except the parallel postulate, and assuming instead that in a plane, given a line  $l$  and a point  $P$  not on  $l$ , there are at least two (in fact, infinitely many) distinct lines through  $P$  that never intersect  $l$ .

Some of the curious properties of hyperbolic geometry include the fact that the angle sum of a triangle is always less than  $\pi$  radians (two right angles); the area of a triangle is proportional to the deficit of its angle sum from  $\pi$ ; and any two similar figures are congruent, from which follows

that there is an absolute measure of length (unlike in Euclidean geometry, where there is an absolute measure of angles but not of length). While Lobachevsky and Bolyai had no notion of *curvature*, the later work of Riemann and Eugenio Beltrami showed that hyperbolic geometry is the geometry of a space of *constant negative curvature*. The spherical geometry studied by Riemann, later modified by Felix Klein into what we today call *elliptic geometry*, is the geometry of a space of constant *positive* curvature, such as the surface of a sphere. Euclidean geometry is the geometry of a flat space of constant *zero* curvature.

An interesting fact about Lobachevsky in view of Peirce's later work is that he took seriously the possibility that hyperbolic geometry—or “imaginary” geometry, as he called it—might be the geometry of physical space, and tried to settle the issue through an analysis of measurements of stellar parallax.<sup>4</sup> This line of thought was already put forth in his first publication on the new geometry, “On the Elements of Geometry” (“O nachalakh geometrii”), which appeared in 1829 in Russian in the *Kazan Messenger* (Lobachevsky 1898–1899, 1:22–24). However, there is no evidence that Peirce ever read this memoir, which was unavailable in any language other than Russian until 1898, when a German translation was published (Lobachevsky 1898–1899, 1:1–66). The only work of Lobachevsky with which Peirce was certainly familiar is the *Geometrische Untersuchungen zur Theorie der Parallelinien* (*Geometrical Researches on the Theory of Parallels*), a short booklet published in German in 1840, but here Lobachevsky only briefly alludes to his earlier astronomical investigations (Lobachevsky 1891:44–45).<sup>5</sup> We therefore cannot assume that Peirce was familiar with Lobachevsky's use of parallax measurements to verify the geometry of physical space, and in any case, the methods employed by Peirce are much more elaborate and wide-ranging than those of Lobachevsky, as can be seen in his “Methods of Investigating the Constant of Space” (W 8:229–230, 1891). Nonetheless, we can say that Peirce was following in Lobachevsky's footsteps in his attempts to determine the curvature of space through astronomical observations.

The pioneering work of Lobachevsky and Bolyai, when it first appeared, fell on deaf ears. Largely neglected by the international community, it was not until the 1860s that mathematicians began rediscovering the new geometry, stimulated by the researches of Riemann, Helmholtz, and Beltrami. By 1870, there were several different non-Euclidean geometries: the hyperbolic geometry of Lobachevsky and Bolyai, the elliptic geometry of Riemann, and projective geometry, which had followed an independent path of development in the 19th century.<sup>6</sup> It was Klein who succeeded in unifying these various geometries, with projective geometry playing the role of the most fundamental geometry. The three metric geometries of constant curvature—the geometries

of Lobachevsky-Bolyai, Riemann, and Euclid—were to be regarded as specializations of projective geometry. It was also Klein who gave the names *hyperbolic*, *elliptic*, and *parabolic* to these three geometries, respectively.

The problem Klein had to overcome was that unlike the metric geometries, projective geometry lacks any notion of distance or angle. What was needed was a way to introduce these metrical notions using only the concepts available in projective geometry. Klein's solution drew its inspiration from Arthur Cayley's paper "A Sixth Memoir upon Quantics" (Cayley 1859). Here, Cayley had shown how one could construct the Euclidean plane, including its familiar metric, from purely projective notions, by introducing in the projective plane a fixed conic "at infinity" which he called the *Absolute*, and measuring distances using this conic. Cayley draws out the implication of this discovery as follows: "[T]he metrical properties of a figure are not the properties of the figure considered *per se* apart from everything else, but its properties when considered in connexion with another figure, viz. the conic termed the Absolute" (Cayley 1859:90). His paper concludes with the striking claim that "[m]etrical geometry is ... a part of descriptive [i.e., projective] geometry, and descriptive geometry is *all* geometry" (Cayley 1859:90). Cayley's interest, however, did not lie in non-Euclidean geometry; he was content in showing that projective geometry is more basic than Euclidean geometry.

Building on Cayley's idea, Klein established the more general result that all three metric geometries of constant curvature can be derived from projective geometry as special cases. His findings were published in his 1871 paper, "Ueber die sogenannte Nicht-Euklidische Geometrie" ("On the So-called Non-Euclidean Geometry," Klein 1871). Here, he shows how one can construct a measure of distances and angles in projective geometry by introducing, in the two-dimensional case, a suitable conic at infinity in the projective plane, and in the three-dimensional case, a suitable quadric surface at infinity in projective space. The conic, which Cayley had termed the "Absolute," is called by Klein the *fundamental conic*, while the quadric is called the *fundamental surface*. Peirce follows Cayley in calling the conic/quadric at infinity the "Absolute" (we shall also follow this terminology in the remainder of this essay). As we will see below, Klein's theory of the Absolute figures prominently in Peirce's philosophy, not only in his theory of space but also in his broader cosmological speculations.

What Klein showed is that the three metric geometries correspond to different choices of the Absolute, as follows:<sup>7</sup>

- (1) Hyperbolic geometry corresponds to the case in which the Absolute is *real*, i.e., the equation of the Absolute has real solutions. In this case, the Absolute has a well-defined "interior" and

“exterior,” and the hyperbolic plane or space lies in the interior. All lines in the hyperbolic plane or space intersect the Absolute at two distinct points.

- (2) Elliptic geometry corresponds to the case in which the Absolute is *imaginary*, i.e., the equation of the Absolute has no real solutions. In this case, the Absolute has no well-defined “interior” and “exterior,” and no line has points at infinity.
- (3) Euclidean geometry (or parabolic geometry, as Klein calls it) corresponds to the limiting transitional case between the above two cases, in which the Absolute degenerates into a pair of imaginary points (called *circular points at infinity*) in the two-dimensional case, or into an imaginary circle (called the *circle at infinity*) in the three-dimensional case. All lines have two coincident points at infinity.

The entities in projective geometry that are posited as lying “at infinity” may seem mysterious at first. However, this is a perfectly natural notion if one considers the way objects appear in our field of vision—which is why Peirce often refers to projective geometry as “geometrical optic(s)” (e.g. RLT 120, EP 2:39, 1898; RLT 243, 1898) or “perspective geometry” (RLT 256, 1898). Imagine, for example, a parallel pair of train rails that appear to meet each other on the horizon. To represent this in projective geometry, one adds *points at infinity* to the familiar Euclidean plane (or better, to the *affine plane*, which is the Euclidean plane with the notions of betweenness and congruence left out). We then say that parallel lines intersect at a point at infinity, just as the parallel train rails appear to meet infinitely far away on the horizon. All lines that are parallel to each other meet at the same point at infinity, while each point at infinity corresponds to a different direction in the plane. The locus of all the points at infinity is called the *line at infinity*, which corresponds to the horizon in our example. The line joining the pair of imaginary points in case (3) above is the line at infinity associated with the Euclidean (or affine) plane.

It turns out that in hyperbolic geometry, we have an entire region at infinity instead of just a line. This is the region exterior to the Absolute, which Peirce calls “the *Jenseits*” or “the other world” (W 8:93–94, 1890).<sup>8</sup> The interior region is the locus of ordinary points of the hyperbolic plane or space. The Absolute itself is the boundary separating the two regions. Asymptotic parallel lines meet on the Absolute, while divergent parallels meet in the exterior region. The *Beltrami-Klein model* or *Klein disk model* of hyperbolic geometry provides a lucid visualization of the Absolute in the hyperbolic plane (Fig. 1).<sup>9</sup> In elliptic geometry, on the other hand, the Absolute is imaginary and there is no longer any distinction between an interior and exterior region. Any two lines meet in the elliptic plane, so there are no coplanar parallel lines in this geometry (in the three-dimensional case, there exist *skew* lines—non-coplanar lines—that do not intersect).

We have dwelt on Klein’s theory of the Absolute at some length not only because of its intrinsic interest, but, more importantly for our purposes, because of the profound influence it had on Peirce’s philosophy. We will take a look at this influence in the next section.

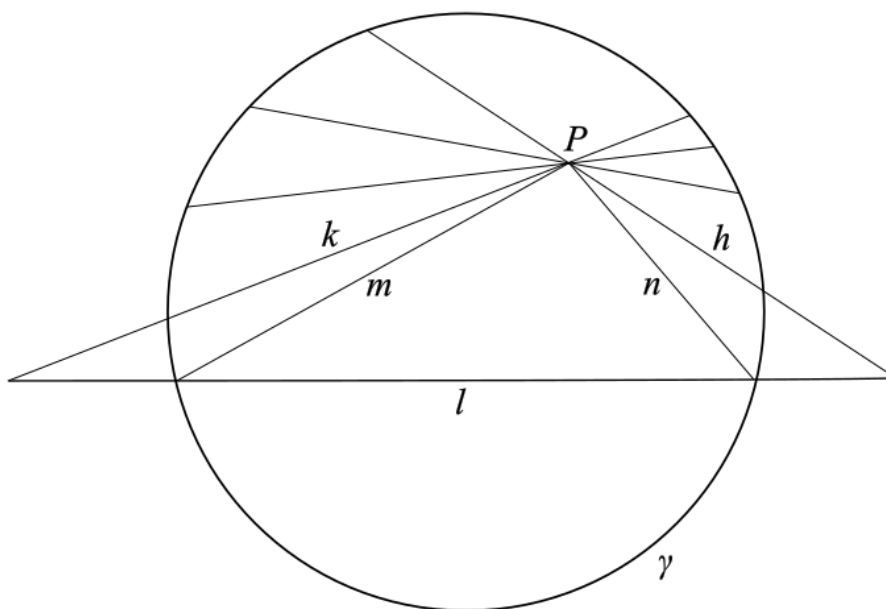


Fig. 1: The Beltrami-Klein model (Klein disk model) of the hyperbolic plane. The circle  $\gamma$  is the Absolute; the hyperbolic plane is represented by the region interior to  $\gamma$ . There are many lines through point  $P$  that are non-intersecting with line  $l$  (in the interior of  $\gamma$ ), reflecting the violation of Euclid’s parallel postulate. Lines  $m$  and  $n$ , which are asymptotically parallel to  $l$ , meet  $l$  on  $\gamma$ , while lines  $k$  and  $h$ , which are divergently parallel to  $l$ , intersect with  $l$  in the region exterior to  $\gamma$ .

## 2. Peirce and Non-Euclidean Geometry

The impact that non-Euclidean geometry had on Peirce ranges from his philosophy of space and hierarchical conception of the different branches of geometry—as elaborated, for example, in his article on “Topical Geometry” (PM 121–123, NEM 2:477–480, 1906)—to his tychism and evolutionary cosmology. Since our main concern in this essay is the connection between Peirce’s tychism and the non-Euclidean nature of physical space, we will not be touching upon all the various aspects of this influence.<sup>10</sup> Instead, we will restrict ourselves to those aspects that are pertinent to our main topic. We will begin by looking at Peirce’s early reception of non-Euclidean geometry, which, interestingly, shows a marked contrast with his more enthusiastic embrace of non-Euclidean geometry in his later years.

Peirce was familiar with the work of Lobachevsky already by 1870. He refers to Lobachevsky

and comments on his work in “Description of a Notation for the Logic of Relatives,” published that year (W 2:416–417, 1870). In a footnote in this paper, Peirce writes:

The researches of Lobatchewsky furnish no solution of the question concerning the apriority of space. For though he has shown that it is conceivable that space should have such properties that two lines might be in a plane and inclined to one another without ever meeting, however far produced, yet he has not shown that the facts implied in that supposition are inconsistent with supposing space to retain its present nature and the properties only of the things in it to change. For example, in Lobatchewsky’s geometry a star at an infinite distance has a finite parallax. But suppose space to have its present properties, and suppose that there were one point in the universe towards which anything being moved should expand, and away from which being moved should contract. Then this expansion and contraction might obey such a law that a star, the parallax of which was finite, should be at an infinite distance measured by the number of times a yard-stick must be laid down to measure off that distance. (W 2:417n11)

Peirce’s argument here seems to be similar to the one later put forward by Henri Poincaré, namely, that empirical facts alone cannot determine which geometry is the “true” one, because empirical facts, whatever they may be, can always be reconciled with the suppositions of Euclidean geometry (or any other geometry), provided that all objects in space are assumed to be distorted in a suitable way (Poincaré 1905, Chaps. 3–5).<sup>11</sup> Unlike Poincaré, however, Peirce does not conclude from this that the choice of which geometry we adopt is a matter of convention. Rather, his view appears closer to that of Kant: he suggests that “the principles of geometry are necessary, and do not result from the specialities of any object cognized, but from the conditions of cognition in general,” where by “conditions of cognition in general” he means “principles which it can be shown cannot become open to doubt from any difficulty in my facts, as long as they continue to be supposed in all logical procedure” (W 2:417n11).

What deserves emphasis, however, is that despite his gesturing towards the Kantian position that space is *a priori*, Peirce is already at this stage open to the possibility that the geometry of space might be non-Euclidean. Indeed, he argues that a further reason why “Lobatchewsky’s conclusions do not positively overthrow the hypothesis that space is *a priori*,” in addition to the one mentioned above, is that the apriority of space is compatible with it having a non-Euclidean geometry: “[Lobachevsky] has only shown that a certain proposition, *not usually believed to be*

*axiomatical*, is conceivably false. That people may be doubtful or even mistaken about *a priori* truth does not destroy all important practical distinction between the two kinds of necessity” (W 2:417n11, emphasis in original). Peirce further notes that while one might argue that “if Lobachevsky’s geometry is the true one, then space involves an arbitrary constant, which value cannot be given *a priori*,” it is quite possible that “the general properties of space, with the general fact that there is such a constant, are *a priori*, while the value of the constant is only empirically determined” (W 2:417n11). The “constant” of space that Peirce is referring to here is the (Gaussian) curvature. In suggesting that the value of this constant might be empirically determined, he is anticipating his later astronomical researches on the curvature of space.

Yet, it should be noted that although Peirce in 1870 is already open to the possibility that space might be non-Euclidean, this is still a far cry from his view in the 1890s, that space *must* be non-Euclidean (which we will examine in §3 below). Furthermore, his attitude towards Lobachevsky’s work at this stage is rather lukewarm, in contrast to the high praise he has for Lobachevsky in the 1890s. In his 1892 review of Halsted’s English translation of Lobachevsky’s *Geometrische Untersuchungen*, for example, Peirce writes that this book “marks an epoch in the history of thought, that of the overthrow of the axioms of geometry,” and that the “philosophical consequences of this are undoubtedly momentous” (W 8:271).<sup>12</sup>

It was through Klein’s theory of the Absolute that Peirce eventually came to appreciate and embrace non-Euclidean geometry. Carolyn Eisele reports that Peirce owned a copy of the lithographed lectures of Klein on *Nicht-Euklidische Geometrie* (Klein 1893), with his notations in the margins (NEM 1:xx). Peirce was also familiar with Klein’s 1871 and 1873 papers on non-Euclidean geometry published in the *Mathematische Annalen* (Klein 1871, 1873); he refers to these as “Klein’s two luminous papers” (NEM 3:979, 1909). However, it is not clear exactly when Peirce became aware of Klein’s theory. A fragment from 1881, where Peirce mentions Klein in connection with a classification of modes of measurement into “elliptic,” “hyperbolic,” and “continuous” (W 4:580, ann. 300.9–12), indicates that he was becoming aware of Klein’s theory by this time.<sup>13</sup> Peirce’s 1885 manuscript on the theory of the Absolute (W 6:248–251) shows that he was definitely familiar with Klein’s classification of geometries by the mid-1880s, although in this manuscript he only mentions Cayley by name. From this period onwards, we begin to see the influence of the Kleinian theory on Peirce’s philosophy.

This influence can be seen, for example, in Peirce’s exposition of projective geometry and the theory of the Absolute in “The Architecture of Theories” (hereinafter abbreviated as “AT”), the first installment of the *Monist* Metaphysical Series (EP 1:293–296, W 8:106–109, 1890). The fact that this discussion occurs in the opening essay of the *Monist* Metaphysical Series is significant,

because it shows how intimately Peirce's considerations of non-Euclidean geometry are intertwined with his cosmological program.

The most obvious indication of Klein's influence, though, is Peirce's classification of philosophies (or rather, cosmologies) into *elliptic*, *parabolic*, and *hyperbolic*. He defines these three philosophies in an unpublished 1890 essay as follows:

1. Elliptic philosophy. Starting-point and stopping-point are not even ideal. Movement of nature recedes from no point, advances toward no point, has no definite tendency, but only flits from position to position.
2. Parabolic philosophy. Reason or nature develops itself according to one universal formula; but the point toward which that development tends is the very same nothingness from which it advances.
3. Hyperbolic philosophy. Reason marches from premises to conclusion; nature has ideal end different from its origin. (W 6:392, 1890)

The same classification also appears in other works from the mid-1880s to the early 1890s (see W6:499, ann. 392.15–25). Peirce's preferred philosophy, of course, is the *hyperbolic philosophy*, according to which the evolution of the world "proceeds from one state of things in the infinite past, to a different state of things in the infinite future" (W 8:386, ann. 111.3–5, 1891). He also had a concomitant predilection for *hyperbolic space*—as we will see below, he hoped to find a negative curvature of space through his astronomical investigations in the 1890s.

Clearly, Peirce's classification of philosophies is an imitation of Klein's classification of the three metric geometries of constant curvature based on the nature of the Absolute. One key point to observe, however, is that while Klein's classification pertains to the geometry of *space*, Peirce's classification pertains to the geometry of *time*. Indeed, Peirce's classification can be obtained by modelling time as a line in projective space and then applying Klein's classification to this line. Recall that according to Klein's classification, if the space is hyperbolic the line has two distinct points at infinity, if the space is elliptic the line has no points at infinity, and if the space is parabolic the line has two coincident points at infinity, matching Peirce's classification of philosophies. Of course, time is not the trajectory of an ordinary *thing* in space, so it is not quite clear what Peirce's justification is for modelling time as a line in projective space (or more specifically, hyperbolic space, since he prefers the hyperbolic philosophy). Murray Murphey has argued that Peirce's preference for hyperbolic space ultimately rests on an *analogy* between space and time: "Although Peirce never envisaged time as the fourth dimension ... nevertheless it seems to have

been the desire to model time upon a one-dimensional continuum in hyperbolic space which governed his preference” (Murphey 1993:384). If this is indeed Peirce’s line of thought, the relation between hyperbolic space and the hyperbolic cosmology would seem to be analogical rather than strictly logical.

Peirce’s preference for hyperbolic space brings us to the topic of his astronomical investigations into the curvature of space. Throughout 1891, and also intermittently in later years, Peirce devoted a considerable portion of his energies to the task of determining the curvature of space (which he also called the “constant of space”). For our purposes, it is well to remember that 1891 is the same year that Peirce wrote DNE—he sent the manuscript off to Paul Carus, editor of *The Monist*, on November 5, 1891. In AT, the essay preceding DNE in the *Monist* Metaphysical Series and published in the January 1891 issue of *The Monist*, Peirce invoked measurements of stellar parallax in connection with the question of the geometry of physical space, but did not draw any positive conclusion as to the geometry of space (EP 1:295–296, W 8:108–109, 1890). While noting that there are two stars that are observed to have negative parallaxes, he ascribed these to “errors of observation” and concluded: “[w]hich of these three hypotheses [i.e., the parabolic, hyperbolic, and elliptic hypotheses] is true we know not” (EP 1:295, W 8:108, 1890).

In March 1891, two months after the publication of AT in *The Monist*, Peirce began corresponding with his computing assistant Allan Risteen on methods of measuring the curvature of space. He had decided to dig deeper into the issue, spurred by Risteen’s article “On the Determination of Parallax by the Spectroscope,” published in the June 20, 1890 issue of *Science* (Risteen 1890). On March 24, Peirce drafted an outline entitled “Methods of Investigating the Constant of Space,” which he apparently intended to send to Risteen (though it appears it was unsent). In the most developed version of the outline, published in the *Writings* (W 8:229–230, 1891), Peirce lists fifteen methods of determining the curvature of space that involve not only stellar parallax but also the proper motions of stars, the distribution of stars of different magnitudes, and Doppler shifts in stellar spectra.

Later that year, at the National Academy of Sciences meeting held on November 10–12, just a few days after having sent out the manuscript of DNE to *The Monist*, Peirce presented a paper on “Astronomical Methods of Determining the Curvature of Space.” While the paper itself has unfortunately been lost, we can get an idea of its content from an abstract published in the *Bulletin of the New York Mathematical Society*: “Professor Peirce presented astronomical evidence tending to show that space possesses a negative curvature, and called attention to various methods of conducting an investigation of this property of space” (Fiske & Jacoby 1892:80; W 8:492, 623).

On December 17 and 21, Peirce wrote to the astronomer Simon Newcomb, inquiring about the

possibility of getting a grant to continue his research on the curvature of space (Eisele 1957:421–422).<sup>14</sup> In the letter of December 17, Peirce gives Newcomb some of his calculations and asks “how encouraging the figures look in regard to my attempt to make out a negative curvature of space” (Eisele 1957:421). But in the December 21 letter, Peirce writes that, having done some calculations with some of the stars in Newcomb’s catalogue, the result he got was “decidedly the reverse” of what he wanted (Eisele 1957:422). In a letter draft written on the same day, Peirce appears to have changed his mind again: “One thing is clear. It is that those figures I sent you lend no support whatever to the idea of a positive curvature, but are rather against it” (Eisele 1957:422). On December 24, Newcomb replied with a discouraging letter that pointed out difficulties involved in Peirce’s methods and advised against his seeking a grant (Eisele 1957:424). It is likely that Peirce hadn’t received this reply yet when he penned another letter to Newcomb on Christmas Day, writing: “I have for the present given up the idea that anything can be concluded with considerable probability concerning the curvature of space” (Eisele 1957:423). The letter concludes: “[A]ll things considered, I can only say that excessively doubtful indications favor a negative curvature. In point of fact, we remain in ignorance” (RL 314:47).<sup>15</sup>

In spite of what he had written in his letter to Newcomb, Peirce continued his investigations in the following years. On November 24, 1894, he gave a talk at the New York Mathematical Society entitled “Rough Notes on Geometry, Constitution of Real Space.” In the paper on which the talk was most likely based, he summarizes his methods of determining the curvature of space and observes that “they seemed to indicate a hyperbolic space with a constant far from insignificant” (NEM 3:707–708, 1894). Peirce’s confidence in the hyperbolicity of space seems to have grown even stronger by 1896. In a 22 March 1896 letter draft to the mathematician William Story, he writes: “[M]y calculations from Auwers’ proper motions leave no reasonable doubt that space is really hyperbolic” (NEM 2:ix). Again, a 1901 manuscript entitled “On Two Map-Projections of the Lobatschewskian Plane” begins with the declaration: “The proper motions of the stars show very strong indications that our space is really hyperbolic” (NEM 3:710).

What I want to re-emphasize here is that Peirce was most actively engaged in his study of the curvature of space at the same time as he was writing DNE. Therefore, as far as timing is concerned, it is quite plausible to suppose that when he speaks of the “observed facts” supporting the hypothesis of chance-spontaneity towards the end of DNE (EP 1:309, W 8:124), he is alluding to his astronomical work on the curvature of space. But the question we have yet to answer is: what exactly is the connection between tychism and the non-Euclidean nature of physical space? Why should Peirce have thought that the discovery of the hyperbolicity of space would lend support to tychism? This will be topic of the next section.

### 3. Tychism and the Non-Euclidean Nature of Physical Space

Peirce's writings make it abundantly clear that in his mind, the issue of tychism was somehow intimately related with the question of the geometry of space. We can see this, for instance, in AT, where a discussion of the overthrow of the axioms of Euclidean geometry serves as a transition into the subject of tychism. Having posed the question of the geometry of space (i.e., whether it is parabolic, hyperbolic, or elliptic), Peirce writes that the axioms of Euclidean geometry "are now exploded," and that geometers "know not the slightest reason for supposing them to be precisely true" (EP 1:296, W 8:109, 1890). He continues:

Now, metaphysics has always been the ape of mathematics. Geometry suggested the idea of a demonstrative system of absolutely certain philosophical principles; and the ideas of the metaphysicians have at all times been in large part drawn from mathematics. The metaphysical axioms are imitations of the geometrical axioms; and now that the latter have been thrown overboard, without doubt the former will be sent after them. It is evident, for instance, that we can have no reason to think that every phenomenon in all its minutest details is precisely determined by law. That there is an arbitrary element in the universe we see,—namely, its variety. This variety must be attributed to spontaneity in some form. (EP 1:296, W 8:109, 1890)

The same line of thought can also be found in "A Guess at the Riddle" (EP 1:273, W 6:203–204, 1887–1888) and an initial version of AT (W 8:89, 1890). The connection between non-Euclidean geometry and tychism is again hinted at in Peirce's 1892 review of Halsted's English translation of Lobachevsky's *Geometrische Untersuchungen*. Here he writes: "there are thinkers who hold that it [i.e., the overthrow of the axioms of geometry] must lead to a new conception of nature, less mechanical than that which has guided the steps of science since Newton's discovery" (W 8:271). Although he speaks of "thinkers" in the third person, he is evidently referring to himself.

Peirce's most explicit statement of the connection between the non-Euclidean nature of space and tychism can be found in his correspondence with Newcomb. In his 17 December 1891 letter to Newcomb, he writes:

The discovery that space has a curvature would be more than a striking one; it would be epoch-making. It would do more than anything to break up the belief in the immutable character of mechanical law, and would thus lead to a conception of

the universe in which mechanical law should not be the head and centre of the whole. (Eisele 1957:422)

It is clear from these passages that for Peirce, tychism was somehow inseparably bound up with the non-Euclidean nature of space—in fact, bound up to such an extent that the discovery of a non-zero curvature of space would “do more than anything to break up the belief in the immutable character of mechanical law.” These statements, I believe, lend strong support to my reading of the passage in DNE about the observed facts supporting tychism as an allusion to his work on the curvature of space.

Unfortunately for us, Peirce does not spell out the exact nature of the relation between tychism and the geometry of space in these passages. Even if we grant that the axioms of Euclidean geometry have been overthrown by the discovery of non-Euclidean geometry, or that astronomical observations have revealed that space has a non-zero curvature, why should that lead us to reject determinism? Let us turn to the Peircean literature to answer this question.

As was mentioned in the introduction, previous commentators have noted the connection between, on the one hand, Peirce’s belief in the non-Euclidean nature of space, and on the other, his evolutionary cosmology and tychism in particular. We already saw in the previous section how Murphey links Peirce’s preference for hyperbolic space with his preference for the hyperbolic cosmology, arguing that it is ultimately based on an analogy between space and time (Murphey 1993:384). A similar account is given by Andrew Reynolds, who speaks of the “geometric analogy” behind Peirce’s classification of the three types of philosophy (Reynolds 2002:115). However, both Murphey and Reynolds confine their attention to Peirce’s broader hyperbolic cosmology; neither of them deals specifically with the issue of tychism and its relation to non-Euclidean geometry.

The link between Peirce’s embracement of non-Euclidean geometry and his rejection of determinism is drawn explicitly by Dea (2008). She explains this link as follows:

Peirce argued that Euclidean geometry encourages the belief that fundamental questions are soluble by reason alone—that, without ever measuring the angles of a triangle, for instance, it is possible to determine the sum of its angles. A universe in which this is possible is as precise and predictable as the sum of those angles ... Absent the parallel postulate and the *a priori* certainty of Euclidean approaches, Peirce saw no good reason to be certain that a triangle’s angles sum to precisely

180 degrees. And, if not even geometrical facts like this one are susceptible of perfect certainty and precision then, Peirce reasoned, there is little reason for confidence that empirical questions of causation are susceptible of perfect certainty and precision. This allowed Peirce to carve out a space for chance and vagueness in the physical universe—a space which more easily conformed to an evolving universe characterized by stochastic causation than the static universe of mechanistic determinism. (Dea 2008:615)

This undoubtedly captures at least one aspect of Peirce's thinking, as can be seen in the various passages (such as the one from AT quoted above) where he argues that metaphysics has always been modelled on geometry, and that non-Euclidean geometry must lead to a rejection of mechanical determinism. Nonetheless, the link between non-Euclidean geometry and tychism as described by Dea seems to boil down, again, to analogy. Strictly speaking, there is no clear reason why the unattainability of perfect certainty and precision in the case of certain geometrical facts should carry over into empirical questions of causation, which belong to an entirely different domain of inquiry. Moreover, Dea's account does not explain why Peirce thought that the discovery of a non-zero curvature would "do more than anything to break up the belief in the immutable character of mechanical law," as he writes in his correspondence with Newcomb. It would seem, according to her account, that the mere discovery of non-Euclidean geometry should be sufficient to bring about that result.

A deeper glimpse into the connection between tychism and the geometry of space is afforded by Peirce's argument that space *must* be non-Euclidean. This argument has been well-studied by Dipert (1977). Peirce advances the argument in an initial version of AT (W 8:88, 1890).<sup>16</sup> As in the published version, he emphasizes that "there is no reason whatever to think the sum of the three angles of a triangle precisely equal to 180°" (W 8:88; note that a value of precisely 180° is a logical consequence of space being precisely Euclidean). He then goes on to argue:

[T]here is an infinite number of different possible values, of which precisely 180° is only one; so that the probability is as 1 to  $\infty$ , or 0 to 1, that the value is just 180°. In other words, it seems for the present impossible to suppose the postulates of geometry precisely true. The matter is reduced to one of evidence; and as absolute precision is beyond the reach of direct observation, so it can never be rendered probable by evidence, which is indirect observation. (W 8:88)

Dipert points out that Peirce is here making two distinct arguments. The first is a “probabilistic argument” that makes use of the idea that since a value of precisely  $180^\circ$  is just one out of infinitely many possible values, the probability of the angle sum having that exact value is zero. Dipert argues that this argument is flawed, because when dealing with infinitely many possibilities, the assignment of a zero probability to a particular case does not entail that it cannot occur (Dipert 1977:406). It is indeed flawed, *if* the conclusion Peirce was trying to draw is that space cannot be Euclidean. However, a more charitable reading of Peirce’s line of thought here is that the two arguments are rather two steps of a single argument leading to that conclusion, and that the first step (what Dipert calls the probabilistic argument) is only intended to show that Euclidean geometry cannot be assumed, on *a priori* grounds, to be precisely true. Note the way Peirce phrases his conclusion: “it seems for the present impossible *to suppose* the postulates of geometry precisely true” (emphasis added), not “it seems for the present impossible that the postulates of geometry are precisely true.” Note also that the upshot of this is that “[t]he matter is reduced to one of evidence.” On this reading, Peirce’s probabilistic argument seems to be free of serious objections.

Peirce’s second argument (or second step, on our reading) is based on the unattainability of absolute precision through observation. This argument is interesting for our purposes because, as Dipert notes, it is “essentially the argument he so forcefully used against the ‘doctrine of necessity’” in DNE (Dipert 1977:407). As a practicing scientist, Peirce was well aware of the inexactitude inherent in measurement. That is why he believed that observation could never warrant a statement of the form “ $x$  is exactly  $a$ ,” where  $x$  is a continuously varying quantity and  $a$  is a specific value of  $x$ . As he puts it in DNE: “any statement to the effect that a certain continuous quantity has a certain exact value, if well-founded at all, must be founded on something other than observation” (EP 1:304, W 8:117, 1891). This is the basis from which Peirce launches his attack on determinism. His other core insight is that “the essence of the necessitarian position is that certain continuous quantities have certain exact values” (EP 1:303, W 8:116, 1891). By “certain continuous quantities,” he presumably has in mind quantities such as the position and momentum of particles. From this Peirce is able to argue that determinism cannot be defended on empirical grounds, because the assumption that certain continuous quantities have certain exact values is a kind of statement that goes beyond what can be warranted by any observation.

Peirce’s argument that space cannot be Euclidean is essentially the same argument. The discovery of non-Euclidean geometry has shown that the question of whether space is precisely Euclidean cannot be settled on *a priori* grounds, and so it must be settled on empirical grounds, e.g. by measuring the sum of the interior angles of a triangle. But the statement that a continuously

varying quantity such as an angle sum has a certain exact value is one that can never be warranted by observation. Therefore, the statement that space is precisely Euclidean cannot be justified on either *a priori* or empirical grounds, and so we must assume that space is non-Euclidean—thus runs Peirce’s argument.

The commonality in argumentative structure between Peirce’s argument that space must be non-Euclidean and his argument against determinism is striking. It allows us to see that his belief in the non-Euclidean nature of space and his tychism are both rooted in a unique epistemological thesis about a certain kind of statement that can never be warranted on observational grounds, and are in this sense intimately linked. Yet, this still does not suffice to explain why, for Peirce, the discovery of a non-zero curvature of space should be the “strongest reason” for his belief in tychism—if, as I am urging, he is indeed alluding to his work on the curvature of space in DNE—or why it “would do more than anything to break up the belief in the immutable character of mechanical law.”

I suggest that there is a deeper, more immediate connection between the two. Namely, what I wish to argue is that the non-Euclidean nature of space can be regarded as an *immediate consequence* of tychism. The key observation here is that according to Peirce, space is a *law* just like any other law of nature: specifically, it is a law that governs how easily a certain state may change into another state, or how easily two things may react with one another. In the account of the origin of the universe presented in “A Guess at the Riddle,” Peirce explains how spatial extension could be supposed to have arisen from the tendency to take habits:

[B]esides flashes genuinely second to others, so as to come after them, there will be pairs of flashes, or, since time is now supposed to be developed, we had better say pairs of states, which are reciprocally second, each member of the pair to the other. This is the first germ of spatial extension. These states will undergo changes; and habits will be formed of passing from certain states to certain others, and of not passing from certain states to certain others. Those states to which a state will immediately pass, will be adjacent to it; and thus habits will be formed which will constitute a spatial continuum, but differing from our space by being very irregular in its connections, having one number of dimensions in one place and another number in another place, and being different for one moving state from what it is for another. (EP 1:278–279, 1887–1888)

While Peirce’s account is admittedly obscure on many points, we can tease out the rough idea of

space he is working with here. We assume that time has already been developed. At each moment in time, we have a number of “states.” Peirce says that “states” is another name for “flashes,” so these are presumably events of some sort. Initially, these states are not in any spatial relations with each other. Yet, they are capable of changing into other states at subsequent moments in time. Peirce’s idea seems to be that space consists of habits that determine how easily a certain state can pass into another state. For example, if a state can immediately pass into another state, then those states are spatially “adjacent” (this assumes, of course, that it makes sense to speak of the same state as somehow existing at different moments in time). On the other hand, if a state A cannot pass into a second state B without passing through a third state C, then C lies between A and B on a line, and so on. It is in this sense that space can be said to be a law of nature: it is a pattern of connectivity among states that determines which transitions are possible or probable. Presumably, once this kind of spatial extension has been developed, the passing of states into other states can be understood as motions.

A similar but slightly different account of space is given in the Cambridge Conferences Lectures of 1898. In the sixth lecture, Peirce suggests that “space is that form of intuition in which is presented the law of the mutual reaction of those objects whose mode of existence consists in mutually reacting” (RLT 212). In the same passage he also speaks of space as “a presentation of law” and as having “the mode of being of a law” (RLT 212). The idea here is that space is a law (or “presentation” of a law) that determines how easily certain objects are able to react with one another. Despite the Kantian language invoked in the first formulation, Peirce makes it clear in his article “On Quantity” (c. 1896) that “space is an objective reality and not merely a form of intuition,” noting that he considers Kant’s argument that space is a form of intuition “illogical” (NEM 3:62). We need not go into the details of Peirce’s 1898 account of space, as it was written several years after DNE. For our purposes, it will suffice to note that despite the differences from his earlier account given in “A Guess at the Riddle,” he is consistent in his view that space has a law-like nature.

Peirce’s tychism is the doctrine that no law of nature is perfectly exact: for any given law, there will always be events or facts that deviate from the law in an irregular fashion. Pure spontaneity, or absolute chance, is a fundamental facet of the universe, “acting always and everywhere though restrained within narrow bounds by law, producing infinitesimal departures from law continually, and great ones with infinite infrequency” (EP 1:308, W 8:122, 1891). This view is part and parcel of Peirce’s theory that the laws of nature we observe are the result of an evolutionary process. Given his conception of space as having a law-like nature, the same can be said with respect to

space. That is, it follows from tychism that the variable properties of space—including its curvature—cannot be perfectly exact. Even if there is an evolutionary tendency for the curvature of space to approach zero (and hence for space to become Euclidean), there will always be slight deviations from that precise value. This is an inevitable consequence of space being the result of an evolutionary process: a non-zero curvature can be regarded as a relic of space’s evolutionary origin. It is in this sense that the non-Euclidean nature of space is directly implied by tychism. Therefore, the passage in DNE where Peirce speaks of the “observed facts” supporting tychism (EP 1:309, W 8:124, 1891) can be read naturally as an allusion to his astronomical research on the curvature of space. A non-zero curvature is one of the “inevitable consequences” of the hypothesis of chance-spontaneity. It is no wonder, then, that Peirce should regard the discovery of the hyperbolicity of space as an empirical confirmation of that hypothesis.<sup>17</sup>

## Concluding Remarks

As was mentioned in the introduction, the impact that non-Euclidean geometry had on Peirce’s philosophy is a topic that remains largely unexplored. The present study has been an attempt to begin filling in this lacuna by focusing on one specific aspect of this influence, namely, the relation between Peirce’s tychism and his belief in the non-Euclidean nature of physical space. There are many other facets of this influence that call for further study. For example, there is the question of whether and to what extent Peirce’s view of the *a priori* nature of space changed throughout his career as a result of his engagement with non-Euclidean geometry (as we saw in §2, his view in 1870 was that non-Euclidean geometry does not challenge the apriority of space). There is also the vexed issue of the relation between non-Euclidean geometry and Peirce’s defense of the Newtonian concept of absolute space, briefly touched upon by Dipert (1973:85–86). Finally, there is the question of why Peirce never doubted that the curvature of space must be uniform throughout the universe, despite the fact that he was well-acquainted with Riemann’s *Ueber die Hypothesen, welche der Geometrie zu Grunde liegen* (*On the Hypotheses Which Lie at the Bases of Geometry*, 1854), the work that inaugurated the study of spaces of variable curvature.

Exploring Peirce’s engagement with non-Euclidean geometry will not only give us a better picture of the development of his thought. It will also help us situate Peirce within the broader intellectual landscape of scientific philosophy in the late 19th and early 20th centuries, when philosophically minded scientists and mathematically inclined philosophers were trying to come to grips with the momentous implications of the non-Euclidean revolution.<sup>18</sup>

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<sup>1</sup> See the list of references at the end of this paper for an explanation of the abbreviations used in referring to Peirce's writings.

<sup>2</sup> This is the form of the postulate often attributed to the Scottish mathematician John Playfair and referred to as *Playfair's axiom*, although essentially the same formulation had already been given by Proclus in the 5th century.

<sup>3</sup> Historical accounts of non-Euclidean geometry often follow Bonola (1955 [1912]) in crediting Gauss as a co-discoverer of non-Euclidean geometry, along with Lobachevsky and Bolyai. This view has been challenged by Jeremy Gray (2005), who argues that while Gauss may have "possessed tantalising hints of a new, non-Euclidean, geometry," there is not enough evidence to claim that he "worked his ideas up into a systematic theory" (Gray 2005:75).

<sup>4</sup> See Daniels (1975) for a detailed account of the astronomical arguments employed by Lobachevsky.

<sup>5</sup> Peirce wrote a review for *The Nation* of George Bruce Halsted's English translation of this booklet (W 8:271–274, 1892). It appears that Peirce was already acquainted with this work by 1870, as we will see below in §2.

<sup>6</sup> To be more precise, *elliptic geometry* is the name given by Klein to his modification of the spherical geometry studied by Riemann. The difference between the two geometries is that in elliptic geometry, antipodal points are identified so as to make Euclid's first postulate hold. See Coxeter (1998:13) for details.

<sup>7</sup> The author is indebted to the account of Klein's theory given in A'Campo & Papadopoulos (2015).

<sup>8</sup> See also Peirce's 1896 letter draft to William Story, where he writes: "I regard the *jenseits* of the real absolute of hyperbolic geometry as real" (NEM 2:vi).

<sup>9</sup> For details, see e.g. Greenberg (2008), Chap. 7.

<sup>10</sup> In particular, we will not be dealing with the concept of *continuity*, despite the prominent place it occupies in Peirce's philosophical system. The reason for this is that curvature is a metrical notion, not a topological notion (although it does presuppose some notion of continuity or smoothness, such as that afforded by a differentiable manifold). The topological (continuity) structure of a geometry lies at a "deeper" level than its metrical and projective structures—a point reflected in Peirce's division of geometry into *topics* (topology), *graphics* (projective geometry), and *metrics* (metrical geometry), topics being the most fundamental (e.g. PM 121–123, NEM 2:477–480, 1906). The question of the curvature of space that concerns us here, by contrast, is a question of metrical structure, and for Peirce it is framed within the setting of the so-called Cayley–Klein geometries. These geometries are all defined with respect to the same ambient projective space, which itself carries a fixed topology. Accordingly, while continuity is undoubtedly a concept of central importance in Peirce's broader metaphysics, as far as the curvature of space is concerned, considerations of continuity do not play an explicit or discriminating role.

<sup>11</sup> See Peirce's 1909 letter to Francis Russell, where he reflects on his 1870 argument: "In 1870, I put forth, perhaps before anybody else, the suggestion—without guaranteeing it, — that it might be possible rationally to consider space as Euclidean whatever discoveries might be made looking to the truth of one or other of the non-Euclidean theories, and the Nancy man whose name escapes me takes that ground strongly" (NEM 3:979). The "Nancy man" that Peirce mentions here is, evidently, Poincaré.

<sup>12</sup> See also Peirce's 1895 review of Alexander Vasiliev's biography of Lobachevsky, where he lauds Lobachevsky as a "profound ... genius" whose writings are all marked by "high strung logic" and

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“extremely accurate thought” (CN 2:99).

<sup>13</sup> Although Peirce writes “continuous” here, he may have meant “parabolic,” as he writes later in the same manuscript.

<sup>14</sup> While the first letter from Peirce to Newcomb is undated and recorded as such in Eisele’s paper (Eisele 1957:421n45), Max Fisch has dated it c. 17 December 1891 based on circumstantial evidence (Max H. Fisch Reference Catalogue at the Peirce Edition Project, Indianapolis).

<sup>15</sup> This portion of the letter is missing from Eisele (1957). Also, while the date of the letter is given by Eisele as “Dec. 23” (Eisele 1957:423), here I follow Fisch, who transcribes the date as “Dec. 25.” “RL” refers to correspondence housed in Harvard University’s Houghton Library. The number immediately following “RL” is the catalogue number assigned by Richard S. Robin in his *Annotated Catalogue of the Papers of Charles S. Peirce* (Robin 1967); the second number is the page number according to the ISP numbering scheme.

<sup>16</sup> Dipert cites the relevant passage from the *Collected Papers* (Dipert 1977:404), where it is described as an “unpaginated fragment, c. 1893” (CP 1.130). However, the passage actually belongs to an initial draft of AT, written in the summer of 1890.

<sup>17</sup> One may well wonder, though: is this a good argument? Space having a non-Euclidean geometry is certainly compatible with determinism, as we now know from the general theory of relativity (although, strictly speaking, it is *space-time* rather than space that is curved according to that theory). What should be borne in mind, however, is that the paradigm of physical law in Peirce’s time was Newtonian mechanics, which presupposes a three-dimensional Euclidean space as the arena in which all motions take place. From that perspective, any deviation from Euclidean space could be naturally construed as a deviation from physical law itself. (That said, it should be noted that this line of thought is by no means straightforward; for example, a Poincaré-style argument of the kind mentioned in §2 above could be invoked to argue that Newton’s laws of motion are compatible with a non-Euclidean space.) Thanks to Aaron Wilson for raising this important question.

<sup>18</sup> This paper was presented at the 53rd annual meeting of the Society for the Advancement of American Philosophy, held at the University of Nevada, Las Vegas on March 12–14, 2026. Many thanks to the audience for their stimulating questions and comments, especially Chiara Ambrosio, David Beisecker, Stephen Houchins, Robert Lane, and Aaron Wilson. I also want to express my gratitude to André De Tienne, not only for providing me with information related to the Peirce-Newcomb correspondence, but also for guiding me during my graduate studies many years ago and initiating me into the world of Peirce’s philosophy. I would not be where I am today without his dedicated mentorship. Finally, special thanks to the Charles S. Peirce Society for awarding this paper the 2025 Peirce Essay Prize. This work was supported by JSPS KAKENHI Grant number JP24K15893.