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# Long-term risk-informed ship design optimization

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## ABSTRACT

To ensure the safety and durability of a ship operating in ocean waves, it is essential to characterize not only the short-term but also the long-term statistical properties of wave-induced responses over the ship's life. Long-term risk is typically evaluated through long-term prediction (LTP), which aggregates numerous short-term predictions (STPs) of extreme responses across sea states and operational conditions.

This paper proposes an efficient design optimization methodology that explicitly incorporates long-term safety. The mean outcrossing rate (MOR) relative to a design threshold is adopted as the target reliability metric. To accelerate the LTP process, a Bayesian Active Learning (BAL) strategy is introduced, where a Gaussian Process Regression model of the MOR is adaptively updated. By treating the MOR as a log-normal process, the non-negativity of the MOR is ensured. Furthermore, the design optimization is executed within a single-loop framework, bypassing the conventional double-loop structure by simultaneously processing Bayesian Optimization and the BAL for LTP. The proposed framework is demonstrated through numerical examples involving the optimization of ship parameters under different nonlinear responses. Finally, a First Order Reliability Method (FORM)-based STP procedure and a multi-fidelity strategy combining low- and high-fidelity response models are introduced, thereby enabling high-fidelity nonlinear simulations while maintaining computational tractability.

## 1. Introduction

Ships in service are subjected to random excitation forces induced by ocean waves. Ideally, the relevant *design variables* of the ship are optimized so as to reduce the risk of experiencing excessive wave-induced responses over its service period. By convention, the statistics of the ocean environment expected to be encountered over the operational period are used to predict long-term extreme responses, which is generally called “long-term prediction” (LTP) [1]. The parameters to describe short-term wave statistics, e.g., the significant wave height and peak wave periods, and those to describe operational variability, i.e., the ship speed and wave encounter angle, are to be interpreted as random variables for conducting LTP [2]. An accurate LTP can be attained by executing Monte Carlo Simulation (MCS) of multiple STPs of extreme values, conditional on the given environmental and operational random variables (hereafter collectively referred to as “environmental random variables”). However, as the numerical model leveraged for short-term predictions (STPs) becomes more computationally expensive to account for the inherent nonlinearity in the response, directly applying MCS

becomes computationally prohibitive. Consequently, the high computational burden associated with LTP precludes the practical incorporation of design optimization for mitigating long-term risk.

Several attempts have been made to accelerate the LTP process up to recently. The Environmental Contour Method (ECM) has been widely used to select critical sea states for the response of interest [3, 4]. While ECM provides an efficient framework for design with the requirement of only a limited number of STPs, its robustness with respect to nonlinear wave-induced responses remains uncertain. To account for the nonlinearity in ship responses, specifically roll motion, Matsui and Oka [5] proposed a Response Amplitude Operator (RAO)-based Translation Process method for efficient LTP. However, the strong nonlinearity in the roll motion, i.e. the high order damping and nonlinear restoring, cannot be addressed. Gramstad et al. [6] developed a sequential learning framework for nonlinear vertical bending moment (VBM), in which Gaussian Process Regression (GPR) acts as a surrogate model to estimate the Weibull distribution parameters for short-term VBM statistics based on given environmental random

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## Nomenclature

|         |                                                     |
|---------|-----------------------------------------------------|
| BAL     | Bayesian Active Learning                            |
| BAL-LTP | Bayesian Active Learning-based Long-term Prediction |
| BO      | Bayesian Optimization                               |
| CABO    | Collaborative and Adaptive Bayesian Optimization    |
| CFD     | Computational Fluid Dynamics                        |
| COV     | Coefficient of Variation                            |
| CPU     | Central Processing Unit                             |
| ECM     | Environmental Contour Method                        |
| EI      | Expected Improvement                                |
| FORM    | First Order Reliability Method                      |
| GP      | Gaussian Process                                    |
| GPR     | Gaussian Process Regression                         |
| ITTC    | International Towing Tank Conference                |
| LTP     | Long-term Prediction                                |
| MC      | Monte Carlo                                         |
| MCS     | Monte Carlo Simulation                              |
| MOR     | Mean outcrossing rate                               |
| MPP     | Most Probable Point                                 |
| NMRIW   | Nonlinear Motion in Regular and Irregular Wave      |
| NSM     | New Strip Method                                    |
| PDF     | Probability Density Function                        |
| PoE     | Probability of Exceedance                           |
| PSO     | Particle Swarm Optimization                         |
| PVC     | Posterior Variance Contribution                     |
| RAO     | Response Amplitude Operator                         |
| ROM     | Reduced Order Model                                 |
| RTP     | RAO-based Translation Process                       |
| STP     | Short-term Prediction                               |
| TRF     | Transfer function                                   |
| VBM     | Vertical bending moment                             |

variables. The method in [6], however, has a limitation in that it cannot address rare events in individual short-term sea states, as the tail behavior of the distribution must be faithfully reproduced during the STP stage. Kim et al. [7] employed an approach that resembles that of [6], while introducing time-domain response computations for evaluating the first-passage probability. They utilized a heteroscedastic GPR framework to account for aleatory uncertainty in the STP results. However, their method necessitates an a priori assumption on the type of the response probability distribution obtained from the STPs, which hinders the accurate estimation of rare events; moreover, its effectiveness diminishes in high-dimensional stochastic spaces. In performing the STPs, the stochastic space is typically high-dimensional in order to represent the (inherent) aleatory randomness associated with ocean waves, typically requiring on the order of 50–200 dimensions to describe the incident wave spectrum [8].

To rigorously assess rare events, the use of the long-term mean outcrossing rate (MOR) of extreme responses offers a robust metric. The MOR is generally interpreted as a metric equivalent to the “Probability of Exceedance (PoE) per unit time”, i.e., the average rate at which a stochastic process crosses a prescribed threshold [9]. Lystad et al. [10] and Castellon et al. [11] demonstrated that a GPR-based sequential learning method is effective for long-term MOR estimation of bridge buffeting responses. While their methodologies show promise for broad application, their frameworks rely on training the GPR to estimate both the variance and the second-order moment of the response, which may

entail substantial computational cost when applied to nonlinear ship responses [12]. In the context of wave-induced ship responses, Nielsen et al. [9] and Nielsen [13] conducted pioneering studies introducing the First Order Reliability Method (FORM) to expedite the estimation of MORs in individual STPs, i.e., conditional MORs. By utilizing FORM for the STPs, an introduction of the sequential active learning process for GPR would further accelerate the LTPs in terms of MOR of the extreme response.

In light of recent progress in reliability engineering, the Bayesian active learning (BAL) [14,15] has attracted attention to achieve efficient training of GPR. In [14,15] where the failure probability in stochastic excitations is addressed, the BAL approach is used to frame the active learning process as a Bayesian inference problem, quantifying the uncertainty in the GPR-estimated failure probability based on the training samples as a posterior distribution. Recently, the authors’ research group demonstrated the effectiveness of the BAL approach for long-term MOR estimation, see Takami et al. [16]. However, the metamodeling of the MOR in [16] still leaves room for improvement, as the MOR typically spans multiple orders of magnitude and must remain strictly non-negative. Furthermore, although BAL approaches have proven effective, as exemplified in [14–17] the target posterior is often not analytically tractable and thus one needs to rely on a conditional Gaussian Process (GP) sampling to approximate the posterior. For this, a vast number of conditional predictions need to be generated from the trained GPR, which not only incurs significant Central Processing Unit (CPU) memory requirements but also potentially introduces sampling-dependent uncertainty into the final results.

Even with the introduction of the BAL approach, the design optimization process requires multiple BAL executions to compute long-term MORs for given design variables, resulting in the so-called “double-loop” problem. This issue can be addressed by adopting a collaborative approach in which BO for design optimization and BAL for long-term MOR prediction are processed simultaneously. In this context, Hong et al. [17] proposed the Collaborative and Adaptive Bayesian Optimization (CABO) methodology, demonstrating the effectiveness and efficiency of a “single-loop” approach for design optimization. While effective, CABO still employs a GP sampling phase to evaluate uncertainties in the probabilistic features of the responses, requiring substantial CPU memory and making the estimation results sensitive to the number of GP samples. In light of these limitations, the development of a GP sampling-free single-loop methodology, combined with a rational metamodeling of the MOR, is urgently needed. Furthermore, to achieve practical computational efficiency, it is essential to accelerate not only the LTP and optimization processes but also the STP process, since the latter requires repeated evaluations of the computational model to estimate the conditional MOR. Accelerating these three components, namely design optimization, LTP, and STP, enables the entire optimization procedure to be completed within a realistic computational time (i.e., within a few hours).

From the above perspective, in this paper, a new active learning methodology for efficient execution of design optimization targeting the long-term MOR is proposed. The present methodology incorporates the Bayesian Active Learning-based Long-Term Prediction (BAL-LTP) process. In contrast to the conventional BAL approaches referenced above, the present BAL-LTP incorporates a sampling-free methodology for posterior uncertainty estimation. To enforce a non-negativity of the MOR, which typically spans multiple orders of magnitude, the MOR is modeled as a log-normal process. Building upon the BAL-LTP framework, this paper incorporates a single-loop approach for design optimization targeting the long-term MOR. The objective is to optimize design variables to either achieve a prescribed long-term MOR or minimize the long-term risk. The proposed single-loop approach avoids the computationally expensive double-loop problem by simultaneously executing BO and BAL-LTP. Owing to its inherent versatility, the framework of the present single-loop approach is applicable to a wide range of ship design optimization problems involving LTP. Two

numerical demonstrations are conducted to illustrate the efficiency and effectiveness of the proposed design optimization procedure: (i) optimization of the metacentric height — a fundamental parameter characterizing the ship's transverse stability and its resonance characteristics — accounting for nonlinear roll motion, and (ii) optimization of the longitudinal weight distribution considering nonlinear VBM. For the latter case, two approaches to expedite the STP process are introduced. The first is the incorporation of FORM into the STP process to derive conditional MORs. The second presents a methodology for incorporating a high-fidelity numerical model of VBM into the optimization framework.

## 2. Problem description

This study addresses wave-induced responses in linear long-crested irregular waves. Let the environmental random variables, such as significant wave height ( $H_s$ ) and mean wave period ( $T_m$ ), be  $\mathbf{X}_0 \in \mathbb{R}^{D_x}$ , where  $D_x$  denotes the dimension. Referring to the realization of  $\mathbf{X}_0$  as  $\mathbf{x}_0$ , the wave elevation at a time instant  $t$  can be expressed by a linear superposition of sinusoidal waves,

$$\eta(t | \mathbf{X}_0 = \mathbf{x}_0) = \sum_{i=1}^M \sqrt{S(\omega_{0,i} | \mathbf{X}_0 = \mathbf{x}_0) d\omega_{0,i}} \{u_i \cos(\omega_{0,i}t) - \bar{u}_i \sin(\omega_{0,i}t)\}, \quad (1)$$

where  $S$  denotes the wave spectrum,  $d\omega_{0,i}$  the frequency increment of the absolute wave frequency  $\omega_{0,i}$ , and  $\mathbf{u} = \{u_i, \bar{u}_i\}$  the uncorrelated standard normal distributed random variables. Notice that the wave spectrum  $S$  is conditional on the environmental random variables (e.g.,  $H_s, T_m$ ). One can convert  $\mathbf{X}_0$  to uncorrelated standard normal random variables  $\mathbf{X} \in \mathbb{R}^{D_x}$  based on an iso-probabilistic transformation, e.g., Rosenblatt transformation and Nataf transformation, see Der Kiureghian and Liu [18]. Henceforth, all variables associated with the environmental random variables are denoted by  $\mathbf{X}$  and their realizations by  $\mathbf{x}$ . The wave elevation described by Eq. (1) inherently follows a stationary Gaussian Process [19]. Let the design variables of a ship  $\boldsymbol{\theta} \in \mathbb{R}^{D_\theta}$ , where  $D_\theta$  denotes the number of variables; examples of design variables could be the *metacentric height* and the ship's *weight distribution*. When the wave-induced response of interest follows a stationary Gaussian Process, the MOR with respect to a certain response threshold  $\zeta$  conditional on the realization  $\mathbf{x}$  and  $\boldsymbol{\theta}$ , namely the conditional MOR, can be given by a Rice's formula [9], given as

$$v(\zeta, \boldsymbol{\theta} | \mathbf{X} = \mathbf{x}) = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}} \exp\left(-\frac{\zeta^2}{2m_0}\right), \quad (2)$$

where  $m_0$  and  $m_2$  are the zeroth- and second-order spectral moments of the response, respectively. This study employs three distinct approaches for estimating the conditional MOR:

- An analytical approximation of the PoE of the peak response, i.e.,  $\exp\left(-\frac{\zeta^2}{2m_0}\right)$ .
- A crude MCS based on multiple realizations of time-domain numerical simulations.
- A FORM-based prediction of conditional MOR, i.e.  $v$  [20].

The first approach is applied to the optimization of metacentric height, while the latter two approaches are employed for ship weight distribution optimization. Details of these approaches are provided in the subsequent sections. Note that the authentic solution can, in principle, be obtained by the crude MCS with a sufficiently large number of realizations; however, this inevitably entails a substantial computational burden.

Using the MOR prediction results conditional on the selected environmental random variables  $\mathbf{x}$  under given design variables  $\boldsymbol{\theta}$ , i.e., STP results, the long-term MOR corresponding to  $\zeta$  can then be derived as

$$\bar{v}(\zeta, \boldsymbol{\theta}) = \mathbb{E}[v(\zeta, \boldsymbol{\theta} | \mathbf{X})] = \int_{\text{all } \mathbf{x}} v(\zeta, \boldsymbol{\theta} | \mathbf{X} = \mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}, \quad (3)$$

where  $f_{\mathbf{X}}(\mathbf{x})$  denotes the joint Probability Density Function (PDF) of  $\mathbf{X}$ . In practice, Eq. (3) can be derived via the following MC integration.

$$\bar{v}(\zeta, \boldsymbol{\theta}) \approx \frac{1}{N_s} \sum_{j=1}^{N_s} v(\zeta, \boldsymbol{\theta} | \mathbf{X} = \mathbf{x}^{(j)}), \quad (4)$$

where  $\mathbf{x}^{(j)}$  denotes the  $j$ th realization of environmental random variables according to the joint PDF  $f_{\mathbf{X}}(\mathbf{x})$ . The number of samples  $N_s$  should be taken sufficiently large to correspond to the voyage period under consideration. The high computational burden is evident from Eq. (4) when direct MCS is used to predict  $\bar{v}$ , thereby motivating the incorporation of the BAL framework to expedite its evaluation.

The design optimization targeting the long-term MOR is then made by identifying the design variables  $\boldsymbol{\theta}$  such that

$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta} \in \Theta_c} J(\boldsymbol{\theta}), \quad (5)$$

where  $\Theta_c$  denotes the admissible design space of  $\boldsymbol{\theta}$ , and  $J$  the cost function. If the objective is to identify the design variables for which a prescribed long-term MOR value, denoted by  $\bar{v}^*$ , is attained, the cost function defined in Eq. (5) can be formulated as

$$J(\boldsymbol{\theta}) = \frac{|\bar{v}^* - \bar{v}(\zeta, \boldsymbol{\theta})|}{\bar{v}^*}. \quad (6)$$

On the other hand, if the objective is to minimize the long-term MOR, the cost function can simply be expressed as

$$J(\boldsymbol{\theta}) = \bar{v}(\zeta, \boldsymbol{\theta}). \quad (7)$$

## 3. Bayesian active learning-based long-term prediction

The central idea of the present BAL-LTP is to replace the conditional MOR,  $v(\zeta | \mathbf{x})$ , by GPR. As discussed in the Introduction, the non-negativity of the conditional MOR must be explicitly addressed. In this context,  $v(\zeta | \mathbf{x})$  is modeled as a log-normal process. Specifically, the log-transformed MOR is assumed to follow a Gaussian Process:

$$\log v(\zeta, \boldsymbol{\theta} | \mathbf{x}) \sim \mathcal{GP}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}')), \quad (8)$$

where  $\mu(\mathbf{x}) = f^T(\mathbf{x})\mathbf{w}$  denotes the mean function with the basis function  $f(\mathbf{x})$ , and  $\mathbf{w}$  the vector of regression coefficients. In this study, the mean function is assumed to be constant, i.e.,  $\mu(\mathbf{x}) = w$ .  $\kappa(\mathbf{x}, \mathbf{x}')$  is the covariance function, and various options for its kernel function are available [21]. Among these options, this study employs the Matern 5/2 kernel proposed by [21].

Let the initial sample pool of the environmental random variables be  $\mathcal{X}_W = \{\mathbf{x}^{(k)}, k = 1, \dots, N_s\}$ , where  $N_s$  denotes the number of samples identical to that in Eq. (4). From a Bayesian perspective, the GPR surrogate of the conditional MOR derived from existing training dataset can be interpreted as the *posterior* of the conditional MOR [15]. The posterior mean and standard deviation of the long-term MOR under given design variables  $\boldsymbol{\theta}$  can be obtained as [22]:

$$\mu_{\bar{v}}(\boldsymbol{\theta}) = \frac{1}{N_s} \sum_{i=1}^{N_s} \exp(\mu_{v,i} + \frac{1}{2} K_{ii}), \quad (9)$$

and

$$\sigma_{\bar{v}}(\boldsymbol{\theta}) = \sqrt{\frac{1}{N_s^2} \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \exp(\mu_{v,i} + \mu_{v,j} + \frac{1}{2}(K_{ii} + K_{jj})) (\exp(K_{ij}) - 1)}, \quad (10)$$

where  $\mu_{v,i}$  denotes the posterior mean value at a given  $\mathbf{x}^{(i)}$  and  $K_{ij}$  denotes the component in the posterior covariance matrix. Note that Eq. (10) requires a full covariance matrix  $K_{ij}$ . The posterior Coefficient of Variation (COV) of  $\bar{v}(\zeta, \boldsymbol{\theta})$  is then given by:

$$\delta_{\bar{v}}(\boldsymbol{\theta}) = \frac{\sigma_{\bar{v}}(\boldsymbol{\theta})}{\mu_{\bar{v}}(\boldsymbol{\theta})}, \quad (11)$$

which serves as the stopping criteria for the GPR training.

As a learning function, the Posterior Variance Contribution (PVC) devised by Wei et al. [23] is employed. The PVC-based learning function aims to quantify the posterior variance of Monte Carlo (MC) integration of GPR model [24]. That is,

$$\mathcal{L}_{\text{PVC}}(\mathbf{x}) = \frac{f_{\mathbf{x}}(\mathbf{x})}{N_s} \sum_{j=1}^{N_s} \exp\left(\mu_v(\mathbf{x}) + \mu_{v,j} + \frac{1}{2}(K_{xx} + K_{jj})\right) (\exp(K_{xj}) - 1). \quad (12)$$

Then, the next training sample is chosen as:

$$\mathbf{x}_{\text{next}} = \arg \max_{\mathbf{x} \in \mathcal{X}_W} \mathcal{L}_{\text{PVC}}(\mathbf{x}). \quad (13)$$

In the present study, the next training sample  $\mathbf{x}_{\text{next}}$  is found within the initial sample pool  $\mathcal{X}_W$ .

A specific procedure is summarized as follows:

1. Draw  $N_s$  sample pool of the environmental random variables  $\mathbf{X}$ , denoted as  $\mathcal{X}_W$ .
2. Take  $N_0$  initial training samples, and let  $k = 0$ .
3. Compute the conditional MORs at each training sample via STP.
4. Calculate the long-term MOR  $\mu_{\bar{v}}$  and its COV  $\sigma_{\bar{v}}$  by Eqs. (9) to (11).
5. Check convergence by examining the value of  $\delta_{\bar{v}}$  and the relative change in  $\mu_{\bar{v}}$ . If the solution has converged, go to Step 7; otherwise, go to Step 6.
6. Identify the next training sample  $\mathbf{x}_{\text{next}}$  according to Eq. (13). Let  $k = k + 1$ , and return to Step 3.
7. Store the long-term MOR  $\mu_{\bar{v}}$  as the final solution of the BAL-LTP.  $N_{\text{STP}} = N_0 + k$  represents the total number of STP computations required.

In Step 5, convergence is judged when  $\delta_{\bar{v}} < \varepsilon$  is satisfied for three successive iterations, where  $\varepsilon = 0.03$  is adopted by convention following Takami et al. [25]. On top of that, the relative change in  $\mu_{\bar{v}}$ , i.e.,

$$\Delta_{\mu} = \frac{|\mu_{\bar{v}}^{(k)} - \mu_{\bar{v}}^{(k-1)}|}{\mu_{\bar{v}}^{(k-1)}} \quad (14)$$

is monitored to ensure convergence. In the present BAL-LTP, convergence is judged when  $\Delta_{\mu} < 0.01$  is satisfied for two successive iterations.

#### 4. Single-loop design optimization implementing BAL-LTP

While the BAL-LTP is designed for efficiency, its integration into standard optimization procedures triggers a double-loop structure, leading to significant computational efforts. To overcome this issue, this study incorporates a “single-loop” structure implementing the BAL-LTP. An overall workflow of the present single-loop procedure is depicted in Fig. 1. This section is devoted to delineating the individual procedures. As mentioned above, the whole procedure is inspired by CABO approach, but the present approach eliminates the need for GP sampling.

As illustrated in Fig. 1, in the present single-loop approach, a joint sample pool of the design variables  $\theta$  and the environmental random variables  $\mathbf{x}$  is initially prepared. Let the joint variable vector  $\boldsymbol{\psi}$  be  $\boldsymbol{\psi} = (\theta, \mathbf{x})$ . The joint sample pool is denoted as  $\mathcal{W}_W = \{\boldsymbol{\psi}^{(k)}, k = 1, \dots, N_s\}$ , and the training dataset, denoted as  $\mathcal{W}_T$ , is initially selected from  $\mathcal{W}_W$ . In the present single-loop approach, the conditional MOR  $v(\zeta, \theta | \mathbf{x})$  is approximated by a GPR as a joint function of the design variables  $\theta$  and the environmental random variables  $\mathbf{x}$ . In this context, using the STP results, the conditional MOR  $v(\zeta, \theta | \mathbf{x})$  is then modeled through a log-transformation with a small positive offset ( $10^{-8}$ ) to avoid numerical issues associated with vanishing values:

$$\log(v(\zeta, \theta | \mathbf{x}) + 10^{-8}) \sim \mathcal{GP}(\mu(\boldsymbol{\psi}), \kappa(\boldsymbol{\psi}, \boldsymbol{\psi}')), \quad (15)$$

where  $\mu(\boldsymbol{\psi}) = \mathbf{f}^T(\boldsymbol{\psi})\mathbf{w}$ , and  $\mathbf{w}$  the vector of regression coefficients as in Eq. (8). A constant mean function and a Matern 5/2 kernel function are employed.

In the present approach, Bayesian Optimization (BO) for identifying a design site and the detection of the next training sample with reference to the PVC are performed in a nested manner. First, the global design site  $\theta^*$  is sought, corresponding to either a prescribed long-term MOR value  $\bar{v}^*$  or the minimum long-term MOR. Specifically, for the former case (target MOR), the global design site is identified as:

$$\theta^* = \arg \min_{\theta} \left\{ \frac{|\mu_{\bar{v}}(\theta) - \bar{v}^*|}{\bar{v}^*} + \alpha_{\text{BO}} \frac{\sigma_{\bar{v}}(\theta)}{\bar{v}^*} \right\}, \quad (16)$$

whereas for the latter case (minimization), the following is employed:

$$\theta^* = \arg \min_{\theta} \{ \mu_{\bar{v}}(\theta) + \alpha_{\text{BO}} \sigma_{\bar{v}}(\theta) \}, \quad (17)$$

where  $\alpha_{\text{BO}}$  is the degree of risk aversion. In this study,  $\alpha_{\text{BO}} = 1$  is employed, following the practice in Ref. [17].  $\theta^*$  is searched for within the training dataset  $\mathcal{W}_T$ . Upon the detection of  $\theta^*$ , then the following learning function, which is fundamentally based on the Expected Improvement (EI) function [26], is defined. That is,

$$\mathcal{L}_{\text{BO}}(\theta) = \max \left[ (\bar{\mu}_{\bar{v}}(\theta^*) - \bar{\mu}_{\bar{v}}(\theta)) \Phi(z(\theta)) + \bar{\sigma}_{\bar{v}}(\theta) \phi(z(\theta)), 0 \right], \quad (18)$$

where

$$\begin{aligned} \bar{\mu}_{\bar{v}}(\theta) &= \frac{|\mu_{\bar{v}}(\theta) - \bar{v}^*|}{\bar{v}^*}, \\ \bar{\sigma}_{\bar{v}}(\theta) &= \frac{\sigma_{\bar{v}}(\theta)}{\bar{v}^*}, \\ z(\theta) &= \frac{\bar{\mu}_{\bar{v}}(\theta^*) - \bar{\mu}_{\bar{v}}(\theta)}{\bar{\sigma}_{\bar{v}}(\theta)}, \end{aligned} \quad (19)$$

is used if  $\bar{v}^*$  is to be attained. When the problem is to attain the minimum long-term MOR value,

$$\mathcal{L}_{\text{BO}}(\theta) = \max \left[ (\mu_{\bar{v}}(\theta^*) - \mu_{\bar{v}}(\theta)) \Phi(z(\theta)) + \sigma_{\bar{v}}(\theta) \phi(z(\theta)), 0 \right], \quad (20)$$

is simply used. Here,  $\Phi(\cdot)$  and  $\phi(\cdot)$  denote the cumulative distribution function and the PDF of the univariate standard normal distribution, respectively. It should be emphasized that this formulation of  $\mathcal{L}_{\text{BO}}(\theta)$  does not require GP sampling, thereby distinguishing it from the methodology employed in CABO [17]. Referring to  $\mathcal{L}_{\text{BO}}(\theta)$ , the design site is then determined by:

$$\theta^+ = \arg \max_{\theta} \{ \mathcal{L}_{\text{BO}}(\theta) \}. \quad (21)$$

To determine  $\theta^+$ , the Particle Swarm Optimization (PSO) is used in this study.

Note that  $\mu_{\bar{v}}(\theta)$  and  $\sigma_{\bar{v}}(\theta)$  in Eqs. (16)–(20) can be provided by Eqs. (9) and (10), respectively. However, calculating  $\sigma_{\bar{v}}(\theta)$  by Eq. (10) may require a large computational burden if the size of  $N_s$  is large, since the full posterior covariance matrix is required. To alleviate this issue,  $\sigma_{\bar{v}}(\theta)$  can be approximated by neglecting the off-diagonal covariance terms as follows:

$$\sigma_{\bar{v}, \text{approx}}(\theta) = \sqrt{\frac{1}{N_s^2} \sum_{i=1}^{N_s} \exp(2\mu_{v,i} + K_{ii}) (\exp(K_{ii}) - 1)}. \quad (22)$$

In the subsequent sections on numerical demonstrations, the results using the full covariance-based  $\sigma_{\bar{v}}(\theta)$  and those using the approximated  $\sigma_{\bar{v}, \text{approx}}(\theta)$  during the BO phase are compared.

Upon determining  $\theta^+$ , the screening process for the next training sample is performed exclusively under  $\theta^+$ , i.e.,  $\boldsymbol{\psi}_{\text{next}} = (\theta_{\text{next}}, \mathbf{x}_{\text{next}})$  where  $\theta_{\text{next}} = \theta^+$ . Hence, with reference to the BAL-LTP procedure described in Section 2, the COV of  $v(\zeta, \theta^+)$  is merely given by

$$\delta_{\bar{v}}(\theta^+) = \frac{\sigma_{\bar{v}}(\theta^+)}{\mu_{\bar{v}}(\theta^+)}. \quad (23)$$

Following this, the PVC learning function  $\mathcal{L}_{\text{PVC}}(\mathbf{x})$  is also computed under  $\theta^+$  as per Eqs. (12) and (13). Note that  $\sigma_{\bar{v}}(\theta^+)$  in Eq. (23) is based on the full covariance one, i.e., Eq. (10). Since the computation of the

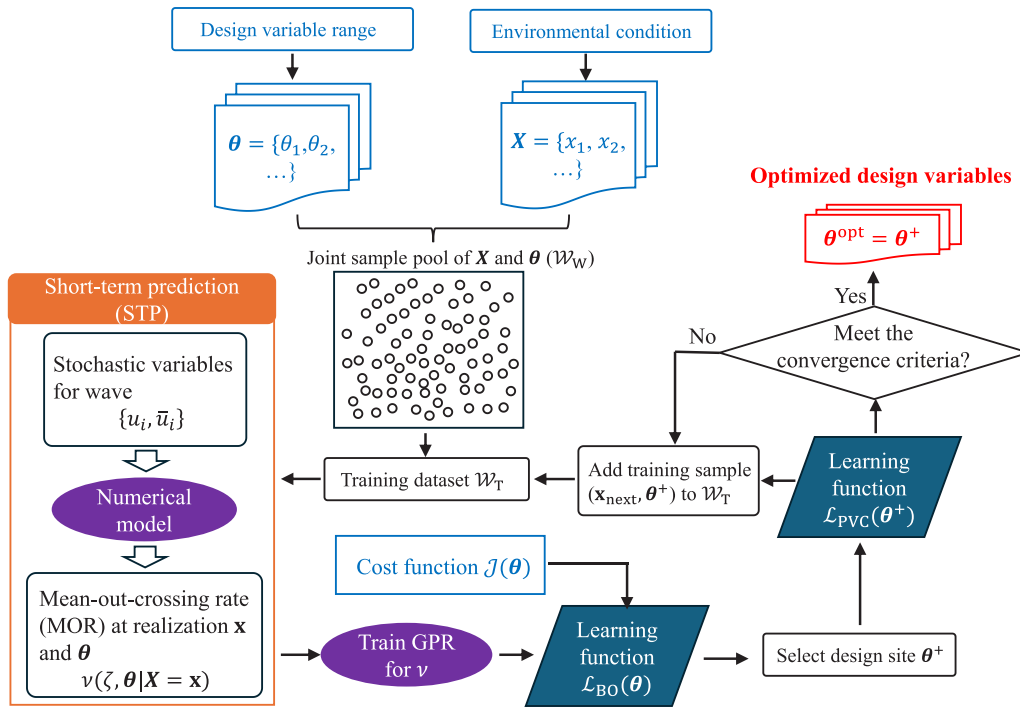


Fig. 1. Framework for single-loop design optimization process with reference to the long-term mean outcrossing rate.

COV  $\delta_v(\theta^+)$  is required exclusively for the current design site  $\theta^+$ , the associated computational burden is not significant.

Convergence is assessed by tracking three quantities, namely  $\delta_v(\theta^+)$ ,  $\mathcal{L}_{BO}(\theta)$ , and  $\Delta_\mu$ . With respect to  $\mathcal{L}_{BO}(\theta)$ , when a prescribed long-term MOR value  $\bar{v}^*$  is sought, the quantity  $\mathcal{L}_{BO}(\theta)$  itself is monitored. Conversely, when the minimum long-term MOR is sought, the ratio  $\mathcal{L}_{BO}(\theta)/\mu_v(\theta^+)$  is monitored. The specific convergence conditions are as follows:

- $\delta_v < 0.03$  for three successive iterations.
- $\max \{\mathcal{L}_{BO}(\theta)\} < 0.01$  or  $\max \{\mathcal{L}_{BO}(\theta)\} / \mu_v(\theta^+) < 0.01$ .
- $\Delta_\mu < 0.01$  for two successive iterations.

where  $\Delta_\mu$  is computed with reference to  $\mu_v(\theta^+)$  at the immediately preceding iteration.

To sum up, the specific procedure of the present single-loop approach is detailed as follows:

1. Draw  $N_s$  joint sample pool of  $\psi = (\theta, x)$ , denoted as  $\mathcal{W}_W$ .
2. Take  $N_0$  initial training samples (i.e.,  $\mathcal{W}_T$ ), and let  $k = 0$ .
3. Compute the conditional MORs for each training sample via STP.
4. Identify the global design site  $\theta^*$  from the current training sample pool  $\mathcal{W}_T$  according to Eq. (16).
5. Identify the design site  $\theta^+$  with reference to  $\mathcal{L}_{BO}(\theta)$  given by Eq. (21).
6. Compute the long-term MOR  $\mu_v$  and its COV  $\delta_v$  under the identified design site by Eqs. (9) to (11).
7. Check convergence by examining the values of  $\delta_v$ ,  $\max \{\mathcal{L}_{BO}(\theta)\}$  (or  $\max \{\mathcal{L}_{BO}(\theta)\} / \mu_v(\theta^+)$ ), and  $\Delta_\mu$ . If the solution has converged, go to Step 9; otherwise, go to Step 8.
8. Identify the next training sample  $\psi_{\text{next}} = (\theta^+, x_{\text{next}})$  with reference to the PVC learning function  $\mathcal{L}_{PVC}(x)$  under the design site  $\theta^+$ . Add  $\psi_{\text{next}}$  to  $\mathcal{W}_T$ . Let  $k = k + 1$ , and return to Step 3.
9. Store the design site  $\theta^{\text{opt}} = \theta^+$  as the final solution that attains the target long-term MOR  $\bar{v}^*$ .  $N_{\text{STP}} = N_0 + k$  represents the total number of STP computations required.

## 5. Numerical demonstration on metacentric height optimization

### 5.1. Numerical model

In this study, a numerical model for the nonlinear roll motion in a ship presented by Maki et al. [27] is utilized for the first numerical demonstration. As per [27], the instantaneous rolling angle in beam seas at time  $t$ , denoted as  $\varphi(t)$  is given by:

$$\begin{aligned} \ddot{\varphi} + \alpha\dot{\varphi} + \beta\varphi|\dot{\varphi}| + \omega_\varphi^2 \left( \varphi + \frac{\text{GZ}_2}{\text{GM}} \varphi^2 + \frac{\text{GZ}_3}{\text{GM}} \varphi^3 + \frac{\text{GZ}_4}{\text{GM}} \varphi^4 + \frac{\text{GZ}_5}{\text{GM}} \varphi^5 \right) \\ = \sum_i \gamma \omega_\varphi^2 \frac{\omega_{0,i}^2}{g} \sqrt{2S(\omega_{0,i})\Delta\omega} \cos(\omega_{0,i}t + \varepsilon_i). \end{aligned} \quad (24)$$

where  $\alpha$  and  $\beta$  denote the linear and nonlinear damping coefficients, respectively,  $\omega_\varphi$  the natural roll frequency, GM the transverse metacentric height,  $\text{GZ}_i$  the  $i$ th coefficient of the polynomial fit of the righting lever,  $g$  the acceleration of gravity,  $\gamma$  the effective wave slope coefficient, and  $\omega_{0,i}$  the discretized absolute wave frequency with the frequency increment  $\Delta\omega$ .  $\varepsilon_i \sim \mathcal{U}(0, 2\pi)$  denotes the phase component of the roll excitation force. The incident wave spectrum  $S$  is characterized by the International Towing Tank Conference (ITTC) type wave spectrum defined by the significant wave height  $H_s$  and mean wave period  $T_m$ . That is,

$$S(\omega) = 0.11 \frac{(2\pi)^4 H_s^2}{T_m^4 \omega^5} \exp \left[ -0.44 \frac{(2\pi)^4}{T_m^4 \omega^4} \right]. \quad (25)$$

$\omega_\varphi$  holds the following relationship with the ship weight  $W_s$  and the roll moment of inertia (including the added moment of inertia)  $I_{xx}$ :

$$\omega_\varphi = \sqrt{\frac{W_s \times \text{GM}}{I_{xx}}}. \quad (26)$$

The conditional MOR of the present roll motion model can, in principle, be estimated by crude MCS through direct numerical solution of Eq. (24); however, this approach is computationally prohibitive. Alternatively, [27] proposed a theoretical approximation to the PoE of

**Table 1**

Model parameters used in the nonlinear roll motion model.

| Parameter                                  | Value                                 |
|--------------------------------------------|---------------------------------------|
| Linear damping coefficient, $\alpha$       | 0.03                                  |
| Nonlinear damping coefficient, $\beta$     | 0.3                                   |
| Ship weight, $W_s$                         | $1.471 \times 10^7$ N                 |
| Roll moment of inertia, $I_{xx}$           | $3.727 \times 10^7$ kg m <sup>2</sup> |
| $GZ_2/GM$                                  | 0                                     |
| $GZ_3/GM$                                  | 7.0                                   |
| $GZ_4/GM$                                  | 0                                     |
| $GZ_5/GM$                                  | -7.0                                  |
| Effective wave slope coefficient, $\gamma$ | 0.9                                   |

roll amplitude. As given in the literature [27], the PDF of the nonlinear roll motion can be represented as:

$$P(\varphi) = \int_{-\infty}^{+\infty} C_0 \exp \left\{ -D \left[ \alpha H + \frac{8\beta}{9\pi} (2H)^{\frac{3}{2}} \right] \right\} dt$$

$$\text{where } H \equiv \frac{1}{2} \dot{\varphi}^2 + \int_{\varphi}^{\varphi} G(\varphi) d\varphi \quad (27)$$

$$G(\varphi) = \omega_0^2 \left( \varphi + \frac{GZ_2}{GM} \varphi^2 + \frac{GZ_3}{GM} \varphi^3 + \frac{GZ_4}{GM} \varphi^4 + \frac{GZ_5}{GM} \varphi^5 \right)$$

Here,  $H$  is the Hamiltonian,  $C_0$  is the normalization constant, and  $D$  is the unknown parameter to be determined from  $\alpha$  and  $\beta$ . For the derivation of  $C_0$  and  $D$ , cf. [27]. Eq. (27) is the PDF for the instantaneous rolling angle  $\varphi$ , but the PDF for the roll amplitude, i.e., peak value of  $\varphi$  between two adjacent zero up-crossing and zero down-crossing, is needed to estimate the conditional MOR. To this end, the following conversion is adopted.

$$P_{\text{amp}}(\varphi) = C_1 P(\varphi) G(\varphi) \mathcal{T}(H), \quad (28)$$

where  $\mathcal{T}(H)$  denotes the expected value of the period of two consecutive roll amplitudes under a given Hamiltonian  $H$ , which is obtained by numerically solving Eq. (24). Further,  $C_1$  denotes the normalization constant. In this study, the Newton method is employed to solve Eq. (24) in this regard. Note that the simulation time required is sufficiently short, since only  $\mathcal{T}(H)$  is to be derived. Following this, the conditional MOR is given by:

$$v(\zeta) = \frac{\omega_{\varphi}}{2\pi} \text{PoE}(\zeta)$$

$$\text{where } \text{PoE}(\zeta) = \int_{\zeta}^{\infty} P_{\text{amp}}(\varphi) d\varphi, \quad (29)$$

where the introduction of the term  $\frac{\omega_{\varphi}}{2\pi}$  is based on the approximation of the zero outcrossing rate being equal to the natural roll frequency, see Choi et al. [28].

In the present study,  $GM$  is selected as the design variable  $\theta$ , while  $H_s$  and  $T_m$  are treated as environmental random variables  $X$ . The other relevant parameters used in the present case are listed in Table 1, based on the representative use cases presented by [27].

## 5.2. Results and discussion

### 5.2.1. BAL-LTP results

Prior to discussing the design optimization results, the present BAL-LTP results on the long-term MOR estimation are validated by comparing to the crude MCS. The statistical properties of  $H_s$  and  $T_m$  are indicated in Table 2. The properties are not intended to reproduce a real ocean environment, but are chosen to facilitate numerical demonstration; nevertheless, the statistical properties of the wave parameters can be calibrated using available ocean measurement data [29,30].

The crude MCS for the long-term MOR using up to 10000 realizations of  $H_s$  and  $T_m$  is performed.  $\zeta = 35$  deg and  $GM = 1.25$  m are introduced. The resulting  $\bar{v}$  with respect to the number of MC samples is shown in Fig. 2. Apparently, the  $\bar{v}$  value shows a tendency toward convergence when the number of MC samples is around 5000.

**Table 2**

Statistical properties of environmental random variables. Nonlinear roll motion case.

| Parameter | Description                 | Mean | Distribution | COV  |
|-----------|-----------------------------|------|--------------|------|
| $H_s$     | Significant wave height (m) | 5.0  | Log-normal   | 0.20 |
| $T_m$     | Mean wave period (s)        | 11.0 | Log-normal   | 0.25 |

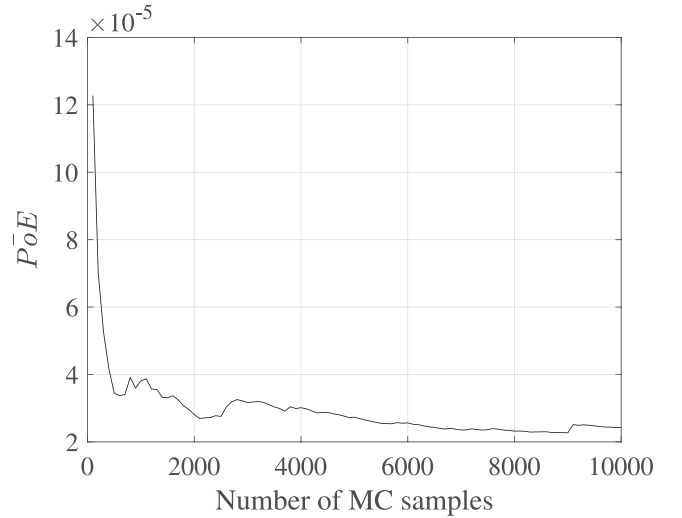


Fig. 2. Crude MCS result on the long-term mean outcrossing rate for nonlinear roll motion.  $\zeta = 35$  deg and  $GM = 1.25$  m.

Following the generation of a sample pool  $\mathcal{X}_W$  of size  $N_s = 10000$ , 20 independent runs of the BAL-LTP are executed. To account for the potential variability arising from the initial sample selection, each run begins with  $N_0 = 10$  training samples, which are randomly and independently selected for each run. The resulting long-term MOR values are presented in Table 3, together with those obtained by crude MCS using the full sample pool of  $N_s = 10000$ . For the long-term MOR estimated by BAL-LTP, the mean value over 20 independent runs is indicated. Evidently, BAL-LTP yields a long-term MOR that is comparable to that obtained by crude MCS, while maintaining a sufficiently small COV. Notably, BAL-LTP demonstrates highly efficient estimation, as manifested by the remarkably small value of  $N_{\text{STP}}$ . The table also includes a comparison of the absolute CPU time required to complete the long-term MOR estimations using the authors' (Core™ i5-1145G7 @ 2.60 GHz). Since BAL-LTP requires additional computations to evaluate  $\sigma_v(\theta)$  via Eq. (10), the absolute CPU time does not scale directly with the ratio of  $E[N_{\text{STP}}]$  between crude MCS and BAL-LTP. Nevertheless, BAL-LTP demonstrates remarkable computational efficiency compared to crude MCS. It is important to emphasize that the computational advantage of BAL-LTP becomes increasingly pronounced as the individual cost of executing each STP increases. In Fig. 3, the histograms of the log-transformed conditional MORs,  $\log(v)$ , obtained from crude MCS and predicted by the GPR model trained via BAL-LTP are compared. As can be seen from the figure, the overall distributions are in good agreement, suggesting that the trained GPR accurately reproduces the long-term characteristics of  $v$ . Nevertheless, some discrepancies remain. This is because the present BAL-LTP framework is formulated to reduce the COV of the estimated mean value of  $v$ , as described in Section 3. Incorporating higher-order statistical moments into the BAL framework could further improve the accuracy of the long-term distribution of  $v$ , albeit at the expense of additional computational cost [31].

### 5.2.2. Optimization results

Next, the design variable optimization case is investigated, namely the identification of  $GM$  value so as to minimize the long-term MOR.

**Table 3**

Long-term mean outcrossing rates estimated by BAL-LTP and crude MCS. The BAL-LTP results are obtained from 20 independent runs with randomized initial training sets.  $N_s = 10000$  and  $N_0 = 10$ . Nonlinear roll motion case, with  $\zeta = 35$  deg and  $GM = 1.25$  m.

| Method    | Long-term MOR ( $\bar{\nu}(\zeta)$ or $E[\mu_{\bar{\nu}}(\zeta)]$ ) | COV( $\mu_{\bar{\nu}}$ ) | $E[N_{STP}]$ | Absolute CPU time |
|-----------|---------------------------------------------------------------------|--------------------------|--------------|-------------------|
| Crude MCS | $2.424 \times 10^{-5}$                                              | –                        | 10 000       | $\approx 8120$ s  |
| BAL-LTP   | $2.321 \times 10^{-5}$                                              | 0.100                    | 113          | $\approx 662$ s   |

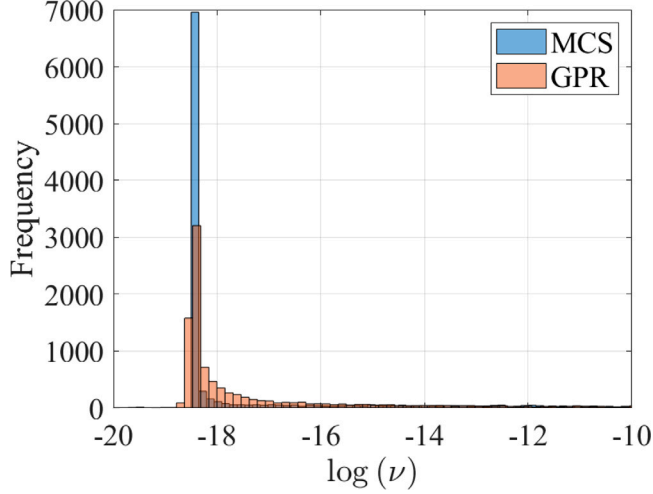


**Fig. 3.** Comparison of histograms of conditional mean outcrossing rates computed by crude MCS and predicted by the GPR model constructed via BAL-LTP.  $\zeta = 35$  deg and  $GM = 1.25$  m.

Using the environmental random variables in Table 2, the design value of  $GM$  is identified, within the range of  $0.5 \leq GM \leq 1.5$ . In computing the present single-loop approach, the sample population size  $N_s = 5000$  is adopted, reflecting the observation that crude MCS generally achieves convergence with approximately 5000 samples, as shown in Fig. 2. To validate the optimization results using the present single-loop scheme, the double-loop optimization incorporating the present BAL-LTP together with  $N_s = 5000$  is adopted; noting that the same initial population of the environmental random variables as the single-loop scheme is utilized. In computing the double-loop scheme, the following cost function is simply defined as:

$$J(\theta) = \mu_{\bar{\nu}}(\theta), \quad (30)$$

and the design parameter yielding  $\min J$  is identified by the PSO. The swarm size and the maximum number of iterations are both set to 15. To maintain consistency, the initial sample population  $\mathcal{X}_W$  is kept unchanged during the optimization phase. For a fair comparison under the same sample population, the subset of environmental variables in the initial sample population  $\mathcal{V}_W$  for the single-loop approach is taken to be identical to  $\mathcal{X}_W$ . Hereafter, the initial training sample size is  $N_0 = 10$  for both the double-loop and single-loop cases, unless otherwise stated.

The design parameter optimization results are summarized in Table 4. For the single-loop approach, 5 independent runs with different initial training sample selections are performed when the full covariance-based  $\sigma_{\bar{\nu}}(\theta)$  is employed in the BO phase. Meanwhile, 10 independent runs are performed when the approximated value of  $\sigma_{\bar{\nu}}(\theta)$ , i.e.,  $\sigma_{\bar{\nu},\text{approx}}$  given by Eq. (22), is employed. In the PSO stage to identify the design site  $\theta^+$  by Eq. (21) using the single-loop approach, the swarm size and the maximum number of iterations are both set to 15 when the full covariance matrix is used, whereas they are set to 20 when  $\sigma_{\bar{\nu},\text{approx}}$  is introduced. Comparing the double-loop and single-loop results, the optimized parameters by the present single-loop approach are close to the double-loop approach with small COV. Noticeably, the required

numbers of STPs are significantly reduced, and the absolute CPU time required to complete the optimization using a standard PC (Core™ i5-1145G7 @ 2.60 GHz) is substantially shorter. Comparing the single-loop results, the mean value of estimated design parameter  $E[\theta^{\text{opt}}]$  somewhat deviates when  $\sigma_{\bar{\nu},\text{approx}}$  is introduced. This is attributed to the approximation of  $\sigma_{\bar{\nu}}$ , which may occasionally mislead the search path. This issue can be mitigated, to some extent, by increasing the number of initial training samples, as evidenced by the results shown in the bottom row of Table 4, where  $N_0$  is increased to 30. With  $N_0 = 30$ , the mean estimated design parameter  $E[\theta^{\text{opt}}]$  and the mean minimum long-term MOR  $E[\min \bar{\nu}]$  become closer to those obtained using the double-loop approach. The benefit of employing  $\sigma_{\bar{\nu},\text{approx}}$  is significant in terms of reducing the absolute CPU time, which is attributed to the absence of computing the off-diagonal terms of the covariance matrix  $K_{ij}$ . Based on the crude MCS results by Fig. 2,  $\bar{\nu} = 2.694 \times 10^{-5}$  is calculated for  $GM = 1.25$  m. Thus, the  $\theta_1^{\text{opt}}$  values and  $\min \bar{\nu}$  values obtained from the double-loop and single-loop approaches are both accurate.

Representative histories of the selected design site  $\theta_1^+$  (corresponding to  $GM$ ) from the single-loop approach are illustrated in Fig. 4. The figure compares two distinct cases: one utilizing the full covariance-based formulation and the other employing the approximation  $\sigma_{\bar{\nu},\text{approx}}$ . In both cases, the design site successfully converges to the optimal value as the training progresses, demonstrating the robustness of the present single-loop approach.

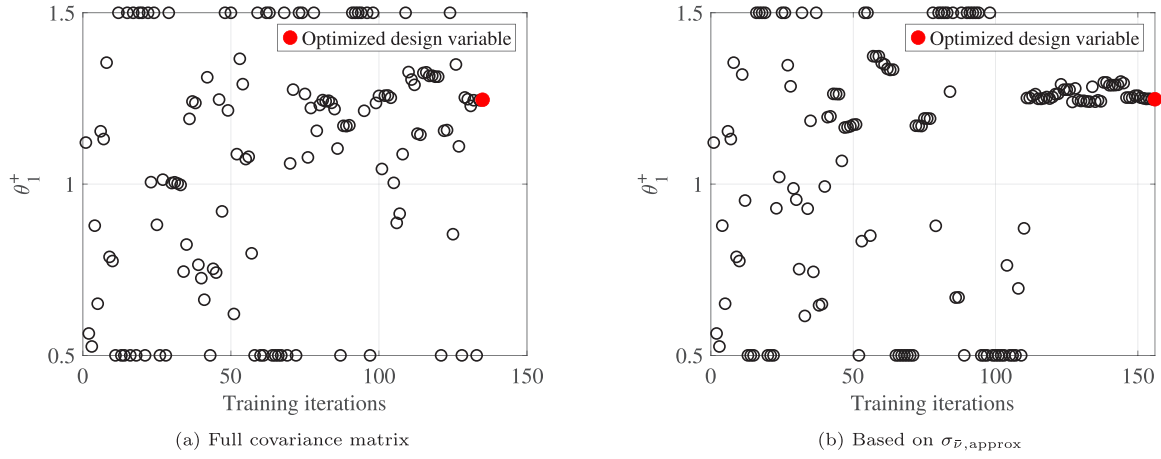
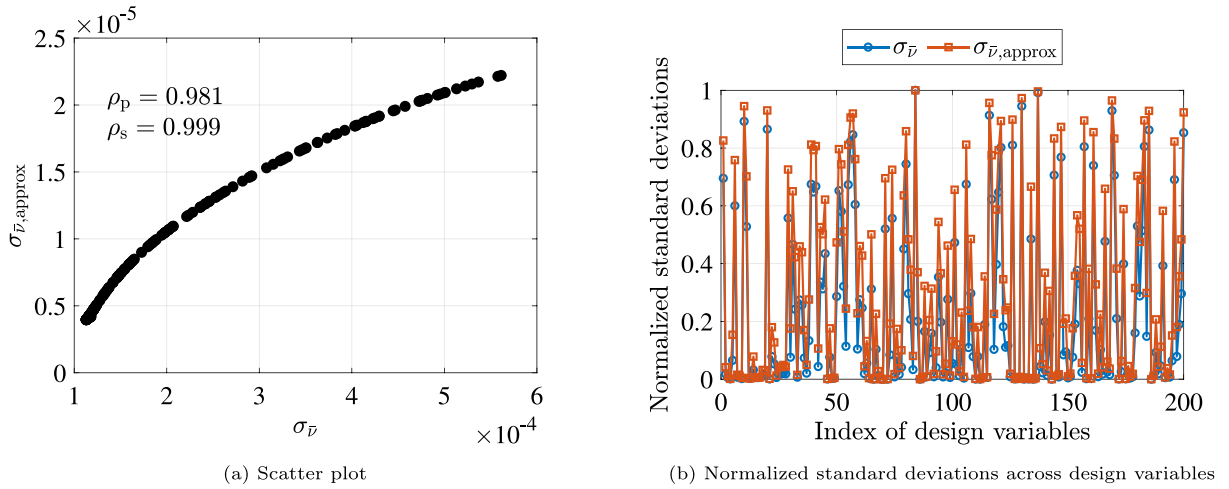
The difference between the full covariance-based standard deviation of the long-term MOR,  $\sigma_{\bar{\nu}}$ , computed using Eq. (10), and its approximation,  $\sigma_{\bar{\nu},\text{approx}}$ , computed using Eq. (22), is examined. To this end, 200 joint random samples of  $\psi = (\theta, \mathbf{x})$  are generated, and a GPR model is constructed based on the corresponding short-term MOR evaluations. Using the trained GPR model,  $\sigma_{\bar{\nu}}$  and  $\sigma_{\bar{\nu},\text{approx}}$  are evaluated for each of the 200 design samples. Fig. 5 presents the comparison results. In Fig. 5(a), the Pearson correlation coefficient,  $\rho_p$ , is indicated to quantify the linear correlation between  $\sigma_{\bar{\nu}}$  and  $\sigma_{\bar{\nu},\text{approx}}$ , while the Spearman correlation coefficient,  $\rho_s$ , is indicated to quantify the consistency of their rankings. As can be seen from Fig. 5(a),  $\sigma_{\bar{\nu},\text{approx}}$  is generally smaller than  $\sigma_{\bar{\nu}}$ , owing to the omission of the off-diagonal covariance terms in the variance calculation. Nevertheless, the Pearson correlation coefficient remains high ( $\rho_p = 0.981$ ). More importantly, the Spearman correlation coefficient is found to be nearly unity ( $\rho_s = 0.999$ ), indicating that  $\sigma_{\bar{\nu},\text{approx}}$  preserves the *ranking* of uncertainty across the design space almost perfectly. Indeed, as observed in Fig. 5(b), the design variables associated with the largest values of  $\sigma_{\bar{\nu}}$  coincide with those associated with the largest values of  $\sigma_{\bar{\nu},\text{approx}}$ . This observation suggests that the contribution of the off-diagonal covariance terms is relatively uniform throughout the design space and therefore does not substantially alter the ordering of the uncertainty estimates. Consequently, although the diagonal approximation introduces a bias in the absolute magnitude of the standard deviation, it retains the information relevant to uncertainty ranking, which is of primary interest during the BO phase. It should be noted that  $\sigma_{\bar{\nu},\text{approx}}$  is introduced solely for the BO phase and should not be used to evaluate  $\delta_{\bar{\nu}}(\theta^+)$  via Eq. (23) in the BAL-LTP framework.

The effectiveness of the diagonal approximation is also reflected in the optimization results. As shown in Table 4, when the number of initial training samples is relatively small ( $N_0 = 10$ ), the use of  $\sigma_{\bar{\nu},\text{approx}}$  occasionally leads to deviations in the estimated optimal design parameters compared with those obtained using the full covariance

**Table 4**

Identified design parameter ( $\theta_1 = \text{GM}$ ) for minimization of the long-term MOR obtained by the single-loop and double-loop approaches. An initial sample population size of  $N_s = 5000$  with  $N_0 = 10$  is used. The initial sample population  $\mathcal{W}_W$  for the single-loop approach is taken to be identical to  $\mathcal{X}_W$ . The results for the single-loop approach with the full covariance matrix are based on five independent runs, while those with  $\sigma_{\bar{v},\text{approx}}$  are based on ten independent runs, with different initial training sample selections.

| Method                                                     | $E[\theta^{\text{opt}}]$ | $\text{COV}[\theta^{\text{opt}}]$ | $E[\min \bar{v}]$      | $\text{COV}[\min \bar{v}]$ | $E[N_{\text{STP}}]$ | Absolute CPU time |
|------------------------------------------------------------|--------------------------|-----------------------------------|------------------------|----------------------------|---------------------|-------------------|
| Double-loop                                                | 1.234                    | –                                 | $2.123 \times 10^{-5}$ | –                          | 23 282              | >48 h             |
| Single-loop ( $\sigma_{\bar{v}}$ )                         | 1.294                    | 0.077                             | $2.462 \times 10^{-5}$ | 0.069                      | 256                 | $\approx 18$ h    |
| Single-loop ( $\sigma_{\bar{v},\text{approx}}$ )           | 1.327                    | 0.093                             | $2.762 \times 10^{-5}$ | 0.153                      | 251                 | $\approx 56$ min  |
| Single-loop ( $\sigma_{\bar{v},\text{approx}}, N_0 = 30$ ) | 1.234                    | 0.119                             | $2.548 \times 10^{-5}$ | 0.114                      | 208                 | $\approx 46$ min  |

**Fig. 4.** Selected design variables (i.e.,  $\theta_1 = \text{GM}$ ) during the process of the single-loop approach.**Fig. 5.** Comparison of the full covariance-based standard deviation,  $\sigma_{\bar{v}}$ , using Eq. (10), and its approximation,  $\sigma_{\bar{v},\text{approx}}$ , computed using Eq. (22). Both quantities are evaluated based on a GPR model trained with 200 joint random samples of  $\psi = (\theta, x)$ .  $\rho_p$  and  $\rho_s$  denote the Pearson and Spearman correlation coefficients, respectively.

formulation. However, these deviations become significantly smaller when the initial training sample size is increased to  $N_0 = 30$ . It should also be noted that if strong local interactions exist among the covariance terms, the omission of the off-diagonal terms may alter the relative ranking of the standard deviations and consequently affect the BO search path. Further investigation is therefore required to clarify the applicability of the present approximation to such problems.

## 6. Numerical demonstration on ship weight distribution optimization

### 6.1. Low-fidelity numerical model for vertical bending moment

To facilitate optimization of the ship's weight distribution under nonlinear VBM exerted on the hull girder, the longitudinal weight

distribution is expressed in a parametric form. Let  $u$  denote the longitudinal coordinate of the ship, where  $u = 0$  m indicates the aft end of the ship (see Fig. 6). In this study, the following weight distribution based on a second-order polynomial with respect to  $u$  is employed:

$$w_u(u) = C \left\{ 1 + \alpha_1 \left( \frac{u - L_{pp}/2}{L_{pp}/2} \right) - \alpha_2 \left( \frac{u - L_{pp}/2}{L_{pp}/2} \right)^2 \right\} \quad (31)$$

where  $L_{pp}$  is the length between perpendiculars, and  $\alpha_1$  and  $\alpha_2$  are the coefficients for the first- and second-order terms, respectively. The coefficient  $C$  can be analytically defined by referencing the total ship weight  $W_s$  as follows:

$$C = \frac{W_s}{L_{pp}(1 - \alpha_2/3)}, \quad (32)$$

which is derived from the relation between the total weight and the longitudinal distribution

$$W_s = \int_0^{L_{pp}} w_u(u) du. \quad (33)$$

In the present study, the coefficients  $\alpha_1$  and  $\alpha_2$  are taken as the design variables; that is,  $\theta = (\alpha_1, \alpha_2)$ . The longitudinal radius of gyration is determined from the weight distribution  $w_u(u)$ . It is worth noting that the present single-loop (and double-loop) frameworks are versatile enough to integrate alternative parameterized weight distribution models, such as the trapezoidal-type distribution [32].

To compute the nonlinear VBM in long-crested waves, the Reduced Order Model (ROM), which is the same in essence as that proposed in [33,34], is leveraged as a low-fidelity model. The subject ship is the 6600TEU size container ship, of which principal particulars are given in Table 5. The weakly nonlinear component of VBM, taking only into account the body nonlinearity, is computed according to the following cubic formula [35]:

$$M_{\text{wave}}(t) = c_1 M_{\text{LN}}(t - t_f) + c_2 M_{\text{LN}}^2(t - t_f) + c_3 M_{\text{LN}}^3(t - t_f), \quad (34)$$

with

$$M_{\text{LN}}(t, \theta | \mathbf{X} = \mathbf{x}) = \sum_{i=1}^{N_{\text{wave}}} \Psi(\omega_{e,i}, \theta) \sqrt{S(\omega_{0,i} | \mathbf{X} = \mathbf{x})} \omega_{0,i} \times \begin{bmatrix} u_i \cos(\omega_{e,i}t + \Gamma(\omega_{e,i}, \theta)) \dots \\ -\bar{u}_i \sin(\omega_{e,i}t + \Gamma(\omega_{e,i}, \theta)) \end{bmatrix},$$

where  $\Psi$  and  $\Gamma$  denote the wave-to-response transfer function (TRF) of the VBM, in which the parameterized weight distribution  $w_u$  is introduced. The coefficients  $c_1$ ,  $c_2$ , and  $c_3$  denote the nonlinear correction factors to account for the body nonlinearity, and  $t_f$  is a calibration parameter for time shift. Recall that  $S$  denotes the wave spectrum,  $\omega_0$  the frequency increment, and  $\mathbf{u} = \{u_i, \bar{u}_i\}$  the uncorrelated standard normal distributed random variables, see Eq. (1).  $\omega_{e,i}$  is the encounter wave frequency, which holds the following relationship with the absolute wave frequency  $\omega_0$ :

$$\omega_e = \left| \omega_0 - \frac{\omega_0^2 U_V}{g} \cos \chi \right| \quad (35)$$

where  $U_V$  denotes the ship forward speed and  $\chi$  the relative wave direction, with  $\chi = 180^\circ$  corresponding to head seas and  $\chi = 0^\circ$  to following seas. In the present study, the linear strip theory incorporating the Lewis form approximation for the ship cross-sections is employed to derive the TRF, based on the so-called ‘‘New Strip Method’’ (NSM) [36].

The whipping vibration is computed using the convolution integral of the impact force  $F_{\text{imp}}$  and an intrinsic impulse response function  $I_r(t)$ , which is characterized by the damping ratio  $\xi_d$  and the natural angular frequency of the two-node vibration  $\omega_{\text{whip}}$  [33,34]. It means,

$$M_{\text{whip}}(t, \theta | \mathbf{X} = \mathbf{x}) = \int_0^t I_r(t - \tau) l_{\text{bow}} F_{\text{imp}}(\tau, \theta | \mathbf{X} = \mathbf{x}) d\tau, \quad (36)$$

where  $l_{\text{bow}}$  denotes the longitudinal distance from the ship’s center of gravity to incorporate the impact force  $F_{\text{imp}}$ . The impulse response function is approximated by using an analogy to one degree-of-freedom system under impact loading:

$$I_r(t) = \omega_{\text{whip}}^2 \exp(-\xi_d \omega_{\text{whip}} t) \left( \frac{-\sin(\omega_d t)}{\omega_d} \right), \quad (37)$$

where  $\omega_d = \omega_{\text{whip}} \sqrt{1 - \xi_d^2}$ . The impact force is evaluated according to von Karman’s momentum theory [37] involving the impact force on a wedge profile:

$$F_{\text{imp}}(t, \theta | \mathbf{X} = \mathbf{x}) = \dot{M}_a \dot{\eta}_{\text{rel}}, \quad (38)$$

where the dot above the variable denotes a differentiation with respect to time  $t$ . The added mass is given by:

$$M_a(t, \theta | \mathbf{X} = \mathbf{x}) = \frac{\rho_f L_0 \pi (\zeta_{\text{rel}} - d_w)^2 \cot^2 \delta_w}{2}, \quad (39)$$

where  $\rho_f = 1000 \text{ kg/m}^3$  is the fluid density,  $L_0$  is the effective longitudinal length for the added mass.  $\delta_w$  is the deadrise angle of the wedge against calm water surface, see Fig. 7. The relative displacement reads

$$\eta_{\text{rel}}(t, \theta | \mathbf{X} = \mathbf{x}) = \eta_{\text{bow}}(t | \mathbf{X} = \mathbf{x}) - [h_s(t, \theta | \mathbf{X} = \mathbf{x}) - l_{\text{bow}} \tan \theta_s(t, \theta | \mathbf{X} = \mathbf{x})]. \quad (40)$$

Here,  $\eta_{\text{bow}}$  denotes the wave elevation at the bow section located a distance  $l_{\text{bow}}$  from the ship’s center of gravity, which is given by:

$$\eta_{\text{bow}}(t | \mathbf{X} = \mathbf{x}) = \sum_{i=1}^{N_{\text{wave}}} \sqrt{S(\omega_{0,i} | \mathbf{X} = \mathbf{x})} \omega_{0,i} \times \begin{bmatrix} u_i \cos \left( \omega_{e,i}t - \frac{\omega_{0,i}^2}{g} l_{\text{bow}} \cos(\chi) \right) \dots \\ -\bar{u}_i \sin \left( \omega_{e,i}t - \frac{\omega_{0,i}^2}{g} l_{\text{bow}} \cos(\chi) \right) \end{bmatrix}. \quad (41)$$

The heave and pitch motions, denoted as  $h_s$  and  $\theta_s$  in Eq. (40), are derived analogously to the linear VBM  $M_{\text{LN}}$  (see Eq. (34)) using their respective TRFs. Consequently, both  $h_s$  and  $\theta_s$  are explicitly formulated as functions of the design variables  $\theta$ . Assuming a wedge profile entering the water,  $\eta_{\text{rel}} - d_w$  represents the height of the wedge below the water surface. If  $\eta_{\text{rel}}$  obtained from Eq. (40) exceeds the freeboard height, it is reset to the freeboard height. The total VBM is finally given by:

$$M_{\text{tot}}(t, \theta | \mathbf{X} = \mathbf{x}) = M_{\text{wave}} + M_{\text{whip}}. \quad (42)$$

The numerical computations (time series reproduction) of  $M_{\text{wave}}$  and  $M_{\text{whip}}$  are performed with a sampling frequency of 10 Hz. In the present study on weight distribution optimization, a Pierson–Moskowitz type wave spectrum is adopted, defined as follows:

$$S(\omega) = \frac{4\pi^3 H_s^2}{T_p^4 \omega^5} \exp \left[ -16\pi^3 \left( \frac{1}{T_p \omega} \right)^4 \right], \quad (43)$$

where  $T_p = 1.4086 T_z$  denotes the peak wave period, with  $T_z$  being the zero-upcrossing period. Note that any type of wave spectrum can be incorporated into the present framework, including the ITTC spectrum defined by Eq. (25), provided that it is parameterized in terms of its statistical properties.

Evidently, the VBM exhibits nonlinear behavior with respect to wave excitation. Thus, the conditional MOR can be estimated in a Monte Carlo fashion based on long-duration numerical simulations. Specifically,

$$v_{\text{MCS}}(\zeta | \mathbf{X} = \mathbf{x}) = \frac{n_{\text{out}}}{N_{\text{mcs}} T_s}, \quad (44)$$

where  $n_{\text{out}}$  denotes the number of out-crossings of a target threshold level,  $T_s$  is the duration of a single numerical simulation using the ROM,

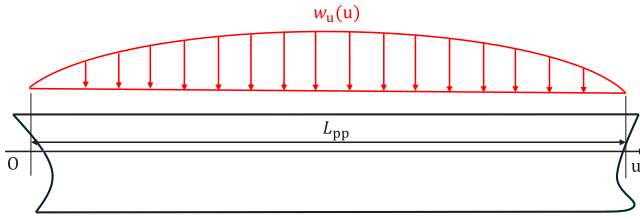


Fig. 6. Schematic of longitudinal weight distribution of a ship.

Table 5

Principal particulars of the container ship used for the nonlinear VBM computation.

| Parameter                                  | Value          |
|--------------------------------------------|----------------|
| Length between perpendiculars ( $L_{pp}$ ) | 283.8 m        |
| Breadth                                    | 42.8 m         |
| Depth                                      | 24.0 m         |
| Draught                                    | 14.0 m         |
| Displacement                               | 107 072.18 ton |

and  $N_{mcs}$  is the number of realizations. In this study,  $N_{mcs} = 100\,000$  realizations of  $T_s = 175$  s each are used to estimate the conditional MOR by MCS.

The MCS-based estimate of the conditional MOR by Eq. (44) is inevitably accompanied by the need for a large number of realizations  $N_{mcs}$ , particularly when estimating rare-event probabilities. To address this issue, this study also employs FORM-based prediction [20]. To introduce FORM, the following performance function w.r.t. the response threshold  $\zeta$  is formulated:

$$G(\zeta, \theta, \mathbf{u} | \mathbf{X} = \mathbf{x}) = 1 - \frac{M_{tot}(t = T_0, \theta, \mathbf{u} | \mathbf{X} = \mathbf{x})}{\zeta}, \quad (45)$$

where  $T_0$  denotes the reference time instance for FORM analysis. In the present study,  $T_0 = 100$  s is used. FORM detects an Most Probable Point (MPP) on the limit state surface characterized by Eq. (45), i.e.,  $G = 0$ , at which the distance from the origin is shortest [38]. The MPP, designated as  $\mathbf{u}^* = \{u_i^*, \bar{u}_i^*\}$ , is accompanied with a reliability index  $\beta_{FORM}$  expressed as

$$\beta_{FORM} = \sqrt{\sum_{i=1}^{N_{wave}} (u_i^{*2} + \bar{u}_i^{*2})}. \quad (46)$$

The MPP yields not only a time-invariant “failure probability”, but also a time-variant MOR, see [20], via the following relation:

$$\nu_{FORM}(\zeta | \mathbf{X} = \mathbf{x}) = \frac{1}{2\pi} \exp\left(-\frac{1}{2}\beta_{FORM}^2\right) \sqrt{\frac{\sum_{i=1}^{N_{wave}} \omega_{0,i}^2 (u_i^{*2} + \bar{u}_i^{*2})}{\beta_{FORM}^2}} \quad (47)$$

FORM requires numerical computation of the gradient vectors of the performance function, yet it is much more computationally efficient than the crude MCS. In this study, the Improved Hasofer–Lind–Rackwitz–Fiessler algorithm [39] is employed to search for the MPP.

## 6.2. Results and discussion based on the low-fidelity model

First, the optimization results obtained using only the low-fidelity model, i.e., the ROM, are presented and discussed. Table 6 summarizes the resulting values of coefficients. A distinct set of  $c_1$ ,  $c_2$ , and  $c_3$  is used for the hogging (i.e.,  $M_{LN} \geq 0$ ) and sagging (i.e.,  $M_{LN} < 0$ ) moments. Table 7 summarizes the parameters used in the present case study. Although the values listed in the table are selected somewhat arbitrarily, they are close to those adopted in previous studies [33,34]. Introducing the Pierson–Moskowitz type wave spectrum with  $H_s = 8$  m and  $T_z = 12$  s, an example of the nonlinear VBM amidship for the

Table 6

Nonlinear correction factors for weakly nonlinear VBM.

|                 | $c_1$  | $c_2$                   | $c_3$                   | $t_f$ |
|-----------------|--------|-------------------------|-------------------------|-------|
| $M_{LN} \geq 0$ | 0.9147 | $5.158 \times 10^{-5}$  | $-7.166 \times 10^{-9}$ | 0 s   |
| $M_{LN} < 0$    | 0.4944 | $-2.929 \times 10^{-4}$ | $-1.889 \times 10^{-8}$ | 0 s   |

Table 7

Parameters for computing whipping vibration in the vertical bending moment used for the numerical demonstration.

| Parameter       | Value      |
|-----------------|------------|
| $L_0$           | 1.79 m     |
| $\xi_d$         | 0.01       |
| $\omega_{whip}$ | 3.14 rad/s |
| $d_w$           | 18.0 m     |
| $l_{bow}$       | 113.52 m   |
| $\delta_w$      | 40 deg     |

relative wave direction  $\chi = 180$  deg, ship speed  $U_V = 6$  m/s,  $\alpha_1 = 1.0$ , and  $\alpha_2 = 0.0$  case is shown in Fig. 8. As observed from the figure, the VBM includes a high-frequency vibration component originating from  $M_{whip}$  and a low-frequency component associated with  $M_{wave}$ .

### 6.2.1. Optimization settings

In the present case, both the environmental variability and operational variability are accounted for in deriving the long-term MOR. The Pierson–Moskowitz type wave spectrum by Eq. (43) is assumed, and the employed statistical properties of  $H_s$ ,  $T_z$ ,  $\chi$ , and  $U_V$  are indicated in Table 8.  $\zeta = 3000$  M Nm is targeted. With respect to the relative wave direction, two distinct cases are considered, namely *all headings* and *head seas*, as indicated in the table.

Looking at the optimization of the ship weight distribution, the design variable optimization of  $\theta = (\alpha_1, \alpha_2)$  values so as to minimize the long-term MOR is performed by the double- and single-loop approaches. The sample population size of  $N_s = 5000$  and the initial training sample size of  $N_0 = 10$  are adopted for the double- and single-loop approaches. Note that the same initial population of the environmental random variables is used among the double- and single-loop schemes. In computing the double-loop scheme the PSO is introduced, in which the swarm size and the maximum number of iterations are both set to 15. To maintain consistency, the sample population  $\mathcal{X}_W$  is kept unchanged during the optimization phase by the double-loop approach. For a fair comparison under the same sample population, the subset of environmental variables in the initial sample population  $\mathcal{W}_W$  for the single-loop approach is taken to be identical to  $\mathcal{X}_W$ . In the PSO stage to identify the design site  $\theta^+$  by Eq. (21) using the single-loop approach, the swarm size and the maximum number of iterations are both set to 15 when the full covariance matrix is used, whereas they are set to 20 when  $\sigma_{\bar{v},approx}$  is introduced. The optimization is conducted subject to the bounds  $-0.1 \leq \alpha_1 \leq 0.1$  and  $0 \leq \alpha_2 \leq 0.9$  for the *all headings* case, while  $-0.5 \leq \alpha_1 \leq 0.5$  and  $0 \leq \alpha_2 \leq 0.5$  for the *head seas* case to ensure that  $w_u$  remains positive. Note that, when the *head seas* case is analyzed using the present single-loop approach, the resulting  $\theta^{opt}$  values are occasionally trapped in local minima, which appear as outliers with still relatively large  $\bar{v}$  values. To alleviate this issue, in identifying the design site  $\theta^+$  by Eq. (21), the PSO algorithm is implemented with a hybrid local search using a gradient-based constrained nonlinear optimization solver, so that the best solution obtained by PSO is further refined through local optimization.

### 6.2.2. Optimization results

The design parameter optimization results together with estimated minimum value of the long-term MOR  $\bar{v}$  are summarized in Tables 9 and 10, for the *all headings* and *head seas* cases, respectively. For the single-loop approach, 5 independent runs with different initial training

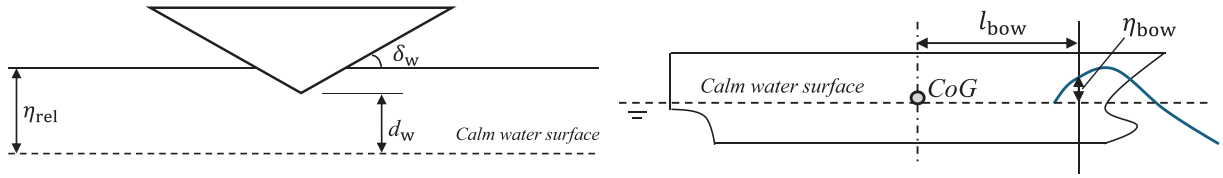


Fig. 7. Parameters in the Reduced Order Model for nonlinear vertical bending moment in a ship.

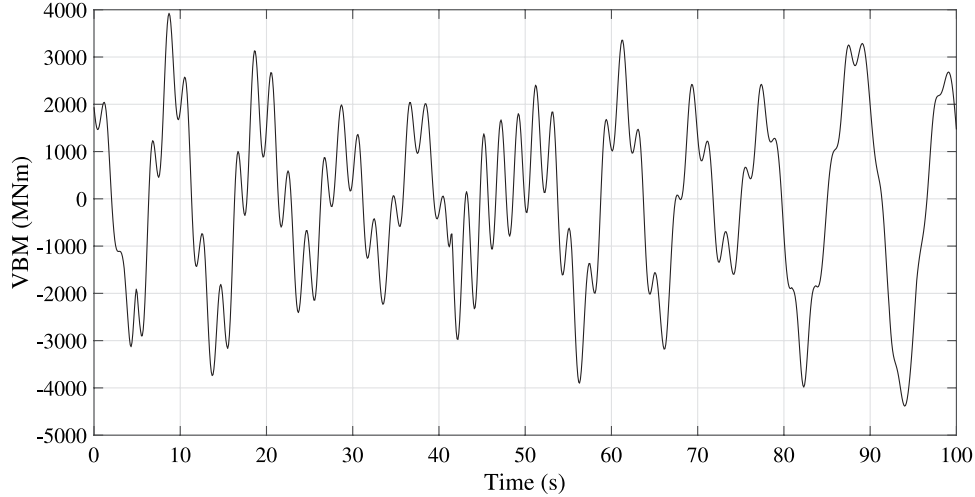


Fig. 8. An example of the nonlinear vertical bending moment in a ship generated by the present ROM. The significant wave height  $H_s = 8$  m, zero-upcrossing period  $T_z = 12$  s, relative wave direction  $\chi = 180$  deg, ship speed  $U_V = 6$  m/s.  $\alpha_1 = 1.0$  and  $\alpha_2 = 0.0$  case.

Table 8

Statistical properties of environmental and operational random variables used for ship weight distribution optimization under nonlinear vertical bending moment.

| Parameter | Description                     | Mean | Distribution     | COV  |
|-----------|---------------------------------|------|------------------|------|
| $H_s$     | Significant wave height (m)     | 8.0  | Log-normal       | 0.20 |
| $T_z$     | Zero-upcrossing wave period (s) | 10.7 | Log-normal       | 0.25 |
| $U_V$     | Ship speed (m/s)                | 6.0  | Log-normal       | 0.20 |
| $\chi$    | Relative wave direction (deg)   |      |                  |      |
|           | – All headings                  | 90   | Uniform (0, 180) | –    |
|           | – Head seas                     | 180  | Normal           | 0.10 |

sample selections are conducted for the full covariance-based case, while 10 runs are conducted when  $\sigma_{\bar{v},\text{approx}}$  is used. From Table 9, the mean values of  $\min \bar{v}$  from the double-loop and single-loop approaches are comparable, despite the slight discrepancy. As evidenced by the COV values of  $\min \bar{v}$ , the single-loop approach yields robust and stable results. While the use of  $\sigma_{\bar{v},\text{approx}}$  leads to a slightly higher COV compared to the full covariance-based formulation, the variability remains sufficiently small for engineering purposes. Furthermore, the computational efficiency of the single-loop approach is remarkable, particularly when the approximation  $\sigma_{\bar{v},\text{approx}}$  is employed. The same observation applies to the *head seas* case, see Table 10. From the perspective of facilitating smooth cargo handling operations at ports, the absolute CPU time of 45 to 150 min is indeed feasible.

The results based on the FORM-based conditional MOR are first discussed. To quantify the variability of the identified optimal design parameters  $\theta^{\text{opt}}$ , a normalized standard deviation is introduced:

$$\text{NStd}(\theta^{\text{opt}}) = \left[ \frac{\text{Std}(\alpha_1^{\text{opt}})}{\alpha_1^U - \alpha_1^L}, \frac{\text{Std}(\alpha_2^{\text{opt}})}{\alpha_2^U - \alpha_2^L} \right], \quad (48)$$

where  $\alpha_i^U$  and  $\alpha_i^L$  denote the upper and lower bounds of the design parameter  $\alpha_i$ , respectively, and  $\text{Std}(\cdot)$  denotes standard deviation.

Looking at  $\theta^{\text{opt}}$  in the *all headings* case, the value of  $\alpha_2$  identified by the single-loop approach is consistently close to zero, whereas  $\alpha_1$  exhibits considerable variation, as indicated by the  $\text{NStd}(\theta^{\text{opt}})$  values, see Table 9. This variation becomes particularly large when  $\sigma_{\bar{v},\text{approx}}$  is employed. Conversely, in the *head seas* case, the variation in  $\alpha_1$  is relatively small, and the expected value of  $\theta^{\text{opt}}$  by the single-loop approach (using  $\sigma_{\bar{v},\text{approx}}$ ) is closed to that by the double-loop approach, see Table 10. The large variation in  $\alpha_1$  in the *all headings* case can be attributed to its relatively low sensitivity to the long-term MOR  $\bar{v}$ . To examine this sensitivity, the long-term MOR  $\bar{v}$  is computed using the BAL-LTP for varying values of  $\alpha_1$  while fixing  $\alpha_2 = 0.0$ . These computations are performed with  $N_s = 5000$  and  $N_0 = 10$ , using a consistent sample population  $\mathcal{X}_W$ . As illustrated in Fig. 9, the  $\bar{v}$  values in the *all headings* case remain nearly constant regardless of  $\alpha_1$ . This insensitivity likely underlies the large  $\text{NStd}(\theta^{\text{opt}})$  associated with  $\alpha_1$  observed in Table 9, as the optimization landscape lacks a distinct gradient. In contrast, the  $\bar{v}$  values in the *head seas* case exhibit a clear sensitivity to changes in  $\alpha_1$ .

Comparing the results based on the MCS- and FORM-based conditional MORs, the identified optimal design variables  $\theta^{\text{opt}}$  are found to be reasonably consistent, whereas noticeable discrepancies are observed in the corresponding values of  $\min \bar{v}$ . Such discrepancies are primarily attributed to the linearization of the limit state surface inherent in the FORM approximation, as discussed in [40]. Nevertheless, the FORM-based conditional MOR appears sufficiently accurate for identifying the optimal design variables, even though the predicted MOR values themselves may exhibit noticeable errors. Thanks to the computational efficiency of the present ROM, the single-loop approach based on the MCS-based conditional MOR can be completed within a practical computational time (at most 120 min). In the present examples, a clear reduction in computational cost by using FORM is not observed, because the expected number of STP evaluations,  $E[N_{\text{STP}}]$ , is smaller for the MCS-based approach than for the FORM-based approach. This is partly attributable to the fact that the resulting long-term MORs obtained with MCS are relatively high, as shown in Tables 9 and 10. In such cases, the conditional MOR distribution over short-term sea

**Table 9**

Identified design parameter  $\theta = (\alpha_1, \alpha_2)$  for minimizing the long-term MOR ( $N_s = 5000$ ), in the *all headings* case. The initial sample population  $\mathcal{W}_w$  for the single-loop approach is taken to be identical to  $\mathcal{X}_w$ . For the single-loop approach, 5 independent runs with different initial training sample selections are conducted for the full covariance-based case, while 10 runs are conducted when  $\sigma_{\bar{v}, \text{approx}}$  is used.

| Conditional MOR   | Method                                            | $E[\theta^{\text{opt}}]$ | NStd( $\theta^{\text{opt}}$ ) | $E[\min \bar{v}]$      | COV[ $\min \bar{v}$ ] | $E[N_{\text{STP}}]$ | CPU time |
|-------------------|---------------------------------------------------|--------------------------|-------------------------------|------------------------|-----------------------|---------------------|----------|
| $v_{\text{FORM}}$ | Double-loop                                       | (0.011, 0.018)           | –                             | $2.494 \times 10^{-4}$ | –                     | 33 339              | >72 h    |
|                   | Single-loop ( $\sigma_{\bar{v}}$ )                | (0.038, 0.000)           | (0.205, 0.000)                | $2.881 \times 10^{-4}$ | 0.013                 | 287                 | 30 h     |
|                   | Single-loop ( $\sigma_{\bar{v}, \text{approx}}$ ) | (−0.009, 0.000)          | (0.465, 0.000)                | $2.843 \times 10^{-4}$ | 0.050                 | 126                 | 45 min   |
| $v_{\text{MCS}}$  | Single-loop ( $\sigma_{\bar{v}, \text{approx}}$ ) | (0.078, 0.000)           | (0.315, 0.000)                | $1.064 \times 10^{-3}$ | 0.126                 | 92                  | 120 min  |

**Table 10**

Identified design parameter  $\theta = (\alpha_1, \alpha_2)$  for minimizing the long-term MOR ( $N_s = 5000$ ) in the *head seas* case. The initial sample population  $\mathcal{W}_w$  for the single-loop approach is taken to be identical to  $\mathcal{X}_w$ . For the single-loop approach, 10 runs are conducted.

| Conditional MOR   | Method                                            | $E[\theta^{\text{opt}}]$ | NStd( $\theta^{\text{opt}}$ ) | $E[\min \bar{v}]$      | COV[ $\min \bar{v}$ ] | $E[N_{\text{STP}}]$ | CPU time |
|-------------------|---------------------------------------------------|--------------------------|-------------------------------|------------------------|-----------------------|---------------------|----------|
| $v_{\text{FORM}}$ | Double-loop                                       | (0.163, 0.008)           | –                             | $1.224 \times 10^{-4}$ | –                     | 10 161              | >72 h    |
|                   | Single-loop ( $\sigma_{\bar{v}, \text{approx}}$ ) | (0.171, 0.000)           | (0.041, 0.000)                | $1.456 \times 10^{-4}$ | 0.064                 | 363                 | 150 min  |
| $v_{\text{MCS}}$  | Single-loop ( $\sigma_{\bar{v}, \text{approx}}$ ) | (0.158, 0.000)           | (0.041, 0.000)                | $3.220 \times 10^{-3}$ | 0.051                 | 68                  | 90 min   |

states may be sufficiently smooth, allowing the MCS-based approach to remain efficient. Nevertheless, FORM is expected to become more advantageous for lower MOR levels, where direct MCS would require a substantially larger number of realizations. To further assess the validity of the identified optima, crude MCS evaluations were performed using the MCS-based conditional MORs, i.e., without incorporating the BAL-LTP framework. Consequently, in the *all headings* case, the long-term MOR at the MCS-based optimum  $\theta^{\text{opt}} = (0.078, 0.0)$  is evaluated as  $\bar{v} = 1.137 \times 10^{-3}$ , while in the *head seas* case,  $\bar{v} = 3.113 \times 10^{-3}$  is obtained at  $\theta^{\text{opt}} = (0.158, 0.0)$ . These values are in good agreement with the corresponding MCS-based single-loop optimization results indicated in Tables 9 and 10, providing additional confidence in the identified  $\theta^{\text{opt}}$  values.

The present single-loop approach is then introduced to find the weight distributions corresponding to  $\bar{v}^* = 5 \times 10^{-4}$ , where  $v_{\text{FORM}}$  is used throughout. Five independent runs of the single-loop approach, together with  $\sigma_{\bar{v}, \text{approx}}$ , are conducted, with the initial sample size of  $N_s = 5000$  for the *all headings* case. In this case, the conditional MOR is evaluated using FORM in place of MCS. The identified weight distributions for  $\bar{v}^* = 5 \times 10^{-4}$  are shown in Fig. 10. In the figure, the black dotted line indicates the weight distribution for minimizing the long-term MOR  $\bar{v}$ , which is based on one of the results indicated in Table 9. The identified design variables corresponding to each colored line for  $\bar{v}^* = 5 \times 10^{-4}$  are indicated in the legends. From the black dotted line, since the  $\alpha_2$  values are identified as zero, the weight is linearly distributed. This result is reasonable, as a non-uniform weight distribution is likely to induce larger VBMs. On the other hand, if  $\bar{v}^* = 5 \times 10^{-4}$  is targeted, the other weight distribution options become available, which allow non-uniform distribution. In the *all headings* case, the weight distributions identified from the five runs are almost equivalent. To validate the accuracy of the optimized design variables, the corresponding long-term MORs are evaluated using BAL-LTP. To account for the variability, 5 independent runs of BAL-LTP are performed by shuffling  $\mathcal{X}_w$  with the size of  $N_s = 5000$ . The results are summarized in Table 11. From the table, it can be confirmed that all identified design variables achieve values close to the target  $\bar{v}^* = 5 \times 10^{-4}$ . In certain instances, the deviation is somewhat larger than in other cases, which may be partially attributed to the approximation  $\sigma_{\bar{v}, \text{approx}}$  introduced during the BO phase. Nevertheless, the computational efficiency of the single-loop approach combined with  $\sigma_{\bar{v}, \text{approx}}$  is substantial, as discussed previously, thereby making it well-suited for practical applications such as loading and unloading operations in ports.

**Table 11**

Long-term MORs calculated by the BAL-LTP with respect to identified design parameters for  $\bar{v}^* = 5 \times 10^{-4}$  as indicated in Fig. 10. 5 independent runs with different initial sample populations  $\mathcal{X}_w$  are carried out.  $N_s = 5000$  and  $N_0 = 10$  are used. *All headings* case.

| $\theta^{\text{opt}}$ | $E[\mu_{\bar{v}}]$     | $\delta_{\bar{v}}$ |
|-----------------------|------------------------|--------------------|
| (0.036, 0.712)        | $4.712 \times 10^{-4}$ | 0.065              |
| (0.100, 0.809)        | $5.191 \times 10^{-4}$ | 0.057              |
| (0.056, 0.730)        | $4.648 \times 10^{-4}$ | 0.104              |
| (0.100, 0.832)        | $5.410 \times 10^{-4}$ | 0.085              |
| (0.047, 0.684)        | $5.464 \times 10^{-4}$ | 0.071              |

### 6.3. Incorporation of high-fidelity numerical model

#### 6.3.1. Methodology

In many applications, it is preferable to compute the VBM using a high-fidelity numerical model, such as Computational Fluid Dynamics (CFD), in order to reproduce complex nonlinear behavior. Although the proposed single-loop approach has been shown to significantly accelerate the LTP process, iterative time-domain evaluations of the VBM are still required to estimate the conditional MOR, which can become computationally expensive when a high-fidelity model is employed. To alleviate this computational burden, a multi-fidelity procedure can be introduced. Specifically, rather than evaluating the conditional MOR entirely using the high-fidelity model, short-duration high-fidelity simulations are first performed and used to calibrate the low-fidelity model (i.e., the ROM). The calibrated low-fidelity model is then employed for the evaluation of the conditional MOR. To ensure that the calibration captures nonlinear response characteristics relevant to extreme events, the high-fidelity simulations are conducted for wave episodes corresponding to the MPP identified by FORM using the uncalibrated low-fidelity model. Since the MPP corresponds to the *design wave* that induces the target large-amplitude response [41], nonlinear effects are expected to be most pronounced in the associated response. Consequently, the resulting high-fidelity simulations provide highly informative calibration data. The parameters embedded in the low-fidelity model are subsequently calibrated so that the low-fidelity response reproduces the corresponding high-fidelity response for the selected design wave episode.

As the high-fidelity model, the nonlinear strip-theory-based code “Nonlinear Motion in Regular and Irregular Wave (NMRIW)” is employed, which is the same model used in [42]. In the present study, only body nonlinearity is taken into account. Accordingly, whipping vibrations are neglected by setting  $M_{\text{tot}} = M_{\text{wave}}$ , and only the parameters  $p = (c_1, c_2, c_3, t_f)$  in the ROM are calibrated. The procedure for

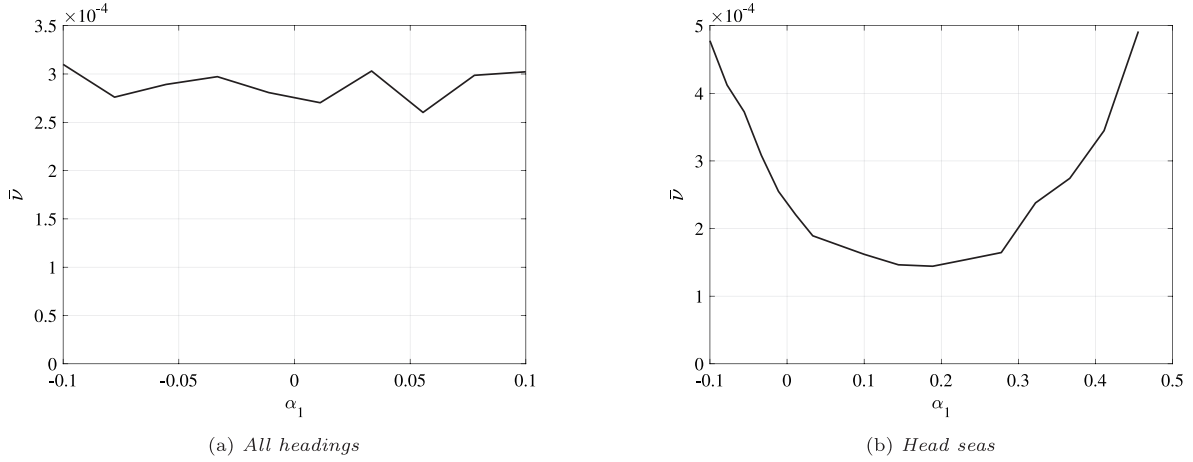


**Fig. 9.** Long-term mean outcrossing rate  $\bar{\nu}$  computed by BAL-LTP for different values of  $\alpha_1$ , where  $\alpha_2 = 0.0$  is introduced. An initial sample population size of  $N_s = 5000$  is used, and the same sample population  $\mathcal{X}_w$  is employed in all plots.

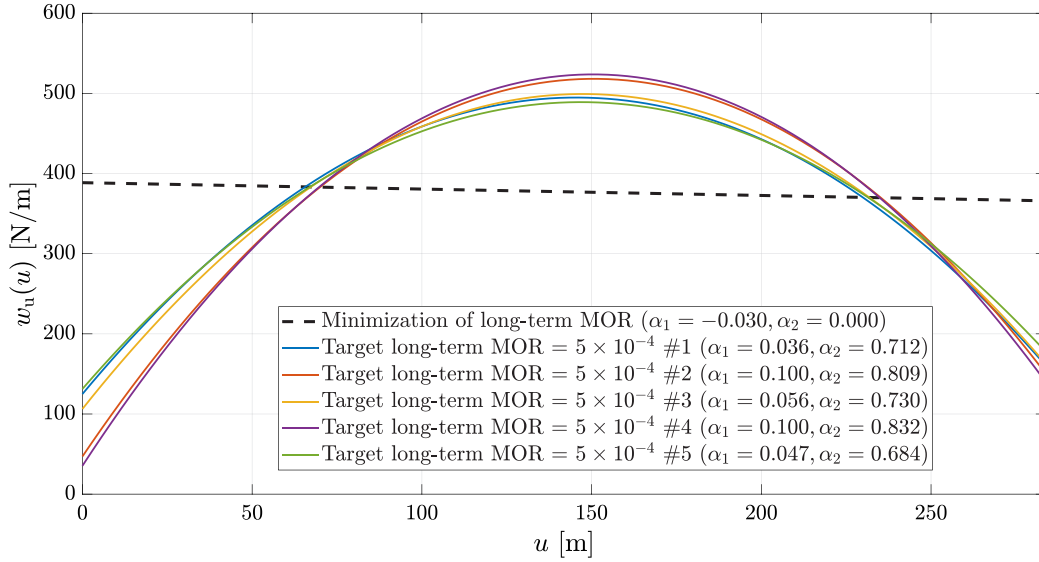


**Fig. 10.** Identified weight distribution curves targeting the long-term MOR of  $\bar{\nu}^* = 5 \times 10^{-4}$ . An initial sample population size of  $N_s = 5000$  is used. The single-loop approach together with  $\sigma_{\bar{\nu}, \text{approx}}$  in the BO phase is used. 5 independent runs with different initial sample populations  $\mathcal{W}_w$  are carried out. *All headings* case.

estimating the conditional MOR while incorporating the high-fidelity model is summarized below:

- Set  $c_1 = 1$  and  $c_2 = c_3 = t_f = 0$ , i.e., assume a fully linear VBM, and perform FORM using the performance function in Eq. (45) to identify the MPP  $\mathbf{u}^* = \{u_i^*, \bar{u}_i^*\}$
- Conduct a short-duration high-fidelity simulation for the design wave episode corresponding to  $\mathbf{u}^*$ . The resulting total VBM time history is denoted by  $M_{\text{tot}}^{\text{HF}}(t)$ .
- Calibrate the parameter vector  $\mathbf{p}$  embedded in the ROM by minimizing the discrepancy between the low- and high-fidelity VBMs over the selected time interval  $\mathcal{T}$ :

$$\mathbf{p}^{\text{opt}} = \arg \min_{\mathbf{p}} \left[ \frac{1}{|\mathcal{T}|} \int_{\mathcal{T}} \{M_{\text{tot}}(t; \mathbf{p}) - M_{\text{tot}}^{\text{HF}}(t)\}^2 dt \right]^{1/2}. \quad (49)$$

The optimization problem is solved using a genetic algorithm. The lower and upper bounds of the calibration parameters are specified as  $0.2 \leq c_1 \leq 2.0$ ,  $-10^{-4} \leq c_2 \leq 10^{-4}$ ,  $-10^{-8} \leq c_3 \leq 10^{-8}$ , and  $-5 \leq t_f \leq 5$ . The calibration interval is selected as  $\mathcal{T} = [50, 175]$  s, which covers the dominant response event associated with the design wave episode while avoiding the initial transient response

- Calculate the conditional MOR using the ROM with the calibrated parameters  $\mathbf{p}^{\text{opt}}$ .

Note that, since the present case study focuses on the extreme hogging moment, only the parameters  $\mathbf{p} = (c_1, c_2, c_3, t_f)$  associated with  $M_{\text{LN}} \geq 0$  are calibrated. The corresponding parameters for  $M_{\text{LN}} < 0$  are kept unchanged from the values listed in Table 6. The present approach shares the same philosophy as the multi-fidelity design wave framework of Dermatis et al. [41], in that high-fidelity simulations are performed only for extreme wave episodes, while the overall probabilistic evaluation relies on an efficient low-fidelity model.

### 6.3.2. Optimization results

A numerical investigation is conducted to demonstrate the effectiveness of the proposed method incorporating the high-fidelity model. The environmental and operational variability of the *all headings* case listed in Table 8 is considered, and the Pierson–Moskowitz wave spectrum is assumed. The target threshold is set to  $\zeta = 3000$  M Nm, and the optimization is performed subject to the bounds  $-0.1 \leq \alpha_1 \leq 0.1$  and  $0 \leq \alpha_2 \leq 0.9$ . The single-loop approach combined with  $\sigma_{\bar{\nu}, \text{approx}}$  is employed, and all other settings are identical to those described in

Section 6.2.1. In this sub-section all the conditional MORs are computed by the MCS, i.e.,  $v_{\text{MCS}}$ .

First, the capability of the ROM with the calibrated parameters  $p^{\text{opt}}$  to reproduce the high-fidelity responses is examined. To this end, the calibration procedure described in Section 6.3.1 is applied to a set of randomly generated joint samples  $\psi = (\theta, x)$ . Subsequently, the VBM time histories predicted by the high-fidelity model and the calibrated ROM are compared for the corresponding design-wave episodes. The results for five representative samples are shown in Fig. 11. In the figures, the black dotted lines denote the VBM time histories obtained using the uncorrected ROM at the corresponding MPPs,  $u^* = \{u_i^*, \bar{u}_i^*\}$ , whose peak values are equal to the target threshold  $\zeta = 3000$  M Nm at the reference time  $T_0 = 100$  s. It is evident that the corrected ROM reproduces the high-fidelity responses with good accuracy for all selected cases. Some discrepancies remain in a few realizations, such as that shown in Fig. 11(c). However, such discrepancies do not necessarily affect the long-term MOR significantly, since the corresponding conditional MOR is extremely small, as indicated by the large reliability index ( $\beta_{\text{FORM}} = 12.66$ ).

For each realization shown in Fig. 11, the conditional MORs with respect to the VBM threshold  $\zeta$  are evaluated using the high-fidelity model, the corrected ROM, and the uncorrected ROM based on the MCS estimator given in Eq. (44). Owing to the high computational cost of the high-fidelity model, only  $N_{\text{mcs}} = 1000$  realizations with a simulation duration of  $T_s = 175$  s are employed to estimate the conditional MORs. The resulting comparison is presented in Fig. 12. As can be seen, the corrected ROM reproduces the conditional MOR distributions obtained by the high-fidelity model with reasonable accuracy, while providing a clear improvement over the uncorrected ROM. Although discrepancies remain in Figs. 12(a) and 12(c), the improvement achieved by the corrected ROM is still significant. Since the present ROM employs the simplified polynomial correction model defined in Eq. (34), further accuracy may be attained by adopting a more sophisticated correction function. Alternatively, the calibration procedure described in Section 6.3.1 could be enhanced by incorporating multiple high-fidelity simulation results, albeit at the expense of additional computational cost.

To further quantify the performance of the calibrated ROM, the extreme VBM values obtained from the high-fidelity model and the corrected ROM are compared using randomly generated joint samples. Only cases with conditional MOR greater than  $10^{-6}$  are retained, resulting in 560 samples. The corresponding scatter plot is shown in Fig. 13. Although a few outliers are observed, a strong correlation is obtained, with a Pearson correlation coefficient of 0.881. This result demonstrates that the corrected ROM via the present calibration process successfully reproduces the high-fidelity extreme-response characteristics.

By incorporating the high-fidelity model together with the present calibration process, the design optimizations for minimizing the long-term MOR and for attaining a prescribed level of  $\bar{v}^* = 10^{-4}$  are carried out. The results are summarized in Table 12. As indicated in the table, the present single-loop approach successfully identifies the design parameters that minimize the long-term MOR, yielding highly consistent solutions characterized by very small  $\text{NStd}(\theta^{\text{opt}})$  values and negligible variation in  $\min \bar{v}$ . Similar observations can be made for the prescribed long-term MOR case. Although  $\alpha_1$  exhibits relatively large variability, this is attributable to the insensitivity of the long-term MOR to  $\alpha_1$ , as discussed in Fig. 9. The small values of  $\text{NStd}(\alpha_2^{\text{opt}})$  and  $\text{COV}[\bar{v}]$  indicate that the proposed single-loop approach incorporating the high-fidelity model yields reasonably consistent optimal solutions and stable estimates of the long-term MOR.

## 7. Discussion and future work

As discussed, the proposed single-loop approach, particularly when integrated with  $\sigma_{\bar{v},\text{approx}}(\theta)$ , demonstrates significant computational efficiency for design optimization aimed at both minimizing the long-term MOR and identifying design variables that achieve a prescribed

long-term MOR. Nevertheless, the computational cost (excluding STP processes) scales with the initial sample size  $N_s$ . For instance, estimating the long-term MOR for a ship's 25-year lifespan, assuming a typical short-term sea state duration of 3 h, would require  $N_s \approx 7.3 \times 10^4$ . In such scenarios, the computational burden of the BAL-LTP within the single-loop approach becomes significant. This is primarily attributed to the evaluation of the standard deviation of the long-term MOR via Eq. (10), which necessitates the manipulation of the full covariance matrix. Nonetheless, as preliminarily demonstrated in [16], the long-term MOR typically converges at an  $N_s$  significantly lower than the total number of sea states, potentially allowing for a reduction in sample size without compromising accuracy. Another promising remedy involves exploiting  $\sigma_{\bar{v},\text{approx}}(\theta)$  to evaluate the COV  $\delta_{\bar{v}}(\zeta, \theta)$  in Eq. (11). However, such an approximation might lead to a sub-optimal selection of training samples; hence, this trade-off warrants careful examination in future work.

In essence, the present single-loop approach can incorporate the random variables required for the STP stage, i.e.,  $u = \{u_i, \bar{u}_i\}$  in Eq. (1). However, since  $u$  in the STP is typically high-dimensional (e.g., 50 to 100 dimensions), its direct incorporation into the single-loop approach is computationally ill-advised. To mitigate this, dimension reduction techniques, such as the Karhunen–Loève expansion [43], active subspaces [44,45], or neural operators [46], could be integrated into the STP stage. That said, a critical prerequisite is that the reduced dimensional space remains consistent across all short-term sea states; otherwise, the present single-loop framework is inapplicable. FORM analysis provides a reliable approximation of the reliability index and the associated conditional MOR at a modest computational cost, even within high-dimensional stochastic spaces. However, a significant caveat is that the conditional MOR yielded by FORM loses its accuracy when the response of interest exhibits strong nonlinearity, as discussed in Section 6.2.2. MCS can provide accurate conditional MOR, but a large number of realizations is required to capture rare events. To circumvent this limitation, one can integrate a surrogate model-based failure probability estimation method, specifically, an adaptive Kriging-based approach, with dimension reduction techniques [44] to evaluate the conditional MOR via Eq. (47). In [44], it was demonstrated that failure probabilities on the order of  $10^{-6}$  could be accurately estimated through a sequential active learning approach. Nevertheless, when extending this framework to long-term MOR estimation, it may become necessary to estimate significantly smaller failure probabilities in estimating the long-term MOR, depending on the specific short-term sea state. Building upon these considerations, a more sophisticated treatment of the STP stage remains a crucial objective for future work.

As demonstrated in Section 6.3, a high-fidelity model can be incorporated into the present single-loop framework together with a low-fidelity model through the proposed conditional-MOR calibration procedure. In the present study, only a single short-duration high-fidelity simulation was employed for calibration in order to maintain computational tractability while making effective use of limited high-fidelity data. As discussed in Section 6.3.2, however, incorporating multiple high-fidelity simulations into the calibration process would be preferable when computational resources permit, as this is expected to improve the accuracy and robustness of the corrected ROM. Furthermore, the present study considered only the body (global) nonlinear contribution to the VBM. Extending the proposed framework to account for whipping-induced VBM responses remains an important topic for future research, for example through CFD and structural response co-simulation [47].

## 8. Conclusion

This study proposed a novel numerical method for design optimization of a ship subjected to nonlinear wave-induced responses accounting for its long-term risk. To quantify the risk of encountering an extreme response, this paper targeted the long-term prediction (LTP)

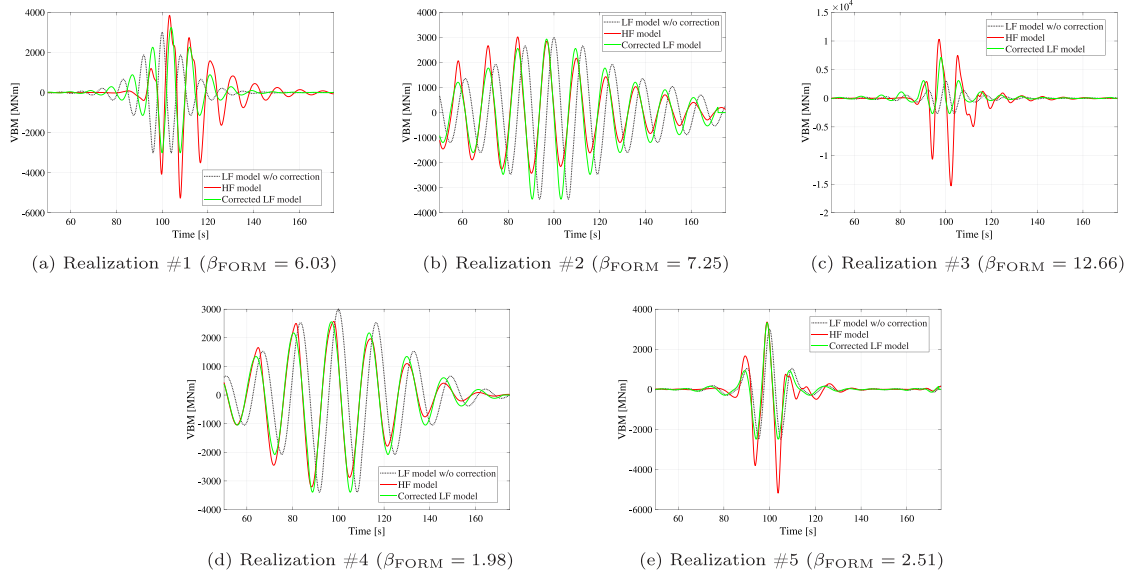


**Fig. 11.** Comparison of VBM time histories computed by the high-fidelity model and the corrected ROM for five randomly selected design-wave episodes. The black dotted lines represent the VBM time histories predicted by the uncorrected ROM with  $c_1 = 1$  and  $c_2 = c_3 = t_f = 0$ .

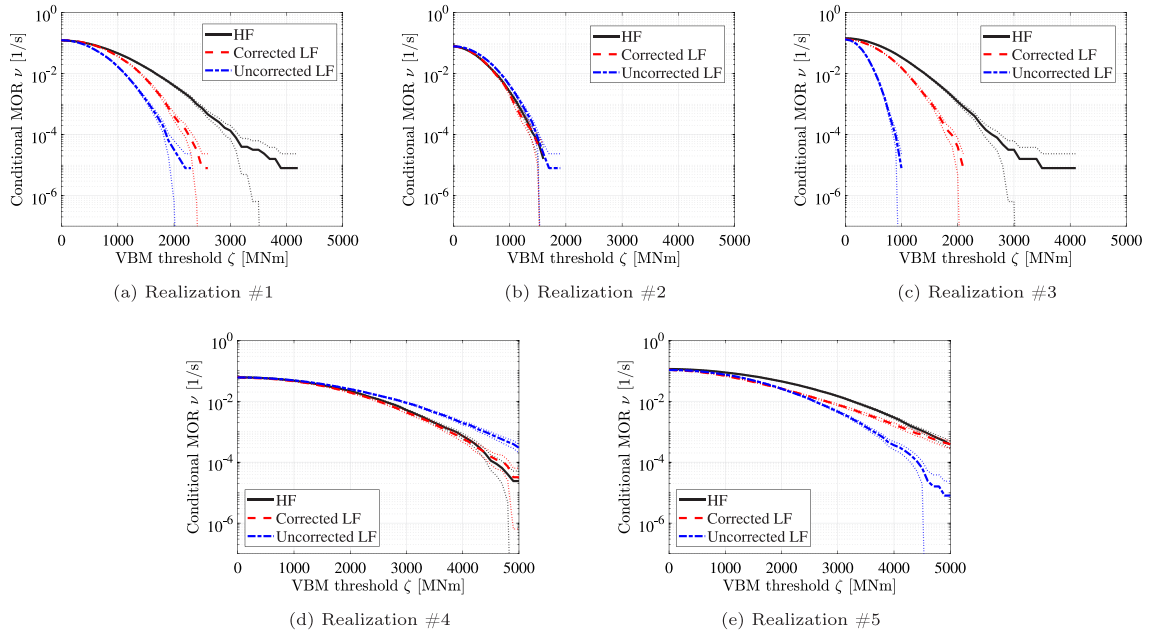


**Fig. 12.** Comparison of conditional mean outcrossing rates estimated using the high-fidelity model and the corrected/uncorrected ROMs. The dotted lines represent the 95% confidence intervals. The conditional MORs are evaluated using the MCS estimator in Eq. (44) with  $N_{MCS} = 1000$  realizations.

**Table 12**

Identified design parameter  $\theta = (\alpha_1, \alpha_2)$  when the high-fidelity model is incorporated, *all headings* case. 10 runs with different initial sample populations of the single-loop approach together with  $\sigma_{v,approx}$  are conducted.

| Target                | $E[\theta^{opt}]$ | $NStd(\theta^{opt})$ | $E[\min \bar{v}]$ or $E[\bar{v}]$ | $COV[\min \bar{v}]$ or $COV[\bar{v}]$ | $E[N_{STP}]$ |
|-----------------------|-------------------|----------------------|-----------------------------------|---------------------------------------|--------------|
| $\min \bar{v}$        | (0.100, 0.000)    | (0.000, 0.000)       | $4.728 \times 10^{-5}$            | 0.123                                 | 419          |
| $\bar{v}^* = 10^{-4}$ | (0.035, 0.346)    | (0.408, 0.064)       | $10^{-4}$                         | 0.004                                 | 220          |

of the mean outcrossing rate (MOR), which is evaluated by accumulating the conditional MOR under a given short-term sea state, namely short-term prediction (STP). An active learning methodology for efficient execution of design optimization targeting the long-term MOR was devised. First, the proposed method incorporated Bayesian Active Learning-based Long-Term Prediction (BAL-LTP), which is built upon the Bayesian cubature framework. Within this framework, Gaussian

Process Regression (GPR) is employed to construct a meta-model of the conditional MOR. Notably, unlike conventional GPR applications, the MOR is modeled as a log-normal process to explicitly ensure its non-negativity. Furthermore, the present BAL-LTP incorporated a sampling-free methodology for posterior uncertainty estimation, thereby eliminating the variability associated with the conventional GP sampling technique. A design optimization method was built upon the BAL-LTP

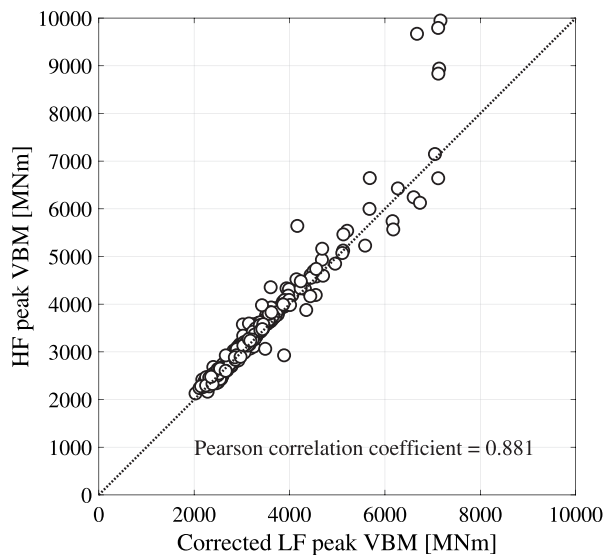


**Fig. 13.** Comparison of extreme values of vertical bending moments derived from high-fidelity and corrected ROM models. The extreme values are evaluated at the MPPs predicted by FORM using the uncorrected ROM. Only samples with a conditional MOR greater than  $10^{-6}$  are plotted (560 samples).

framework, while the double-loop problem is avoided by nesting the Bayesian optimization and BAL-LTP. Two numerical demonstrations are conducted to illustrate the efficiency and effectiveness of the proposed design optimization procedure based on the single-loop approach.

The first numerical example was the optimization of the metacentric height to minimize the long-term MOR while accounting for nonlinear roll motion. Theoretical formulae for nonlinear roll motion estimation [27] was introduced herein. Through the numerical investigation, the significant computational efficiency and accuracy in the single-loop approach was demonstrated, by comparing with the double-loop approach where the Particle Swarm Optimization is integrated with the BAL-LTP. Moreover, the single-loop computation can be further accelerated by incorporating an approximated formula for the variance of the long-term MOR, without compromising estimation accuracy.

The second numerical example focused on the optimization of ship weight distribution, accounting for nonlinear vertical bending moment (VBM). In this study, a computationally efficient low-fidelity model for VBM, referred to as a Reduced Order Model (ROM), was employed. In this analysis, the STP process, incorporating a high-dimensional stochastic problem, was evaluated using Monte Carlo Simulation (MCS) or the First-Order Reliability Method (FORM) to derive the conditional MORs. The results demonstrated the effectiveness of the proposed single-loop approach in identifying the optimal weight distribution that minimizes the long-term MOR. Furthermore, the exceptional computational efficiency achieved by the single-loop approach, combined with the approximated variance formula, confirmed its feasibility for practical applications during port-side loading and unloading operations. In addition, target long-term MOR-based optimization was shown to be feasible using the present single-loop approach. Although FORM yielded less accurate estimates of the long-term MOR than MCS, the resulting optimal designs through the present single-loop approach were comparable. This finding indicates that FORM can be employed as a viable alternative to MCS in the STP stage. Its computational advantage is expected to become more pronounced for lower target MOR levels, where direct MCS would require a substantially larger number of realizations.

For the latter numerical example involving nonlinear VBM, an additional study was conducted using a high-fidelity nonlinear VBM model that explicitly accounts for body nonlinearity. The ROM was

employed as a low-fidelity model and calibrated using a limited number of high-fidelity simulation results obtained for design wave episodes identified by FORM, thereby accelerating the STP process. The results demonstrated that the proposed multi-fidelity framework can efficiently leverage high-fidelity information through only a limited number of high-fidelity simulations, enabling the determination of optimized weight distribution parameters at a fraction of the computational cost required by direct high-fidelity analyses.

The present single-loop approach, integrating BAL-LTP, is highly versatile and applicable to a wide range of *long-term risk-informed* design optimization problems where stochastic excitations govern the inherent randomness of the response of interest. The present study focused on simplified mathematical examples and a weight-distribution optimization problem incorporating a high-fidelity nonlinear VBM model, but the proposed framework is sufficiently general to accommodate more realistic ship design optimization problems. In particular, structural scantlings, hull-form parameters, and other engineering design variables can, in principle, be incorporated within the same long-term risk-informed optimization framework, provided that the relevant design variables and constraints can be appropriately parameterized. For instance, one promising application is structural optimization that accounts for the ultimate strength of the hull girder, which may be attained by integrating a progressive collapse analysis [48]. Furthermore, by stochastically modeling wind loads, the scope of the present methodology could be extended to offshore structures, such as floating offshore wind turbines, where the coupling between wind and wave excitations is of importance. Regarding the STP process, there remains significant potential for further refinement; for instance, replacing FORM with more sophisticated reliability methods in conjunction with effective dimensionality reduction techniques. In addition, extending the proposed multi-fidelity framework to highly nonlinear VBM problems involving whipping responses remains an important topic for future research.

#### CRediT authorship contribution statement

**Tomoki Takami:** Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Masaru Kitahara:** Writing – review & editing, Validation, Software, Methodology, Investigation, Conceptualization. **Atsuo Maki:** Writing – review & editing, Validation, Software, Formal analysis. **Ulrik Dam Nielsen:** Writing – review & editing, Validation, Methodology, Conceptualization. **Jørgen Juncher Jensen:** Writing – review & editing, Validation, Methodology, Conceptualization.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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#### Data availability

Data will be made available on request.

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