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Induced Scattering of Strong Waves in Pair Plasmas

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We study induced (stimulated) scattering of linearly polarized, strong electromagnetic waves in pair plasmas, which is crucial for understanding the propagation of fast radio bursts (FRBs). Magnetars are the most promising progenitors of FRBs, and FRBs propagate through the magnetar wind and successfully escape before being significantly scattered. We revisit the steady-state solution of linearly polarized electromagnetic waves in pair plasmas with arbitrary amplitude, and demonstrate that the nonlinearity is characterized by the nonlinearity parameter $a_0\omega_{pe}/\omega_0$ rather than the dimensionless amplitude a_0 , where ω_{pe} is the electron plasma frequency and ω_0 is the wave frequency in the frame in which the plasma is at rest when the wave fields vanish. We follow the time evolution of the steady-state solution for the linear regime $a_0\omega_{pe}/\omega_0 \ll 1$ by performing one-dimensional particle-in-cell simulations, and show that the conventional linear analysis of induced scattering assuming $a_0 \ll 1$ is applicable even for $a_0 > 1$ when the Lorentz boost due to the plasma motion in the incident wave is considered. The saturation level is controlled by $a_0\omega_0/\omega_{pe}$, which corresponds to the ratio of the wave energy to the plasma energy, and the incident wave is hardly scattered for $a_0\omega_0/\omega_{pe} \gg 1$. We discuss the application of our results to FRBs.

I. INTRODUCTION

Strong electromagnetic waves are ubiquitous in the universe, with fast radio bursts (FRBs) being the most prominent example. FRBs are millisecond-long bright flashes of radio waves, mostly from extragalactic distances [1–3]. Some FRBs are known to burst repeatedly, and such repeating FRBs often show the high degree of linear polarization [4–6]. Magnetars are the most likely progenitors of repeating FRBs [7–11]. The mechanism by which FRBs are generated in magnetars remains a topic of debate, and numerous theoretical models have been proposed, including coherent curvature radiation in the magnetosphere [12–17], an expanding fireball [18, 19], fast magnetosonic waves via reconnection [20–22], and relativistic magnetized shocks in pair (electron-positron) plasmas [23–29] or electron-ion plasmas [30–33], among others. In all of these models, FRBs must propagate through plasmas surrounding their sources and successfully escape. The radio waves are strong in the sense that the normalized wave electric field, called the strength parameter, exceeds unity,

$$a_0 = \frac{eE_0}{m_e c \omega_0} > 1 \quad (1)$$

for distances from the source $R \lesssim 10^{13} \text{cm}$ [34], where e and m_e are the electron charge and mass, E_0 is the amplitude of the wave electric field, and ω_0 is the wave frequency. Such strong waves inevitably suffer from induced (or stimulated) scattering¹ [35, 36], which could

hinder their propagation and constrain the emission region [37–45]. On the other hand, the effect of induced scattering on FRB propagation remains controversial, especially for $a_0 > 1$ [46–48].

One of the main challenges in studying induced scattering in FRBs lies in the analytical intractability of the self-consistent equations for linearly polarized electromagnetic plane waves with arbitrary amplitude, even in unmagnetized plasmas [49–54]. Although a similar problem has been addressed in the context of pulsars [55–57], it has received little attention in FRBs. Previous analyses of linearly polarized electromagnetic waves in pair plasmas [58, 59] were limited to the regime $a_0 \ll 1$, where relativistic effects are negligible and the steady-state solution can be expressed solely in terms of elementary functions. Recently, Ref. [60] studied the stability of wave packets and demonstrated that nonlinear effects remain negligible when the nonlinearity parameter is sufficiently smaller than unity,

$$a_0 \frac{\omega_{pe}}{\omega_0} \ll 1, \quad (2)$$

where ω_{pe} is the plasma frequency and ω_0 is the wave frequency in the frame in which the plasma is at rest when the wave fields vanish. This behavior arises because, in this regime, the plasma current can be approximated as a linear function of the vector potential. Since the FRB frequency is sufficiently high $\omega_0 \gg \omega_{pe}$ [61], the condition $a_0\omega_{pe}/\omega_0 \ll 1$ can be satisfied and the linear treatment can remain valid for $a_0 > 1$. This result implies that the previous studies on induced scattering can be extrapolated to the regime $a_0 > 1$ when $a_0\omega_{pe}/\omega_0 \ll 1$.

In this paper, we revisit the self-consistent equations of linearly polarized electromagnetic waves with arbitrary amplitude and study the steady-state solution. We also perform particle-in-cell (PIC) simulation and follow the time evolution of the steady-state solution for

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¹ Induced scattering and stimulated scattering are essentially the same process and are terms often used interchangeably.

$a_0\omega_{pe}/\omega_0 \ll 1$. This paper is organized as follows. In Section II, we derive the self-consistent equations following the previous studies and investigate the parameter dependence of the solution. Section III describes our simulation results. We compare them with the linear analysis of induced scattering and discuss the saturation. In Section IV, we apply our results to FRBs. We finally summarize our results in Section V.

II. ANALYTICAL FORMULATION

A. Basic Equations

We derive the self-consistent equations for linearly polarized electromagnetic waves with arbitrary amplitude following previous works [49–54, 56, 57]. Throughout this paper, unless explicitly specified, we use the frame in which the plasma would be at rest in the absence of the incident electromagnetic wave and refer to this frame as the laboratory frame. Basic equations are the relativistic, cold two-fluid equations and Maxwell equations in the laboratory frame,

$$\frac{\partial}{\partial t}(\gamma_{\pm}n_{\pm}) + \nabla \cdot (\gamma_{\pm}n_{\pm}\mathbf{v}_{\pm}) = 0, \quad (3)$$

$$\frac{\partial \mathbf{u}_{\pm}}{\partial t} + (\mathbf{v}_{\pm} \cdot \nabla)\mathbf{u}_{\pm} = \pm \frac{e}{m_e c} \left[\mathbf{E} + \frac{\mathbf{v}_{\pm}}{c} \times \mathbf{B} \right], \quad (4)$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad (5)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (6)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (7)$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (8)$$

$$\rho = e(\gamma_+n_+ - \gamma_-n_-), \quad (9)$$

$$\mathbf{j} = e(\gamma_+n_+\mathbf{v}_+ - \gamma_-n_-\mathbf{v}_-), \quad (10)$$

where the plus (minus) index denotes positron (electron), $\mathbf{v}_{\pm}$ is the particle three velocity, $\mathbf{u}_{\pm} = \gamma_{\pm}\mathbf{v}_{\pm}/c$ is the four velocity normalized by the speed of light c , and $\gamma_{\pm} = 1/\sqrt{1 - |\mathbf{v}_{\pm}|^2/c^2} = \sqrt{1 + |\mathbf{u}_{\pm}|^2}$ is the particle Lorentz factor. Unless otherwise stated, all unprimed fluid and field quantities are evaluated in the laboratory frame, except that $n_{\pm}$ denotes the proper density measured in the local plasma rest frame. The density measured in the laboratory frame is therefore $\gamma_{\pm}n_{\pm}$. We consider a monochromatic plane electromagnetic wave propagating in the x direction with a superluminal phase velocity, linearly polarized in the y direction. We assume that all physical quantities can be expressed as a function of the phase,

$$\phi = \omega_0 t - k_0 x, \quad (11)$$

where k_0 and ω_0 denote the wavenumber and frequency of the incident electromagnetic wave in the laboratory frame, respectively. We consider a superluminal wave

satisfying $\omega_0/k_0 > c$. The basic equations are then written as

$$\frac{d}{d\phi}[(\gamma_{\pm} - \beta_g u_{x\pm})n_{\pm}] = 0, \quad (12)$$

$$(\gamma_{\pm} - \beta_g u_{x\pm}) \frac{du_{x\pm}}{d\phi} = \pm \frac{eB_z u_{y\pm}}{m_e c \omega_0}, \quad (13)$$

$$(\gamma_{\pm} - \beta_g u_{x\pm}) \frac{du_{y\pm}}{d\phi} = \pm \left(\frac{eE_y \gamma_{\pm}}{m_e c \omega_0} - \frac{eB_z u_{x\pm}}{m_e c \omega_0} \right), \quad (14)$$

$$\frac{dB_z}{d\phi} = \beta_g \frac{dE_y}{d\phi}, \quad (15)$$

$$\frac{dE_y}{d\phi} = -\frac{4\pi e c \gamma_g^2 (n_+ u_{y+} - n_- u_{y-})}{\omega_0}, \quad (16)$$

where

$$\beta_g = \frac{ck_0}{\omega_0}, \quad (17)$$

$$\gamma_g = \frac{1}{\sqrt{1 - \beta_g^2}} = \frac{\omega_0}{\sqrt{\omega_0^2 - c^2 k_0^2}}. \quad (18)$$

The parameter $c\beta_g$ corresponds to the velocity of a reference frame moving relative to the laboratory frame, in which the spatial dependence of both particle and field variables vanishes [53]. This velocity can be interpreted as the group velocity of the wave [57]. It is convenient to introduce the normalized electric field $y = E_y/E_0$, where the wave electric field E_y is assumed to take the maximum value E_0 at $\phi = 0$, i.e. $y = 1$ at $\phi = 0$. Since we can set $\gamma \equiv \gamma_+ = \gamma_-$, $u_x \equiv u_{x+} = u_{x-}$, $u_y \equiv u_{y+} = -u_{y-}$, and $n \equiv n_+ = n_-$ [54], γ , u_x , u_y , and n are expressed in terms of y ,

$$\gamma = 1 + \frac{\alpha a_0^2}{2}(1 - y^2), \quad (19)$$

$$u_x = \frac{\alpha \beta_g a_0^2}{2}(1 - y^2), \quad (20)$$

$$u_y = a_0 \int_0^{\phi} y d\phi, \quad (21)$$

$$n = n_0 \left[1 + \frac{2}{q}(1 - y^2) \right]^{-1}, \quad (22)$$

where

$$\alpha = \frac{\omega_0^2}{2\gamma_g^2 \omega_{pe}^2} = \frac{\omega_0^2 - c^2 k_0^2}{2\omega_{pe}^2}, \quad (23)$$

$$q = \frac{4\gamma_g^2}{\alpha a_0^2}. \quad (24)$$

The parameter a_0 is the strength parameter defined above and is Lorentz invariant. The value of γ at $\phi = 0$ can be chosen arbitrarily [53]. Here, we choose $\gamma = 1$ at $\phi = 0$, so that the proper density at this phase is equal to the reference density n_0 of the plasma in the absence of the electromagnetic wave. The reference electron plasma

frequency is defined from this density as

$$\omega_{pe} = \sqrt{\frac{4\pi n_0 e^2}{m_e}}. \quad (25)$$

Throughout this paper, ω_{pe} denotes this reference plasma frequency and is not redefined in Lorentz-transformed frames. This definition of n_0 and ω_{pe} is consistent with that used in Ref. [60].

Eq. 21 represents the conservation law of canonical momentum. The parameters α and β_g originate from the plasma dispersion, and Eqs. 19, 20, and 21 describe the motion of test particles in transverse electromagnetic waves when these factors are neglected [62]. The normalized electric field y is determined from the differential equation with the boundary condition $y = 1$ at $\phi = 0$ ²,

$$\frac{\alpha^2 a_0^2}{\gamma_g^2} \left(\frac{dy}{d\phi} \right)^2 = \frac{(1-y^2)(1-y^2+q)}{(1-y^2+q/2)^2}. \quad (26)$$

The dispersion relation follows from the fact that the phase shifts by $\pi/2$ after a quarter-cycle,

$$\int_0^1 \frac{1}{|dy/d\phi|} dy = \frac{\pi}{2}. \quad (27)$$

By substituting Eq. 26 into this dispersion relation, one can find

$$\frac{\alpha a_0}{\gamma_g} \frac{2E(m) - (1-m)K(m)}{2\sqrt{m}} = \frac{\pi}{2}, \quad (28)$$

where $m = 1/(1+q)$ and $0 < m < 1$. $K(m)$ and $E(m)$ are the complete elliptic integral of the first and second kind with modules m ,

$$\begin{aligned} K(m) &= \int_0^1 \frac{1}{\sqrt{(1-y^2)(1-my^2)}} dy \\ &= \int_0^{\pi/2} \frac{1}{\sqrt{1-m\sin^2\theta}} d\theta, \end{aligned} \quad (29)$$

$$\begin{aligned} E(m) &= \int_0^1 \sqrt{\frac{1-my^2}{1-y^2}} dy \\ &= \int_0^{\pi/2} \sqrt{1-m\sin^2\theta} d\theta. \end{aligned} \quad (30)$$

Our final set of equations consists of Eqs. 19, 20, 21, 22, 26, and 28 for a given normalized amplitude a_0 and frequency ω_0/ω_{pe} .

B. Steady-State Solution

We demonstrate that the steady-state solution of linearly polarized electromagnetic waves is controlled by the nonlinearity parameter $a_0\omega_{pe}/\omega_0$ and that the linear approximation holds for $a_0\omega_{pe}/\omega_0 \ll 1$. Here, ω_0 is measured in the laboratory frame, while ω_{pe} is the reference plasma frequency defined from n_0 . Therefore, $a_0\omega_{pe}/\omega_0$ is not Lorentz invariant and it should be understood as the nonlinearity parameter defined in the laboratory frame.

Considering Eqs. 23 and 24, the coefficient $\alpha a_0/\gamma_g$ is expressed as

$$\frac{\alpha a_0}{\gamma_g} = \left(\frac{32}{q^3} \right)^{\frac{1}{4}} \left(a_0 \frac{\omega_{pe}}{\omega_0} \right)^{-\frac{1}{2}}. \quad (31)$$

By substituting this into Eq. 28 and using $q = (1-m)/m$, we obtain

$$\frac{m[2E(m) - (1-m)K(m)]^4}{(1-m)^3} = \frac{1}{2} \left(\frac{\pi}{2} \right)^4 \left(a_0 \frac{\omega_{pe}}{\omega_0} \right)^2, \quad (32)$$

indicating that m (and thus q) can be expressed as a function of the nonlinearity parameter $a_0\omega_{pe}/\omega_0$. For $m \ll 1$, $K(m)$ and $E(m)$ can be expanded as

$$K(m) \simeq \int_0^{\pi/2} \left(1 + \frac{1}{2} m \sin^2 \theta \right) d\theta = \frac{\pi}{2} \left(1 + \frac{m}{4} \right), \quad (33)$$

$$E(m) \simeq \int_0^{\pi/2} \left(1 - \frac{1}{2} m \sin^2 \theta \right) d\theta = \frac{\pi}{2} \left(1 - \frac{m}{4} \right). \quad (34)$$

By substituting these into Eq. 32 and keeping the lowest-order of m , we obtain

$$m \simeq \frac{1}{2} \left(a_0 \frac{\omega_{pe}}{\omega_0} \right)^2. \quad (35)$$

The validity condition $m \ll 1$ now becomes

$$a_0 \frac{\omega_{pe}}{\omega_0} \ll 1. \quad (36)$$

On the other hand, for $1-m \ll 1$, $K(m)$ and $E(m)$ can be expanded as [63],

$$K(m) \simeq \ln 4 - \frac{1}{2} \ln(1-m), \quad (37)$$

$$E(m) \simeq 1 + \frac{\ln 4 - 1}{2} (1-m) - \frac{1}{4} (1-m) \ln(1-m). \quad (38)$$

Keeping the lowest-order of $1-m$, we obtain

$$1-m \simeq \frac{8}{\pi^{\frac{4}{3}}} \left(a_0 \frac{\omega_{pe}}{\omega_0} \right)^{-\frac{2}{3}}. \quad (39)$$

The validity condition $1-m \ll 1$ can be rewritten as

$$a_0 \frac{\omega_{pe}}{\omega_0} \gg 1. \quad (40)$$

² Eq. 26 is equivalent to Eq. 83 of Ref. [53], but is written with a different dependent variable and frame convention. Ref. [53] writes the equation in the frame where the spatial dependence vanishes, and uses the transverse four-velocity component η (u_y in our notation). Here, we use the normalized electric field $y = E_y/E_0$, which is related to u_y by Eq. 21.

One can find the wave electric field y after determining the solution of Eq. 32. By substituting Eq. 31 into Eq. 26, we obtain

$$\left(\frac{dy}{d\phi}\right)^2 = a_0 \frac{\omega_{pe}}{\omega_0} \frac{q^{\frac{3}{2}}}{4\sqrt{2}} \frac{(1-y^2)(1-y^2+q)}{(1-y^2+q/2)^2}. \quad (41)$$

This demonstrates that y is well-characterized by the nonlinearity parameter $a_0\omega_{pe}/\omega_0$. For $a_0\omega_{pe}/\omega_0 \ll 1$ (i.e. $q = (1-m)/m \gg 1$), we consider the zeroth order of $a_0\omega_{pe}/\omega_0$ and Eq. 41 can be written as

$$\frac{dy}{d\phi} = \pm\sqrt{1-y^2}. \quad (42)$$

Note that $y^2 \leq 1$ is satisfied by definition. This derivative equation is easily solved for the boundary condition $y = 1$ at $\phi = 0$,

$$y = \cos \phi. \quad (43)$$

For $a_0\omega_{pe}/\omega_0 \gg 1$ (i.e. $q = (1-m)/m \ll 1$), the zeroth-order equation is expressed as

$$\frac{dy}{d\phi} = \pm \frac{2}{\pi}. \quad (44)$$

The periodic solution is given by

$$y = \begin{cases} -\frac{2}{\pi}\phi + 4n + 1 & \text{if } 2n\pi \leq \phi \leq (2n+1)\pi, \\ \frac{2}{\pi}\phi - 4n - 3 & \text{if } (2n+1)\pi \leq \phi \leq 2(n+1)\pi, \end{cases} \quad (45)$$

where $n = 0, 1, 2, \dots$ is an integer. We have numerically determined m from Eq. 32 and solved Eq. 41. Fig. 1 shows the numerical solutions for y at various values of $a_0\omega_{pe}/\omega_0$: 10^{-1} (yellow), 1 (green), 10 (blue), 10^2 (magenta), and 10^3 (red). The steady-state solution asymptotically approaches the linear one $y = \cos \phi$ for $a_0\omega_{pe}/\omega_0 \ll 1$. For $a_0\omega_{pe}/\omega_0 \gg 1$, the wave electric field has a sawtooth-like profile, which is consistent with previous studies [51, 52]

The parameter α can be determined from the solution of Eq. 32 as well. Considering $a_0^2/\gamma_g^2 = 4/q\alpha$ and $q = (1-m)/m$, Eq. 28 is rewritten as

$$\alpha = \left(\frac{\pi}{2}\right)^2 \frac{1-m}{[2E(m) - (1-m)K(m)]^2}. \quad (46)$$

This demonstrates that α also depends solely on the nonlinearity parameter $a_0\omega_{pe}/\omega_0$. We retain the lowest-order of m when $a_0\omega_{pe}/\omega_0 \ll 1$ (i.e. $m \ll 1$),

$$\alpha \simeq 1 - \frac{3}{2}m \simeq 1 - \frac{3}{4} \left(a_0 \frac{\omega_{pe}}{\omega_0}\right)^2, \quad (47)$$

and $1-m$ when $a_0\omega_{pe}/\omega_0 \gg 1$ (i.e. $1-m \ll 1$),

$$\alpha \simeq \frac{1}{4} \left(\frac{\pi}{2}\right)^2 (1-m) \simeq \frac{\pi^{\frac{2}{3}}}{2} \left(a_0 \frac{\omega_{pe}}{\omega_0}\right)^{-\frac{2}{3}}. \quad (48)$$

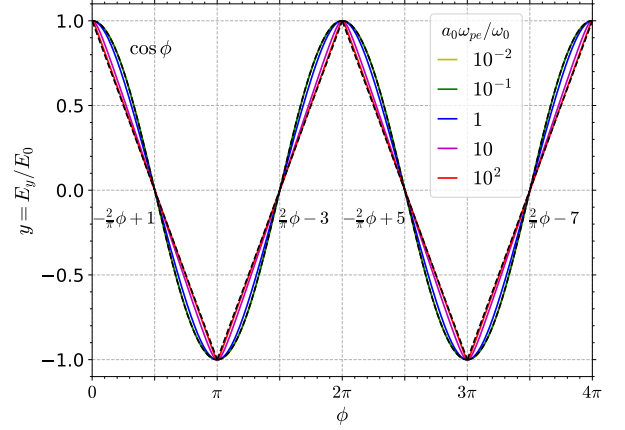


FIG. 1. The wave electric field $y = E_y/E_0$ as a function of the phase ϕ . The color indicates the numerical solutions for different values of the nonlinearity parameter $a_0\omega_{pe}/\omega_0$. The black dashed lines correspond to the analytical asymptotic solutions for $a_0\omega_{pe}/\omega_0 \ll 1$ and $a_0\omega_{pe}/\omega_0 \gg 1$.

Fig. 2 shows the numerical solution α as a function of $a_0\omega_{pe}/\omega_0$ in the red solid line and the asymptotic solutions in the dashed black lines. In the limit $a_0\omega_{pe}/\omega_0 \ll 1$, the parameter α asymptotically approaches unity $\alpha = 1$, thereby recovering the linear dispersion relation $\omega_0^2 = 2\omega_{pe}^2 + c^2k_0^2$.

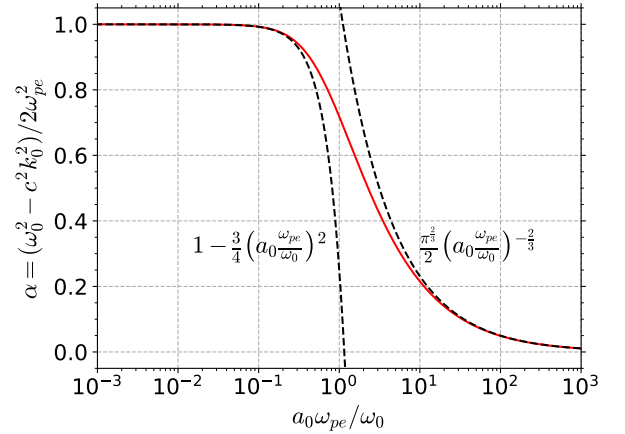


FIG. 2. The parameter $\alpha = (\omega_0^2 - c^2k_0^2)/2\omega_{pe}^2$ as a function of the nonlinearity parameter $a_0\omega_{pe}/\omega_0$ (red solid line). The black dashed lines correspond to the asymptotic solutions for $a_0\omega_{pe}/\omega_0 \ll 1$ and $a_0\omega_{pe}/\omega_0 \gg 1$.

The Lorentz factor γ_g can be determined from Eq. 23 after the dispersion relation α is obtained. One can find

$$\gamma_g \simeq \frac{\omega_0}{\sqrt{2}\omega_{pe}} \left[1 + \frac{3}{8} \left(a_0 \frac{\omega_{pe}}{\omega_0}\right)^2 \right], \quad (49)$$

for $a_0\omega_{pe}/\omega_0 \ll 1$, and

$$\gamma_g \simeq \frac{\omega_0}{\pi^{\frac{1}{3}}\omega_{pe}} \left(a_0 \frac{\omega_{pe}}{\omega_0} \right)^{\frac{1}{3}}, \quad (50)$$

for $a_0\omega_{pe}/\omega_0 \gg 1$. When $a_0\omega_{pe}/\omega_0 \ll 1$, the Lorentz factor γ_g is approximately equal to $\omega_0/\sqrt{2}\omega_{pe}$ at the lowest order. This is the same as the group velocity Lorentz factor of the wave packet in Ref. [60], indicating that our treatment is consistent with the wave packet analysis in the linear regime.

In the vacuum limit, $\omega_{pe}/\omega_0 \rightarrow 0$, which corresponds to $a_0\omega_{pe}/\omega_0 \rightarrow 0$ for fixed a_0 , the plasma feedback on the electromagnetic wave is absent, and the waveform is prescribed as $y = \cos\phi$. In this limit, $\alpha = 1$ and $\beta_g = 1$, and Eqs. 19–21 reduce to

$$\gamma = 1 + \frac{a_0^2}{2} \sin^2 \phi, \quad (51)$$

$$u_x = \frac{a_0^2}{2} \sin^2 \phi, \quad (52)$$

$$u_y = a_0 \sin \phi. \quad (53)$$

For $a_0 \ll 1$, these equations further give the usual non-relativistic quiver orbit.

The nonlinear feedback of the plasma on the electromagnetic wave is mediated by the plasma current,

$$j_y = 2encu_y. \quad (54)$$

For $a_0\omega_{pe}/\omega_0 \ll 1$ (i.e., $q \gg 1$), Eq. 22 gives $n \simeq n_0$ at leading order. The conservation of canonical momentum (Eq. 21) then implies $u_y = eA_y/m_e c$, where A_y is the vector potential, so that the current reduces to

$$j_y \simeq 2en_0cu_y \propto A_y. \quad (55)$$

Hence, the source term in Maxwell's equation remains linear in the wave amplitude, and the plasma response reduces to the test-particle limit. In particular, no additional amplitude-dependent coupling is generated between the wave and the plasma, so the waveform does not undergo nonlinear distortion. Nonlinear feedback is therefore negligible for $a_0\omega_{pe}/\omega_0 \ll 1$. We thus conclude that the steady-state solution of linearly polarized electromagnetic waves is controlled by the nonlinearity parameter $a_0\omega_{pe}/\omega_0$, and that the linear treatment remains valid in this regime, consistent with Ref. [60].

C. Induced Scattering

Induced scattering can be understood as a parametric instability, which has been studied through the stability analysis of plasma waves [64–68]. Linearly polarized electromagnetic waves traveling through unmagnetized pair plasmas are subject to stimulated Brillouin scatter-

ing (SBS)³. SBS is often referred to as induced Compton scattering when kinetic effects are important, as is always the case for unmagnetized pair plasmas [69]. Previous studies [58, 59] evaluated the linear growth rate of SBS $\Gamma = \text{Im } \omega_1$ and the wavenumber of the scattered wave k_1 under the assumption that the incident wave is weak $a_0 \ll 1$ and the plasma temperature is non-relativistic. We assume that the SBS operates in a frame where the averaged longitudinal moment vanishes, and that the linear analysis of SBS remains valid in this center-of-momentum frame. The drift velocity v_D of the center-of-momentum frame relative to the laboratory frame is given by [70]

$$v_D = \frac{c\langle u_x \rangle}{\langle \gamma \rangle} = \frac{c\beta_g \alpha a_0^2 (1 - \langle y^2 \rangle)/2}{1 + \alpha a_0^2 (1 - \langle y^2 \rangle)/2}, \quad (56)$$

with Eqs. 19 and 20. This drift velocity depends not only on the nonlinearity parameter $a_0\omega_{pe}/\omega_0$ but also on a_0 itself. Here $\langle \dots \rangle$ denotes an average taken over a time interval much longer than the wave period but much shorter than the SBS growth timescale. Note that this drift velocity is different from the averaged three velocity $c\langle u_x/\gamma \rangle$. One can find $\langle y^2 \rangle \simeq 1/2$ for $a_0\omega_{pe}/\omega_0 \ll 1$ and $\langle y^2 \rangle \simeq 1/3$ for $a_0\omega_{pe}/\omega_0 \gg 1$ with Eqs. 43 and 45, respectively. In the center-of-momentum frame, we assume that the maximum growth rate Γ'_{max} and the corresponding wavenumber $k'_{1,\text{max}}$ can be derived from the linear theory [58, 59],

$$\frac{\Gamma'_{\text{max}}}{\omega'_0} = \sqrt{\frac{\pi}{32e}} \left(a_0 \frac{\omega_{pe}}{\omega'_0} \right)^2 \frac{1}{\beta_{th0}^2}, \quad (57)$$

$$\frac{k'_{1,\text{max}}}{k'_0} = -(1 - 2\beta_{th0}), \quad (58)$$

for the weak coupling regime $\beta_{th0} \gg (a_0\omega_{pe}/\omega'_0)^{2/3}$. Here the primed quantities are defined in the center-of-momentum frame and $\beta_{th0} = \sqrt{k_B T_0/m_e c^2} \ll 1$ is the initial thermal velocity defined by the proper temperature T_0 . Note that a_0 is Lorentz invariant and ω_{pe} is defined by the reference density n_0 . The negative wavenumber indicates the backward scattering. For $\beta_{th0} \ll (a_0\omega_{pe}/\omega'_0)^{2/3}$, which is the strong coupling regime [65], they are given by

$$\frac{\Gamma'_{\text{max}}}{\omega'_0} = \frac{\sqrt{3}}{2} \left(a_0 \frac{\omega_{pe}}{\omega'_0} \right)^{\frac{2}{3}}, \quad (59)$$

$$\frac{k'_{1,\text{max}}}{k'_0} = -1, \quad (60)$$

The strong-coupling scaling $(a_0\omega_{pe}/\omega'_0)^{2/3}$ is closely connected to the scaling recently discussed for pair plasmas

³ We refer to this process as “stimulated” Brillouin scattering rather than “induced” Brillouin scattering, as the former term is more widely used in the literature.

by Ref. [71]. These results indicate that the SBS is well characterized by the nonlinearity parameter in the center-of-momentum frame. We also assume that the linear theory can be extrapolated to the regime $a_0 > 1$ as long as the nonlinearity parameter is sufficiently small, $a_0\omega_{pe}/\omega_0 \ll 1$. Since the incident wave strongly drives the plasma in the $+x$ direction for $a_0 > 1$ and the Lorentz boost effect due to v_D is not negligible (see Eq. 56), the maximum growth rate $\Gamma_{\max}$ in the laboratory frame should satisfy the Lorentz transformation [72]

$$\Gamma'_{\max} = \frac{\Gamma_{\max}}{\gamma_D(1 - \beta_g\beta_D)}, \quad (61)$$

where

$$\beta_D = \frac{v_D}{c}, \quad (62)$$

$$\gamma_D = \frac{1}{\sqrt{1 - \beta_D^2}}. \quad (63)$$

Here we have assumed that the back-scattered wave has a phase velocity of $\omega_1/k_1 \simeq -\omega_0/k_0$ in the laboratory frame. The wavenumber of the back-scattered wave satisfies

$$k'_{1,\max} = \gamma_D \left(1 + \frac{\beta_D}{\beta_g}\right) k_{1,\max}. \quad (64)$$

The Lorentz transformation of the frequency and wavenumber of the incident wave is given by

$$\omega'_0 = \gamma_D(1 - \beta_g\beta_D)\omega_0, \quad (65)$$

$$k'_0 = \gamma_D \left(1 - \frac{\beta_D}{\beta_g}\right) k_0. \quad (66)$$

We finally obtain the maximum growth rate and the wavenumber of the scattered wave in the laboratory frame as

$$\frac{\Gamma_{\max}}{\omega_0} = \sqrt{\frac{\pi}{32e}} \left(a_0 \frac{\omega_{pe}}{\omega_0}\right)^2 \frac{1}{\beta_{th0}^2}, \quad (67)$$

$$\frac{k_{1,\max}}{k_0} = -\frac{\beta_g - \beta_D}{\beta_g + \beta_D} (1 - 2\beta_{th0}), \quad (68)$$

for the weak coupling regime and

$$\frac{\Gamma_{\max}}{\omega_0} = \frac{\sqrt{3}}{2} \left(a_0 \frac{\omega_{pe}}{\omega_0}\right)^{\frac{2}{3}} [\gamma_D(1 - \beta_g\beta_D)]^{\frac{4}{3}}, \quad (69)$$

$$\frac{k_{1,\max}}{k_0} = -\frac{\beta_g - \beta_D}{\beta_g + \beta_D}, \quad (70)$$

for the strong coupling regime. When $a_0\omega_{pe}/\omega_0 \ll 1$ and $\omega_0 \simeq ck_0$ are satisfied, one can find

$$\frac{\beta_g - \beta_D}{\beta_g + \beta_D} \simeq \left(1 + \frac{a_0^2}{2}\right)^{-1}, \quad (71)$$

$$\gamma_D(1 - \beta_g\beta_D) \simeq \left(1 + \frac{a_0^2}{2}\right)^{-\frac{1}{2}}. \quad (72)$$

These equations indicate that both the growth rate and the wavenumber of the scattered wave in the simulation frame depend not only on the nonlinearity parameter $a_0\omega_{pe}/\omega_0$ but also on the incident wave amplitude a_0 through β_D and γ_D , except for the maximum growth rate in the weak coupling case (Eq. 67). In the above analysis, we neglect relativistic mass effects associated with the rapid quiver motion of the particles. This approximation is justified in the regime $a_0\omega_{pe}/\omega_0 \ll 1$, where the wave dynamics is well approximated by the linear solution and the SBS growth timescale is much longer than the incident wave period, i.e., $\Gamma_{\max} \ll \omega_0$, so that a secular description is well defined. When this condition is not satisfied, relativistic mass effects become important, which can significantly reduce the instability [36]. We perform numerical simulations to test the above analysis.

III. KINETIC SIMULATIONS

A. Simulation Setup

We use a fully kinetic PIC code, WumingPIC [73]. The simulation setup is based on previous studies [58, 59] and the physics of SBS is fully captured. We consider one-dimensional (1D) spatial domain in the x direction and the periodic boundary condition is applied for both particles and fields. All three components of fields and velocities are tracked in our simulations. The simulation frame corresponds to the laboratory frame in Section II. We initially introduce a strong wave as described below. The wave amplitude and frequency are given as $(a_0, \omega_0/\omega_{pe}) = (0.05, 5)$, $(0.1, 10)$, $(1, 100)$, $(2, 200)$, and $(4, 400)$. ω_0 is defined in the simulation frame. We focus on the linear regime $a_0\omega_{pe}/\omega_0 \ll 1$, and fix $a_0\omega_{pe}/\omega_0 = 0.01$ throughout this study. The initial wavenumber k_0 is numerically determined by the dispersion relation (Eq. 28). The wavelength of the incident wave is resolved by 200 computational cells, $\lambda_0 = 200\Delta x$. The time step is set as $\Delta t = \Delta x/c$. Our code employs an implicit Maxwell solver, which is not constrained by the CFL condition. The size of simulation domain is set as $L_x = 100\lambda_0$. The initial spatial profiles of wave electric field $E_y = E_0 y(\phi = -k_0 x)$ and magnetic field $B_z = \beta_g E_y$ are numerically determined by the self-consistent equation, Eq. 26.

Particle motion must be consistent with the wave fields. Eqs. 19, 20, and 21 indicate that the bulk Lorentz

factor $\bar{\gamma}$ and bulk four velocity $\bar{\mathbf{u}}$ at $t = 0$ satisfy

$$\bar{\gamma} = 1 + \frac{\alpha a_0^2}{2} \{1 - [y(\phi = -k_0 x)]^2\}, \quad (73)$$

$$\bar{u}_x = \frac{\alpha \beta_g a_0^2}{2} \{1 - [y(\phi = -k_0 x)]^2\}, \quad (74)$$

$$\bar{u}_y = a_0 \int_0^{-k_0 x} y d\phi, \quad (75)$$

$$\bar{u}_z = 0. \quad (76)$$

We generate Maxwellian particles with a thermal spread β_{th0} in the plasma rest frame, and then compute the particle Lorentz factors and four velocities in the simulation frame by performing the inverse Lorentz transformation. We carry out our simulations for both strong- and weak-coupling cases: $\beta_{th0} = 0.01$ and 0.1 , respectively. As described in Section II C, increasing the thermal velocity moves the system from the strong-coupling regime to the weak-coupling regime. The weak-coupling regime is particularly sensitive to kinetic effects, including Landau damping. The growth rates and corresponding wavenumbers used in our comparison are derived from a fully kinetic treatment of SBS and therefore include finite-temperature effects. The fully kinetic PIC method also self-consistently captures these kinetic effects in the numerical results.

The Debye length in units of Δx is $\lambda_D/\Delta x = (100/\pi)\beta_{th0}(\omega_0/\omega_{pe})$, which ranges from $\simeq 1.6$ to $\simeq 1273$ for the parameters used here. Thus, the stability condition $\Delta x \lesssim \lambda_D$ is satisfied. The same numerical resolution was tested in Ref. [59], where SBS in pair plasmas was shown to be accurately captured.

The particle position in the simulation frame is determined by the laboratory density N , which is related to the incident electric field as (see Eqs. 19 and 22),

$$N = \gamma n = \frac{1 + \alpha a_0^2(1 - y^2)/2}{1 + 2(1 - y^2)/q} n_0. \quad (77)$$

Since the laboratory density depends on both a_0 and ω_0/ω_{pe} , we evaluate it for each set of parameters. Fig. 3 shows an enlarged view of the initial spatial profile of the laboratory density, normalized by the averaged density $N_0 = \langle N \rangle$. For $a_0\omega_{pe}/\omega_0 \ll 1$ (i.e., $q \gg 1$), one can find

$$N_0 \simeq \left(1 + \frac{a_0^2}{4}\right) n_0 \quad (78)$$

for the zeroth-order of $a_0\omega_{pe}/\omega_0$. We distribute particle positions accordingly to represent the density profile. To ensure adequate resolution of the density fluctuations, the averaged number of particles per species per cell is set to $N_0\Delta x = 100$.

B. Linear Stage

We first focus on the linear stage of the wave evolution and verify our assumption in Section II C. Fig. 4

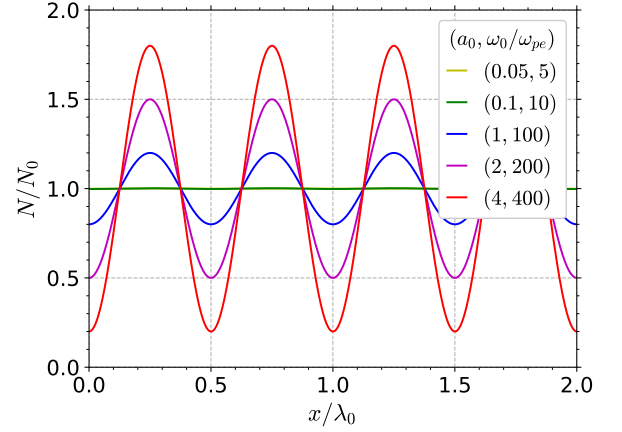


FIG. 3. Enlarged view of the initial spatial profile of laboratory density N , normalized by the average density N_0 , for various combinations of $(a_0, \omega_0/\omega_{pe})$. The case $(a_0, \omega_0/\omega_{pe}) = (0.05, 5)$ (yellow) is overlapped by $(0.1, 10)$ (green) and indistinguishable in the plot.

shows the time evolution of the power spectrum of the Poynting flux S_x normalized by its initial value S_0 for $(a_0, \omega_0/\omega_{pe}) = (0.1, 10)$ (left) and $(2, 200)$ (right). The initial thermal velocity in the proper frame is $\beta_{th0} = 0.1$ (weak coupling) in both cases. We perform a spatial Fourier transform of S_x at each snapshot and distinguish between oppositely propagating wave components ($k_x > 0$ and $k_x < 0$), following the method described in Ref. [58]. The back-scattered waves ($k_x < 0$) are generated by SBS for both cases, and the absolute wavenumber of the fastest-growing mode for $a_0 = 2$ (left) is smaller than that for $a_0 = 0.1$ (right), which is due to the Lorentz boost effect as discussed in Section II C. The theoretical prediction given by Eq. 68 (blue lines) is indeed consistent with the simulation results.

Fig. 5 shows the time evolution of the Poynting flux S_x in the simulation frame associated with the fastest-growing mode $k_{\max}$ for $\beta_{th0} = 0.1$ (left) and $\beta_{th0} = 0.01$ (right), for various combinations of $(a_0, \omega_0/\omega_{pe})$: $(0.05, 5)$ (yellow), $(0.1, 10)$ (green), $(1, 100)$ (blue), $(2, 200)$ (magenta), and $(4, 400)$ (red). The maximum growth rates for $\beta_{th0} = 0.1$ are independent of a_0 , whereas those for $\beta_{th0} = 0.01$ decrease with increasing a_0 . The black dashed lines represent exponential growth with the theoretical maximum growth rate $\propto e^{2\Gamma_{\max}t}$, where $\Gamma_{\max}$ is calculated from Eqs. 67 and 69. The linear growth rates are well reproduced by the theoretical predictions for both β_{th0} . We also checked the density perturbation as a consistency check. The density spectrum shows power around the wavenumber expected from the matching condition, $k_{\text{SBS}} = k_0 - k_{1,\max}$. On the other hand, the saturation levels vary significantly depending on the combination of $(a_0, \omega_0/\omega_{pe})$. The saturation behavior will be discussed in detail in the next section.

Figure 6 shows the maximum growth rate (top) and the corresponding wavenumber (bottom) of the scattered

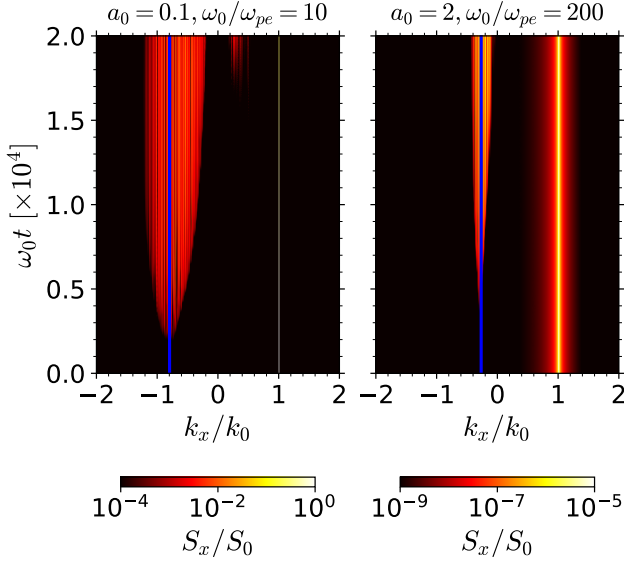


FIG. 4. Time evolution of the power spectrum of the Poynting flux for $(a_0, \omega_0/\omega_{pe}) = (0.1, 10)$ (left) and $(2, 200)$ (right). The initial thermal velocity in the plasma rest frame is $\beta_{th0} = 0.1$ in both cases. The blue lines represent the theoretical fastest-growing modes (Eq. 68).

wave as a function of a_0 for $\beta_{th0} = 0.1$ (red circles) and $\beta_{th0} = 0.01$ (blue circles). In the weak coupling regime ($\beta_{th0} = 0.1$), the maximum growth rates are independent of a_0 , whereas they decrease with increasing a_0 in the strong coupling regime ($\beta_{th0} = 0.01$). In both cases, the wavenumbers of the backscattered wave approach zero as a_0 increases. These behaviors are well explained by the theoretical predictions from Eqs. 67 and 68 for the weak coupling regime ($\beta_{th0} = 0.1$), and Eqs. 69 and 70 for the strong coupling regime ($\beta_{th0} = 0.01$). The theoretical predictions are represented by dashed lines in the corresponding colors, and they are in good agreement with the simulation results. The agreement between the simulation results and theoretical predictions confirms that the a_0 dependence of the growth rate and wavenumber is due to the Lorentz boost effect. This result indicates that the linear analysis of SBS is extrapolated to the regime $a_0 > 1$ as long as the nonlinearity parameter is sufficiently small, $a_0\omega_{pe}/\omega_0 \ll 1$.

A recent related study of strong electromagnetic waves in unmagnetized pair plasmas found a scattered-wavenumber scaling consistent with our result, while the growth-rate behavior is treated in a different setup [74]. In particular, the related study considers wave packets rather than plane waves, and does not address the weak-coupling regime considered here. Developing a comprehensive theory that connects these complementary cases is an important subject for future work.

C. Nonlinear Stage

We now discuss the nonlinear stage of SBS. The saturation levels of the fastest-growing modes (Fig. 5) tend to decrease with increasing a_0 while keeping the nonlinearity parameter $a_0\omega_{pe}/\omega_0$ constant for both β_{th0} , indicating that the incident waves are less affected by SBS for larger a_0 . Fig. 7 shows the time evolution of the incident Poynting flux $S_x(k_0)$ (solid lines) and scattered Poynting flux δS_x (dashed lines) for $\beta_{th0} = 0.1$ (left) and $\beta_{th0} = 0.01$ (right). The scattered Poynting flux δS_x is defined as the sum of the Poynting fluxes over all scattered modes,

$$\delta S_x = \sum_{k_x \neq k_0} S_x(k_x). \quad (79)$$

The color represents the results for various combinations of $(a_0, \omega_0/\omega_{pe})$, following the same convention as in Fig. 5. For $(a_0, \omega_0/\omega_{pe}) = (0.05, 5)$ (yellow) and $(0.1, 10)$ (green), a significant fraction of the incident Poynting flux is dissipated, and the scattered Poynting flux becomes comparable to the incident flux by the end of the simulation for both values of β_{th0} . As a_0 increases, the incident Poynting flux is less affected by SBS, and the scattered Poynting flux decreases. This behavior can be explained as follows.

The incident wave energy is dissipated via Landau damping of the acoustic-like modes and subsequently converted into plasma kinetic energy. When the incident wave energy is sufficiently large, it requires a considerable amount of time to transfer a significant fraction of this energy to the plasma. The ratio of the incident wave energy density to the electron rest mass energy density is given by

$$\frac{E_0^2}{4\pi n_0 m_e c^2} = \left(a_0 \frac{\omega_0}{\omega_{pe}} \right)^2. \quad (80)$$

For the cases $(a_0, \omega_0/\omega_{pe}) = (0.05, 5)$ (yellow) and $(0.1, 10)$ (green), this ratio is smaller than or comparable to unity. Consequently, the incident wave energy is dissipated relatively quickly, and SBS effects become significant within the simulation timescale. In contrast, for $(a_0, \omega_0/\omega_{pe}) = (1, 100)$ (blue), $(2, 200)$ (red), and $(4, 400)$ (magenta), this ratio is much larger than unity, meaning that the incident wave energy is hardly converted into plasma energy during the simulation. This argument qualitatively explains the dependence of the saturation levels; thus, the energy ratio $a_0\omega_0/\omega_{pe}$ likely governs the saturation behavior of SBS. Note a different combination of parameters from the nonlinearity parameter $a_0\omega_{pe}/\omega_0$.

Although the incident wave is less affected by SBS at larger ratio $a_0\omega_0/\omega_{pe}$, the plasma is more significantly modified in this regime. This occurs because, when wave energy dominates ($a_0\omega_0/\omega_{pe} \gg 1$), even the dissipation of a small fraction of the incident energy is sufficient to perturb the velocity distribution substantially. Figure 8 shows the time evolution of the longitudinal four-velocity u_x distribution in the simulation frame for

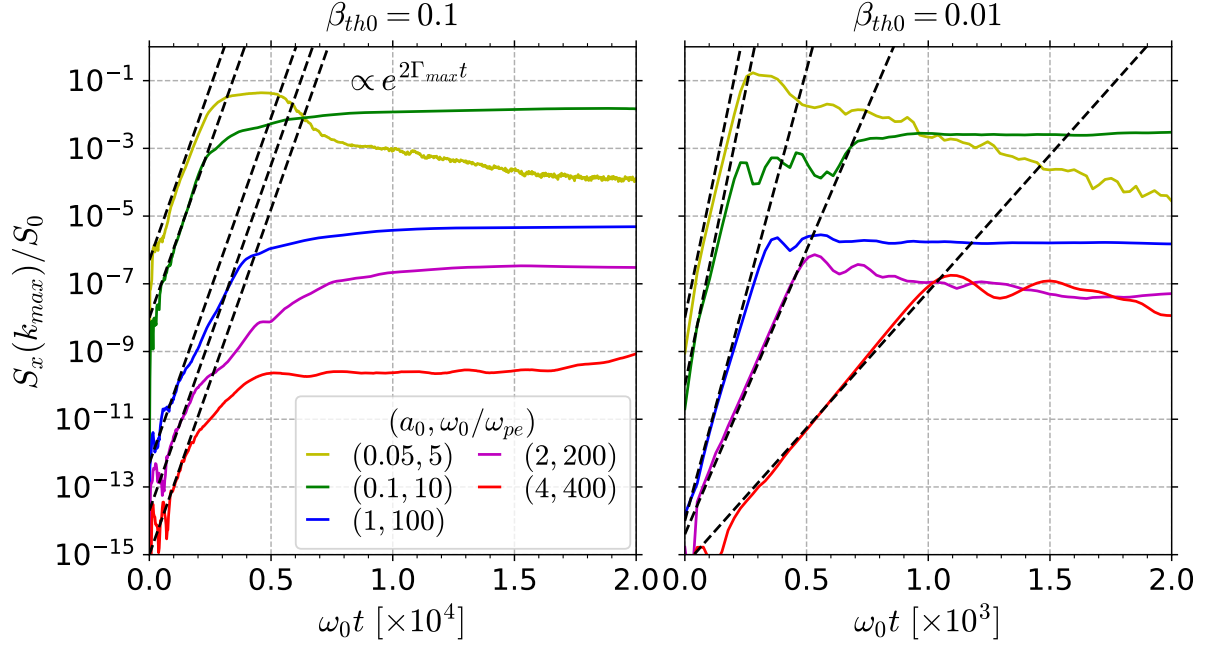


FIG. 5. Time evolution of the Poynting flux $S_x(k_{\max})$ associated with the fastest-growing mode for $\beta_{th0} = 0.1$ (left) and $\beta_{th0} = 0.01$ (right) for various combinations of $(a_0, \omega_0/\omega_{pe})$. The black dashed lines represent $\propto e^{2\Gamma_{\max}t}$, where $\Gamma_{\max}$ is the theoretical maximum growth rate in the simulation frame (Eqs. 67 and 69).

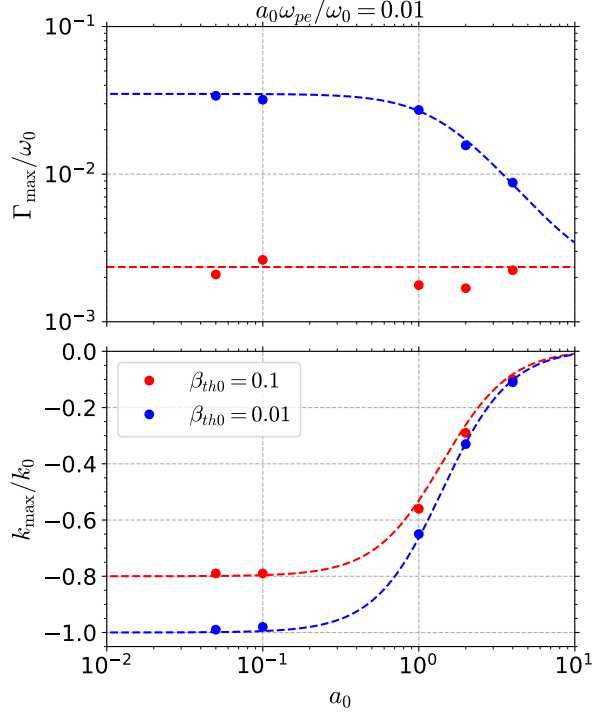


FIG. 6. Maximum growth rate (top) and corresponding wavenumber (bottom) of the scattered wave as a function of a_0 while keeping the nonlinearity parameter $a_0\omega_{pe}/\omega_0 = 0.01$. The color represents the results for $\beta_{th0} = 0.1$ (red) and $\beta_{th0} = 0.01$ (blue). The theoretical predictions from Eqs. 67, 68, 69, and 70 are shown by the dashed lines.

($a_0, \omega_0/\omega_{pe}$) = (0.1, 10) (left) and (2, 200) (right) with $\beta_{th0} = 0.1$. Landau damping of the acoustic-like modes excited by SBS results in the formation of a plateau in the u_x distribution, which is a characteristic feature of SBS-induced heating reported in previous studies [67]. The development of such a plateau triggers the saturation of SBS because the resonant coupling, which depends on the gradient of the velocity distribution function, is reduced as the distribution flattens [44, 45]. For the ($a_0, \omega_0/\omega_{pe}$) = (0.1, 10) case, a distinct plateau forms at early stages around $u_x \sim 0.1$, which is comparable to the initial thermal velocity $\beta_{th0} = 0.1$, as in the previous studies [44, 45]. This plateau subsequently expands toward both larger and smaller u_x . In the nonlinear stage, scattered waves trigger secondary SBS, further modifying the distribution.

In contrast, for the ($a_0, \omega_0/\omega_{pe}$) = (2, 200) case, the initial velocity distribution is heavily influenced by the incident wave; consequently, a standard Maxwellian distribution is not observed in the simulation frame even at $\omega_0 t = 0$. While particles are initialized with a Maxwellian distribution ($\beta_{th0} = 0.1$) in the plasma rest frame, the large-amplitude incident wave ($a_0 > 1$) significantly shifts the distribution in the simulation frame (see Eq. 20). Although the Landau damping process is more complex in this case because of the relativistic oscillatory motion of the particles, the u_x distribution is nonetheless significantly modified by SBS, with the high-energy tail extending to extremely large u_x at later times. The inset in the right panel of Fig. 8 shows the corresponding particle energy spectra on a log-log scale, demonstrat-

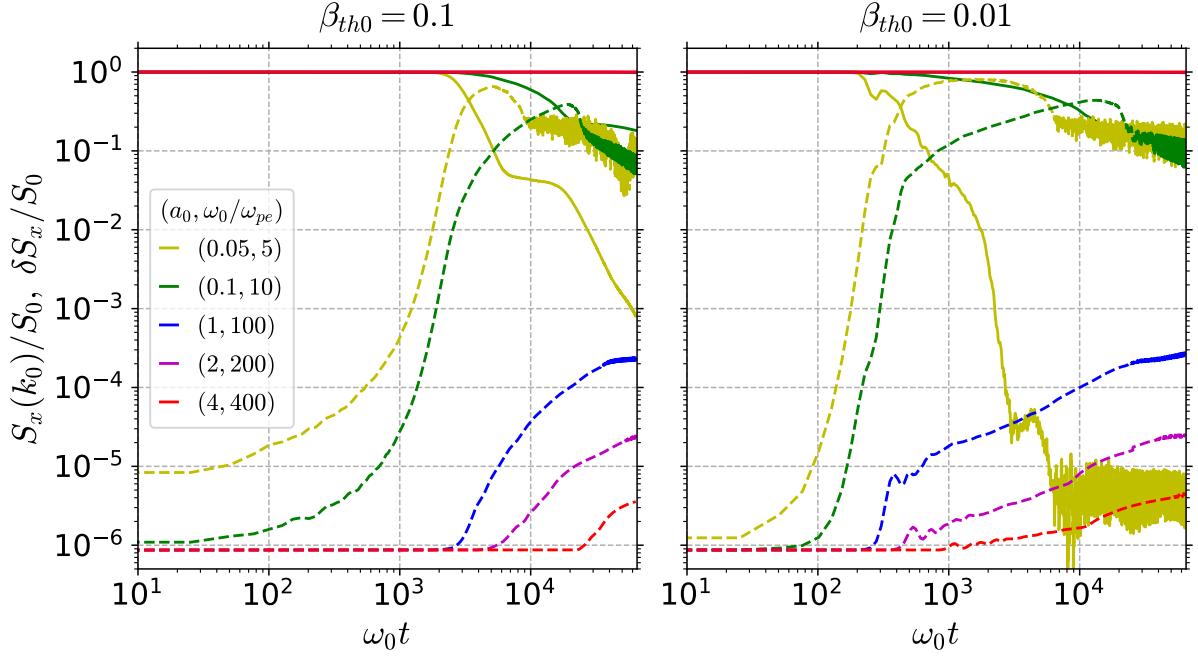


FIG. 7. Time evolution of the incident Poynting flux $S_x(k_0)$ (solid lines) and scattered Poynting flux δS_x (dashed lines) for $\beta_{th0} = 0.1$ (left) and $\beta_{th0} = 0.01$ (right). The color represents the results for various combinations of $(a_0, \omega_0/\omega_{pe})$ in the same manner as Fig. 5.

ing significant energy gain. However, a clear power-law distribution or a distinct hot component is not observed by the end of the simulation under the present physical conditions (cf. Refs. [75–77]). These results demonstrate that particle heating becomes more pronounced as $a_0\omega_0/\omega_{pe}$ increases, even when the backreaction on the incident wave remains small.

IV. DISCUSSION

We discuss the implications of our results for FRBs. The radio pulses propagate through the magnetar wind, and the laboratory (simulation) frame in our analysis corresponds to the frame where the magnetar wind is at rest prior to the arrival of the pulse. Note that the large-amplitude incident waves drive the plasma in the wave propagation direction (see Eq. 56). Given that the typical frequency of FRBs is $\nu_{\text{obs}} \sim 1$ GHz in the observer frame [2], the incident wave frequency in the wind rest frame is given by

$$\omega_0 = \frac{\pi\nu_{\text{obs}}}{\gamma_{\text{wind}}} \sim 3 \times 10^7 \nu_{\text{obs},9} \gamma_{\text{wind},2}^{-1} \text{ rad s}^{-1}, \quad (81)$$

where $\gamma_{\text{wind}} \sim 10^2$ is the Lorentz factor of the magnetar wind [26], $\nu_{\text{obs},9} = \nu_{\text{obs}}/10^9$ Hz, and $\gamma_{\text{wind},2} = \gamma_{\text{wind}}/10^2$. Hereafter, we use the notation $Q_x = Q/10^x$. The isotropic radio luminosity of FRBs is $L_{\text{obs}} = 2c\gamma_{\text{wind}}^2 E_0^2 R^2 \sim 10^{42} \text{ erg s}^{-1}$, where R is the distance from the magnetar, and thus the strength parameter is

estimated as

$$a_0 \sim 20 L_{\text{obs},42}^{\frac{1}{2}} \nu_{\text{obs},9}^{-1} R_{12}^{-1}. \quad (82)$$

The electron number density in the wind rest frame is

$$n_0 = \frac{\dot{N}}{8\pi\gamma_{\text{wind}} R^2 c} \sim 10 \dot{N}_{39} \gamma_{\text{wind},2}^{-1} R_{12}^{-2} \text{ cm}^{-3}, \quad (83)$$

where $\dot{N} \sim 10^{39} \text{ s}^{-1}$ is the particle flux of the magnetar wind [26]. Note that $\dot{N}$ is still uncertain. The electron plasma frequency in the wind rest frame is then given by

$$\omega_{pe} \sim 2 \times 10^5 \dot{N}_{39}^{\frac{1}{2}} \gamma_{\text{wind},2}^{-\frac{1}{2}} R_{12}^{-1} \text{ rad s}^{-1}. \quad (84)$$

The nonlinearity parameter is estimated as

$$a_0 \frac{\omega_{pe}}{\omega_0} \sim 10^{-1} L_{\text{obs},42}^{\frac{1}{2}} \dot{N}_{39}^{\frac{1}{2}} \nu_{\text{obs},9}^{-2} \gamma_{\text{wind},2}^{\frac{1}{2}} R_{12}^{-2}. \quad (85)$$

This indicates that linear SBS analysis remains applicable for $R \sim 10^{12} \text{ cm}$ despite the large amplitude $a_0 > 1$. Assuming that the thermal velocity of the magnetar wind is controlled by adiabatic expansion and Compton heating by X-rays emitted from the magnetar [61], we estimate

$$\beta_{th0} \sim 10^{-3} R_{12}^{-\frac{1}{2}}. \quad (86)$$

Thus, the strong coupling regime is relevant at $R \sim 10^{12} \text{ cm}$, where the plasma temperature does not significantly

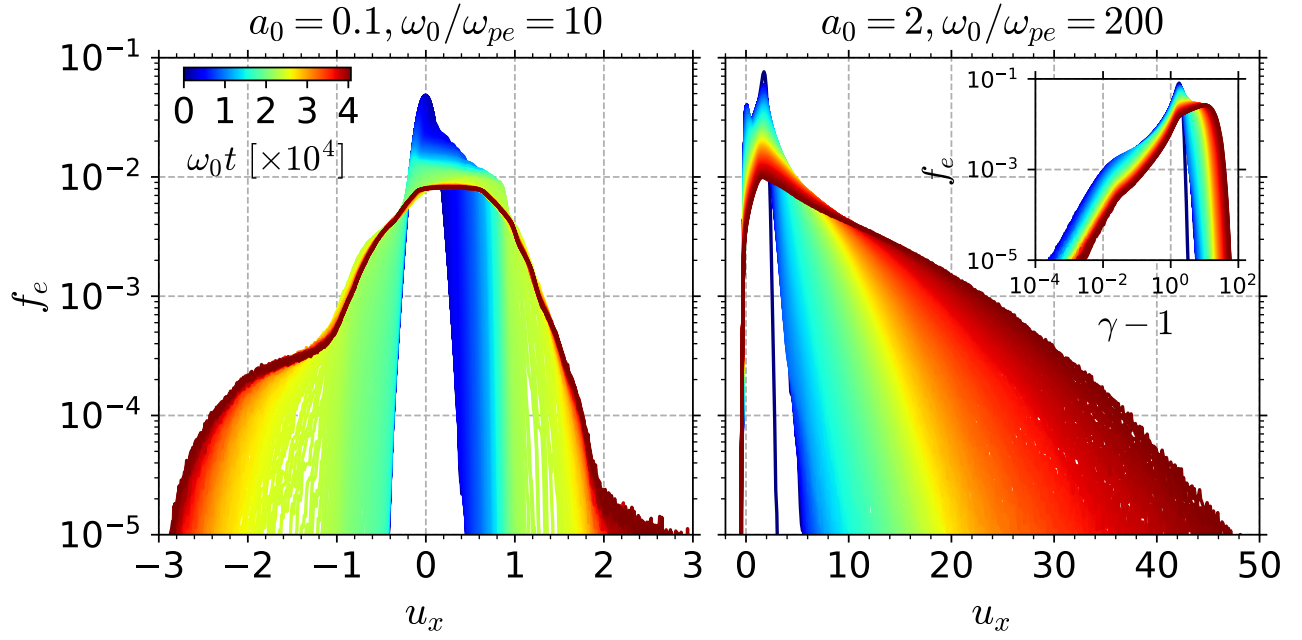


FIG. 8. Time evolution of the longitudinal four velocity u_x distribution in the simulation frame for $(a_0, \omega_0/\omega_{pe}) = (0.1, 10)$ (left) and $(2, 200)$ (right) with $\beta_{th0} = 0.1$ (weak coupling). The inset in the right panel displays the corresponding particle energy spectra on a log-log scale.

570 affect the SBS growth rate or wavenumber. The linear
571 growth timescale of SBS is given by

$$\tau_{\text{SBS}} = \frac{1}{\Gamma_{\text{max}}} \sim 3 \times 10^{-6} L_{\text{obs},42}^{\frac{1}{3}} \dot{N}_{39}^{-\frac{1}{3}} \nu_{\text{obs},9}^{-1} \gamma_{\text{wind},2}^{\frac{2}{3}} \text{ s}, \quad (87)$$

572 We use Eqs. 69, 72, 81, 82, and 85 to derive the above
573 estimate. We compare τ_{SBS} with the time duration
574 of the characteristic timescale of FRBs. The time duration
575 of the radio pulse in the wind rest frame τ_{pulse} is

$$\tau_{\text{pulse}} = 2\gamma_{\text{wind}}\tau_{\text{obs}} \sim 2 \times 10^{-1} \gamma_{\text{wind},2}\tau_{\text{obs},-3} \text{ s}, \quad (88)$$

576 where $\tau_{\text{obs}} \sim 10^{-3}$ s is the observed pulse duration. The
577 dynamical time in the wind rest frame is

$$\tau_{\text{dyn}} = \frac{R}{2\gamma_{\text{wind}}c} \sim 2 \times 10^{-1} R_{12}\gamma_{\text{wind},2}^{-1} \text{ s}. \quad (89)$$

578 Since $\tau_{\text{pulse}} \lesssim \tau_{\text{dyn}}$ for $R \gtrsim 10^{12}$ cm, the relevant
579 timescale for the growth of SBS during the FRB propa-
580 gation is τ_{pulse} . The pulse duration is much longer than
581 the linear growth timescale: $\tau_{\text{pulse}} \gg \tau_{\text{SBS}}$, indicating
582 that SBS can grow during the FRB propagation in the
583 magnetar wind.

584 Regarding nonlinear evolution, the saturation level of
585 SBS is characterized by the ratio of the incident wave
586 energy to the rest mass energy, $a_0\omega_0/\omega_{pe}$ as discussed in
587 Section III C. This ratio for FRBs is estimated as

$$a_0 \frac{\omega_0}{\omega_{pe}} \sim 3 \times 10^3 L_{\text{obs},42}^{\frac{1}{2}} \dot{N}_{39}^{-\frac{1}{2}} \gamma_{\text{wind},2}^{-\frac{1}{2}}, \quad (90)$$

588 which is comparable to the largest value in our simu-
589 lations, $a_0\omega_0/\omega_{pe} = 1600$. For such a large value of

590 $a_0\omega_0/\omega_{pe}$, the incident wave is barely affected by SBS
591 by the end of our simulation,

$$\omega_0\tau_{\text{end}} \sim 6 \times 10^4. \quad (91)$$

592 On the other hand, the characteristic FRB timescale in
593 units of ω_0 is given by

$$\omega_0\tau_{\text{pulse}} \sim 6 \times 10^6 \nu_{\text{obs},9}\tau_{\text{obs},-3}. \quad (92)$$

594 Although $\omega_0\tau_{\text{pulse}}$ significantly exceeds the simulation
595 duration $\omega_0\tau_{\text{end}}$, several factors likely suppress the dissi-
596 pation rate in realistic astrophysical scenarios. First, our
597 periodic boundary conditions prevent scattered waves
598 from escaping the interaction region, which may lead to
599 an artificial enhancement of SBS growth and energy dissi-
600 pation. In actual FRB propagation, scattered waves
601 naturally escape from the pulse region, while the leading
602 edge of the incident wave continuously encounters unper-
603 turbed plasma. Consequently, the growth of SBS is lim-
604 ited by the convection of scattered waves, thereby reduc-
605 ing the net dissipation rate of the incident pulse. We plan
606 to address these propagation effects in future work using
607 open-boundary simulations, following Ref. [74]. Second,
608 SBS is known to be suppressed for broadband incident
609 waves [42, 58], a state characteristic of FRB signals. Fur-
610 thermore, the filamentation instability [61, 78, 79], which
611 is not captured in our current 1D model, typically grows
612 faster than SBS [58]. Since filamentation is relatively in-
613 sensitive to Landau damping [59], it may further inhibit
614 the SBS energy loss channel. In addition, as plasma
615 heating progresses, the system may transition from a
616 strong coupling to a weak coupling regime, where the SBS

growth rate is significantly lower. Finally, for $a_0 \gg 1$, the ponderomotive force can trigger bulk acceleration of the magnetar wind. An increase in the wind Lorentz factor γ_{wind} effectively shortens the dynamical timescale τ_{dyn} in the wind rest frame, such that the SBS cannot grow sufficiently during the FRB propagation. Therefore, we conclude that FRBs can propagate through the magnetar wind at $R \gtrsim 10^{12}$ cm without substantial energy loss for our fiducial parameters, even if the signals undergo spectral or temporal modifications.

V. SUMMARY

We have investigated the induced scattering of linearly-polarized, large-amplitude electromagnetic waves in unmagnetized pair plasmas using analytical theory and PIC simulations. Our results demonstrate that the steady-state solution is governed by the nonlinearity parameter $a_0\omega_{pe}/\omega_0$ rather than a_0 itself. In the regime where $a_0\omega_{pe}/\omega_0 \ll 1$, the plasma current follows the test-particle limit, allowing the solution to remain essentially linear even for $a_0 > 1$.

We showed that the linear growth rate and wavenumber of SBS in the simulation frame depend on both $a_0\omega_{pe}/\omega_0$ and a_0 , a consequence of the Lorentz boost from the incident-wave-driven bulk motion. PIC simulations confirm that conventional linear theory can be extrapolated to the $a_0 > 1$ regime, provided the nonlinearity parameter remains small. Furthermore, the SBS saturation level is found to be controlled by the energy ratio $a_0\omega_0/\omega_{pe}$. When $a_0\omega_0/\omega_{pe} \gg 1$, incident wave dissipation is minimal despite significant particle heating via Landau damping.

Applying these results to FRBs in magnetar winds, we find that $a_0 \sim 20$ and $a_0\omega_{pe}/\omega_0 \sim 10^{-1}$ at $R \sim 10^{12}$ cm for our fiducial parameters. These values indicate that linear SBS analysis remains applicable for FRBs, and the growth timescale of SBS is much shorter than the pulse duration. However, the saturation level of SBS is expected to be low due to the large value of $a_0\omega_0/\omega_{pe} \sim 10^3$, suggesting that FRBs can propagate through the

magnetar wind without substantial energy loss, even if they undergo spectral or temporal modifications.

In this work, we neglect the effects of the background magnetic field, an approximation that remains valid in regions far from the magnetar. In highly magnetized plasmas $\omega_L \gg \omega_{pe}$, where ω_L is the cyclotron frequency, the nonlinearity parameter is effectively defined by $a_0\omega_L/\omega_0$ rather than $a_0\omega_{pe}/\omega_0$ [41, 60]. Under the condition $a_0\omega_L/\omega_0 \ll 1$, the incident wave can be treated within a linear framework; consequently, linear analyses of induced scattering in magnetized pair plasmas [42–45] might remain applicable even for $a_0 > 1$. The influence of the background magnetic field on the propagation of large-amplitude waves will be investigated in a future publication.

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