Universal Nucleation Length for Slip-Weakening Rupture Instability Under Nonuniform Fault Loading

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We consider the nucleation of instability on a slip-weakening fault subjected to a non-uniform, locally peaked "loading" stress. That loading stress is the stress which would be sustained if there was no slip on the fault; it is assumed to gradually increase due to tectonic loading but to retain its peaked character. The case of a linear stress versus slip law, i.e., with a constant slip-weakening rate, is considered in the framework of two-dimensional quasi-static elasticity (in-plane or anti-plane strain) for a planar fault. Slip initiates when the "loading" stress first reaches the strength level of the fault to start slip weakening. Then the size of the slipping region on the fault grows under increased loading stress until finally a critical nucleation length is reached at which no further quasi-static solution exists for additional increase of the loading. That marks the onset of a dynamically controlled instability. We prove that the nucleation length is independent of the shape of the "loading" stress distribution. Its universal value is proportional to an elastic modulus and inversely proportional to the slip-weakening rate, and is given by the solution to an eigenvalue problem. That is the same eigenvalue problem as introduced by Campillo, Ionescu and collaborators for dynamic slip nucleation under spatially uniform pre-stress on a fault segment of fixed length; the critical length we derive is the same as in their case. To illustrate the nucleation process, and its universal feature, in specific examples, we consider cases for which the loading stress is peaked symmetrically or non-symmetrically, and employ a numerical approach based on a Chebyshev polynomial representation. Laboratory-derived and earthquake-inferred data are used to evaluate the nucleation size.

Keywords: Earthquake nucleation, Slip weakening, Fault instability, Earthquake dynamics and mechanics

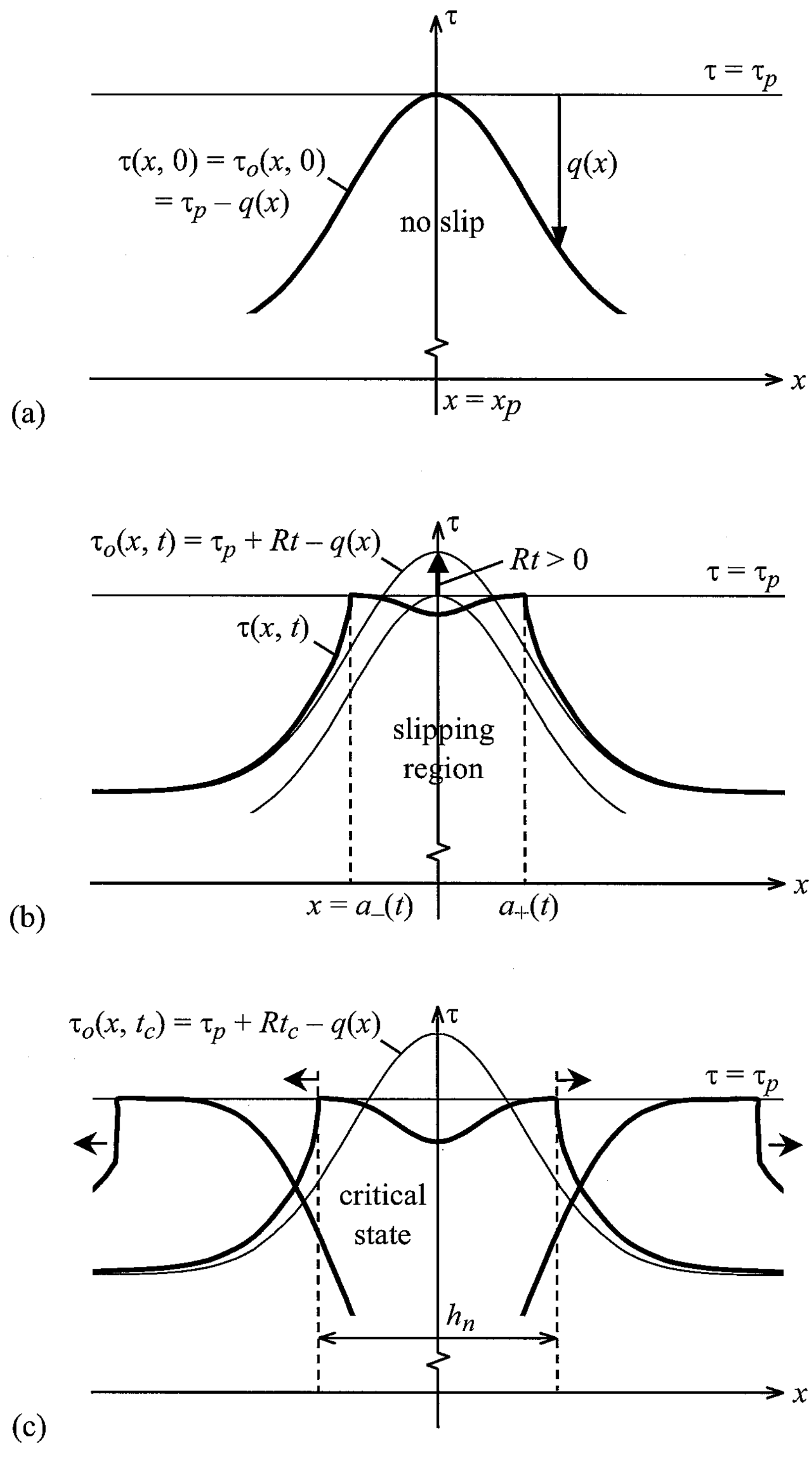

Figure 1. Development of the slipping region induced by the increasing loading stress. (a) At time $t = 0$, the peak of the stress distribution reaches the peak strength of the fault, τ_p. Prior to this stage, no slip has occurred; (b) At $t > 0$, part of the fault slips and the stress inside the slipping region drops according to the slip-weakening friction law; and (c) At a later stage, when the length of the slipping region reaches a critical value, h_n, the fault system becomes unstable and the slipping region will expand even without any increase of the loading stress. We shall show that h_n is independent of R and $q(x)$.

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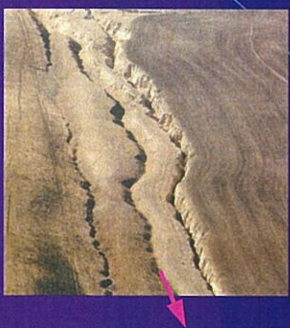
Earthquakes?

Are catfish guilty??

In Japan earthquakes were believed to be triggered by catfish (*Bolt*, 1993).

Earthquakes?



Earthquake

Fault slip:

Quasi-static → Dynamic

Typical Parameters

Stress on Fault: τ

Slip (displacement discontinuity): δ

Stress-Slip Constitutive Law

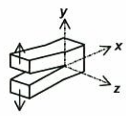
Slip-weakening

Rate- and state-dependent

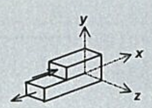
⇓

Instabilities

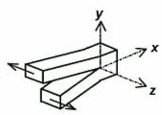
Three Fundamental Modes of Fracture



(a) Mode I: tensile or opening mode



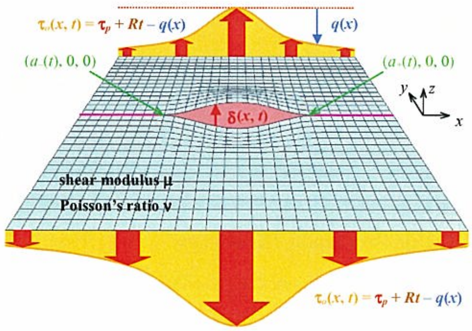
(b) Mode II: in-plane shear or sliding mode



(c) Mode III: anti-plane shear or tearing mode

Problem Statement

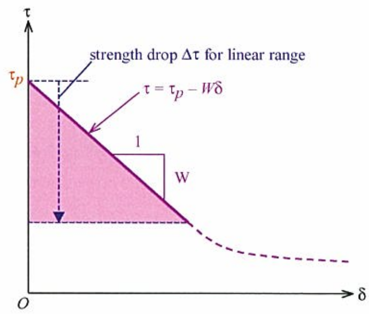
We consider the nucleation of instability on a slip-weakening fault subjected to a non-uniform, locally peaked "loading" stress.



A displacement field associated with anti-plane shear (mode III) rupture in an infinite, homogeneous, linear elastic space. The "loading" shear stress τ_o is locally peaked in space and increases gradually with time, but retains its peaked character. Similarly, we can define the problem for in-plane shear (mode II), or for tensile (mode I) rupture.

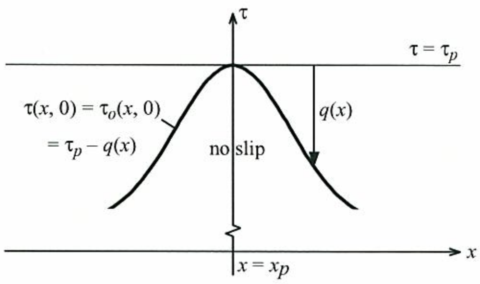
Constitutive Law

The linear stress versus slip law, i.e., with a constant slip-weakening rate, is employed. We consider only slips which lie on the linear portion.

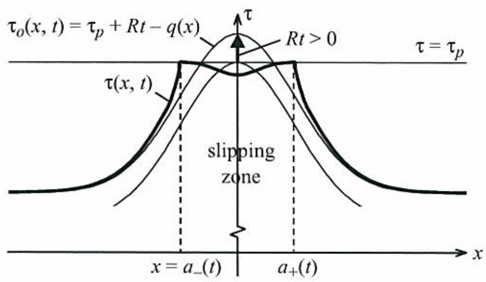


The (initially) linear slip-weakening constitutive law. The stress inside the slipping region of the fault obeys the linear relation $\tau = \tau_p - W\delta$, at least when the strength drop is less than $\Delta\tau$. The slip-weakening rate W is a constant ($W > 0$).

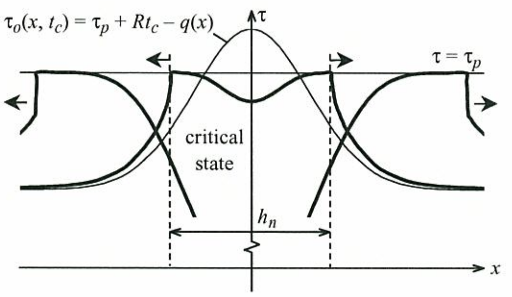
Slip Development



Development of the slipping region induced by the increasing loading stress. (a) At time $t = 0$, the peak of the stress distribution reaches the peak strength of the fault, τ_p. Prior to this stage, no slip has occurred.



(b) At $t > 0$, part of the fault slips and the stress inside the slipping region drops according to the slip-weakening friction law.



(c) At a later stage, when the length of the slipping region reaches a critical value, h_n, the fault system becomes unstable and the slipping region will expand even without any increase of the loading stress.

Basic Equations

- The elastic equilibrium condition:

$$\tau(x,t) = \tau_o(x,t) - \frac{\mu^*}{2\pi}\int_{a_-(t)}^{a_+(t)} \frac{\partial\delta(\xi,t)/\partial\xi}{x-\xi}\, d\xi$$

where $\mu^* = \mu$ for mode III; and $\mu/(1-\nu)$ for modes I and II.

- The locally peaked, quasi-statically increasing loading stress:

$$\tau_o(x, t) = \tau_p + Rt - q(x)$$

- The slip-weakening friction law:

$$\tau(x, t) = \tau_p - W\delta(x, t)$$

$$\Downarrow$$

$$\boxed{\tau_p - W\delta(x,t) = \tau_p + Rt - q(x) - \frac{\mu^*}{2\pi}\int_{a_-(t)}^{a_+(t)} \frac{\partial\delta(\xi,t)/\partial\xi}{x-\xi}\, d\xi}$$

for $a_-(t) < x < a_+(t)$

$$-W\delta(x,t) = Rt - q(x) - \frac{\mu^*}{2\pi}\int_{a_-(t)}^{a_+(t)} \frac{\partial\delta(\xi,t)/\partial\xi}{x-\xi}\, d\xi$$

for $a_-(t) < x < a_+(t)$.

(Careful) differentiation with respect to time:

$$-WV(x,t) = R - \frac{\mu^*}{2\pi}\int_{a_-(t)}^{a_+(t)} \frac{\partial V(\xi,t)/\partial\xi}{x-\xi}\, d\xi$$

Here, $V(x, t)$ is slip rate: $V(x, t) \equiv \partial\delta(x, t)/\partial t$.

By introducing

$$a(t) \equiv [a_+(t) - a_-(t)]/2, \quad b(t) \equiv [a_+(t) + a_-(t)]/2,$$

$$X \equiv [x - b(t)]/a(t), \text{ and } \nu(X, t) \equiv V(x, t)/(\sqrt{2}V_{rms}(t)),$$

with the root-mean-square slip rate being

$$V_{rms}(t) \equiv \sqrt{\frac{1}{a_+(t) - a_-(t)}\int_{a_-(t)}^{a_+(t)} V^2(x,t)\,dx}$$

and suppressing explicit reference to the time-dependence, we can normalize the above equation as

$$-\frac{aW}{\mu^*}\nu(X) = \frac{a}{\sqrt{2}\mu^*}\frac{R}{V_{rms}} - \frac{1}{2\pi}\int_{-1}^{+1}\frac{\nu'(s)}{X-s}\,ds$$

for $-1 < X < +1$.

Nucleation Length

$$\frac{aW}{\mu^*}\nu(X) = -\frac{a}{\sqrt{2}\mu^*}\frac{R}{V_{rms}} + \frac{1}{2\pi}\int_{-1}^{+1}\frac{\nu'(s)}{X-s}\,ds$$

for $-1 < X < +1$.

As $a \to a_c$ (critical half-length), $aR/(\mu^* V_{rms}) \to 0$. So,

$$\boxed{\frac{aW}{\mu^*}\nu(X) = \frac{1}{2\pi}\int_{-1}^{+1}\frac{\nu'(s)}{X-s}\,ds.}$$

Nontrivial solution for $\nu(X)$ is given when

$a(t)W/\mu^* = a_c W/\mu^* = \lambda_0 \approx 0.579$ (smallest eigenvalue) and

$\nu(X) = \nu_0(X) \approx (0.925 - 0.308X^2)\sqrt{1-X^2}$

(associated eigenfunction).

Thus, the nucleation length is given by:

$$\boxed{h_n = 2a_c \approx 1.158\ \mu^*/W}$$

Chebyshev Polynomials

$x_o \equiv x - [a_+(t) + a_-(t)]/2 = a(t)\cos\omega \quad (0 < \omega < \pi)$

$$\delta(x_o, t) = \sum_{n=1}^{\infty} u_n(t)\sin(n\omega)$$

Equilibrium condition

$$b_m = \sum_{n=1}^{\infty} A_{mn} u_n(t)$$

where

$$A_{mn} = \frac{n\mu^*}{2a(t)} \int_0^{\pi} \frac{\sin(m\omega)\sin(n\omega)}{\sin\omega} d\omega - W\frac{\pi}{2}\delta_{m,n}$$

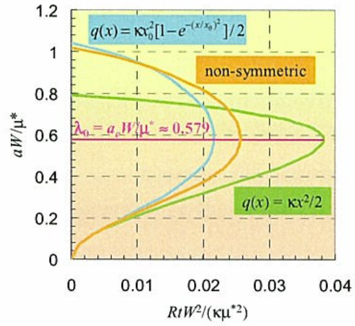
$$b_m = \frac{Rt}{m}[1-(-1)^m] - \int_0^{\pi} q(a(t)\cos\omega + b(t))\sin(m\omega)d\omega$$

and $\delta_{m,n}$ is 1 if $m = n$ and 0 if $m \neq n$.

Finiteness condition

$$\sum_{n=1,3,5,\ldots}^{\infty} n u_n(t) = 0 \quad \text{and} \quad \sum_{n=2,4,6,\ldots}^{\infty} n u_n(t) = 0$$

Examples



The development of the slipping region half-length a for three different loading distributions. The parameter $x_0 W/\mu^* = 1/3$.

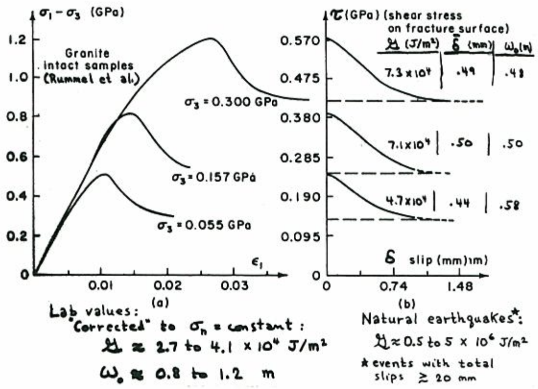
Laboratory Testing

$$\boxed{h_n = 2a_c \approx 1.158\, \mu^*/W}$$

1. Slip-weakening process in the post- failure stage of laboratory testing of shear fracture of initially intact Fichtelbirge granite specimens at different confining pressures (7.5 to 300MPa) in a stiff, servo-controlled triaxial apparatus

$\Rightarrow \mu = 30$GPa and $\nu = 0.25$ ($\mu^* = 40$GPa)

$W \approx 50$ to 90GPa/m and $\underline{h_n \approx 0.5 \text{ to } 0.9\text{m}}$



(*Rice*, *Eos* 2000)

2. A laboratory-derived rate- and state-dependent friction law with the Dieterich-Ruina-type slowness (or ageing) state evolution

$$\tau = \bar{\sigma}[f_o + a\ln(V/V_o) + b\ln(V_o\theta/L)]$$
$$d\theta/dt = 1 - V\theta/L$$

$\bar{\sigma}$: effective normal stress, V: slip rate

θ: state variable, L: characteristic slip distance

a, b: frictional parameters

f_o: friction coefficient at reference rate V_o

If $V\theta/L >> 1$ and $\bar{\sigma}$ is constant and V varies by less than a few powers of 10 (since $a \sim 0.01$ and $f_o \sim 0.6$)

$$\tau = \tau_p - (b\bar{\sigma}/L)\delta = \tau_p - W\delta$$

$\Rightarrow$ $\mu^* = 40$GPa, $\bar{\sigma} = 140$MPa, $b = 0.015$ and $L = 5$ to 100μm

$W \approx 20$ to 420GPa/m and $\underline{h_n \approx 0.1 \text{ to } 2.3\text{m}}$

Seismic Inversions?

$W = \Delta\tau/D_c$

$\Delta\tau$: strength drop

D_c: characteristic slip-weakening distance; linear weakening for slips $\delta < D_c$ and no further weakening for $\delta > D_c$

Laboratory-derived value of D_c:

- Of the order of up to 1mm

A much larger value of D_c is widely inferred in seismic inversions as well as in numerical calculations:

$D_c = 0.1$ to 1m and $\Delta\tau = 5$ to 15MPa

$\Rightarrow$ $\underline{h_n \approx 0.3 \text{ to } 9\text{km}}$ with $\mu^* = 40$GPa

Incompatible with existence of small-magnitude earthquakes !

D_c inferred from seismic inversion is likely to be biased large:

- Effects of spatial and temporal smoothing constraints
- Short periods are filtered out of the instrumental records
- Moreover, solution of D_c is not uniquely given by seismic inversions !

(*Guatteri and Spudich*, *BSSA* 2000)

From the forward modeling point of view, D_c tends to be large:

- Difficulty of simulating earthquakes
  - Fault region at least tens to hundreds of kilometers
  - Slip and rapid stress change at the tips of propagating ruptures

(*Lapusta et al.*, *JGR* 2000)

Conclusions

1. The purpose of this study:

   $\Rightarrow$ To show the universal critical length that is relevant to instabilities of a linear slip-weakening fault under locally peaked, quasi-statically increasing stress field;

2. The problem has been reduced to an eigenvalue problem:

   $\Rightarrow$ The critical length can be expressed in terms of the smallest eigenvalue, the elastic modulus and the slip-weakening rate only;

3. The fundamental nature of slip instabilities remains the same even when the type (mode I, II, or III) and the shape of the loading change.