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The Graph of a Monotonic Reaction Function is a von Neumann-Morgenstern Stable Set for a Game with Preplay Negotiations

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February, 2002

Running Head: Monotonic reaction function and the vN-M stable set

Key words: Monotonic reaction function, von Neumann-Morgenstern stable set, Supermodularity, Preplay negotiation, Social situations

JEL Classification: C72, C79, D74.

Abstract

We consider a n-player game with preplay negotiations, in which each player can deviate from a currently proposed combination of actions unilaterally but can not do so jointly with other players. The negotiation among the players is formulated as an "individual contingent threats situation" within the framework of the theory of social situations. We show that the graph of a monotonic reaction function derived from a payoff function satisfying the supermodularity together with some technical conditions is a von Neumann-Morgenstern stable set for this game and it includes at least one Pareto-efficient outcome.

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1 Introduction

We examine a n-player game with preplay negotiations, in which each player can deviate from a currently proposed combination of actions unilaterally but can not do so jointly with other players.[1] Among those possible approaches to construct a formal model of such a game, the theory of social situations (TOSS) developed by Greenberg (1990) provides us with a simple but powerful framework. In particular, the "individual contingent threats situation" (ICT situation) in TOSS is suitable for analyzing the above mentioned game. In the ICT situation in TOSS, the negotiation among players is formalized by means of the inducement correspondence, which describes how each player can change the current combination of actions to other combinations.

The solution concept in this paper is the von Neumann-Morgenstern stable set (vN-M stable set), which, in our context, is a subset of the set of all possible combinations of actions (the strategy space) that satisfies both the internal stability and the external stability. Roughly speaking, the internal stability requires that no player can make herself better-off by deviating—according to the inducement correspondence—from an outcome (i.e., a combination of actions) in the vN-M stable set to another outcome in it; On the other hand, the external stability requires that for an outcome *not in* the vN-M stable set there must be a player who can make herself better-off by deviating—again, according to the inducement correspondence—from the very outcome to another outcome *included in* the vN-M stable set. Although the solution concept in TOSS is the "optimistic stable standard of behavior" (OSSB), which is a certain *mapping* from the strategy space to itself, we do not adopt this as the solution concept, because it can be shown that an OSSB (if it exists) yields a corresponding subset of the strategy space that can be regarded as a vN-M stable set with respect to an appropriately defined binary relation on the strategy space and also because it is much easier to find an appropriate subset of the strategy space than to find an appropriate mapping.[2]

Some studies have applied the notion of the ICT situation and its variants to some well-known games and examined the existence and the efficiency property of the vN-M stable set.[3] Arce (1994) has examined a three-player two-strategy prisoners' dilemma game as an ICT situation and shown the

[1] Kalai (1981) has examined a game with a different scenario of the negotiation, which allows *joint* actions by the players.

[2] See Greenberg (1990, Chap. 4).

[3] Using different solution concepts, Bhaskar (1989) and Muto (1993) have examined a price-setting duopoly game and some other games with a similar (essentially the same) negotiation procedure as the ICT situation.

existence of the vN-M stable set, which includes some Pareto-efficient outcomes.[4] Muto and Okada (1996) have applied the ICT situation to a price-setting duopoly game and shown the existence of the vN-M stable set, which includes the Pareto-efficient monopoly pricing outcome. In a companion paper, Muto and Okada (1998) have examined a Cournot duopoly game as an ICT situation. They have shown that some vN-M stable sets can include Pareto-efficient outcomes on one hand, but there can be other vN-M stable sets that include no Pareto-efficient outcome on the other. In international trade, Nakanishi (1999) has examined the international export quota game between two countries as an ICT situation and shown that the existence of the vN-M stable sets and that every vN-M stable set includes at least one Pareto-efficient combination of quotas and, conversely, every Pareto-efficient combination of quotas can be supported by a vN-M stable set.[5]

Up to the present, although there are many specific examples of the ICT situations as shown above, only a few results concerning more general conditions for the existence of the vN-M stable set for the ICT situation have been known. In view of two theorems established by Greenberg (1990), we can say that a "two"-player ICT situation with each player's strategy set being finite or a n-player ICT situation in which each player's strategy set contains no more than "two" elements admits the existence of the vN-M stable set.[6] Recently, relaxing the twoness requirements in Greenberg's theorems, Nakanishi (2001) has shown that there exist vN-M stable sets for the n-player ICT situation in which each player's strategy set is a closed interval on the real line and each player's payoff function is monotonically decreasing in its own argument. In this paper, we show other sufficient conditions that guarantee the existence as well as the efficiency property of the vN-M stable set for n-player ICT situations with each player's strategy set being a closed interval on the real line.

The rest of the paper is organized as follows. In Section 2 we construct a formal model of the n-player ICT situation and give formal definitions of the inducement correspondence and the vN-M stable set. In Section 3, we describe conditions imposed on the payoff function and the reaction function and prove theorems concerning the existence and the efficiency property of the vN-M stable sets. Section 4 includes some remarks.

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[4] In fact, Arce (1994) has not used the notion of the ICT situation, but it is quite obvious that his notion of "Nash play" is identical to the ICT situation.

[5] What Nakanishi (1999) has shown actually is the existence and the efficiency of the OSSBs. As noted in the text, the set of outcomes supported by an OSSB can be seen as a vN-M stable set.

[6] See Theorems 7.4.5 and 7.4.6 in Greenberg (1990), pp.100–101.

2 Model

Consider the following normal form game:

$$G \equiv (N, \{X_i\}_{i \in N}, \{u_i\}_{i \in N}), \tag{1}$$

where $N \equiv \{1, 2, \ldots, n\}$ is the finite set of players, X_i is the set of actions for player i, which is assumed to be a closed interval on $\mathbf{R}$, and $u_i : X \to \mathbf{R}$ is the payoff function for player i, where $X \equiv \prod_{i \in N} X_i$ denotes the strategy space. For notational convenience, we write $X_{-i} \equiv \prod_{j \neq i} X_j$. We shall call a typical element of X (or that of X_{-i}) an *outcome* and write it as x, y, or z and so on (x_{-i}, y_{-i}, or z_{-i}, for outcomes in X_{-i}). For each $x_{-i} \in X_{-i}$, let us define the set of player i's best response against x_{-i} as follows:

$$\psi_i(x_{-i}) \equiv \arg\max_{x_i \in X_i} u_i\left(x_i, x_{-i}\right). \tag{2}$$

Naturally, ψ_i determines a correspondence from X_{-i} to X_i and we shall call it the **reaction function** of player i. (Although ψ_i is not singleton-valued and, hence, is not a function in general, we shall use here a simpler term "reaction function," because we will place restrictions on the payoff function so that ψ_i becomes indeed singleton-valued in the next section.) We shall write the graph of the reaction function of player i as follows: For $i \in N$,

$$\Psi_i \equiv \left\{x = (x_i, x_{-i}) \in X \middle|\, x_i \in \psi_i(x_{-i})\right\}. \tag{3}$$

The set of Pareto-efficient outcomes is defined as follows:

$$E \equiv \left\{ x \in X \;\middle|\; \begin{array}{l} \text{There is no } y \in X \text{ such that} \\ u_i(y) \geq u_i(x) \text{ for all } i \in N \text{ and} \\ u_j(y) > u_j(x) \text{ for some } j \in N. \end{array} \right\}. \tag{4}$$

The normal form game G does not yet capture the notion of negotiation among the players. To formalize the negotiation procedure, we need to introduce the notion of the **inducement correspondence** for the ICT situation. The negotiation in the ICT situation goes as follows. Suppose that an outcome $x = (x_1, \ldots, x_i, \ldots, x_n)$ is proposed to the players. If all the players openly consent to follow x, then x will be adopted and played. If player i objects to x, she has to declare that if all the other players will stick to x, then she will employ x_i' instead of x_i. Then, the current outcome changes to another, say, $y = (x_1, \ldots, x_i', \ldots, x_n)$. In turn, another player j may object to y and say (or, threaten by saying) that she will employ x_j' instead of x_j. Then the outcome y changes to another, say, $z = (x_1, \ldots, x_i', \ldots, x_j', \ldots, x_n)$. The negotiation continues in this manner until an outcome that will be followed by all the players is reached.

As shown above, at any point of the negotiation process, each single player can object to the prevailing outcome and can threaten the others by saying that she will employ another strategy. When player i changes the current outcome x to another y in such a way, we say that "player i induces y from x." We denote the set of outcomes that player i can induce from x as follows: For $i \in N$,

$$\gamma_i(x) \equiv \{y \in X|\ y_i \in X_i \text{ and } y_j = x_j \text{ for all } j \neq i,\ j \in N\}. \tag{5}$$

γ_i determines a correspondence from X into itself and we call it the inducement correspondence. With γ_i in hand, we can now define the ICT situation associated with G as follows:

$$G_\gamma \equiv (N, X, \{u_i\}_{i\in N}, \{\gamma_i\}_{i\in N}). \tag{6}$$

We define a binary relation $\leq$ on $\mathbf{R}^n$ such that, for $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in X$, $x \leq y$ if $x_i \leq y_i$ for all $i \in N$. In addition, we write $x < y$ if $x \leq y$ and $x \neq y$; $x \ll y$ if $x_i < y_i$ for all $i \in N$. Since the binary relation $\leq$ is reflexive (i.e., $x \leq x$ for each $x \in X$), antisymmetric (i.e., $x \leq y$ and $y \leq x$ imply $x = y$ for all $x, y \in X$), and transitive (i.e., $x \leq y$ and $y \leq z$ imply $x \leq z$ for all $x, y, z \in X$), then $\mathbf{R}^n$ endowed with $\leq$ is a partially ordered set. For two outcomes $x, y \in X$, we define their **join** denoted by $x \vee y$ and **meet** denoted by $x \wedge y$ as follows: $x \vee y \equiv (\max\{x_1, y_1\}, \dots, \max\{x_n, y_n\}) = (\max\{x_i, y_i\})_{i\in N}$ and $x \wedge y \equiv (\min\{x_1, y_1\}, \dots, \min\{x_n, y_n\}) = (\min\{x_i, y_i\})_{i\in N}$, respectively. For any $x, y \in \mathbf{R}^n$, we have both $x \vee y \in X$ and $x \wedge y \in X$. Therefore, $\mathbf{R}^n$ with $\leq$ is a **lattice**. For $x, y \in \mathbf{R}^n$ with $x \leq y$, $[x, y]$ denotes a closed interval on $\mathbf{R}^n$ with respect to $\leq$, that is, $[x, y] \equiv \{z \in \mathbf{R}^n|\ x \leq z \leq y\}$. Since the strategy space X is a subset of $\mathbf{R}^n$ and, for any $x, y \in X$, we have both $x \wedge y \in X$ and $x \vee y \in X$, then X with $\leq$ can be seen as a **sublattice** of $\mathbf{R}^n$. By the same token, $\mathbf{R}^{n-1}$ with $\leq$ is a lattice and we can regard X_{-i} as a sublattice of $\mathbf{R}^{n-1}$. We write a closed interval on $\mathbf{R}^{n-1}$ with respect to $\leq$ as $[x_{-i}, y_{-i}]$ for $x_{-i}, y_{-i} \in \mathbf{R}^{n-1}$ with $x_{-i} \leq y_{-i}$.[7]

Let us turn to define the solution concept, that is, the **von Neumann-Morgenstern stable set** (vN-M stable set). A subset K of X is said to be a vN-M stable set for G_γ if it satisfies the following two conditions.

[7] Strictly speaking, we have to distinguish the relation $\leq$ defined on $\mathbf{R}^n$ and the relation $\leq$ defined on $\mathbf{R}^{n-1}$. But, since the contexts are clear in this paper, we do not make use of distinct symbols for these relations.

Internal stability: If $x \in K$, then there does not exist $y \in K$ such that $y \in \gamma_i(x)$ and $u_i(y) > u_i(x)$ for some $i \in N$;

External stability: If $x \in X \setminus K$, then there exists $y \in K$ such that $y \in \gamma_i(x)$ and $u_i(y) > u_i(x)$ for some $i \in N$.

For $x, y \in X$, we say that "y dominates x through i" if $y \in \gamma_i(x)$ and $u_i(y) > u_i(x)$ for some $i \in N$, or, we say more simply that "y dominates x" if there exists at least one such player. The internal stability means that an outcome in the vN-M stable set is never dominated by another outcome in it. On the other hand, the external stability means that an outcome outside the vN-M stable set must be dominated by some outcome in it.

3 Theorems

We first introduce some conditions placed on the payoff function and on the reaction function and, then, we show a series of lemmas that follow those conditions.

Condition 1 (upper semi-continuity of $\mathbf{u_i}$ on $\mathbf{X_i}$) *For each $x_{-i} \in X_{-i}$, if the upper-contour sets $\{x_i \in X_i | \ u_i(x_i, x_{-i}) \geq \alpha\}$ are closed for all $\alpha \in \mathbf{R}$, then the payoff function u_i is said to be upper semi-continuous on X_i.*

Condition 2 (strict quasi-concavity of $\mathbf{u_i}$ on $\mathbf{X_i}$) *For each $x_{-i} \in X_{-i}$, if the payoff function u_i satisfies*

$$\min \left\{u_i\left(y_i, x_{-i}\right), u_i\left(z_i, x_{-i}\right)\right\} < u_i\left(\lambda y_i + (1-\lambda) z_i, x_{-i}\right) \tag{7}$$

for any distinct $y_i, z_i \in X_i$ and for any real number $\lambda \in (0, 1)$, then it is said to be strictly quasi-concave on X_i.

It is well known that, for each $x_{-i} \in X_{-i}$, the upper semi-continuity of u_i on X_i guarantees the non-emptiness of $\psi_i(x_{-i})$ and, in addition, the strict quasi-concavity of u_i on X_i guarantees that $\psi_i(x_{-i})$ is singleton-valued. Therefore, with these conditions, it is justified to call ψ_i the "reaction function."

Condition 3 (supermodularity of $\mathbf{u_i}$ on $\mathbf{X}$) *If the payoff function u_i satisfies*

$$u_i(x) + u_i(y) \leq u_i(x \vee y) + u_i(x \wedge y) \tag{8}$$

for any $x, y \in X$, then it is said to be supermodular on X.

Condition 4 (local responsiveness of the reaction function) *The reaction function ψ_i is said to be locally responsive at $x_{-i} \in X_{-i}$ if there exists a $(n-1)$-vector $\varepsilon \gg 0$ such that $\psi_i(x_{-i}) \neq \psi_i(y_{-i})$ for any distinct $y_{-i} \in [x_{-i} - \varepsilon, x_{-i} + \varepsilon] \cap X_{-i}$. ψ_i is said to be locally responsive on X_{-i} if it is locally responsive at each and every $x_{-i} \in X_{-i}$.*

The following lemma is a direct consequence of the supermodularity of u_i on X, which is a simplified version of Theorem 2.6.1 in Topkis (1998). The property of u_i shown in the lemma is known as "increasing differences."

Lemma 1 *Suppose that the payoff function u_i is supermodular on X. Then, for $x = (x_i, x_{-i}), y = (y_i, y_{-i}) \in X$ such that $x_i < y_i$ and $x_{-i} < y_{-i}$, we have*

$$u_i(y_i, x_{-i}) - u_i(x_i, x_{-i}) \leq u_i(y_i, y_{-i}) - u_i(x_i, y_{-i}). \tag{9}$$

Proof. Let us define two outcomes $z, w \in X$ such that $z = (x_i, y_{-i})$ and $w = (y_i, x_{-i})$. By definition, we have $z \wedge w = x$ and $z \vee w = y$. Then, by the supermodularity of u_i on X, we have

$$u_i(z) + u_i(w) \leq u_i(z \vee w) + u_i(z \wedge w) = u_i(y) + u_i(x). \tag{10}$$

Rearranging the above inequality, we have $u_i(w) - u_i(x) \leq u_i(y) - u_i(z)$, which is equivalent to eq. (9). **Q.E.D.**

The supermodularity of u_i on X places a restriction on the shape of the reaction function ψ_i as shown below, in addition, the local responsiveness of ψ_i on X_{-i} strengthens this result. For any $x_{-i}, y_{-i} \in X_i$ with $x_{-i} < y_{-i}$, if $\psi_i(x_{-i}) \leq \psi_i(y_{-i})$, then ψ_i is said to be *increasing*; if $\psi_i(x_{-i}) < \psi_i(y_{-i})$, then *strictly increasing.*

Lemma 2 *Suppose that the reaction function ψ_i is well-defined and singleton-valued.* (i) *If u_i is supermodular on X, then ψ_i is increasing;* (ii) *If, in addition, ψ_i is locally responsive on X_{-i}, then ψ_i is strictly increasing.*

Proof. [**Part (i)**] Suppose, in negation, that there exists a pair of $x_{-i}, y_{-i} \in X_{-i}$ such that $x_{-i} < y_{-i}$ and $\psi_i(x_{-i}) > \psi_i(y_{-i})$. By lemma 1, we have

$$u_i\left(\psi_i(x_{-i}), x_{-i}\right) - u_i\left(\psi_i(y_{-i}), x_{-i}\right) \leq u_i\left(\psi_i(x_{-i}), y_{-i}\right) - u_i\left(\psi_i(y_{-i}), y_{-i}\right). \tag{11}$$

By the definition and singleton-valuedness of ψ_i, the left-hand-side of the above inequality is strictly positive. Then, the right-hand-side becomes strictly positive; This contradicts to the definition of ψ_i.

[**Part (ii)**] Let us take any pair of $x_{-i}, y_{-i} \in X_{-i}$ with $x_{-i} < y_{-i}$. Since we have $\psi_i(x_{-i}) \leq \psi_i(y_{-i})$ by Part (i) above, then it suffices to show that this inequality is strict. Due to the local responsiveness, there exists a $(n-1)$-vector $\varepsilon \gg 0$ such that $\psi_i(x_{-i}) \neq \psi_i(z_{-i})$ for any distinct $z_{-i} \in [x_{-i} - \varepsilon, x_{-i} + \varepsilon] \cap X_{-i}$. If y_{-i} is in $[x_{-i} - \varepsilon, x_{-i} + \varepsilon] \cap X_{-i}$, we have $\psi_i(x_{-i}) \neq \psi_i(y_{-i})$ by the local responsiveness; If not, then we can choose another outcome $w_{-i} \in [x_{-i} - \varepsilon, x_{-i} + \varepsilon] \cap X_{-i}$ such that $x_{-i} < w_{-i} < y_{-i}$. Part (i) and the local responsiveness together imply $\psi_i(x_{-i}) < \psi_i(w_{-i}) \leq \psi_i(y_{-i})$. **Q.E.D.**

We are now in a position to state the first theorem, which asserts the existence of the vN-M stable set for G_γ. The following theorem provides us with a set of sufficient conditions for the existence of the vN-M stable set for the n-player ICT situation, which are completely different from those conditions described in Nakanishi (2001).

Theorem 1 *Suppose that there exists a player (say, player k) whose payoff function u_k is upper semi-continuous on X_k, strictly quasi-concave on X_k, and supermodular on X, in addition, its reaction function ψ_k is locally responsive on X_{-k}. Then, the graph Ψ_k of player k's reaction function is a vN-M stable set for G_γ.*

Proof. (Note that, by the upper semi-continuity and the strictly quasi-concavity of u_k on X_k, $\psi_k(x_{-k})$ is well-defined and singleton-valued for each and every $x_{-k} \in X_{-k}$.)

[**Internal stability**] Take any $x = (x_k, x_{-k}) \in \Psi_k$. Consider player k, and take any $y \in \gamma_k(x)$ with $y \neq x$, that is, $y_k \neq x_k$ and $y_{-k} = x_{-k}$. We have $u_k(x_k, x_{-k}) = u_k(\psi_k(x_{-k}), x_{-k}) \geq u_k(y_k, x_{-k}) = u_k(y_k, y_{-k})$ by the definition of Ψ_k. Thus, x can not be dominated through k.

Next, consider an arbitrary player $j \in N \backslash \{k\}$ and take any $z = (z_k, z_{-k}) \in \gamma_j(x)$ with $z \neq x$, that is, $z_j \neq x_j$, $z_k = x_k$, and $z_i = x_i$ for all $i \in N \setminus \{j, k\}$. We have either $x_{-k} < z_{-k}$ or $x_{-k} > z_{-k}$. Since ψ_k is strictly increasing in each of its arguments by lemma 2, then we have $\psi_k(z_{-k}) \neq \psi_k(x_{-k}) = x_k = z_k$, which implies $z \notin \Psi_k$. There is no outcome in Ψ_k that dominates x through $j \in N \setminus \{k\}$. Hence, Ψ_k is internally stable.

[**External stability**] Take any $x = (x_k, x_{-k}) \in X \setminus \Psi_k$. Let us define $y = (y_k, y_{-k}) \in X$ such that $y_k = \psi_k(x_{-k})$ and $y_{-k} = x_{-k}$. Clearly, we have $y_k \neq x_k$, $y \in \gamma_k(x)$, and $y \in \Psi_k$. Further, by the definition and singleton-valuedness of ψ_k, we have $u_k(y) > u_k(x)$. That is, y dominates x through k. Hence, Ψ_k is externally stable. **Q.E.D.**

Nakanishi (2001) has shown the efficiency property of the vN-M stable set for the n-player prisoners' dilemma game. Unfortunately, the conditions required in our theorem 1 do not guarantee such an efficiency result. To establish the efficiency result, we have to modify some of the conditions required in theorem 1 as follows.

Condition 5 (upper semi-continuity of $\mathbf{u_i}$ on X) *If the upper-contour sets* $\{x \in X|\ u_i(x) \geq \alpha\}$ *are closed for all* $\alpha \in \mathbf{R}$*, then the payoff function* u_i *is said to be upper semi-continuous on* X*.*

Condition 6 (strict quasi-concavity of $\mathbf{u_i}$ on X) *If the payoff function* u_i *satisfies*

$$\min\{u_i(x), u_i(y)\} < u_i(\lambda x + (1-\lambda)y) \tag{12}$$

for any distinct $x, y \in X$ *and for any real number* $\lambda \in (0,1)$*, then it is said to be strictly quasi-concave on* X*.*

Theorem 2 *Suppose that there exists a player (say, player k) whose payoff function* u_k *is upper semi-continuous on* X*, strictly quasi-concave on* X*, and supermodular on* X *and, in addition, its reaction function* ψ_k *is locally responsive on* X_{-k}*. Then, the graph* Ψ_k *of player k's reaction function is a vN-M stable set for* G_γ *and it includes at least one Pareto-efficient outcome.*

Proof. It is easy to verify that the upper semi-continuity of u_k on X implies the upper semi-continuity of u_k on X_k for each $x_{-k} \in X_{-k}$ and that the strict quasi-concavity of u_k on X implies the strict quasi-concavity of u_k on X_k for each $x_{-k} \in X_{-k}$. By theorem 1, Ψ_k is a vN-M stable set for G_γ. Then, we only have to show that Ψ_k includes a Pareto-efficient outcome.

Because u_k is upper semi-continuous and strictly quasi-concave on X (which is a compact subset of $\mathbf{R}^n$ with respect to the Euclidean topology), there exists a maximum of u_k on X and its maximizer is determined uniquely, which we denote as $x^* = (x_k^*, x_{-k}^*) \in X$. By definition, $u_k(x^*) > u_k(y)$ for any distinct $y \in X$; Therefore, x^* is Pareto-efficient. On the other hand, by the definition of ψ_k, we have $x_k^* = \psi_k(x_{-k}^*)$. Hence, $x^* \in \Psi_k \cap E$. **Q.E.D.**

4 Remarks

We have shown the existence (theorem 1) and the efficiency (theorem 2) of the vN-M stable set for the n-player ICT situation G_γ associated with a normal form game G. Some remarks are in order.

The conditions stated in theorem 1 do not guarantee the continuity of the reaction function ψ_k on X_{-k}. Therefore, the vN-M stable set Ψ_k may not be a closed and/or connected subset of X. In general, we can not expect the vN-M stable set for G_γ to have some *nice*—topologically as well as algebraically—features such as connectedness, closedness, compactness, convexity, finiteness, and so forth.

Unlike Nakanishi (2001), theorem 2 does not assert that *every* vN-M stable set includes at least one Pareto-efficient outcome; It only asserts that a particular vN-M stable set can include a particular Pareto-efficient outcome. In view of Muto and Okada (1998), there can be a vN-M stable set that is completely separated from the set of Pareto-efficient outcomes.

The local responsiveness of ψ_i by itself does not necessarily imply the monotonicity of ψ_i; It is possible to have $\psi_i(x_{-i}) = \psi_i(y_{-i})$ for two outcomes x_{-i} and y_{-i} sufficiently distant from each other. The local responsiveness may seem a rather strong condition, but it is satisfied automatically in many practical problems and theoretical examples. For example, the reaction function of each player in a Cournot duopoly game with linear demand function and linear cost functions satisfies the local responsiveness.[8]

Lastly, let us briefly mention the relation between the Nash equilibrium for G and the ICT situation G_γ associated with G. Even under the conditions stated in theorem 1 and/or theorem 2, the Nash equilibrium for G may not exist. If the set of the Nash equilibria for G is non-empty, then, as shown in theorem 7.4.1 in Greenberg (1990), it is contained in every vN-M stable set for G_γ. If all the conditions in our theorem 2 are satisfied by all the players, then the graph of the reaction function of each player $i \in N$ is a vN-M stable set for G_γ and, further, the intersection of the graphs of the reaction functions of all the players (i.e., $\bigcap_{i \in N} \Psi_i$) coincides with the set of the Nash equilibria for G. It should be noted, however, that the set of the Nash equilibria for G thus obtained does not necessarily constitute a vN-M stable set for G_γ, unless all the graphs of the reaction functions are identical. In general, we can show that neither an intersection of any distinct vN-M stable sets nor a union of them is a vN-M stable set for G_γ.

[8] The payoff function of a player in this Cournot duopoly game does not satisfy the supermodularity in its original form. But, we can modify the strategic variables as in Fudenberg and Tirole (1992, Chap. 12, Sec. 3) appropriately so that the payoff function satisfies the supermodularity. Hence, this Cournot duopoly game admits the existence of the vN-M stable set.

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