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Abstract

This paper shows that we can not deal with multiple public goods contributed by many persons in the theory of private provision of public goods. It is only possible that only one individual contributes multiple public goods.

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1 Introduction

We will establish in this paper a fundamental fact that the number of public goods contributed by many individuals must be *unity* in almost all the models on the private provision of public goods.

It may appear easy for the economists in the theory of private provision of public goods to expand the model containing one public good to that containing multiple public goods. Kemp(1984) constructed a model where every individual contributed multiple public goods in the equilibrium. A situation was considered in Bergstrom-Blume-Varian(1986) where multiple individuals contributed multiple public goods. Our result, on the other hand, shows that these economies are almost vacuous. That is, let H and D be a group consisting of two or more individuals and an arbitrary set of multiple public goods respectively. Then we can not deal with the economies where each individual in H contributes every public good in D. It is established by a simple fact that the number of equations are strictly greater than the number of unknowns.

It is Warr(1983) who pointed out that the number of equations exceeded that of unknowns when all the individuals were contributors to every public good. Our result is a generalization of his in two respects. One is that non-contributors to the public goods can exist in our setting. The other is that no operations concerning the differentiability of functions are exploited.

2 The Number of Public Goods

Consider an economy which contains one private good and m kinds of public goods. Denote the number of individuals, the index set of individuals and the set of public goods by n, $N \stackrel{\text{def}}{=} \{1, 2, \ldots, n\}$ and $M \stackrel{\text{def}}{=} \{1, 2, \ldots, m\}$ respectively[1]. An individual i's

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[1]The symbol "$\stackrel{\text{def}}{=}$" implies that the left hand side is defined by the right hand side.

utility function is represented by $u_i : (x_i, G_1, \ldots, G_m) \in \mathbb{R}_+^{m+1} \mapsto u_i(x_i, G_1, \ldots, G_m) \in \mathbb{R}$, where x_i and $G_k, k \in M$ are the amount of private good and that of the k-th public good respectively[2]. Let $I \stackrel{\text{def}}{=} (I_1, \ldots, I_n) \in \mathbb{R}_{++}^n$ be a given income distribution. A list $(N, (u_i, I_i)_{i \in N})$ is an *economy*. Producing one unit of each public good requires one unit of private good. The price of the private good is unity.

We assume:

Assumption 1 *The utility function $u_i(x_i, G_1, \ldots, G_m)$ is continuous, increasing, quasi-concave in $\mathbb{R}_+^{m+1}$, strictly increasing and strictly quasi-concave in $\mathbb{R}_{++}^{m+1}$, for $i \in N$.*

Suppose that a vector of all the individuals' contributions to each public good $g_k \stackrel{\text{def}}{=} (g_k^1, \ldots, g_k^n)$, $k \in M$ is given where g_k^j is the amount of contribution to the k-th public good by the individual j. We consider a maximization problem for each $i \in N$:

$$\left.\begin{array}{c} \max_{x_i, g_1^i, \ldots, g_m^i} u_i(x_i, \sum_{j \neq i} g_1^j + g_1^i, \ldots, \sum_{j \neq i} g_m^j + g_m^i) \\ \text{sub. to} \\ x_i + g_1^i + \cdots + g_m^i = I_i, \; g_k^i \geqq 0, k \in M \end{array}\right\}. \tag{1}$$

Denote the solution to (1) by $(x_i(g_1, \ldots, g_m), (\psi_k^i(g_1, \ldots, g_m))_{k=1}^m)$, $i \in N$. $x_i(g_1, \ldots, g_m)$ is the amount of private good of individual i and $\psi_k^i(g_1, \ldots, g_m)$ is the k-th public good.

Let us consider a following artificial maximization problem for each i:

$$\max_{x_i, G_1, \ldots, G_m} u_i(x_i, G_1, \ldots, G_m) \;\; \text{sub. to } x_i + G_1 + \cdots + G_m = Y_i, \tag{2}$$

where Y_i is a positive real. The solution to the problem (2) is denoted by $(\xi^i(Y_i), \phi_k^i(Y_i))$, $k \in M, i \in N$, where the values of ξ^i and ϕ_k^i correspond to the amount of private good and the amount of k-th public good respectively.

Definition 1 [Nash Equilibrium]
An allocation $((x_i^*)_{i \in N}, (g_k^*)_{k=1}^m)$ is a Nash equilibrium in an economy $(N, (u_i, I_i)_{i \in N})$ when

$$\psi_k^i(g_1^*, \ldots, g_m^*) = g_k^{i*}, \;\; \forall i \in N, \;\; k \in M \tag{3}$$

$$x_i^* = x_i(g_1^*, \ldots, g_m^*), \;\; \forall i \in N. \tag{4}$$

2.1 Multiple Public Goods Economy

Consider simultaneous equations with respect to unknowns $Y_i, i \in N$:

$$\phi_k^i(Y_i) = \phi_k^j(Y_j), \;\; \forall i, j \in N, \;\; k \in M \tag{5}$$

$$\phi_1^1(Y_1) + \cdots + \phi_m^1(Y_1) = \sum_{i=1}^n \left(I_i - \xi^i(Y_i)\right) \tag{6}$$

[2]The sets $\mathbb{R}$, $\mathbb{R}_+^\ell$ and $\mathbb{R}_{++}^\ell$ are the set of real numbers, ℓ dimensional non-negative vectors and ℓ dimensional strictly positive vectors, respectively.

Then we establish a theorem which is stated as follows:

Theorem 1 *Suppose that Assumption* 1 *holds. A necessary and sufficient condition for a Nash equilibrium* $((x_i^*)_{i\in N}, (g_k^*)_{k=1}^m)$ *satisfying* $x_i^* > 0$ *and* $g_k^{i*} > 0, k \in M, i \in N$ *to exist, is that there exists a solution* $(Y_i^*)_{i\in N}$ *satisfying* $\xi^i(Y_i^*) > 0$ *and* $\phi_k^i(Y_i^*) > 0$*,* $k \in M$*,* $i \in N$ *to the system* (5) *and* (6)*.*

Proof. [**Necessity**] Let $((x_i^*)_{i=1}^n, (g_k^*)_{k=1}^m) \in \mathbb{R}_{++}^{nm+n}$ be a Nash equilibrium. Define $Y_i^* \stackrel{\text{def}}{=} I_i + \sum_{k=1}^m \sum_{j\neq i} g_k^{j*},\ i \in N$ and $G_k^* \stackrel{\text{def}}{=} \sum_{i\in N} g_k^{i*},\ k \in M$. We have $Y_i^* = x_i^* + G_1^* + \cdots + G_m^*$. Let $(x_i^\#, G_1^\#, \ldots, G_m^\#)$ be a solution to the problem (2) when $Y_i = Y_i^*$. Then the inequality $u_i(x_i^*, G_1^*, \ldots, G_m^*) \leqq u_i(x_i^\#, G_1^\#, \ldots, G_m^\#)$ holds. Suppose that $u_i(x_i^*, G_1^*, \ldots, G_m^*) < u_i(x_i^\#, G_1^\#, \ldots, G_m^\#)$. Define $x_i(\lambda)$ and $G_k(\lambda)$ for λ satisfying $0 < \lambda < 1$ as

$$x_i(\lambda) \stackrel{\text{def}}{=} \lambda x_i^\# + (1-\lambda)x_i^*,$$
$$G_k(\lambda) \stackrel{\text{def}}{=} \lambda G_k^\# + (1-\lambda)G_k^*,\ k \in M.$$

We have $x_i(\hat\lambda) > 0$ and $G_k(\hat\lambda) > \sum_{j\neq i} g_k^{j*} > 0, k \in M$ for a sufficiently small $\hat\lambda$, since $x_i^* > 0$ and $G_k^* > \sum_{j\neq i} g_k^{j*} > 0, k \in M$. Therefore both points $(x_i^*, G_1^*, \ldots, G_m^*)$ and $(x_i(\hat\lambda), G_1(\hat\lambda), \ldots, G_m(\hat\lambda))$ are interior points of the domain of the utility function. Due to strict quasi-concavity of utility functions, we have

$$u_i(x_i^*, G_1^*,\ \ldots, G_m^*) < u_i(x_i(\hat\lambda), G_1(\hat\lambda), \ldots, G_m(\hat\lambda)).$$

Moreover, since $x_i(\hat\lambda) + G_1(\hat\lambda) + \cdots + G_m(\hat\lambda) = Y_i^*$, we have

$$\hat g_k^i \stackrel{\text{def}}{=} G_k(\hat\lambda) - \sum_{j\neq i} g_k^{j*} > 0, k \in M$$
$$x_i(\hat\lambda) + \sum_{k=1}^m \left(G_k(\hat\lambda) - \sum_{j\neq i} g_k^{j*} \right) = I_i,$$
$$u_i(x_i(\hat\lambda), G_1(\hat\lambda), \ldots, G_m(\hat\lambda)) = u_i(x_i(\hat\lambda), \sum_{j\neq i} g_1^{j*} + \hat g_1^i, \ldots, \sum_{j\neq i} g_m^{j*} + \hat g_m^i).$$

This contradicts the fact that $((x_i^*)_{i=1}^n, (g_k^*)_{k=1}^m)$ is a Nash equilibrium. Therefore the $m+1$-tuple $(x_i^*, G_1^*, \ldots, G_m^*)$ is a solution to the problem (2) when $Y_i = Y_i^*$. This is true for any i. Therefore, $(Y_i^*)_{i\in N}$ is the solution to the simultaneous equations (5) and (6).
[**Sufficiency**] Let $(Y_i^*)_{i\in N}, i \in N$ be a solution to (5) and (6). Note that

$$\left(T \stackrel{\text{def}}{=}\right) \sum_{i=1}^n (I_i - \xi^i(Y_i^*)) = \sum_{k=1}^m \phi_k^1(Y_1^*) > 0.$$

Define for each $i \in N$ and $k \in M$

$$g_k^{i*} \stackrel{\text{def}}{=} \frac{1}{T}\left(I_i - \xi^i(Y_i^*)\right)\phi_k^1(Y_1^*) > 0, \quad x_i^* \stackrel{\text{def}}{=} \xi^i(Y_i^*) > 0.$$

By definition, we have

$$\begin{aligned} x_i^* + g_1^{i*} + \cdots + g_m^{i*} &= \xi^i(Y_i^*) + \sum_{k=1}^m \frac{1}{T}\left(I_i - \xi^i(Y_i^*)\right)\phi_k^1(Y_1^*) \\ &= \xi^i(Y_i^*) + I_i - \xi^i(Y_i^*) = I_i, \ i \in N, \\ G_k^* \stackrel{\text{def}}{=} \sum_{j \in N} g_k^{j*} &= \frac{1}{T}\sum_{j=1}^n \left(I_j - \xi^j(Y_j^*)\right)\phi_k^1(Y_1^*) = \phi_k^i(Y_i^*), \ i \in N, k \in M. \end{aligned}$$

This implies that $(x_i^*, g_1^{i*}, \ldots, g_m^{i*})$ satisfies the budget constraint in (1). Let $(x_i^{\#}, g_1^{i\#}, \ldots, g_m^{i\#})$ be a solution to the following problem:

$$\max u_i\left(x_i, G_1^{-i*} + g_1^i, \ldots, G_m^{-i*} + g_m^i\right) \text{ sub. to } \ x_i + g_1^i + \cdots + g_m^i = I_i,$$

where $G_k^{-i*} \stackrel{\text{def}}{=} \sum_{j \neq i} g_k^{j*}$, $k \in M$. It is obvious that $u_i(x_i^{\#}, G_1^{-i*} + g_1^{i\#}, \ldots, G_m^{-i*} + g_m^{i\#}) \geqq u_i(x_i^*, G_1^{-i*} + g_1^{i*}, \ldots, G_m^{-i*} + g_m^{i*})$. Suppose that the strict inequality were true, i.e. $u_i(x_i^{\#}, G_1^{-i*} + g_1^{i\#}, \ldots, G_m^{-i*} + g_m^{i\#}) > u_i(x_i^*, G_1^{-i*} + g_1^{i*}, \ldots, G_m^{-i*} + g_m^{i*})$. Define $x_i(\lambda) \stackrel{\text{def}}{=} \lambda x_i^{\#} + (1-\lambda)x_i^*$, $G_k(\lambda) \stackrel{\text{def}}{=} G_k^{-i*} + \lambda g_k^{i\#} + (1-\lambda)g_k^{i*}$, $k \in M$ for any λ satisfying $0 < \lambda < 1$. It is obvious that $x_i(\lambda) > 0$ and $G_k(\lambda) > 0$ since $x_i^* > 0$ and $g_k^{i*} > 0$, $k \in M$. By the strict quasi-concavity in the interior of the domain of utility function, we have $u_i(x_i(\lambda), G_1(\lambda), \ldots, G_m(\lambda)) > u_i(x_i^*, G_1^*, \ldots, G_m^*)$. On the other hand, we obtain:

$$\begin{aligned} x_i(\lambda) + \sum_{k=1}^m G_k(\lambda) &= \lambda x_i^{\#} + (1-\lambda)x_i^* + \sum_{k=1}^m \left(G_k^{-i*} + \lambda g_k^{i\#} + (1-\lambda)g_k^{i*}\right) \\ &= \lambda(x_i^{\#} + \sum_{k=1}^m g_k^{i\#}) + (1-\lambda)(x_i^* + \sum_{k=1}^m g_k^{i*}) + \sum_{k=1}^m G_k^{-i*} \\ &= I_i + \sum_{k=1}^m G_k^{-i*} = x_i^* + \sum_{k=1}^m \sum_{j=1}^n g_k^{j*} = Y_i^*. \end{aligned}$$

This is a contradiction. And thus, $(x_i^*, g_1^{i*}, \ldots, g_m^{i*})$ is the solution to (1) when a list $(g_1^*, \ldots, g_m^*)$ is given. Therefore, $((x_i^*)_{i \in N}, (g_k^*)_{k=1}^m)$ is a Nash equilibrium. ∎

By the above theorem, it suffices for studying the system (5) and (6) for us to scrutinize the properties of the Nash equilibrium. In the simultaneous equations (5) and (6), there are n kinds of unknowns: $Y_1, \ldots, Y_n$. On the other hand, there are $mn - m + 1$ equations in (5) and (6). When $m = 1$, the number of equations coincides with that of unknowns. In the general case that $m > 1$, the number of unknowns is strictly less than that of equations. There are no solutions to the system (5) and (6) without exceptional cases.

We are to show the essence of the above theorem by an example. Let $n = m = 2$ and let utility functions of individuals be of Cobb-Douglas type:

$$\begin{aligned}
&u_i(x_i, G_1, G_2) = (x_i)^{\alpha^i}(G_1)^{\beta^i}(G_2)^{\gamma^i}, \\
&\alpha^i + \beta^i + \gamma^i = 1, \alpha^i > 0, \beta^i > 0, \gamma^i > 0, i = 1, 2.
\end{aligned}$$

The equations (5) and (6) are represented by:

$$\begin{aligned}
\beta^1 Y_1 &= \beta^2 Y_2 \\
\gamma^1 Y_1 &= \gamma^2 Y_2 \\
\beta^1 Y_1 + \gamma^1 Y_1 &= I_1 + I_2 - \alpha^1 Y_1 - \alpha^2 Y_2.
\end{aligned}$$

Clearly, there exist no solutions unless $\beta^1/\beta^2 = \gamma^1/\gamma^2$.

We can summarize the above arguments as follows.

> **We can not necessarily depict an economy where many (more than or equal to two) individuals contribute two or more public goods in Nash equilibrium.**

2.2 Two Public Goods Economy

In this section we scrutinize further the problem on the number of public goods in the private provision of public goods. The remaining problem is to answer whether one individual can contribute many public goods.

We can simplify an economy $(N, (u_i, I_i)_{i\in N})$ containing m kinds of public goods into the one containing two public goods. Let $((x_i^*)_{i\in N}, (g_k^*)_{k=1}^m)$ be a Nash equilibrium in the economy $(N, (u_i, I_i)_{i\in N})$. Define:

$$\begin{aligned}
v_i(x_i, G_1, G_2) &\stackrel{\text{def}}{=} u_i\left(x_i, G_1, G_2, \sum_{j=1}^n g_3^{j*}, \ldots, \sum_{j=1}^n g_m^{j*}\right), \ i \in N \\
y_i &\stackrel{\text{def}}{=} I_i - \sum_{k\neq 1,2} g_k^{i*}
\end{aligned}$$

The economy $(N, (v_i, y_i)_{i\in N})$ thus derived is an economy with two public goods where $((x_i^*)_{i\in N}, (g_k^*)_{k=1}^2)$ is a Nash equilibrium.

Define three types of the contributors to public goods as:

$$\begin{aligned} J &\stackrel{\text{def}}{=} \{i \in N \mid g_1^{i*} > 0, g_2^{i*} > 0\}, \\ J_1 &\stackrel{\text{def}}{=} \{i \in N \mid g_1^{i*} > 0, g_2^{i*} = 0\}, \\ J_2 &\stackrel{\text{def}}{=} \{i \in N \mid g_1^{i*} = 0, g_2^{i*} > 0\}. \end{aligned}$$

The key problem (2) can be rewritten according to the sets of contributors.

$$\max_{x_i, G_1, G_2} v_i(x_i, G_1, G_2) \text{ sub. to } x_i + G_1 + G_2 = Y_i, \; i \in J, \tag{7}$$

$$\max_{x_i, G_1} v_i(x_i, G_1, G_2^*) \text{ sub. to } x_i + G_1 = Y_i, \; i \in J_1, \tag{8}$$

$$\max_{x_i, G_2} v_i(x_i, G_1^*, G_2) \text{ sub. to } x_i + G_2 = Y_i, \; i \in J_2. \tag{9}$$

Denote the solutions to these problems by $\xi^i(Y_i), \phi_1^i(Y_i)$ and $\phi_2^i(Y_i)$. Consider simultaneous equations with respect to unknowns $Y_i, i \in J \cup J_1 \cup J_2$ as follows:

$$\phi_1^i(Y_i) = \phi_1^j(Y_j), \; \forall i, j \in J \cup J_1 \tag{10}$$

$$\phi_2^i(Y_i) = \phi_2^j(Y_j), \; \forall i, j \in J \cup J_2 \tag{11}$$

$$\phi_1^i(Y_i) + \phi_2^i(Y_i) = \sum_{j \in J \cup J_1 \cup J_2} \left(y_j - \xi^j(Y_j)\right), \; \forall i \in J \tag{12}$$

$$\phi_1^i(Y_i) = \sum_{j \in J} \left(y_j - \xi^j(Y_j) - g_2^{j*}\right) + \sum_{j \in J_1} \left(y_j - \xi^j(Y_j)\right), \; \forall i \in J_1 \tag{13}$$

$$\phi_2^i(Y_i) = \sum_{j \in J} \left(y_j - \xi^j(Y_j) - g_1^{j*}\right) + \sum_{j \in J_2} \left(y_j - \xi^j(Y_j)\right), \; \forall i \in J_2. \tag{14}$$

We establish a theorem which is stated as follows:

Theorem 2 *Suppose that Assumption* 1 *holds. If there exist a Nash equilibrium* (x_i^*, g_k^{i*}), $k = 1, 2$, $i \in N$ *in an economy* $(N, (v_i, y_i)_{i \in N})$, *then there exists a solution* $Y_i^*, i \in J \cup J_1 \cup J_2$ *to the simultaneous equations* (10), (11), (12), (13) *and* (14).

Proof. Let $((x_i^*)_{i \in N}, (g_k^*)_{k=1}^2)$ be a Nash equilibrium. Define

$$\begin{aligned} Y_i^* &\stackrel{\text{def}}{=} y_i + \sum_{j \neq i} (g_1^{j*} + g_2^{j*}), \; i \in J \\ Y_i^* &\stackrel{\text{def}}{=} y_i + \sum_{j \neq i} g_k^{j*}, \; i \in J_k, k = 1, 2. \end{aligned}$$

Since $G_k^* \stackrel{\text{def}}{=} \sum_{i \in J \cup J_k} g_k^{i*}$, $k = 1, 2$, it is clear that

$$\begin{aligned} Y_i^* &= x_i^* + G_1^* + G_2^*, && \text{if } i \in J \\ &= x_i^* + G_1^*, && \text{if } i \in J_1 \\ &= x_i^* + G_2^*, && \text{if } i \in J_2. \end{aligned}$$

We can repeat the same argument as in the proof of Theorem 1 to show that (x_i^*, G_1^*, G_2^*) is a solution to the maximization problem (7) for the individual $i \in J$ when $Y_i = Y_i^*$. By the similar procedure, for the individual $i \in J_1$, (x_i^*, G_1^*) is a solution to the maximization problem (8) when $Y_i = Y_i^*$. Similarly for the individuals $i \in J_2$, (x_i^*, G_2^*) is a solution to the problem (9) when $Y_i = Y_i^*$. The above discussion implies that Y_i^*, $i \in J \cup J_1 \cup J_2$ form a solution to the simultaneous equations (10), (11), (12), (13), and(14). ∎

By Theorem 2, a Nash equilibrium is characterized by the equations (10), (11), (12), (13) and (14). The equations depend on the sets J, J_1 and J_2 which is determined by the Nash equilibrium. In this sense, they may be impractical when we are to find a Nash equilibrium. However they are pertinent to our present objective, i.e. for us to make it clear whether one individual can contribute many public goods.

The numbers of equations in (10) and (11) are $\#J + \#J_1 - 1$ and $\#J + \#J_2 - 1$ respectively[3]. The number of equations in (12), (13) and (14) is three. In short, the number of unknowns $Y_i, i \in J \cup J_1 \cup J_2$ is $\#J + \#J_1 + \#J_2$ whereas the number of equations is $2 \times \#J + \#J_1 + \#J_2 + 1$.

Let us examine whether the numbers of these equations and unknowns are identical. We distinguish three cases (i) $\#J = 0$, (ii) $\#J = 1$ and (iii) $\#J \geqq 2$.

[Case 1: $\#J = 0$] In this case the equation (12) is null. Therefore the total number of the equations is equal to $\#J_1 + \#J_2$ which is the number of the unknowns. In other words, Nash equilibria can exist when all the individuals contribute to a single public good.

[Case 2: $\#J = 1$] If either $J_1 \neq \emptyset$ or $J_2 \neq \emptyset$ holds, the number of equations in (10), (11) and (12) is equal to $(1 + \#J_1 - 1) + (1 + \#J_2 - 1) + 1 = \#J + \#J_1 + \#J_2$. Therefore at least one of the equations (13) and (14) is redundant. In other words, there exist no Nash equilibria when $J_1 \neq \emptyset$ or $J_2 \neq \emptyset$. The remaining case is $J_1 = \emptyset$ and $J_2 = \emptyset$. In this case, the equations (10) and (11) are null. In addition to this, the two equations (13) and (14) are equivalent to (12). Hence the number of unknowns coincides with that of equations.

[Case 3: $\#J \geqq 2$] The total number of equations in (10), (11) and (12) is $2 \times \#J + \#J_1 + \#J_2 - 1$ which is strictly greater than $\#J + \#J_1 + \#J_2$. Therefore the equations (13) and (14) are redundant. Thus there do not necessarily exist Nash equilibria.

In the above discussion, the choice of public goods 1 and 2 is arbitrary. Above inference applies to any pair of two public goods. Therefore, we can conclude as follows.

[3]Let A be a set. The symbol "$\#A$" represents the cardinality of the set A.

Two cases are possible in the models of private provision of public goods. One is that one individual contributes to multiple public goods. The other is that many individual contribute to one particular public good.

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