The Preference Parameters and Cointegration Approach: The Case of the United States

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Abstract

In this paper, we estimate the intertemporal elasticity of substitution (hereafter IES) of durable and non-durable consumption expenditure in the United States taking a cointegration approach. We apply several types of cointegration and compare the results. The results show that some tests can support the existence of cointegration. However, the estimated IES does not have the correct sign. These results indicate that the *intra*temporal elasticity of substitution ignored in this paper might play an important role in estimating IES, as Ogaki and Reinhart (1998) states.

Keywords intertemporal elasticity of substitution, cointegration

1. Introduction

In this paper, we estimate the preference parameters in consumption, intertemporal elasticity of substitution (hereafter IES), using the durable and non-durable consumption data in the United States through the 1980s and 1990s. Here, we attempt to estimate the IES of the expenditure on each consumption good with a standard utility function whose form is additive separable.

The estimating method employed in previous studies is GMM (Generalized Method of Moment) proposed by Hansen (1982). Of course, GMM is an important and useful procedure from a statistical point of view and for issues in financial macroeconomics. However, the problem with the estimation Euler equation remains with this procedure. The problem is that the estimated preference parameters might not be consistent estimator because of the stationary nuisance factors included in the model, for example, liquidity constraint and preference shock. Ogaki (1992), Ogaki and Park (1997) and Nishiyama (2005) employ a cointegration approach and they treat these nuisance factors as the stationary error term. As a result, the estimated preference parameters become a super-consistent estimator. However,

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the model used in this approach is a very simple two-good model –based on the additive separable CRRA (Constant Relative Risk Aversion) utility function. Ogaki and Reinhart (1998) mentions that the *intra*temporal factor is important. They also obtain a positive value of the intertemporal elasticity of substitution with the model considering the *intra*temporal factor, not with the simple two-good model.

Here, we verify the robustness of this simple specification of consumer behavior with the macroeconomics data in the United States. We employ two types of cointegration tests; tests with null of *no* cointegration (Engle and Granger 1987,Gregory and Hansen 1996) and tests with null of cointegration (Shin 1994). These are the residual-based cointegration tests.

In this study, we examine the preference parameters and the robustness of the model through the several kinds of cointegration test. In Section 2, we show the derivation of cross-Euler equation. In Section 3, we perform the empirical analysis. Section4 is conclusion.

2. The Model

The utility function is

$$u(C_t, S_t) = a\frac{C_t^{1-1/\sigma}}{1-1/\sigma} + \frac{S_t^{1-1/v}}{1-1/v} \quad (1)$$

Here, C_t is the consumption expenditure on non-durable goods, S_t is the service flow of the durable goods expenditure, and σ and v are the intertemporal elasticity of substitution of non-durable and durable goods. a is the weight. Here, S_t is defined as

$$S_t = \sum_{i=0}^{\infty} \delta^i D_{t-i}$$

D_t is the consumption expenditure on durable goods at period t and δ is the depreciation rate. Then, the utility maximization problem can be written as follows:

$$U = E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i}, S_{t+i}) \quad (2)$$

$s.t. \quad A_{t+1} = (1 + r_{t+1})A_t + Y_t - P_t^C C_t - P_t^D D_t \quad (3)$

A_t is the stock price and r_t is the return or interest rate. P_t^C and P_t^D are the price level of non-durable and durable goods. The *intra*temporal optimization condition in respect of C_t and D_t is

$$\frac{P_t^D}{P_t^C} = \frac{\partial U/\partial D_t}{\partial U/\partial C_t} = \frac{E_t \sum_{i=0}^{\infty} \beta^i \delta^i S_{t+i}^{-1/v}}{aC_t^{-1/\sigma}} \quad (4)$$

Then, multiplying $S_t^{-1/v} \Big/ E_t \sum_{i=0}^{\infty} \beta^i \delta^i S_{t+i}^{-1/v}$ on both sides of (4) and taking the logarithm,

we can transform the equation as follows:

$$\ln\left[\frac{P_t^D}{P_t^C}\right] = const. + \frac{1}{\sigma}\ln C_t - \frac{1}{\nu}\ln S_t + e_{1t} \tag{5}$$

where e_t is the error term and it consisting of the growth rate of S_t,

$$e_{1t} = \ln\left(\frac{E_t\sum_{i=0}^{\infty}\beta^i\delta^i S_{t+i}^{-1/\nu}}{S_t^{-1/\nu}}\right) \tag{6}$$

Usually, we can presume that (6) is the stationary process because of the growth rate of S_t. In other words, the residual of (5) is stationary and a cointegrating vector might exist.

When we multiply $D_t^{-1/\nu}\Big/E_t\sum_{i=0}^{\infty}\beta^i\delta^i S_{t+i}^{-1/\nu}$ on both sides of (4), we obtain

$$\ln\left[\frac{P_t^D}{P_t^C}\right] = const. + \frac{1}{\sigma}\ln C_t - \frac{1}{\nu}\ln D_t + e_{2t} \tag{7}$$

where

$$e_{1t} = \ln\left(\frac{E_t\sum_{i=0}^{\infty}\beta^i\delta^i S_{t+i}^{-1/\nu}}{D_t^{-1/\nu}}\right) \tag{8}$$

In this case, Equation (8) can be regarded as stationary, because the service flow of durable goods expenditures S_t is the discounted sum of D_t. In addition, Equation (7) also might contain the cointegrating vector.

3. Empirical Analysis

Then, we perform the empirical analysis with Equation (5) and (7). We use the quarterly personal real consumption data (chained) and the implicit deflator in the National Income and Production Accounts (NIPAs). We use the population including the armed forces overseas published by the Census Bureau. The sample period is 1980:Q1-2001:Q4. We use the FFR (Federal Fund Rate) as the real rate of interest. Non-durable consumption deflators deflate the FFR.

The results of the stationarity test, KPSS test (Kwiatkowski et al. 1992), show that all variables in Equation (5) and (7) are I (1) variables (see Table 1 for detail)[1]. Then, we employ two types of residual-based cointegration test: Engle and Granger (1987)'s test and Shin (1994)'s test.

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[1] We set the bandwidth of Newey and West (1987)'s covariance estimator as 8.

First, we perform the Engle-Granger test. This test is the Dickey and Fuller (1979)'s unit root test for residuals. The results show the acceptance of the null in both cases (see Table 2 -1 for detail). Next, we perform the Shin (1994) test with null of cointegration. This test consists of the unit root test for residuals with KPSS. Here, we set the bandwidth of the Newey-West estimator as 8, same as in the KPSS unit root test. We consider a case with a constant term and the a case with constant and trend. Here, we utilize a technique incorporating leads and lags following Stock and Watson (1993)'s dynamic OLS (DOLS) in order to avoid a biased distribution in the estimated cointegrating vector. When we perform Shin (1994)'s test, we can utilize this *dynamic* equation. The *dynamic* equations we test and estimate here are as follows:

$$\ln\left[\frac{P_t^D}{P_t^C}\right] = const. + \frac{1}{\sigma}\ln C_t - \frac{1}{\nu}\ln S_t + \sum_{j=-p}^{p}\gamma_j^C \Delta\ln C_{t+j} + \sum_{j=-p}^{p}\gamma_j^S \Delta\ln S_{t+j} + e_{1t} \quad (9)$$

$$\ln\left[\frac{P_t^D}{P_t^C}\right] = const. + \frac{1}{\sigma}\ln C_t - \frac{1}{\nu}\ln D_t + \sum_{j=-p}^{p}\gamma_j^C \Delta\ln C_{t+j} + \sum_{j=-p}^{p}\gamma_j^S \Delta\ln S_{t+j} + e_{2t} \quad (10)$$

where Δ is the difference operator. In this case, we set $p = 4$. Here, we can analyze the specification with the linear trend in (9) and (10).

According to the results of the Shin's tests shown in Table 2-2, we cannot reject the null hypothesis of existence of cointegration in Equation (9) and (10) with a 5% significant level. When we employ the model with a linear trend, we obtain the same results (see Table 2-2) [2].

Then, we consider the existence of unknown structural change in the cointegration vector. Here, we perform the cointegration test developed by Gregory and Hansen (1996) (hereafter GH) that considers the structural break in the cointegrating vector based on Engle and Granger (1987)'s test. The null hypothesis of the GH test is no cointegration, and alternative is the existence of cointegration with possible structural break in the cointegrating vector. In this study, we set the possible point of structural change τ as $\tau \in (0.3T, 0.7T)$ where T is the size of a full sample. In this paper, we compare the following model[3]:

$$\ln\left[\frac{P_t^D}{P_t^C}\right] = const. + \kappa_0 D_\tau + \frac{1}{\sigma}\ln C_t - \frac{1}{\nu}\ln S_t + e_{1t} \text{ (C-model)} \quad (11)$$

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[2] The maximum eigenvalue test developed by Johansen (1988) and Johansen and Juselius (1991) also shows that cointegration does not exist between three variables or that the sign of the estimated parameters is not correct.

[3] The specification of the model is followed by Gregory and Hansen (1996).

$$\ln\left[\frac{P_t^D}{P_t^C}\right] = const. + \kappa_0 D_\tau + \frac{1}{\sigma}\ln C_t - \frac{1}{\nu}\ln S_t + \gamma_t t + e_{1t} \quad \text{(CT-model)} \tag{12}$$

The cases with the purchase of durable goods are as follows:

$$\ln\left[\frac{P_t^D}{P_t^C}\right] = const. + \kappa_0 D_\tau + \frac{1}{\sigma}\ln C_t - \frac{1}{\nu}\ln D_t + e_{2t} \tag{13}$$

$$\ln\left[\frac{P_t^D}{P_t^C}\right] = const. + \kappa_0 D_\tau + \frac{1}{\sigma}\ln C_t - \frac{1}{\nu}\ln D_t + \gamma_t t + e_{2t} \tag{14}$$

Here, D_τ is defined as follows:
$$D_\tau = 0 \qquad (t < \tau)$$
$$\quad = 1 \qquad (t \geq \tau)$$

The results of the GH test show that the null hypotheses of no cointegration are accepted with Equations (11), (12), (13) and (14) with a 5 % significant level (see Table 2-3 in detail)[4]. We obtain the results that support the existence of cointegration only by Shin's test. Then, the estimated parameters with the equations (9) and (10) can be regarded as the cointegrating vector. However, the results of estimation show that the preference parameters, ν and σ are not positive in all cases. In some cases, the preference parameters are not significant (see Table 3 in detail).

4. Concluding Remarks

In this paper, we perform the estimation of the preference parameters for durable and non-durable goods expenditure in the United States. We employ the cointegration test, which is a desirable method with nuisance factors considered. However, as shown in the previous section, the results of the Engle-Granger test and Gregory-Hansen test support the null hypothesis of no cointegration. The results of the tests of Shin (1994) show that the null hypothesis of cointegration is accepted. However, the estimated preference parameters have the incorrect sign in all cases.

Numerous attempts show that the estimated IES in the United States has an incorrect sign and that the model is not identified. Ogaki and Reinhart (1998) states that considering

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[4] The maximum number of the augmented terms is 8, the same as the Engle-Granger test.

the *intra*temporal elasticity of substitution between two kinds of goods is important in order to avoid the problem shown in previous works. Ogaki and Reinhart (1998) shows that IES tends to be negative when we ignore the *intra*temporal elasticity of substitution and treat two kinds of consumption goods together. The model we estimate in this paper is a two-good model, but the *intra*temporal elasticity of substitution is not considered. This might be the main cause of inability to obtain desirable results in the estimation of IES. The results of this paper support the idea that the *intra*temporal elasticity of substitution is an important factor in estimating IES, as Ogaki and Reinhart (1998) states. However, cointegration is an important and valid method for estimation of the preference parameter, as we mentioned before. As a future work, we should reconsider the utility function and empirical methods that will resolve the specification problems.

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Tables

Note: ** denotes the rejection of the null hypothesis with a significance level of 1%, and * denotes 5%.

Table 1 Unit root test (KPSS test, constant term only)

Variables	Test statistics	Variables	Test statistics
$\ln S_t$	1.793**	$\Delta \ln S_t$	0.314
$\ln D_t$	0.985**	$\Delta \ln D_t$	0.109
$\ln C_t$	1.750**	$\Delta \ln C_t$	0.391
$\ln\left(P_t^D / P_t^C\right)$	0.995**	$\Delta \ln\left(P_t^D / P_t^C\right)$	0.438

We set the bandwidth of the Newey and West (1987)'s covariance estimator as 8.

The critical values are 0.739(1 %) and 0.463(5%) (Kwiatkowski et al. 1992).

Table 2 –1 Cointegration test (Engle-Granger test)

Model	Test statistics (number of lags)	
Service flow (Const.)	-1.205(1)	1
Purchase of durable (Const.)	-0.811(0)	1
Service flow (Const. and trend)	-1.029(1)	2
Purchase of durable (Const. and trend)	-0.536(0)	2

1 The critical values are -4.31(1 %) and -3.77(5%) (Phillips and Ouliaris 1990).

2 The critical values are -4.65(1 %) and -4.16(5%) (Phillips and Ouliaris 1990).

Table 2-2 Cointegration test (Shin's test)

Model	Test statistics
Service flow (const.)	0.120
Purchase of durable (const.)	0.065
Service flow (const. and trend)	0.070
Purchase of durable (const. and trend)	0.051

We set the bandwidth of the Newey and West (1987)'s covariance estimator as 8. The critical values are 0.380 (1 %) and 0.221 (5 %) with constant, 0.150 (1 %) and 0.101 (5 %) with constant and trend (Shin 1994,m=2).

Table 2-3 Cointegration test (Gregory-Hansen test)

Model	Test statistics (break point)
Service flow (C-model)	-3.331 (82:4)
Purchase of durable (C-model)	-1.704(82:2)

The critical values are –5.44(1 %) and –4.92 (5%) (Gregory and Hansen1996).

Model	Test statistics
Service flow (CT-model)	-4.287 (82:4)
Purchase of durable (CT-model)	-1.517(82:2)

The critical values are –5.80 (1 %) and –5.29 (5%) (Gregory and Hansen1996).

Table 3 Cointegration Vector

With constant

	(Service flow)		(Purchase)
Parameter	Estimated (standard error)	Parameter	Estimated (standard error)
1/	-1.599(0.284)	1/	-1.852 (0.118)
/	-0.248 (0.121)	/	-0.352 (0.049)

With constant and trend

	(Service flow)		(Purchase)
Parameter	Estimated (standard error)	Parameter	Estimated (standard error)
1/	-2.582 (0.305)	1/	-2.117 (0.244)
/	-0.133 (0.104)	/	-0.388　(0.057)