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Oshio, Takashi
Yasuoka, Masaya

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How long should we stay in education if ability is screened?

Takashi Oshio and Masaya Yasuoka

Graduate School of Economics, Kobe University

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Abstract

We examine how ability screening affects demand for education and the shape of the optimal education system. Explicitly incorporating gradual screening by education into the model, we illustrate how individuals of different abilities decide to complete or leave education. Public education encourages low-ability individuals to stay longer in education, but this might cause their over-education, and reduce the efficiency of education for a society as a whole. A mixed education system, in which public education is provided before private education, can provide more insurance against the low-ability risk than a fully public system without any loss in efficiency compared to a fully private system.

Keywords: ability screening, drop-out, public education

JEL classification: I21, I22, I28

Correspondence: Takashi Oshio, oshio@econ.kobe-u.ac.jp, 2-1 Rokkodai-cho, Nada-ku, Kobe, Hyogo 657-8501, Japan, tel/fax +81-78-803-6818. We are grateful for financial support from Kobe and Hitotsubashi Universities.

1. Introduction

People are not informed of their abilities before receiving education, and the outcome of their education is always uncertain. Human capital theory (see Becker, 1964) argues that education serves to enhance productivity, but the benefits of education are exposed to uncertainties, and are dependent on individual innate abilities, which are not fully known. The signaling model (see Spence, 1973) argues that education serves as a signal of an individual's ability, assuming asymmetric information; i.e., the individual knows his/her own ability more precisely than others. Information about individual ability is, however, not available to everyone especially at an early stage of education, and it tends to be more commonly shared through a series of tests and sorting (see Burdett, 1978).

Therefore, it is not surprising to see a growing number of research studies discussing demand for education under uncertainty or risk, including Groot and Oosterbeek (1992), Altonji (1993), Dominitz and Manski (1996), and others. Individuals make education decisions under uncertainty about post-education earnings and innate abilities. The conventional view is that higher uncertainty and/or risk about abilities and educational outcomes would discourage people from investing in human capital, as is the case of investment in non-human capital. This is why public support in cash or in kind is often considered to be a desirable policy measure to encourage education, although it is arguable which type of support is most appropriate.

We should not ignore, however, that uncertainty—at least part of which is a *dream*—might give people an incentive to invest in human capital, and stay longer in education. If fully informed of their innate abilities, low-ability individuals who cannot expect positive gains from education are not inclined to receive education. In the same way, risk—at least part of which is a *chance*—about educational outcome might rather encourage individuals to stay in education. Risk about post-education earnings includes the opportunity to get high returns from education, which cannot be realized once one drops out. This is clearly pointed out by Hogan and Walker (2002), who apply the theory of real options to the problem of education choice. Equally important, a series of ability screenings by education reduce uncertainty and

risk about individual abilities by identifying them more precisely and make only those whose abilities turn out to be high stay in education. In this sense, education itself determines demand for itself.

The role of education to screen individual ability also has important implications for education policy. For example, screening ability by education can be justified from the viewpoint of efficiency, in that with educational attainments as a screen individuals can be more efficiently allocated jobs than otherwise would be possible (see Arrow, 1973). At the same time, ability screening is likely to widen inequalities in future income by revealing individual abilities more precisely. If so, the rationale of tax-financed public education, which could be regressive rather than progressive, will be controversial from the viewpoint of equity or income distribution (see Stiglitz, 1975). However, the possibility that public education reduces the net benefits for low-ability individuals cannot be ruled out, because it might make them inclined to stay too long in education due to financial support from the government.

This paper illustrates how screening ability by education affects demand for education, and determines the shape of the optimal education system, based on a simplified model. We first incorporate the mechanism of gradual screening by education into the model, to examine how individuals of different ability levels react to screening. In particular, we focus on an individual's choice between completion (staying in education) and drop-out (leaving education), by comparing the expected net benefits from the two choices. Given the individuals' choices, we assess both efficiency and equity aspects of private, public, and mixed education systems. Under uncertainty about individual ability, the equity aspect can be interpreted at an individual's level as insurance against the risk of turning out to be of low ability.

In the remainder of the paper, Section 2 first presents the basic model on which our theoretical analysis is based. Section 3 compares private, public, and mixed education systems in terms of efficiency, based on the net benefit from education for society as a whole. Section 4 compares equity, or insurance aspect of each education system, judging from the net benefit for low-ability individuals. To illustrate the key results of the theoretical analysis, Section 5

presents some simulation results. Section 6 provides a conclusion.

2. Basic model

2.1 Drop-out and completion

We consider a society consisting of two types of individual: high-ability individuals ($i=H$) and low-ability individuals ($i=L$), who account for p and $1-p$ ($0 < p < 1$), respectively, of the total population. Each type of individual obtains educational output worth $a_i x_i^\varepsilon$ ($0 < a_L < a_H$, $0 < \varepsilon \leq 1$) after receiving education of length x_i , where a_i is a parameter of individual innate ability and constant input elasticity of educational output is assumed. If individuals are fully informed of their abilities in advance, and precisely evaluate their educational outcomes, they maximize the net benefit from education, $a_i x_i^\varepsilon - c x_i$, where c is the unit cost of education. The optimal length of education for each type, denoted by x_H and x_L , is given by $x_H = (\varepsilon a_H / c)^{1/(1-\varepsilon)}$, $x_L = (\varepsilon a_L / c)^{1/(1-\varepsilon)}$, respectively. The social net benefit from education is maximized when the two types of individual receive education of lengths x_H and x_L , respectively.

Assume the government establishes an education system, the total length of which is equal to x_H , in a society. This education system, which consists of, for example, elementary and secondary schools, colleges, and graduate schools, is linear in that it has no multiple routes toward completion. Individuals can drop out (leave education) at any time rather than completing it, if they wish. Normalizing x_H as one to make calculations simple,⁴ we assume:

$$c = \varepsilon a_H, \tag{1}$$

which means $x_L = (a_L / a_H)^{1/(1-\varepsilon)} (< 1)$.

In our model, individuals do not know their abilities before receiving education, and under screening by education they become more informed of their education and choose at each time whether to complete education (i.e., stay in education until time one) or to drop out at that time, by comparing the expected net benefits from the two options.¹ Also, an individual who drops out is always treated as a low-ability individual,² and cannot return to education. Finally, we assume for simplicity that all individuals, including those who drop out, enter the job

market after the total education period.

Denote the expected net benefits from completion and drop-out evaluated at time x as $Q_i(x)$ ($i=H, L$) and $D(x)$, respectively, given ability screening by education. Individuals, who are assumed to be risk-neutral, make education choices based on the following rules.

Rules of education choices:

- As far as the expected net benefit from completion remains higher than that from drop-out ($Q_i(x) > D(x)$), one decides to stay in education.
- If the net benefit from drop-out is expected to be higher than that from completion and to rise ($Q_i(x) < D(x)$ and $D'(x) > 0$), one decides to stay in education.
- If the net benefit from dropping out is expected to be higher than that from completion and to fall ($Q_i(x) < D(x)$ and $D'(x) < 0$), one decides to drop out.

Manski (1984) takes another approach to drop-outs, explicitly considering an individual's decision about enrollment as well, although he does not explicitly incorporate a screening process in education. Also, there has been a rich literature of empirical analysis which find that family background, community diversity, and other socio-economic factors affect drop-outs as well as individuals' abilities and educational attainments.³ Our model ignores other factors than expected net benefits from completion and drop-out, and assumes that risk-neutral individuals make education choices in continuous rather than discrete time.

2.2 Screening

In our model, education gradually distinguishes individual abilities. An individual or a teacher has no information about individual abilities before education, so all individuals are conjectured to be of high ability with probability p and of low ability with probability $1-p$ at the beginning of education. Hence, their innate abilities are estimated to be the weighted average of low and high abilities: $pa_H + (1-p)a_L$. Through various educational achievements, communication with teachers, and so on, high-ability individuals will come to be more

precisely judged as such. Hence, they will put more weight on high ability when calculating their expected ability from p to one. In the same manner, low-ability individuals will put more weight on low ability when calculating their expected ability from $1-p$ toward one.

To trace these processes and make them tractable, we assume that at time x , the innate abilities of high- and low-ability individuals are estimated as $\theta_H(x)a_H+(1-\theta_H(x))a_L$ and $(1-\theta_L(x))a_H+\theta_L(x)a_L$, respectively, where

$$\theta'_H(x)>0, \theta''_H(x)<0, \theta_H(0)=p, \theta_H(1)=1, \text{ and}$$

$$\theta'_L(x)>0, \theta''_L(x)<0, \theta_L(0)=1-p, \theta_L(1)=1.$$

This mechanism of screening ability implies that individual abilities are fully distinguished at the end of education (time one) and that the marginal power of screening by education is decreasing. All individuals have ability screened by education as given, and they cannot know how additional education will affect screening of their abilities.

In addition, to make the social average of innate abilities precisely and consistently estimated as $pa_H+(1-p)a_L$ at any time x , we add the constraint:

$$p\theta_H(x)+(1-p)(1-\theta_L(x))=p, \tag{2}$$

which is equivalent to

$$p(1-\theta_H(x))+(1-p)\theta_L(x)=1-p. \tag{2}'$$

for any $x \in [0,1]$.

3. Efficiency

3.1 Private system

We first compare private and public economic systems in terms of efficiency by focusing on the net benefit from education for society as a whole. We start with a private education system, in which all individuals directly pay education costs. Before receiving education, neither type of individual knows his or her ability, and conjectures it as the weighted average of high- and low-abilities. We assume that they want to complete education when they start to receive education; otherwise, the total length of education set by the government would be too long to be justified.⁵ Because $D(x)$ is maximized at $D(x_L)$, this assumption is satisfied if:

$$pa_H+(1-p)a_L-c \geq D(x_L).$$

Under this condition, we can show the following proposition regarding completion and drop-outs in a private system.

Proposition 1. *In a private education system, high-ability individuals complete education, while low-ability individuals drop out if individual abilities have been distinguished to a sufficient degree by the end of the overall education period.*

Proof. In the private system, high-ability individuals always complete education because:

$$Q_H(x) = \theta_H(x)a_H + (1-\theta_H(x))a_L - c \geq pa_H + (1-p)a_L - c \geq D(x_L) \geq D(x)$$

for any $x \in [0, 1]$, meaning that their expected net benefit from completion is always higher than that from dropping out. For low-ability individuals, we compare $Q_L(x) = (1-\theta_L(x))a_H + \theta_L(x)a_L - c$ and $D(x) = a_Lx^\varepsilon - cx$, and denote the difference between the two as $Z_L(x) \equiv Q_L(x) - D(x)$. Because $Z_L(0) = pa_H + (1-p)a_L > 0$ and $Z_L(1) = 0$, low-ability individuals drop out at a certain point between 0 and 1, as far as $Z'_L(1) = -\theta'_L(1)(a_H - a_L) + (c - \varepsilon a_L) > 0$, which ensures that there exists $x_D \in (0, 1)$ such that $Z_L(x_D) = 0$. Using (1), this condition is rewritten as

$$\theta'_L(1) < \varepsilon \tag{3}$$

This condition (3) is likely to hold, if individual abilities have been distinguished to a sufficient degree by the end of the overall education period. *Q.E.D.*

We assume hereafter that condition (3) is satisfied, so low-ability individuals drop out, but the rules of education choices indicate that they do not drop out before time x_L , when their net benefit is maximized. Figure 1 illustrates the expected net benefits from completion and dropping out for both types of individual and compares two types of drop-outs of low-ability individuals. The curves $Q_H(x)$ and $Q_L(x)$ are sloping upward and downward, respectively, starting with the same value, $pa_H + (1-p)a_L - c$, at time zero, and ending with $a_H - c$ and $a_L - c$, respectively at time one⁵. The curve $Q_H(x)$ is always located above the curve $D(x)$ from (4) and $D(x)$ peaks at time x_L . The left part of Figure 1 illustrates the case in which low-ability

individuals drop out at time x_L , whereas the right part illustrates the case in which they drop at time x_D , later than at time x_L .

To sum up, under condition (3), the timing of low-ability individuals dropping out, which is denoted as x^* , is given by

$$x^* = \max(x_L, x_D).$$

Correspondingly, the social net benefit from education, which is expressed by the weighted average of the net benefits for both types of individual, is given by

$$W^{priv} = p(a_H - c) + (1-p)(a_L x^* - c x^*).$$

If low-ability individuals drop out at time x_L , the net benefit from education for both them and a society as a whole is maximized. If they drop out at time $x_D (> x_L)$, they receive over-education, which causes a loss in efficiency for society as a whole, as they realize their low ability too late.⁶ As implied in Figure 1, the more slowly education screens individual abilities, the more likely low-ability individuals are to receive over-education.

3.2 Public system

We now move to a public education system, where education costs are financed by income-proportional tax, with the rate, t , under which all individuals take it as given that they have an education choice. We assume no difference in quality and cost between private and public education. We can first confirm that both types of individual want to complete education before receiving it, because:

$$(1-t)[p a_H + (1-p) a_L] > (1-t) a_L \geq (1-t) a_L x^e = D(x_L),$$

for any $x \in [0, 1]$ and given the tax rate, t . Moreover, we can state as follows.

Proposition 2. *In a public education system, both high- and low-ability individuals complete education.*

Proof. For high-quality individuals, the expected net benefit from completion and drop-out are given by $Q_H(x) = (1-t)[\theta_H(x) a_H + (1-\theta_H(x)) a_L]$ and $D(x) = (1-t) a_L x^e$, respectively. As in the private

system, but for a different reason, they always complete education, because we have:

$$Q_H(x) \geq (1-t)[pa_H + (1-p)a_L] > (1-t)a_L \geq (1-t)a_L x^e = D(x),$$

for $x \in [0, 1]$ and given t . For low-ability individuals, the net benefit from completion is given by $Q_L(x) = (1-t)[(1-\theta_L(x))a_H + \theta_L(x)a_L]$. They complete education, because:

$$Q_L(x) \geq (1-t)a_L > (1-t)a_L x^e = D(x). \quad Q.E.D.$$

For low-ability individuals, the net benefit from completion remains higher than that from dropping out even if their low ability becomes more precisely identified in a public education system. This is because tax paid by high-ability individuals reduces the education cost of low-ability individuals. Then, the social net benefit is given by:

$$W^{pub} = pa_H + (1-p)a_L - c = p(a_H - c) + (1-p)(a_L - c),$$

which leads to the following proposition.

Proposition 3. *A private education system is more efficient than a public education system in terms of the net benefit for a society as a whole.*

Proof. Because $a_L x^e - cx$ is a decreasing function of x for $x_L < x < 1$ and $x_L \leq x^*$, we have:

$$W^{priv} = p(a_H - c) + (1-p)(a_L x^{*e} - cx^*) \geq p(a_H - c) + (1-p)(a_L - c) = W^{pub} \quad Q.E.D.$$

The inferiority of the public system in terms of the efficiency is caused by too much education received by low-ability individuals, reflecting the lowered education costs.

3.3 Mixed system

Next, we examine whether a combination of public and private education can produce more net benefits for society as a whole than the private system. Assume that the public sector partially provides public education of length λ ($0 \leq \lambda \leq 1$), in which all individuals can receive tax-financed education between 0 and λ . This education system is not far from the reality in which education is publicly financed more at an earlier stage, so the following analysis

roughly assesses the rationale of the current education system.

Individuals can receive additional, private education after completing public education if they wish, and drop out at any time,⁷ although they are not allowed to move to private education before completing public education. We continue to assume that individuals are treated as low-ability once they drop out at any time before completing the whole education period. Then, the expected net benefit from completion at time x is expressed as

$$Q_H(x) = (1-t)[\theta_H(x)a_H + (1-\theta_H(x))a_L] - c(1-\lambda), \text{ and}$$

$$Q_L(x) = (1-t)[(1-\theta_L(x))a_H + \theta_L(x)a_L] - c(1-\lambda)$$

for high- and low-ability individuals, respectively, given ability screening by education. The expected net benefit from dropping out at time x is given by

$$\begin{aligned} D(x) &= (1-t)a_Lx^\varepsilon && \text{for } 0 \leq x \leq \lambda, \\ &= (1-t)a_Lx^\varepsilon - c(x-\lambda) && \text{for } \lambda < x \leq 1, \end{aligned}$$

which peaks at time λ , or later (at $[(1-t)a_L/a_H]^{1/(1-\varepsilon)} < (a_L/a_H)^{1/(1-\varepsilon)} = x_L$ using (1)) if λ is low. Hence, given t , no individual drops out before time λ according to the rules of education choices. Then, we can show the following properties of the mixed system.

Proposition 4. *In a mixed education system, (i) high-ability individuals complete education, while low-ability individuals complete public education only or drop out later; and (ii) the government can set the length of public education so that low-ability individuals drop out no later than in the private system.*

See Appendix for proof. As a corollary of this proposition, we can also state the superiority of the mixed system to the private system in terms of efficiency as follows.

Proposition 5. *A mixed education system can be more efficient than a private education system in terms of the net benefit from education for a society as a whole.*

Proof. Given that low-ability individuals drop out at time x^+ , social net benefit from education

in this mixed system is given by

$$W^{mix} = p[(1-t)a_H - c(1-\lambda)] + (1-p)[(1-t)a_L x^{+\varepsilon} - c(x^+ - \lambda)] = p(a_H - c) + (1-p)(a_L x^{+\varepsilon} - c x^+),$$

based on the tax rate, which is calculated as $t = c\lambda / [pa_H + (1-p)a_L x^{+\varepsilon}]$, and we have:

$$W^{mix} - W^{priv} = (1-p)[(a_L x^{+\varepsilon} - c x^+) - (a_L x^{*\varepsilon} - c x^*)].$$

Considering that $a_L x^{+\varepsilon} - c x$ is a decreasing function of x ($\geq x_L$), the mixed system can yield a higher social net benefit from education than the private system if the government adjusts the length of public education so that $x_L \leq x^+ \leq x^*$. *Q.E.D.*

It is because the mixed system can reduce over-education of low-ability individuals that it can be more efficient than even a fully private system. The mixed system makes low-ability individuals drop out earlier than in the private system for two reasons: first, low-ability individuals have to pay directly the education cost after shifting to private education; and second, tax-financed public education reduces their education cost, which is *sunk* once they drop out. Moreover, the government might want to maximize W^{mix} by adjusting the length of public education to λ^* such that it makes low-ability individuals to drop out just at x_L ($x^+ = x_L$), which maximizes their net benefit (and the social net benefit as well).

4. Insurance against the low-ability risk

In the previous Sections 2 and 3, we concentrated mainly on the social net benefit from education, which is expressed as the weighted average of the net benefits obtained by the two types of individual. It is not, however, appropriate to assess education policy solely in terms of efficiency. This section examines to what extent education can provide insurance against the risk that one turns out to be of low ability through screening by education (referred to as low-ability risk hereafter). The more risk-averse people are, the more important the insurance aspect of education becomes. This issue is potentially important in terms of the economic equity of education, even though redistribution policy outside education can at least partly offset any regressive outcome of education.

To highlight the insurance aspect of education, we focus on the net benefit from education

for low-ability individuals and how education policy can affect it. In the private system, the net benefit for low-ability individuals is given by $W_L^{priv} = a_L x^{\varepsilon} - c x^*$, because they drop out at x^* . By contrast, that in the public system is given by $W_L^{pub} = (1-t)a_L$, where $t = c/[pa_H + (1-p)a_L] = \varepsilon a_H/[pa_H + (1-p)a_L] > \varepsilon$.

The public system is more effective for insuring against low-risk ability than the private system, because:

$$W_L^{priv} = a_L x^{\varepsilon} - c x^* \leq a_L x_L^{\varepsilon} - c x_L = (1 - c x_L^{1-\varepsilon}/a_L) a_L x_L^{\varepsilon} = (1 - \varepsilon) a_L x_L^{\varepsilon} < (1 - \varepsilon) a_L < (1 - t) a_L = W_L^{pub}.$$

This result is intuitively reasonable but adds nothing new to the conventional idea that public education acts as a redistribution policy transferring income from high-ability individuals to low-ability ones. It is of more interest to compare the mixed and public systems in terms of insurance against low-ability risk.

Proposition 6. *A mixed education system can be more effective for insuring against low-ability risk than a public education system.*

Proof. We focus on the case that low-ability individuals drop out just at λ ; in other words, they complete public education only. The sufficient condition for this is $\lambda \geq x^*$.⁸ Under this condition—i.e., as far as $x^* \leq \lambda \leq 1$ —the net benefit from education for low-ability individuals is expressed as

$$W_L^{mix} = (1-t)a_L \lambda^{\varepsilon} = \{1 - \lambda c/[pa_H + (1-p)a_L \lambda^{\varepsilon}]\} a_L \lambda^{\varepsilon}. \quad (4)$$

Differentiating (4) with respect to λ , and evaluating it at $\lambda=1$ (completion), we have:

$$dW_L^{mix}/d\lambda|_{\lambda=1} = -\varepsilon^2 p a_H a_L / [pa_H + (1-p)a_L] < 0,$$

which confirms that the mixed system can give higher net benefits for low-ability individuals than the public system. *Q.E.D.*

The intuitive interpretation of this proposition is that the mixed system, combined with ability screening, succeeds in reducing both over-education and education costs of low-ability individuals, a desirable outcome for them. To insure most against low-ability risk, the

government may want to search for the length of public education, λ_L^* , that maximizes W_L^{mix} within the range between x^* and one. We cannot algebraically solve it in general, and rule out a corner solution, so we have to rely on numerical simulations.⁹ Moreover, if the value of W_L^{mix} evaluated at $\lambda = x^*$ turns out to be higher than that evaluated at $\lambda = 1$, the government can insure against low-ability risk without any loss of the social net benefit obtained in the private system. We examine this possibility in the next section based on numerical simulations.

5. Numerical examples

5.1 Simulation methodology and results

This section presents some numerical examples to show how the length of public education in the mixed system affects the timing of low-ability individuals dropping out, the social net benefit from education, and insurance against low-ability risk. We have presented two key findings: (i) the mixed system can yield a higher net benefit from education for a society as a whole than the private one; and, (ii) the mixed system can provide more insurance against low-ability risk than the public system. We confirm these two findings with simple simulations, and examine whether there is any trade-off between the two policy targets.

We first specify $\theta_L(x)$ such that $\theta_L(x) = 1 - p + p x^\sigma$, $0 < \sigma < 1$.¹⁰ In addition, $\theta_L(x)$ must satisfy the condition (3), so a simple calculation shows that the inequality:

$$\varepsilon > p\sigma \quad (5)$$

must hold. We tentatively assume alternative parameter values such that: $\varepsilon = 0.4, 0.6$; $a_H/a_L = 1.5, 2.0$; $p = 0.25, 0.5$; and $\sigma = 0.75$ to examine eight cases in total. All combinations of ε and σ satisfy (5), and the education cost is given from (1).

In the simulations, we first solve x^* , the timing of low-ability individuals dropping out in the private system: $x^* = \max(x_D, x_L)$, where x_D is such that

$$Q_L(x_D) = D(x_D), \text{ i.e., } (1 - \theta_L(x_D))a_H + \theta_L(x_D)a_L - c = a_L x_D^\varepsilon - c x_D.$$

If the government sets $\lambda \geq x^*$ in the mixed system, low-ability individuals drop out at λ , the tax rate is given by $t = c\lambda / [p a_H + (1 - p)a_L \lambda^\varepsilon]$, and the net benefits from education are calculated as $W^{mix} = p(a_H - c) + (1 - p)(a_L \lambda^\varepsilon - c\lambda)$ and $W_L^{mix} = (1 - t)a_L \lambda^\varepsilon$ for society as whole and low-ability

individuals, respectively. If the government sets: $\lambda < x^*$, the timing of low-ability individuals dropping out, x^+ , is given by solving:

$$Q_L(x^+) = D(x^+), \text{ i.e., } (1-t)[(1-\theta_L(x^+))a_H + \theta_L(x^+)a_L] - c(1-\lambda) = (1-t)a_Lx^{+\varepsilon} - c(x^+ - \lambda)$$

where $t = c\lambda / [pa_H + (1-p)a_Lx^{+\varepsilon}]$. Then, the net benefits from education are calculated as $W^{mix} = p(a_H - c) + (1-p)(a_Lx^{+\varepsilon} - cx^+)$ and $W_L^{mix} = (1-t)a_Lx^{+\varepsilon} - c(x^+ - \lambda)$ for society as whole and low-ability individuals, respectively.

We iteratively put zero to one with a small ridge to λ , and investigate how it determines these variables, given the parameter values. Table 1 summarizes the simulation results (cases I to VIII) corresponding to eight combinations of values of ε , a_H/a_L , and p , which are presented in the leftmost three columns.¹¹ Column (1) reports x^* , the timing of low-ability individuals dropping out in the private system. Column (2) reports λ^* , the length of public education that maximizes social net benefit. Finally, column (3) reports λ_L^* , the length of public education that maximizes net benefit for low-ability individuals. Given the parameter values, we find that in all cases low-ability individuals drop out just when completing public education in the mixed system.¹² So, figures reported in columns (1)-(3) indicate the timings of low-ability individuals dropping out as well. Columns (4)-(8) compare social net benefits from education among the private, mixed, and public systems, with three cases reported for the mixed system corresponding to different levels of public education. In the same way, columns (9)-(13) compare the net benefits for low-ability individuals.

From this table we find the following, all of which are consistent with the results of the theoretical analysis. First, regarding the length of public education, we notice that:

- the length of public education that maximizes the social net benefit is no longer than that low-ability individuals receive in the private system [(2) vs. (1)];
- the length of public education that insures most against low-ability risk is no shorter than that low-ability individuals receive in the private system (but shorter than one) [(3) vs. (1)]; and accordingly,
- the length of public education that insures most against the low-ability risk is longer than that which maximizes the social net benefit [(3) vs. (2)].

Regarding the net benefit from education, we notice that:

- the private system yields a more net benefit than the public system, while the public system insures more against the low-ability risk than the private system [(4) vs. (8) and (9) vs. (13)]
- the mixed system yields the same or higher social net benefit than the private system [(6) vs. (4)] (by making low-ability individuals drop out no later than in the private system)¹³;
- the mixed system can insure more against the low-ability risk than the public system [(12) vs. (13)];
- the mixed system can insure more against the low-ability risk than the private system without any loss of social net benefit [(10) vs. (9) and (5) vs. (4)] (by making low-ability individuals drop out at the same timing as in the private system, x^*); and,
- there is a trade-off between pursuing efficiency for society as a whole and raising insurance against low-risk ability [(6) vs. (7) and (11) vs. (12)].

As for sensitivity to parameter values, we notice that:

- a lower input elasticity of educational output (ε) and a smaller gap in individual abilities (a_H/a_L) delay the timing of low-ability individuals dropping out, and extend both types of optimal public education, λ^* and λ_L^* ; and
- a higher share of high-ability individuals also delays the timing of drop-out, and extends λ_L^* but not λ^* .

5.2 Graphical illustration

Figure 3 graphically illustrates the simulation results by showing the patterns of net benefits from education for society as a whole (above) and for low-ability individuals (below), along with different lengths of public education in the mixed system. In this figure we take case I ($\varepsilon=0.6$, $a_H/a_L=1.5$, and $p=0.5$, together with $\sigma=0.75$) as a typical example. The points A , B , C , D , and E on the curve for a society and the points A' , B' , C' , D' , and E' on the curve for low-ability individuals correspond to the cases in which the government sets the length of public education as 0, λ^* ($=x_L$), x^* , λ_L , and 1, respectively.

With full private education ($\lambda=0$), the net benefits for society and for low-ability

individuals are shown as the heights of points A (0.408) and A' (0.215), respectively. With full public education ($\lambda=1$), they are shown as the heights of points E (0.350) and E' (0.280), respectively. Comparing the heights of points A and E confirms the superiority of the private system in terms of efficiency, while comparing the heights of points A' and E' confirms the superiority of the public system in terms of insurance against low-ability risk.

It is more interesting to see the mixed system with $0 < \lambda < 1$. The curve of the net benefits for society peaks at point B ($\lambda = \lambda^*$) with height 0.409, which is slightly higher than at point A (0.408). The heights at points A and C are the same, indicating that the mixed system can yield the same net benefits for society as the private system by providing public education of length x^* . We add point b , which is located between points A and B , and whose height (0.409) is the same as that of point B . This means that if the government provides public education of a very short length 0.030 (not reported in table and figure), it can make low-ability individuals drop out just at time x_L (0.363) and maximize social net benefit.¹⁴ The government, however, will not choose point b (i.e., public education of length 0.030), because it is inferior to point B (public education of length $\lambda^* = x_L = 0.363$), which yields the same net benefit for society and a higher net benefit to low-ability individuals than at point b' . The curve for low-ability individuals peaks at point D' with height 0.385, higher than that (0.280) at point E' which corresponds to the public system ($\lambda=1$).

This figure has interesting implications for the choice of education systems. A shift from the private system to the mixed system with public education of length x^* (which makes low-ability individuals drop out at the same time as in the private system) is easily accepted, because it strengthens insurance against low-ability risk (an upward shift from A' to C') with no loss of efficiency (a horizontal shift from point A to C). A shift from the public system to this mixed system is even easier to justify because it strengthens insurance against low-ability risk (an upward shift from E' to C') with an improvement of efficiency (an upward shift from point E to C).

At point C and C' with public education of length x^* , however, the government will face a trade-off. On the one hand, if the government tries to raise the efficiency of education (an

upward (albeit slight) shift to point C to B), then it has to allow insurance against the low-ability risk to weaken (a downward shift to point C' to B'). On the other hand, if it tries to raise insurance against low-ability risk (an upward shift to point C' to D'), it has to accept a loss of efficiency (a downward shift to point C to D).

Of course, we can draw different figures for other cases. As shown in Table 1, in the private system low-ability individuals might drop out just when social net benefit is maximized (cases II, IV, VI, and VIII), indicating no over-education in terms of social efficiency. Low-ability individuals might also drop out just when the net benefit of low-ability individuals is maximized (cases V and VII). The bottom line, however, is that the mixed system is superior to both fully private and public systems in terms of both efficiency for society and insurance against low-ability risk.

6. Concluding remarks

We have investigated how ability screening by education affects the demand for education and the shape of optimal education system, based on a simplified theoretical model with some numerical examples. Key results are summarized as follows.

First, ability screening by education reduces demand for education, as low-ability individuals drop out from education at a certain point. Through education, they become more aware of their low ability, and less confident in the net benefit from completion. High-ability individuals, by contrast, tend to complete education, as their high ability becomes more precisely identified. This screening mechanism, which gradually selects only those who are of sufficient high-ability to get the net positive benefit from education, can be justified in terms of efficiency for society as a whole. Even if such is the case, however, low-ability individuals may receive over-education if ability screening is delayed.

Second, a public education system makes low-ability individuals stay in education longer due to lower education costs thanks to taxes paid by high-ability individuals, and give them a higher net benefit than the private system. However, it leads to a greater loss of education efficiency for a society as a whole due to over-education of low-ability individuals.

Third, a mixed education system, in which public education is provided before private education can be a desirable solution. By adjusting the length of public education, the government can provide more insurance against the low-ability risk (in other words raise equity in earnings) than with a fully public system without any loss in efficiency of education compared to a fully private system.

This result gives an additional rationale to the mixed system, which tends to be interpreted just as a compromise in pursuing the two opposing targets of efficiency and equity. Combined with ability screening, the mixed system can reduce both over-education and education costs of low-ability individuals, which is a favorable outcome for a society as a whole as well as these individuals.

Appendix: Proof of Proposition 4.

To prove (i) of Proposition 5, first suppose that $D(x)$ peaks at time $x_P (\geq \lambda)$, noting that we have already shown that it peaks at time λ or later. As in the cases of both private (and public) systems, it is reasonable to assume that before receiving education both types of individual want to complete education. In particular, comparing the expected net benefits from completing and from drop-out at time x_P , we assume that the inequality:

$$(1-t)[pa_H+(1-p)a_L]-c(1-\lambda) \geq (1-t)a_Lx_P^\varepsilon - c(x_P-\lambda), \quad (\text{A1})$$

holds. Regarding high-ability individuals, the only possible timing for them to drop out is x_P according to the rules of education choices, considering that $D(x)$ peaks at time x_P and that $Q_H(x)$ is an increasing function of x . Suppose that they drop out at time x_P . Then, the inequality: $Q_H(x_P) < D(x_P)$, i.e.,

$$(1-t)[\theta_H(x_P)a_H+(1-\theta_H(x_P))a_L]-c(1-\lambda) < (1-t)a_Lx_P^\varepsilon - c(x_P-\lambda) \quad (\text{A2})$$

must hold. If that is the case, the inequality $D(x_P) \geq Q_L(x_P)$ also holds for low-ability individuals because $Q_H(x) \geq Q_L(x)$ for any $x \in [0, 1]$. Hence, we have:

$$(1-t)[(1-\theta_L(x_P))a_H+\theta_L(x_P)a_L]-c(1-\lambda) < (1-t)a_Lx_P^\varepsilon - c(x_P-\lambda) \quad (\text{A3})$$

Multiplying both (A2) and (A3) by p and $1-p$, respectively, adding them up, and using (2) and

(2)', we have:

$$(1-t)[pa_H+(1-p)a_L]-c(1-\lambda)<(1-t)a_Lx_P^\varepsilon-c(x_P-\lambda),$$

which contradicts assumption (A1). Hence, high-ability individuals complete education.

Next, consider low-ability individuals. Define:

$$Z_L(x) \equiv Q_L(x) - D(x) = (1-t)[(1-\theta_L(x))a_H + \theta_L(x)a_L] - c(1-\lambda) - [(1-t)a_Lx^\varepsilon - c(x-\lambda)],$$

for $\lambda < x \leq 1$. Then, we have $Z_L(1) = 0$ and

$$Z'_L(1) = c - (1-t)\varepsilon a_L - (1-t)(a_H - a_L)\theta'_L(1) > \varepsilon a_H - (1-t)\varepsilon a_L - (1-t)\varepsilon(a_H - a_L) = \varepsilon t a_H > 0$$

from (1) and (3) and given t , meaning that there exists $x \in (0, 1)$ such that $Z_L(x) < 0$. Combined with the fact that low-ability individuals do not drop before time λ , this confirms that low-ability individuals drop out at a certain time, x^+ , between time λ and one.

To prove (ii), first define:

$$\begin{aligned} Z_L^{mix}(x) &= [Q_L(x) - D(x)] / (1-t) \\ &= [(1-\theta_L(x))a_H + \theta_L(x)a_L - c] - [a_Lx^\varepsilon - c\lambda / (1-t) + ct / (1-t)] \quad \text{for } 0 \leq x \leq \lambda \\ &= [(1-\theta_L(x))a_H + \theta_L(x)a_L - c] - [a_Lx^\varepsilon - cx / (1-t) + ct / (1-t)] \quad \text{for } \lambda < x \leq 1. \end{aligned}$$

and compare this $Z_L^{mix}(x)$ with

$$Z_L(x) = Q_L(x) - D(x) = [(1-\theta_L(x))a_H + \theta_L(x)a_L - c] - (a_Lx^\varepsilon - cx)$$

in the private system. Graphically, we examine where the curve $(1-\theta_L(x))a_H + \theta_L(x)a_L - c$, which is sloping downward, crosses the curves $a_Lx^\varepsilon - cx / (1-t) + tc / (1-t)$ and $a_Lx^\varepsilon - cx$ between time λ and one (see Figure A), to compare the timings of drop-out between private and mixed systems. It is straightforward to show that the curve $a_Lx^\varepsilon - cx / (1-t) + tc / (1-t)$ is located above the curve $a_Lx^\varepsilon - cx$ for $\lambda < x \leq 1$. In addition, we know that both the two curves are sloping downward for $x \geq x_L$, because the curve $a_Lx^\varepsilon - cx$ peaks at time x_L and the curve $a_Lx^\varepsilon - cx / (1-t) + tc / (1-t)$ peaks at time $[(1-t)a_L / a_H]^{1/(1-\varepsilon)}$, earlier than at time x_L from (1).

Then, suppose that the government sets the length of public education, λ , such that $x_L \leq \lambda \leq x^*$, and focus on the situation between λ and one considering that low-ability individuals do not drop out before λ . There can be two cases. The first case (Case I in Figure A) is that the curve $(1-\theta_L(x))a_H + \theta_L(x)a_L - c$ crosses with the curve $a_Lx^\varepsilon - cx / (1-t) + tc / (1-t)$ earlier than at time x^* , when it crosses with the curve $a_Lx^\varepsilon - cx$ (see . Hence, low-ability individuals drop out between

time λ and x^* . The second case (Case II in Figure A) is that the curve $(1-\theta_L(x))a_H+\theta_L(x)a_L-c$ crosses with the curve $a_Lx^\varepsilon-cx$ only, when the curve $(1-\theta_L(x))a_H+\theta_L(x)a_L-c$ is located below the curve $a_Lx^\varepsilon-cx/(1-t)+tc/(1-t)$ at time λ . In this case, low-ability individuals drop out at time λ , when their net benefit from dropping out is maximized. In both cases, low-ability individuals drop out no later than at time x^* (and no earlier than time λ): i.e., $\lambda \leq x^+ \leq x^*$.

Even if the government makes the length of public education so short that $\lambda < x_L$, the results mentioned above do not change. The curve $a_Lx^\varepsilon-c\lambda/(1-t)+ct/(1-t)$ is located above the curve $a_Lx^\varepsilon-cx$ for $\lambda < x \leq 1$, and the former peaks earlier than the latter because $[(1-t)a_L/a_H]^{1/(1-\varepsilon)} < x_L$, so low ability individuals drop out no later than at time x^* according to the rule of education choices. *Q.E.D.*

Notes

1. This approach is in line with that of Wilson *et al.* (2005), which highlights the effects of individual student perceptions regarding the returns from graduating from high schools versus dropping out in their empirical analysis of drop-outs.
2. This assumption makes sense in our model, because high-ability individuals always complete education and only low-ability ones may drop out as shown later.
3. See Koshal *et al.* (1995), Bickel and Lange (1995), Warren and Jenkins (2005), Van Dorn *et al.* (2006) for recent examples.
4. Another possible assumption is that the government makes the total length of education equal to that which maximizes the expected social net benefit from education, $pa_H+(1-p)a_L-c$, rather than x_H . However, this alternative assumption does not affect the main results in this paper.
5. We assume $a_L < c$, in Figure 1 (as well as Figure A1), but this assumption does not affect the discussion in the text.
6. Our model ignores the labor market and the impact of one's education decision on the wage and employment perspectives of others. Charlot and Decreuse (2005) discuss the possibility of over-education in a general equilibrium framework with heterogeneous individuals.
7. Conceptually, individuals are allowed to drop out even before completing public education at λ . However, they do not do so because the net benefit from dropping out peaks at λ as discussed below.
8. If the government sets $\lambda < x^*$, we cannot clearly state anything algebraically, but this education policy, which reduces over-education of low-ability individuals, is likely to have a positive impact on their net benefit.
9. It should be noted that λ_L^* is independent of the shape of the ability distinction function, if it is an inner solution.
10. We can confirm: $\theta_L(0)=1-p$, $\theta_L(1)=1$, $\theta'_L(x)=p\sigma x^{\sigma-1} > 0$, and $\theta''_L(x)=p\sigma(\sigma-1)x^{\sigma-2} < 0$. We also derive $\theta_H(x)=p+(1-p)x^\sigma$ from (3) or (3)'.

11. High-ability individuals complete education in all cases.
12. More precisely, low-ability individuals also drop out later than at completion of public education, if the government sets up public education of a very short length. In particular, the government can make low-ability individuals drop out just at x_L by providing public education of a length shorter than x_L . But, we do not report the result of this policy in Table 1, because it gives a lower net benefit to low-ability individuals than the latter policy. We discuss it later, using Figure 2.
13. This result implies that over-education of low-ability individuals in the private system is quite limited under the given parameter values.
14. When moving from point A to b , the timing of low-ability individuals dropping out becomes earlier from x^* to x_L . When moving from point b to B , it becomes earlier until a certain point and then returns to x_L .

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Figure 1. Expected net benefits from completion and drop-out in the private system

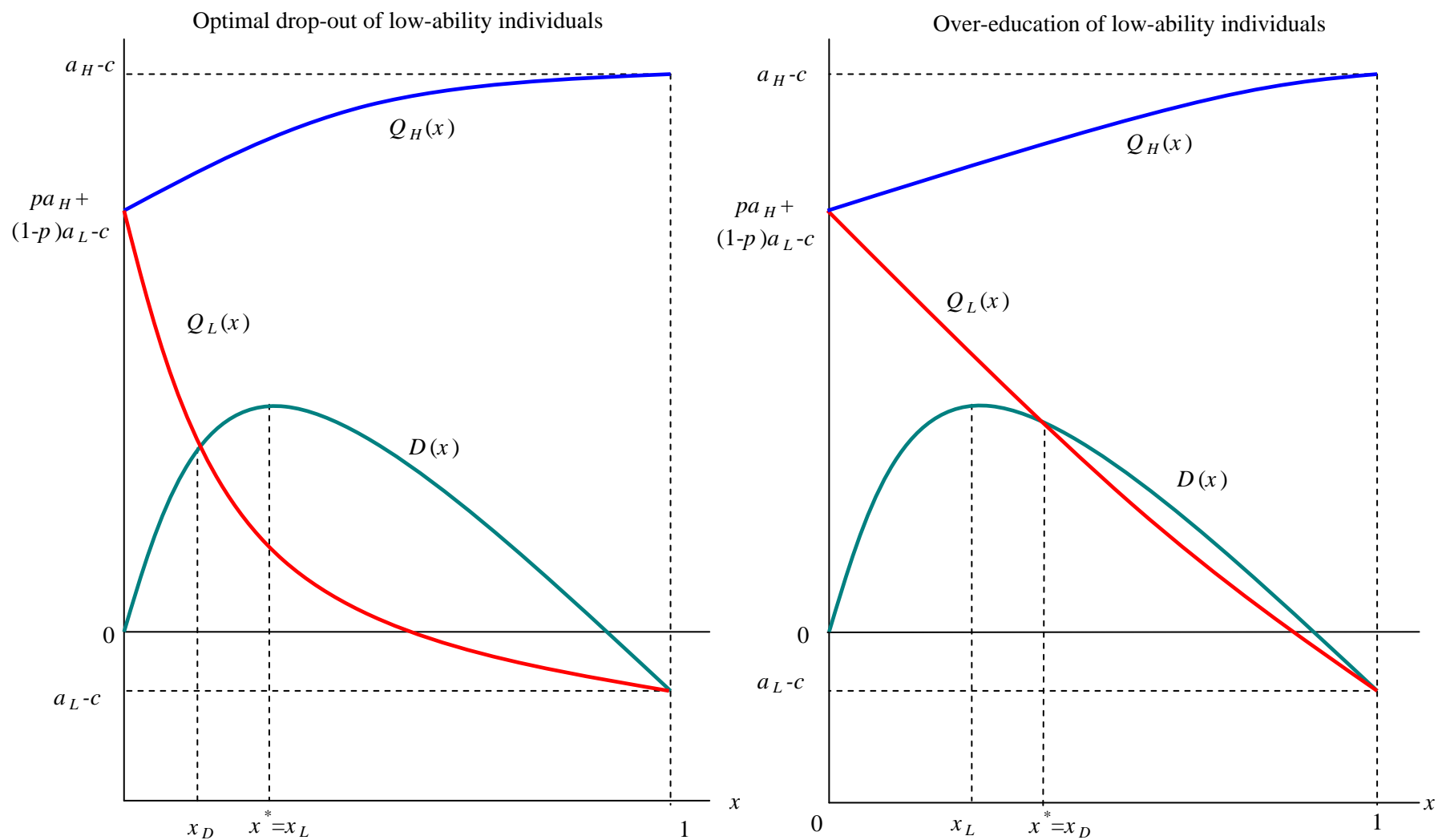
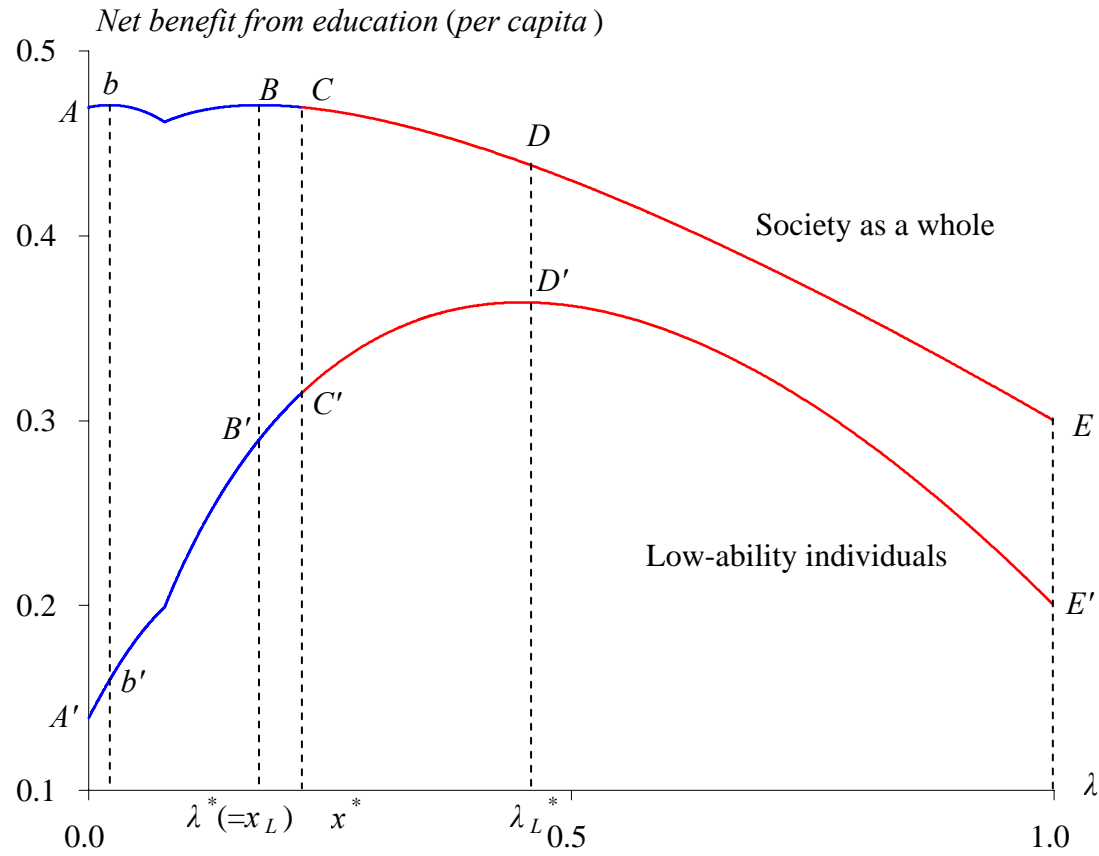


Table 1. Simulation results

Parameter assumptions				x^*	λ^* ($=x_L$)	λ_L^*	W^{priv}	W^{mix} with public education of:				W^{pub}	W_L^{priv}	W_L^{mix} with public education of:				W_L^{pub}
No.	ε	a_H / a_L	p					x^*	λ^*	λ_L^*				x^*	λ^*	λ_L^*		
				(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)		
I	0.6	2.0	1/2	0.221	0.177	0.449	0.470	0.470	0.471	0.440	0.300	0.139	0.315	0.290	0.364	0.200		
II	0.6	2.0	1/3	0.177	0.177	0.374	0.361	0.361	0.337	0.361	0.133	0.141	0.271	0.314	0.271	0.100		
III	0.6	2.5	1/2	0.117	0.101	0.416	0.550	0.550	0.551	0.483	0.250	0.101	0.241	0.225	0.352	0.143		
IV	0.6	2.5	1/3	0.101	0.101	0.330	0.401	0.401	0.346	0.401	0.000	0.101	0.214	0.298	0.214	0.000		
V	0.5	2.0	1/2	0.474	0.250	0.493	0.607	0.607	0.625	0.605	0.500	0.214	0.446	0.400	0.446	0.333		
VI	0.5	2.0	1/3	0.250	0.250	0.422	0.500	0.500	0.485	0.500	0.333	0.250	0.375	0.400	0.375	0.250		
VII	0.5	2.5	1/2	0.365	0.160	0.456	0.699	0.699	0.725	0.678	0.500	0.148	0.427	0.345	0.433	0.286		
VIII	0.5	2.5	1/3	0.160	0.160	0.371	0.550	0.550	0.514	0.550	0.250	0.200	0.327	0.381	0.327	0.167		

Note: $\sigma=0.75$ is assumed.

Figure 2: Net benefit from education in the mixed system



(Note) $\varepsilon=0.6$, $a_H/a_L=2.0$, $p=0.5$, $\sigma=0.75$ are assumed. The heights at points A and C , and those at points b and B are the same, respectively.

Figure A: Proof of $\lambda \leq x^+ \leq x^*$ in the case of $x_L \leq \lambda \leq x^*$

