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Endogenous fertility and human capital accumulation*

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Abstract

The recent sharp decline in the birthrate has attracted considerable interest in Japan. Many researchers and policy makers have tackled the essential question of what kind of policy can reverse the downward trend of the birthrate. However, some attempt has been made to examine the policy implications of this issue by using a dynamic general equilibrium model. On the basis of the overlapping generations model, in which households determine both the quality and quantity of children simultaneously, this paper compares the effects of two different types of government support: one for educational expenses and the other for child care. As a result, it is shown that the support for child care leads to a temporary increase in the fertility rate but results in decreased investment in education, which in turn decreases human capital accumulation; as a result, the household income decreases. Consequently, this policy decreases the fertility rate over time after returning it to the original level. In other words, in the long run, the support for child care is unrelated to the fertility rate. On the contrary, the support for educational expenses increases the fertility rate in the long run because of the increase in the household income resulting from the increase in human capital.

JEL classification: D91, I2, J13

Key words: child-care cost, educational expenses, fertility rate, human capital

1 Introduction

In the medium or long run, one of the greatest concerns for the Japanese economy is the continued decline in the birthrate, which affects not only the sustainability of the social security system but also the fate of the country. However, this problem is not unique to the Japanese economy. As shown in Fig.1, most advanced countries have the same kind of downward trend in their birthrates. In addition, the present-day developing economies are bound to face the same problem at some point in time. Hence, the question of what type of policies can reverse this downward trend is very important not only for the Japanese economy but also for most other economies.

[Fig.1 Inserted]

According to the most recent statistics (2005), the fertility rate in Japan has reached 1.25, which is undoubtedly the lowest in the postwar period. A decline in the number of children leads to many serious problems regarding social security and a shortage in the labor supply and tax revenue; hence, it is absolutely essential to take measures to check a further decline in the number of children. The Japanese governments, including the central and local governments, have taken several steps to deal with this problem in earnest. For example, we find that the provision for the allowance for dependent children increases every year; this is shown in Fig.2.

[Fig.2 Inserted]

Although there have been numerous policy debates, a few theoretical investigations to evaluate the policies have been carried out using a dynamic general equilibrium model. On the basis of the overlapping generations model, in which households determine the quality and number of their children simultaneously, this paper compares the effects of two different types of government support: one for educational expenses and the other for child care.

Oshio (2001) and Yasuoka (2006), whose studies ignore the quality of children, show that the support for child care increases the fertility rate in both the short run and the long run. Furthermore, Galor and Weil (1996) show that the decline in the opportunity cost resulting from the support for child care raises the fertility rate. On the other hand, the model employed in this paper considers a quality-quantity trade-off in parents' decisions on children, which is related

to the study by Croix and Doepke (2003, 2004), and the abovementioned result no longer holds. The reason is as follows. First, the reduction in the child-care cost due to the support policy leads to an increase in the number of children and a decrease in the quality of children due to relatively high prices. Second, the decline in the quality of children decreases the human capital, which in turn decreases the income. Hence, the fertility rate reverses in the long run regardless of this support policy. On the contrary, the support for educational expenses leads to an increase in income due to an increase in human capital; hence, the fertility rate rises. The result that the support for child care is valid in the long run is paradoxical.

This paper is structured as follows. Section 2 presents the model and analyzes it. Section 3 derives the equilibrium. Section 4 shows the effects of the policies of support for child care and educational expenses by using comparative statics. Section 5 analyzes the transitional path after policy shocks, and Section 6 provides some conclusions.

2 The Model

Consider an economy that is populated by overlapping generations of people who live for two periods, young and old. All decisions are made in the young period. The people care about the consumption in the old period c_{t+1} , the number of their children n_t , and the human capital of children h_{t+1} . The utility function is given by

$$U_t = \alpha \ln n_t h_{t+1} + (1 - \alpha) \ln c_{t+1}, \quad 0 < \alpha < 1. \quad (1)$$

The parameter α represents the altruism factor. The parents care about both the number n_t and the quality h_{t+1} of their children. Croix and Doepke (2004) assume the same utility function. The consumption in the old period generates an endogenous supply of physical capital. In the young period, the parents need a fraction of their time ϕ to raise their children. For the young period, we choose the consumption c_{t+1} (the saving s_t), the number of children n_t , and the education level e_t . The budget constraint for the young period with human capital h_t is

$$x e_t n_t + z n_t + \frac{c_{t+1}}{1 + r_{t+1}} = (1 - \phi n_t) w_t h_t \leftrightarrow x e_t n_t + (z + \phi w_t h_t) n_t + \frac{c_{t+1}}{1 + r_{t+1}} = w_t h_t, \quad (2)$$

where z is the child-care cost per child and x is the educational expense per education level e_t .

Let us consider the human capital accumulation. The human capital of the children h_{t+1} depends on the human capital of the parents h_t and the education level e_t :

$$h_{t+1} = B e_t^\epsilon h_t^{1-\epsilon}, 0 < \epsilon < 1, B > 0. \quad (3)$$

Glomm and Ravikumar (1992) assume the same human capital accumulation process. The individual chooses his or her consumption, number of children, and education level so as to maximize his or her lifetime utility subject to the lifetime budget constraint (2) and the equation of human capital (3). The optimal allocations are determined by

$$c_{t+1} = (1 + r_{t+1})(1 - \alpha)w_t h_t, \quad (4)$$

$$n_t = \frac{\alpha(1 - \epsilon)w_t h_t}{\phi w_t h_t + z}, \quad (5)$$

$$e_t = \frac{\epsilon(\phi w_t h_t + z)}{x(1 - \epsilon)}. \quad (6)$$

Clearly, from the above equations, if the child-care cost $z + \phi w_t h_t$ increases, then the parents reduce the number of children and increase the educational expenses.

The production of the final good is given by

$$Y_t = K_t^\theta N_t^{1-\theta}, 0 < \theta < 1, \quad (7)$$

where K_t is the aggregate physical capital and N_t is the aggregate effective labor supply that equals $h_t L_t$. L_t is the population that comprises people born in period t . Assuming perfect competition and the full depreciation of physical capital within one period, the wages and the interest rate are determined by

$$w_t = (1 - \theta)k_t^\theta, \quad (8)$$

$$1 + r_t = \theta k_t^{\theta-1}, \quad (9)$$

where $k_t \equiv \frac{K_t}{L_t}$ is the capital-labor ratio in period t . L_{t+1} is equivalent to $n_t L_t$.

3 Equilibrium

The equilibrium condition in the physical capital market is given as follows:

$$k_{t+1} h_{t+1} = \frac{(1 - \alpha)w_t h_t}{n_t}. \quad (10)$$

From (3), (6), (8), and (10), we obtain the following equation:

$$k_{t+1} = \frac{1-\alpha}{\alpha(1-\epsilon)B} \left(\frac{x(1-\epsilon)}{\epsilon} \right)^\epsilon \left(\phi(1-\theta)k_t^\theta + \frac{z}{h_t} \right)^{1-\epsilon}. \quad (11)$$

Defining $\Delta k_t = k_{t+1} - k_t$,

$$\Delta k_t = \frac{1-\alpha}{\alpha(1-\epsilon)B} \left(\frac{x(1-\epsilon)}{\epsilon} \right)^\epsilon \left(\phi(1-\theta)k_t^\theta + \frac{z}{h_t} \right)^{1-\epsilon} - k_t. \quad (12)$$

The equation that satisfies $\Delta k_t = 0$ is

$$h_t = \frac{z}{\left(\frac{k_t}{X}\right)^{\frac{1}{1-\epsilon}} - \phi(1-\theta)k_t^\theta}, \quad (13)$$

where $X = \frac{1-\alpha}{\alpha(1-\epsilon)B} \left(\frac{x(1-\epsilon)}{\epsilon} \right)^\epsilon$. (13) must satisfy the condition because h_t cannot be nonpositive: $k_t > \left(\phi(1-\theta)X^{\frac{1}{\epsilon}} \right)^{\frac{1-\epsilon}{1-\theta(1-\epsilon)}}$. Substituting (6) and (8) into (3), we obtain the following equation:

$$h_{t+1} = B \left(\frac{\epsilon}{(1-\epsilon)x} \right)^\epsilon (z + \phi(1-\theta)k_t^\theta h_t)^\epsilon h_t^{1-\epsilon} \quad (14)$$

Defining $\Delta h_t = h_{t+1} - h_t$,

$$\Delta h_t = h_{t+1} - h_t = B \left(\frac{\epsilon}{x(1-\epsilon)} \right)^\epsilon (z + \phi(1-\theta)k_t^\theta h_t)^\epsilon h_t^{1-\epsilon} - h_t. \quad (15)$$

The equation that satisfies $\Delta h_t = 0$ is

$$h_t = \frac{z}{\frac{x(1-\epsilon)}{B^{\frac{1}{\epsilon}} \epsilon} - \phi(1-\theta)k_t^\theta}. \quad (16)$$

This equation must also satisfy the conditions because h_t cannot be nonpositive:

$$0 < k_t < \left(\frac{x(1-\epsilon)}{\phi(1-\theta)B^{\frac{1}{\epsilon}} \epsilon} \right)^{\frac{1}{\theta}} \text{ and } \frac{B^{\frac{1}{\epsilon}} \epsilon z}{x(1-\epsilon)} < h_t.$$

Considering the steady state, k_t and h_t are constant. Denoting them as k and h , respectively, in the steady state, (13) and (16) are written as

$$k = \frac{(1-\alpha)x}{\alpha \epsilon B^{\frac{1}{\epsilon}}}, \quad (17)$$

$$h = \frac{z}{\frac{(1-\epsilon)x}{B^{\frac{1}{\epsilon}} \epsilon} - \phi(1-\theta) \left(\frac{(1-\alpha)x}{\alpha \epsilon B^{\frac{1}{\epsilon}}} \right)^\theta}. \quad (18)$$

The long-run equilibrium is shown in Fig.3 (see Appendix A for details).

[Fig.3 Inserted]

In this figure, $k_A = \left(\frac{(1-\epsilon)x}{B^{\frac{1}{\epsilon}} \epsilon \phi (1-\theta)} \right)^{\frac{1}{\theta}}$, $k_B = (\phi(1-\theta)X^{\frac{1}{1-\epsilon}})^{\frac{1-\epsilon}{1-\theta(1-\epsilon)}}$, $k_C = \left(\frac{(1-\epsilon)x}{B^{\frac{1}{\epsilon}} \epsilon \phi (1+\theta)} \right)^{\frac{1}{\theta}}$, $h_A = \frac{zB^{\frac{1}{\epsilon}} \epsilon}{(1-\epsilon)x}$, and k_C is an inflection point. The fertility rate in the steady state is given by

$$n = \frac{\alpha(1-\epsilon)}{\phi + \frac{z}{(1-\theta)k^\theta h}}. \quad (19)$$

4 Policy Comparison

In this section, we analyze the effects of two different policies, namely, the support for education and the support for child care, using comparative statics.

1

4.1 The Support for Education

The support for education implies a decrease in price of education x . The decrease in x shifts down the $\Delta k_t = 0$ locus, while shifts up the $\Delta h_t = 0$ locus (see Fig.4).

[Fig.4 Inserted]

From (17) and (18), we can derive $\frac{dh}{dx} < 0$ $\frac{dk}{dx} > 0$, respectively. Due to the decrease in x , the human capital increases but the capital-labor ratio decreases. From (19), we derive the following relationship:

$$\begin{aligned} \frac{dn}{dx} &= \frac{\alpha(1-\epsilon)}{\left(\phi + \frac{z}{(1-\theta)k^\theta h} \right)^2} \frac{zk^{\theta-1}}{(1-\theta)(k^\theta h)^2} \left(\theta h \frac{dk}{dx} + k \frac{dh}{dx} \right), \\ \theta h \frac{dk}{dx} + k \frac{dh}{dx} &= -\frac{hk}{x} \frac{\frac{(1-\epsilon)x}{B^{\frac{1}{\epsilon}} \epsilon} (1-\theta)}{\frac{(1-\epsilon)x}{B^{\frac{1}{\epsilon}} \epsilon} - \phi(1-\theta) \left(\frac{(1-\alpha)x}{\alpha \epsilon B^{\frac{1}{\epsilon}} \epsilon} \right)^\theta} < 0. \end{aligned}$$

The fertility rate rises; hence, one can arrive at the following.

Proposition 1 Due to the policy of support for educational expenses, the capital-labor ratio decreases, while both the human capital and the fertility rate increase.

¹This paper focuses on the effects of support. Thus, it does not consider the means employed to collect tax revenue.

The policy of support for educational expenses raises the education level, so human capital accumulation is enhanced. The fertility rate increases because of the increase in household income resulting from the increased human capital.

4.2 Indirect Support for Child Care

The policy of indirect support for child care decreases the child-care cost z . The decrease in z shifts down both the $\Delta k_t = 0$ and the $\Delta h_t = 0$ loci (see Fig.5).

[Fig.5 Inserted]

From (17) and (18), we can derive $\frac{dk}{d\phi} = 0$ $\frac{dh}{d\phi} > 0$, respectively. Due to the decrease in z , the human capital decrease and the capital-labor ratio does not change. Thus, from (19), the fertility rate does not change as follows:

$$\frac{dn}{dz} = -\frac{\alpha(1-\epsilon)\left(h - z\frac{dh}{dz}\right)}{(1-\theta)k^\theta h^2 \left(\phi + \frac{z}{(1-\theta)k^\theta h}\right)^2} = 0.$$

We establish the following proposition.

Proposition 2 The policy of indirect support for child care decreases the human capital. However, the capital-labor ratio and the fertility rate are not affected by the policy.

The policy of indirect support for child care does not affect the fertility rate in the long run. The reason is as follows. First, the support decreases educational investment and increases the number of children; however this increase is only temporary. Second, this effect decreases the human capital. Finally, household income decreases due to the decrease in human capital. This, in turn, decreases the fertility rate gradually. Thus, in the long run, the short-run effect is completely offset, so the fertility rate is ultimately unaffected.

4.3 Direct Support for Child Care

In this subsection, we consider the support policy that can lead to a decrease in the opportunity cost of child care ϕwh . The policy of decreasing the opportunity cost of child care implies a decrease in ϕ . The decrease in ϕ shifts down both the $\Delta k_t = 0$ and the $\Delta h_t = 0$ loci (see Fig.6).

[Fig.6 Inserted]

From (17) and (18), we can derive $\frac{dk}{d\phi} = 0$ $\frac{dh}{d\phi} > 0$, respectively. Thus, the decrease in ϕ leads to a decrease in the human capital, and the capital-labor ratio is invariant. From (19), we derive the following relationship:

$$\frac{dn}{d\phi} = \frac{\alpha(1-\epsilon) \left(\frac{z}{(1-\theta)k^\theta h^2} \frac{dh}{d\phi} - 1 \right)}{\left(\phi + \frac{z}{(1-\theta)k^\theta h} \right)^2} = 0.$$

We can establish the following proposition.

Proposition 3 The policy of decreasing the opportunity cost of child care is the same as that of decreasing the child-care cost.

The short-run and long-run effects of this policy are the same as those of the policy discussed in the previous subsection.

5 The Transitional Path

We analyze the transitional path after introducing the policy of support for child care. The policies that lead to a decrease in z and/or ϕ shift the equilibrium point from A to B, and the transitional path is shown by the dashed line in Fig.7.

[Fig.7 Inserted]

The decrease in z and/or ϕ due to policy shocks increases the fertility rate and decreases the capital-labor ratio in the short run because of the increase in population. However, the decline in the accumulation of human capital decreases the household income at the same time; hence, the fertility rate decreases. Since the population decreases, the capital-labor ratio also increases.

6 Conclusions

This paper has analyzed the effects of two different types of support policies-the support for child care and the support for educational expenses-on the basis of the overlapping generations model, in which households determine the quality and the number of their children simultaneously. Among the results that we have presented in this paper, the following two are noteworthy.

First, the support for child care increases the fertility rate in the short run. However, this support is invalid in the long run because the household income diminishes due to the decrease in human capital.

Second, although the policy of support for educational expenses increases the human capital, the fertility rate is constant in the short run. However, in the long run, the increase in human capital leads to an increase in the household income, which leads to an increase in the fertility rate.

In the existing literature, which does not take into account the quality of children, the support for child care raises the fertility rate (see Appendix A). However, once the quality of children is also considered with the number of children, this result no longer holds. In this sense, the second main result is quite unique to the model employed in this paper.

Appendix A

In this appendix, we explain the effect of the policy of support for child care by using a model that does not consider human capital. The problem that each household must solve is as follows:

$$\begin{aligned} \max U_t &= \alpha \ln n_t + (1 - \alpha) \ln c_{t+1}, \\ \text{s.t. } zn_t + \frac{c_{t+1}}{1 + r_{t+1}} &= (1 - \phi n_t)w_t. \end{aligned}$$

The optimal plans are $n_t = \frac{\alpha w_t}{z + \phi w_t}$ and $c_{t+1} = (1 + r_{t+1})(1 - \alpha)w_t$, respectively. Considering the capital market clearing condition, we can derive $k_{t+1} = \frac{(1 - \alpha)w_t}{n_t}$. Substituting the plan of n_t into this equation, we can obtain the following equation:

$$k_{t+1} = \frac{1 - \alpha}{\alpha} (z + \phi(1 - \theta)k_t^\theta). \quad (20)$$

The decrease in z or ϕ caused by the policy of support for child care leads to a decrease in the capital-labor ratio. Regarding the fertility rate, it can be shown by

$$\begin{aligned} \frac{dn}{dz} &= -\frac{\alpha^2}{\left(\frac{z}{(1-\theta)k^\theta} + \phi\right)^2} \frac{1}{\alpha} \frac{1}{(1-\theta)k^\theta} \left(1 - \frac{z\theta}{k} \frac{dk}{dz}\right), \\ &= -\frac{n^2}{\alpha} \frac{1}{(1-\theta)k^\theta} \frac{k - \frac{1-\alpha}{\alpha}\phi(1-\theta)\theta k^\theta - z\theta \frac{1-\alpha}{\alpha}}{k - \frac{1-\alpha}{\alpha}\phi(1-\theta)\theta k^\theta} < 0, \\ \frac{dn}{d\phi} &= -n^2 \frac{1}{\alpha(1-\theta)k^\theta} \left((1-\theta)k^\theta - \frac{z\theta}{k} \frac{dk}{d\phi}\right) < 0, \\ (1-\theta)k^\theta - \frac{z\theta}{k} \frac{dk}{d\phi} &= \frac{(1-\theta)^2 k^\theta \left(k - \phi \frac{1-\alpha}{\alpha} (1-\theta)k^\theta\right)}{k - \phi \frac{1-\alpha}{\alpha} (1-\theta)\theta k^\theta} > 0. \end{aligned}$$

The policy of support for child care increases the fertility rate.

Appendix B

In this appendix, we prove that the equilibrium is locally stable. We begin by taking first-order Taylor approximations to the equations (13) and (16) around $k = k^*$ and $h = h^*$. (13) and (16) may be reduced to the following pair of differential equations in k and h :

$$\begin{pmatrix} \Delta k_t \\ \Delta h_t \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} k_t - k^* \\ h_t - h^* \end{pmatrix}$$

where

$$\begin{aligned}
a_{11} &= \frac{\phi\theta(1-\theta)(1-\epsilon)k^{*\theta}}{\phi(1-\theta)k^{*\theta} + \frac{z}{h^*}} - 1 < 0, \\
a_{12} &= -\frac{(1-\epsilon)\frac{z}{h^{*2}}}{\phi(1-\theta)k^{*\theta} + \frac{z}{h^*}} < 0, \\
a_{21} &= \frac{\phi\epsilon\theta(1-\theta)k^{*\theta-1}h^{*2}}{z + \phi(1-\theta)k^{*\theta}h^*} > 0, \\
a_{22} &= -\frac{\epsilon z}{z + \phi(1-\theta)k^{*\theta}h^*} < 0.
\end{aligned}$$

These two differential equations satisfy $a_{11} + a_{22} < 0$ and $a_{11}a_{22} - a_{12}a_{21} > 0$, respectively; thus, this equilibrium is locally stable.

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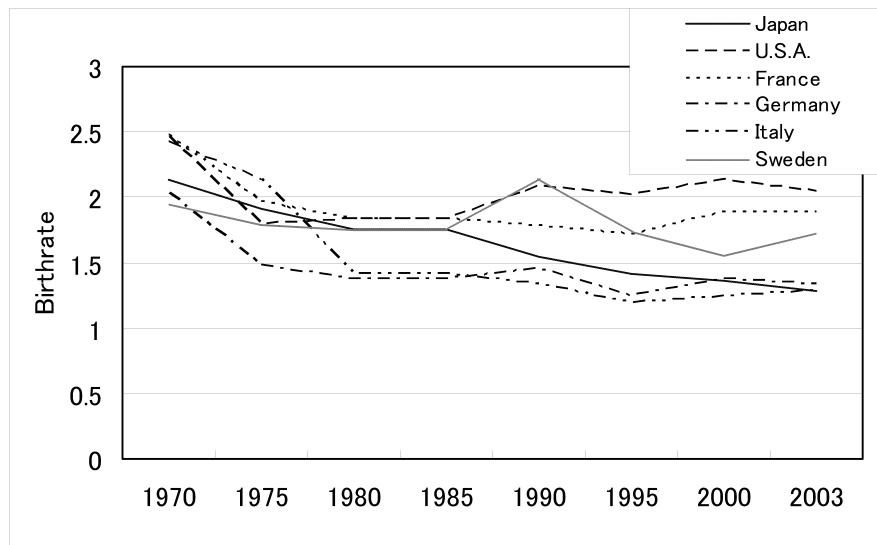


Fig.1 Birthrates in developed countries

(Data : Japan, Dynamic Statistics of the Population. UN, Demographic Yearbook. Council of Europe, Recent demographic developments in Europe, 2003. U.S. Department of Health and Human Services, National Vital Statistics Report, vol.51-2, 53-9. Eurostat Statistics in Focus: Population and Social Conditions 13/2004)

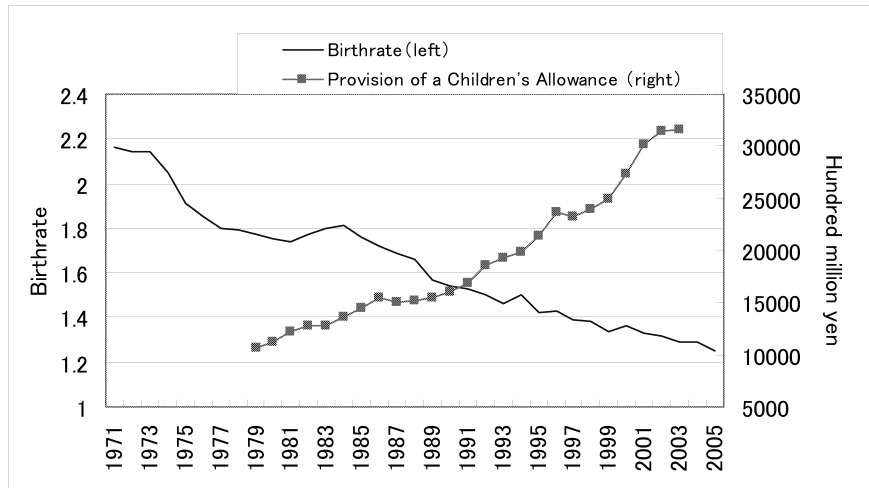


Fig. 2 Birthrate and provision of a children's (family) allowance in Japan (Data: Dynamic Statistics of the Population (the Ministry of Health, Labor and Welfare), Change of the provision of a children's allowance (National Institute of Population and Social Security Research))

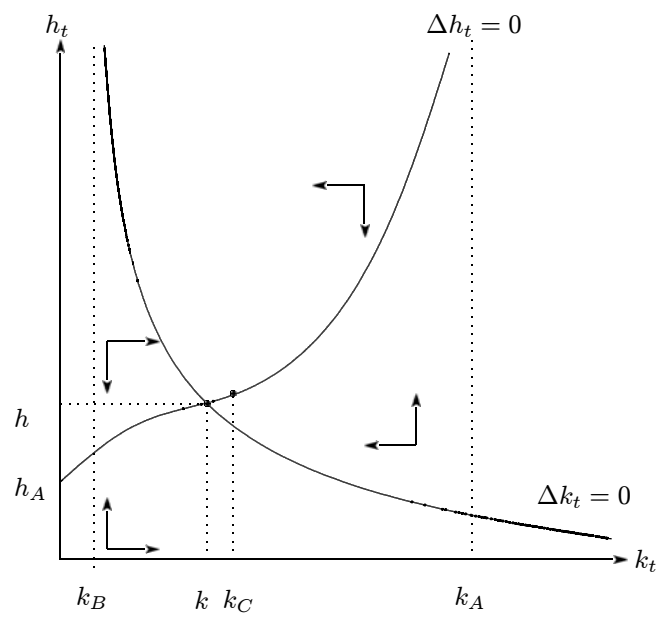


Fig.3 Equilibrium

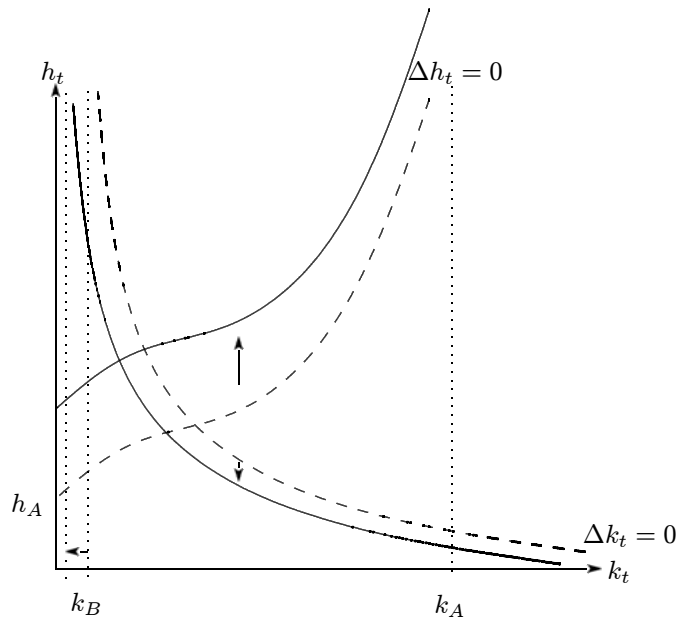


Fig.4 The policy of support for educational expenses (the decrease in x)

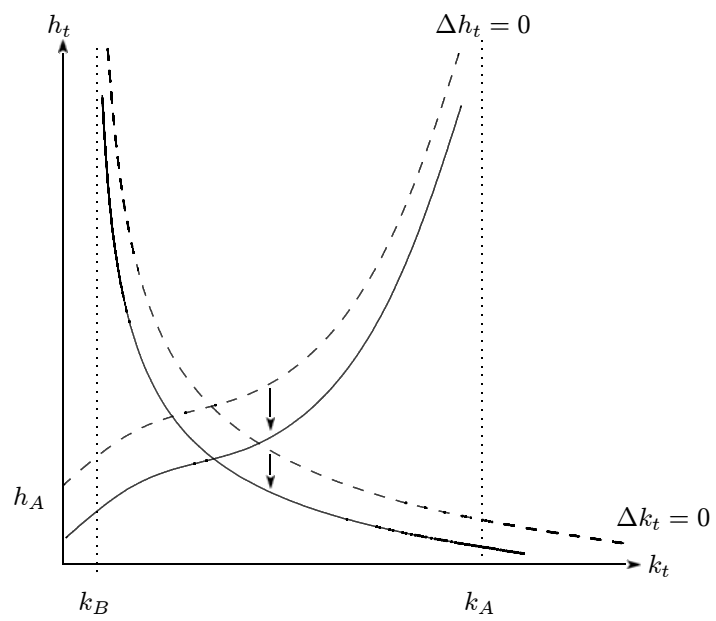


Fig.5 The policy of support for child-care cost (the decrease in z)

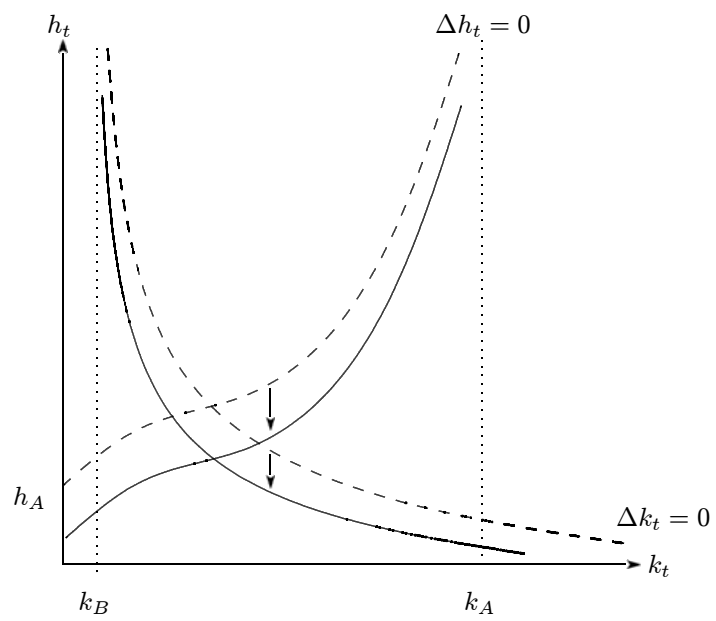


Fig.6 The policy of decreasing in the opportunity cost of child care (the decrease in ϕ)

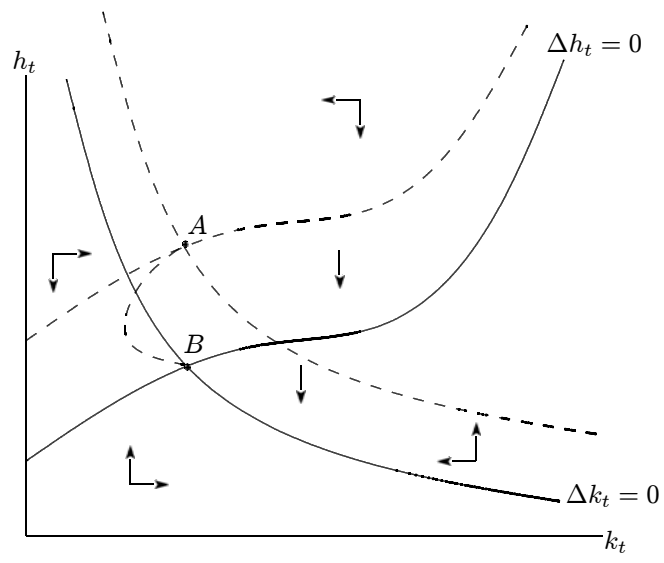


Fig.7 The transitional path of the effects of policy