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(Citation)
神戸大学経済学研究科 Discussion Paper,619

(Issue Date)
2006-10

(Resource Type)
technical report

(Version)
Version of Record

(URL)
https://hdl.handle.net/20.500.14094/80200040

An Intergenerational Childcare Support and the Fluctuating Fertility Rate

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October, 2006

Abstract

This paper examines the relationship between fertility rate and intergenerational childcare support provided by grandparents in the model based on the overlapping generation model with endogenous fertility rate. As a result, this paper reaches the following conclusions. First, intergenerational childcare support causes the fertility rate to fluctuate. Second, intergenerational childcare support exposes the market failure.

JEL Classification: D11, J13

Keywords: fluctuational fertility rate, intergenerational childcare support

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1 Introduction

Intergenerational cooperations is often observed in a family. Parents (or grandparents) assist in the childcare or household works, while the children (grandchildren) provide financial support to their parents (grandparents), eg., Scultz (1988). Such intergenerational cooperation could alleviate the burden of childcare and promote the upbringing of a child. According to a survey conducted by the Ministry of Land, Infrastructure and Transport, the fertility rate of a nuclear family is less than that of a three generational family. This implies that the type of family is one of the factors that determines the fertility rate in a family. In this paper, we consider the overlapping generations model with the intergenerational cooperation.

In our model, the fertility rate could occur endogenously. Eastelin (1973) suggests the possibility of self-generating fluctuations in population growth. A large population will face stiffer economic competition, lower income, congestions, and crowding if other means of production as well as the social infrastructure do not expand simultaneously. The result may be a decline in fertility levels as parents attempt to maintain on adequate standard of living for themselves. Moreover, using the framework developed by Barro and Becker (1989), Benhabib and Nishimura (1989) demonstrate that the fertility rate could fluctuate with the interaction with capital per capita. Our paper presents an alternative mechanism of fertility rate fluctuation. This paper shows that intergenerational cooperation produces the fluctuation of the fertility rate and that the magnitude of fluctuation is determined by the relative altruism within the intergenerations.

The structure of the paper is as follows. In the next section, we describe the intergenerational cooperation and develop the overlapping generation model with cooperation. In section 3, we show that the fluctuation of the fertility rate could be a result of the relative altruism. In section 4, we prove that there is a the relationship between intergenerational childcare support and the relative altruism. In section 5, we conduct a welfare analysis. Finally, we conclude the paper.

2 The Model

We consider the most popular overlapping generations (OLG) framework that assumes that each agent lives for only two periods. We refer to the cohort that is born at $t-1$ as generation t; agents of this generation are young in period t and the old in period $t+1$. At each point in time, there are two generations; the young and old generations. Consider the lifetime of agents of generation t. We assume that all agents have an identical utility function, which depends on the number of children in the young period and on consumption and leisure in the old period. Thus, we assume that the utility function is as follows.

$$U_t = \alpha \ln n_t + \beta \ln c_{t+1} + (1-\alpha-\beta)\ln l_{t+1},\ 0<\alpha<1,\ 0<\beta<1, \qquad (1)$$

where n_t is the number of children of generation t in the young period, and c_{t+1}, l_{t+1} denote the consumption and leisure of this generation in the old period.

Next, let us consider the economic behavior of each agent of generation t. They are endowed with a unit of time in both their young and old periods. In the young period t, they decide on labor times and the number of children they have n_t, that is, they divide a unit of time between labor supply and their child rearing. Child rearing requires ϕ units of time per child; therefore, the time given to child rearing and labor income are denoted as ϕn_t and $(1 - \phi n_t)w_t$, respectively. The wage rate is denoted as w_t. Thus, we can obtain the budget constraint when each agent is young as follows:

$$s_t = (1 - \phi n_t)w_t + z_t(w_t - h), \tag{2}$$

where s_t denotes the savings of the young in period t. z_t denotes the times of child rearing assistance from the previous generation (old when the period is t); this is defined as $(1 - l_t)/n_{t-1}$ and h denotes the reward per unit of support time for the old generation in period t. h is not determined through the market; it depends on the negotiation between the young and the old generations. In other words, h denotes the degree of relative altruism in the economy. Large h implies that the young generation is more (less) altruistic (selfish) than the old generation. Moreover, a size of z_t can be intrepreted as a time of intergenerational support in the society.

In the old period $t + 1$, they use their time for leisure and to support their children's childcare. Let l_{t+1} denote the leisure times of an agent of generation t in their old period $t+1$; thus, the reward from childcare assistance for their children is $(1-l_{t+1})h$. Thus, the budget constraints for each t-generation agent in the old period $t+1$ is

$$c_{t+1} = (1+r_{t+1})s_t + (1-l_{t+1})h, \tag{3}$$

where r_{t+1} is the interest rate.

Each agent treats w_t, r_{t+1}, and $z_t(=(1-l_t)/n_{t-1})$ as given and chooses n_t, c_{t+1}, and l_{t+1} to maximize utility from equation (1), subject to equations (2) and (3). The optimal allocations are shown as follows.

$$n_t = \frac{\alpha}{\phi w_t} I, \tag{4}$$

$$c_{t+1} = (1+r_{t+1})\beta I, \tag{5}$$

$$l_{t+1} = \frac{(1+r_{t+1})(1-\alpha-\beta)}{h} I, \tag{6}$$

where $I \equiv w_t + (w_t - h)z_t + \frac{h}{1+r_{t+1}}$.

3 The Equilibrium

This paper assumes a small open economy; therefore, interest rate, the wage rate, and the capital-labor ratio are constant. Substituting the optimal allocation of n_t into $z_{t+1} = \frac{1-l_{t+1}}{n_t}$, we obtain the following equation:

$$z_{t+1} = \frac{\phi w}{\alpha}\left(\frac{1}{w+(w-h)z_t+\frac{h}{1+r}} - \frac{(1+r)(1-\alpha-\beta)}{h}\right). \tag{7}$$

The locus of this equation is depicted in Fig. 1.

[Fig. 1 Inserted]

This equation shows the dynamic change in the degree of intergenerational childcare support provided by grandparents. If z_t is low, z_{t+1} is high, and vice versa. A higher z_t produces a high fertility rate. However, in the next period, the support that households receive declines, and thus, the fertility rate also decreases. Therefore, in the subsequent period, this support increases due to the decrease in households, which results in an increase in the fertility rate. This process continues further. As a results, z_t fluctuates over time as does the fertility rate.

Considering the steady state $z_{t+1} = z_t = z$, z is given as,

$$z = \frac{-B + (B^2 - 4AC)^{\frac{1}{2}}}{2A}, \tag{8}$$

where $A \equiv w - h$, $B \equiv w + \frac{h}{1+r} + \frac{\phi w}{\alpha} \frac{(1+r)(1-\alpha-\beta)(w-h)}{h}$, $C \equiv \frac{(1+r)(1-\alpha-\beta)}{h} w - (\alpha + \beta)$. z is the positive value; hence, the following conditions are needed $z_t \geq 0$ and $z_{t+1} \geq 0$, that is, $h \geq \frac{(1-\alpha-\beta)(1+r)(w+(w-h)z)}{\alpha+\beta}$. The stable condition is $-1 < \frac{dz_{t+1}}{dz_t}$ around z, and $\frac{dz_{t+1}}{dz_t}$ is given as follows:

$$\frac{dz_{t+1}}{dz_t} = -\frac{\phi w(w-h)}{\alpha \left(w + (w-h)z + \frac{h}{1+r} \right)^2}. \tag{9}$$

This stable condition can be written by $\alpha \left(w + (w-h)z + \frac{h}{1+r} \right)^2 > \phi w(w-h)$. z, which satisfies the stable condition, is given as follows:

$$z > z^0 \equiv \frac{-D + (D^2 - 4A^2 E)^{\frac{1}{2}}}{2A^2}, \tag{10}$$

where $D \equiv 2(w-h)\left(w+\frac{h}{1+r}\right)$, $E \equiv w^2+\frac{h^2}{(1+r)^2}+w\left(\frac{2h}{1+r}-\frac{\phi(w-h)}{\alpha}\right)$. If $z > z^0$, the stable condition is satisfied. Therefore, z_t converges with the fluctuation. However, if $z < z^0$, the stable condition is not satisfied. Hence, z_t deverges with the fluctuation. Fig. 2 illustrates the dynamic path.

[Fig. 2 Inserted]

Due to this analysis, the following proposition is established.

Proposition 1 If the degree of intergenerational support in the steady state z^* is larger than z^0, then z_t converges to z^* with fluctuation. Further, if z^* is smaller than z^0, then z_t^* diverges with fluctuation. However, if z^* is equal to z^0, then z_t is cyclical.

If the z_t that agents in generation t receive is small, then z_{t+1} rises due to the agents supply of additional childcare support in order to gain more income. However, if z_t is large, then z_{t+1} declines due to the fact that agents spend more time in leisure because of sufficient income. As a result, z_t oscillates over time and so does the fertility rate n_t. If the amount of change in z_t is equal to that in z_{t+1}, then z_t and n_t oscillate to infinity.

If the childcare cost ϕ is large, intergenerational childcare support z_t diverges over time. Since the higher ϕ lowers the fertility rate, the childcare support provided by grandparents to their children is relatively large.

4 The analysis

This section presents the relationship between an increase in the reward for the childcare support offered by the old generation denoted by h and that support per household in the steady state denoted by z. Differentiating z with respect to h, we obtain:

$$\frac{dz}{dh} = \frac{\frac{\phi w}{\alpha}\left(\frac{(1+r)(1-\alpha-\beta)}{h^2} - \frac{I_h}{I^2}\right)}{1 + \frac{\phi w}{\alpha}\frac{I_z}{I^2}}, \tag{11}$$

where $I_h \equiv \frac{\partial I}{\partial h}$, $I_z \equiv \frac{\partial I}{\partial z}$. The sign of this equation is ambiguous. The term of $\frac{(1+r)(1-\alpha-\beta)}{h^2}$ in the numerator represents the price effect. The rise in h indicates an increase in the oppotunity cost for leisure in the old, therefore the rise in h, that is, the price effect, increases z. On the one hand, the term $-\frac{I_h}{I^2}$ in the numerator represents the income effect. If $z < \frac{1}{1+r}$, the rise in h increases households' income, and thus, the leisure and the fertility rate also rise. Consequently, the income effect in the case of $z < \frac{1}{1+r}$ decreases z. If the price effect is higher than the income effect, then the rise in h increases z.

However, if $z > \frac{1}{1+r}$, the sign of I_h is negative, and thus, the rise in h always increases z. Due to abovementioned analysis, the following proposition is established.

Proposition 2 If $z < \frac{1}{1+r}$, the rise in h pulls up z when the price effect in higher than the income effect. However, if $z > \frac{1}{1+r}$, then the rise in h always pulls up z.

When $z < \frac{1}{1+r}$, that is, when the intergenerational childcare support z is relatively small, the rise in h does not always increase z.

5 Welfare analysis

This section examines the allocations maximizing the social welfare subject to the resource constraint.

Due to the above conditions, the fertility rate consumption ratio $\frac{n_t}{c_{t+1}}$ is shown as follows:

$$\frac{n_t}{c_{t+1}} = \frac{\alpha}{\beta w \left(\phi(1+r) + \frac{z_{t+1}}{n_t} \right)}. \tag{12}$$

The fertility rate consumption ratio $\frac{n_t}{c_{t+1}}$ is different from the ratio in the decentralized economy due to the term $\beta w \frac{z_{t+1}}{n_t}$. If z_{t+1} is zero, then the ratio in the decentralized economy corresponds to the ratio in command optimum. $z_t = 0$ indicates the absence of intergenerational childcare support. Intergenerational support divorces the fertility rate from the socially optimal rate.

The abovementioned analysis provides the following proposition.

Proposition 3 The fertility rate consumption ratio $\frac{n_t}{c_{t+1}}$ in a command optimum economy is smaller than the ratio in a decentralized economy.

The fertility rate n_t provides the next generation with an externality. In a decentralized economy, the current parents decide the fertility rate without considering the decline in the childcare support per household provided by the old in the next period. Proposition 3 shows that the social welfare is improved by decreasing the fertility rate in the present and increasing the childcare support per household supplied by the old.

6 Concluding Remarks

This paper has analyzed the relationship between the fertility rate and the intergenerational childcare support (the childcare support per household offered by the old) based on the overlapping generations model in which households determine the number of children (the fertility rate) and the childcare support in the old. Among the results that have been put forward in this paper, the following three are especially noteworthy.

First, if the intergenerational support is introduced into the endogenous fertility rate model, the fertility rate fluctuates. Furthermore, the breadth of the fluctuation is determined by the parameter.

Second, a rise in the degree of relative altruism does not always increase the fertility rate, although the support increases the intergenerational childcare support.

Third, the existence of selfish parents causes market failure. If the parents consider the utility of future generations, the market failure vanishes.

This paper can refer to the recent decreasing fertility rate as follows. In Japan, the fertility rate is decreasing with every passing year. The fertility rate in 1947 was 4.54, but the rate in 2005 was 1.25 (Data: Ministry of Health, Labour and Welfare "the dynamic statistics of the population"). The high fertility rate in the past causes the decline in the intergenerational support per households, therefore, the fertility rate declines in recent years.

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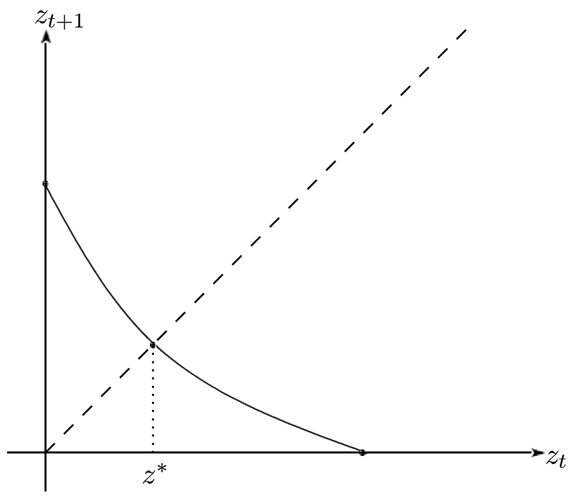

Fig. 1 z in the steady state

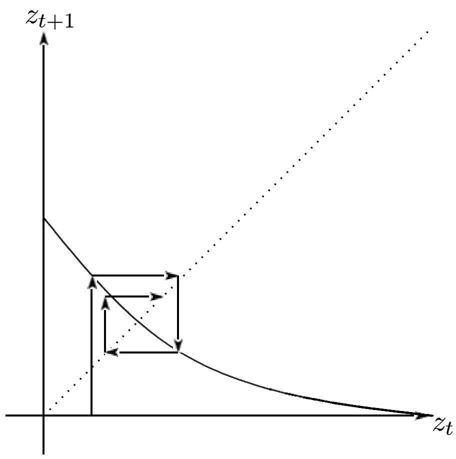

Fig. 2-1 Convergence

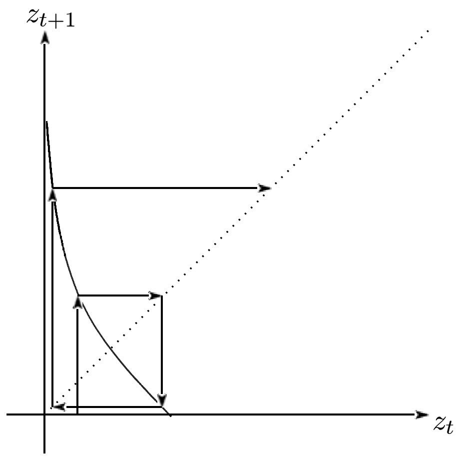

Fig. 2-2 Divergence

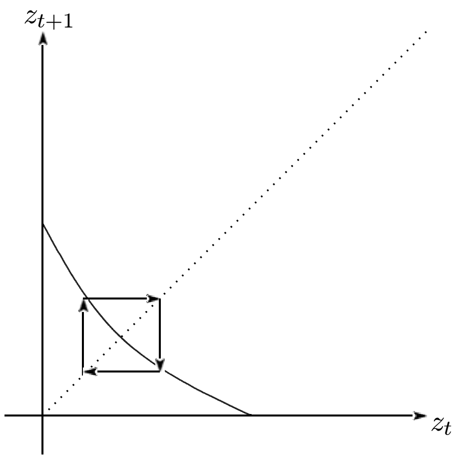

Fig. 2-3 Cycle

Mathematical Notes

Solution for optimal allocations The Lagrange function is set as follows,

$$
\begin{aligned}
L &\equiv \alpha \ln n_t + \beta \ln c_{t+1} + (1-\alpha-\beta)\ln l_{t+1}, \\
&+ \lambda\left(w_t(1+z_t-\phi n_t) - hz_t + \frac{h}{1+r_{t+1}}(1-l_{t+1}) - \frac{c_{t+1}}{1+r_{t+1}}\right).
\end{aligned}
$$

where λ is the Lagrange multiplier. The first order conditions are shown as follows:

$$
\begin{aligned}
\frac{\partial L}{\partial n_t} &= \frac{\alpha}{n_t} - \lambda\phi w_t = 0, && (13)\\
\frac{\partial L}{\partial c_{t+1}} &= \frac{\beta}{c_{t+1}} - \frac{\lambda}{1+r_{t+1}} = 0, && (14)\\
\frac{\partial L}{\partial l_{t+1}} &= \frac{1-\alpha-\beta}{l_{t+1}} - \frac{\lambda h}{1+r_{t+1}}, && (15)\\
\frac{\partial L}{\partial \lambda} &= w_t(1+z_t-\phi n_t) - hz_t + \frac{h}{1+r_{t+1}}(1-l_{t+1}) - \frac{c_{t+1}}{1+r_{t+1}} = 0. && (16)
\end{aligned}
$$

Substituting (13)∼(15) into (16), we obtain $\lambda = \frac{1}{w_t+(w_t-h)z_t+\frac{h}{1+r_{t+1}}}$. By substituting λ into (13)∼(15), the optimal allocations (4)∼(6) are shown.

Social welfare problem The Lagrange function to be achieved in order to maximize the social welfare function shown by $W \equiv \sum_{t=1}^{\infty} U_t$ subject to the resource constraints $(1+r)s_{t-1} + w(1+z_t) - \phi w n_t - c_t - s_t = 0$ and

$z_t = \frac{1-l_t}{n_{t-1}}$ is set as follows:

$$
\begin{aligned}
L &\equiv \sum_{t=1}^{\infty} \rho^t \left(\alpha \ln n_t + \beta \ln c_{t+1} + (1-\alpha-\beta) \ln l_{t+1}\right), \\
&+ \sum_{t=1}^{\infty} \mu_{1t} \left((1+r)s_{t-1} + w(1+z_t) - \phi w n_t - c_t - s_t\right), \\
&+ \sum_{t=1}^{\infty} \mu_{2t} \left(1 - l_t - z_t n_{t-1}\right), \\
\textit{where} \quad & s_0 \textit{ is given},
\end{aligned}
$$

where s_t represents the savings in period t and ρ denotes the discount rate. μ_{1t} and μ_{2t} are Laglange multipliers, respectively. The first order conditions are shown as follows,

$$n_t \qquad \rho^t \alpha \frac{1}{n_t} - \phi w \mu_{1t} - \mu_{2t+1} z_{t+1} = 0, \tag{17}$$

$$c_{t+1} \qquad \rho^t \beta \frac{1}{c_{t+1}} - \mu_{1t+1} = 0, \tag{18}$$

$$l_{t+1} \qquad \rho^t (1-\alpha-\beta) \frac{1}{l_{t+1}} - \mu_{2t+1} = 0, \tag{19}$$

$$s_t \qquad \mu_{1t+1}(1+r) - \mu_{1t} = 0, \tag{20}$$

$$z_t \qquad \mu_{1t} w - \mu_{2t} n_{t-1} = 0. \tag{21}$$

From the abovementioned conditions, (12) is obtained.