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Iritani, Jun

Lee, Tae-Yeoun

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A General Equilibrium Model with Tradable Emission Permits: Efficiency and Coase Property *

Jun Iritani
Graduate School of Economics, Kobe University

and

Tae-Yeoun Lee
Faculty of Economics, Ryukoku University

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Abstract

Recently, much attention has been paid to tradable emission permits (TEP) which many countries contemplate introducing as a key instrument of reducing emissions of greenhouse gases. In this paper, we construct a general equilibrium model in which the price of TEP is determined through the global market trading. We deal with two countries. There are one household and one firm in each country. Both households and firms are allowed to sell and buy TEP at a market price. Our model exhibits two characteristic features. One is the *inefficiency*, i.e., the quantity of emissions in equilibrium is *excessive* from the Paretean viewpoint. The other is a *Coase Property*, i.e., the allocation of goods and emissions attained in equilibrium does not depend on the initial distribution of TEP nor on the aggregate amount of TEP.

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Correspondence address: Jun Iritani, Graduate School of Economics, Kobe University, 2-1, Rokko-dai-cho, Nada-ku, Kobe 657-8501, Japan (e-mail:*iritani@econ.kobe-u.ac.jp*).
Tae-Yeoun Lee, Faculty of Economics, Ryukoku University, 67 Tsukamoto-cho, Fukakusa, Fushimi-ku, Kyoto 612-8577, Japan (e-mail:*tlee@econ.ryukoku.ac.jp*).

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1 Introduction

Recently, much attention has been paid to tradable emission permits (abbreviated as TEP hereafter) which many countries contemplate introducing as a key instrument of reducing emissions of greenhouse gases. The main objectives of this paper are to construct a general equilibrium model containing a global market for tradable emission permits and to establish some characteristic features of the equilibrium, especially, the inefficiency of the tradable emission permits system and a Coase property which asserts that the equilibrium allocation is independent of the initial distribution of the tradable emission permits.

The UK emissions trading scheme toward the greenhouse gases was established and has been implementing since 2001 before the Kyoto Protocol becomes effectual. An EU-wide emissions trading scheme (EU ETS) consisting of twenty five countries of the EU has been carried out since January 2005. In the United sates an emissions trading program has been put in practice tentatively since December 2003, where firms participate voluntarily and the private sector takes the initiative. Furthermore, in December 2005, seven states of the east part of United States announced the introduction of an emissions trading system on the power plants from 2009. Also, the Japanese Environmental Agency is experimentally carrying out an emissions trading system from 2005.

The underlying idea common to these systems is for them to set up a market for trading TEP and to make use of the efficiency of the market. In fact, the EU ETS, which is currently the biggest in market scale, introduces the following scheme:[1]

1. The EU-Allowances are allocated to firms by way of grandfathering.
2. Households are not given EU-Allowances, but they can purchase the allowance through auctioning.

It is Coase (1960) who first contrives the concept of tradable emission permits. He obtains the so-called Coase theorem which consists of two sub-theorems. One is the efficiency theorem stating that an efficient resource allocation can be achieved through voluntary bargains between the agents concerned. And the other is the Coase property stating that the resulting resource allocation does not depend on an initial state of the vested rights. Therefore, we can focus the discussion on emissions of carbon dioxide and the global warming in two points, i.e., (i) the efficiency and (ii) the Coase property of resource allocation.

Coase's discussion is based on the voluntary bargains struck among interested parties to internalize the social cost of an externality. It is not necessarily based on the trade in market. Therefore, it is often pointed out that the efficiency is not necessarily attained when the transactions costs are very high or when the information is incomplete.[2] While, it is expected that the transactions costs can be made small if the private negotiations can be replaced by the exchange in the TEP market. For example, Crocker (1966) and Dales (1968) applied the

[1] See European Commission (2000, 2001, 2003, 2005) for details.

[2] Schweizer (1988) develops a suitable theoretical framework to analyze the Coase theorem and externality. For discussion on the transactions costs, see Stavins (1995), Andersson (1997) and Netusil and Braden (2001).

Coase's idea especially to the environmental problem, and they support a TEP system where the permits right is allocated to emitting pollutants properly and where the permits right can be traded. Montgomery (1972) and Tietenberg (1985) consider the theoretical features of the TEP system. In particular, Montgomery (1972) shows that the efficiency is achieved from the viewpoint of cost minimization of a firm and that the initial distribution of TEP does not have effects on the producer's plan on the target level of total emissions, that is, he shows that the Coase property holds true for the behavior of firm.

On the other hand, the competitiveness of the TEP market is frequently questioned. In fact, Misiolek and Elder (1989), Sartzetakis (1997) and Joskow and Schmalensee (1998) scrutinize the problem with paying special attention to the strategic aspect of agents' behavior to show that Pareto efficiency is not always achieved. Furthermore, Hahn (1984), Malueg (1990), Maeda (2003) and Eshel (2005) show that the efficiency is not necessarily achieved when firms have market power. Even more, Malueg (1990) and Eshel (2005) show that the Coase property does not hold.

Recently, the role of the households in the TEP market is highlighted. Ahlheim and Schneider (2002) stress the importance of taking households into consideration. This is because the households suffer from the negative externality induced by the emissions. The externality cannot be fully internalized without considering the behavior of households. In particular, it is noteworthy that they consider a case in which the households are given all the initial TEP distribution. This case corresponds to auctioning of the TEP because the government revenues from auctioning are returned to households by way of reducing taxes or by transfer payments. We will pick this case in the present paper.

Although the above literature allows us to understand the functioning of the TEP market, most of the researches base their conclusions on a partial equilibrium analysis. It is extremely important to establish a model containing the TEP market in the framework of a general equilibrium setting in order to fully comprehend the characteristics of the TEP market. In this paper, we will construct a general equilibrium model with two countries, two firms, two households, two products and three factors of production. We will deal with a market where firms and households can buy and sell the TEP. Both firms and households have incentives to buy (resp. sell) the TEP when they feel the amount of emissions is too much (resp. little) in comparison with the market price of TEP. An important point in constructing a general equilibrium model with TEP is the number of households. No problems can arise when the number is unity. We must consider the problem of how many TEP one household buys in response to other household's purchases of TEP when the number is plural. That is to say, a household has to be given beforehand the level of the other households' purchases of TEP. This is because that the emissions have a property of "public bads" from which all households suffer ineludibly. In the present paper we assume a Nash equilibrium concept. That is, given the other household's amount of purchase or selling of TEP, one household determines her size of purchase or selling in TEP. An equilibrium has to contain an equality between the amounts of TEP given beforehand and the amounts determined. This kind of equilibrium is assumed in various fields, e.g., in theory of voluntary contribution of public goods and in game theory.[3]

The main results of this paper can be summarized as:

[**Excessiveness of total emissions**] The allocation determined in equilibrium is not efficient and, in particular, the total amount of emissions is excessive from the Paretean viewpoint.

[**Coase Property**] The resulting allocation of private goods and total emissions in equilibrium is independent of the initial distribution of TEP and furthermore of the summation of TEP initially allocated over nations.

It may be straightforward that the allocation in equilibrium is not efficient since it is described á la Nash. Our result obtained here is, however, strict in the sense that the amount of emissions in equilibrium is excessive.

Our Coase property is remarkable since the two methods of distributing TEP such as grandfathering and auctioning have an identical effect on the total amount of emissions. In fact, our model contains a case where all the TEP are initially given to the firms. Therefore, the method of grandfathering is in our scope. On the other hand, the method of auctioning is also represented by a case where all the TEP are given to households, since the government revenues due to auctioning of the TEP belong finally to the households. In addition to this, our Coase property holds regardless of the volume of summation of initially given amounts of TEP over all the nations.

This paper is organized as follows. In section 2, we present the model and study the efficiency property of the equilibrium. In section 3, we establish the Coase property under grandfathering. In section 3.3, we prove more comprehensive Coase property.

2 The Model

In this paper, we establish a simple general equilibrium model with two products and three factors of production containing labor, materials and emissions. There are two countries. In each country there are one household and one firm. We call a household and a firm in the country i the household i and the firm i respectively, $i = 1, 2$. The firm i produces the commodity i, $i = 1, 2$ and commodities 1 and 2 are distinct.

The utility function of household i is the following:

$$u^i(x_1^i, x_2^i, E), i = 1, 2,$$

where $x_j^i (i, j = 1, 2)$ and E are her consumption of the goods j and the total emissions she suffers from, respectively. We assume that the utility function is decreasing with E and that it satisfies usual properties such as quasi concavity and continuity.[4]

[3] See, e.g., Bergstrom, Blume and Varian (1986) and Warr (1983).

[4] Denote the maximum permissible emissions level of a household as $\hat{E}$, we can express utility function as $u^i(x_1^i, x_2^i, E) = U^i(x_1^i, x_2^i, \hat{E} - E)$. It is reasonable to assume usual quasi-concavity and continuity for U^i.

The firm i produces the i-th good y_i by using materials m_i, labor L_i and emissions e_{fi}. The production technology is represented as:

$$y_i = f^i(m_i, e_{fi}, L_i),\ \ i = 1, 2.$$

We assume that the production function f^i is increasing in m_i, e_{fi}, and L_i and satisfies usual properties such as continuity and concavity. We suppose that materials represent a general carbon energy such as fossil fuel. The total emissions E are equal to the sum $e_{f1} + e_{f2}$ of emissions of each firm e_{fi}, $i = 1, 2$.

In the literature, emissions and pollution are often distinguished. For example, in Cropper and Oates (1992) utility function and production technology are represented as:

$$v^i(x_1^i, x_2^i, Q),\ \ f^i(M, E, Q, L),\ \ Q = \psi(E),$$

where E and Q denote emissions and pollution respectively and where ψ is a function transforming emissions to pollution. In this paper, however, we suppose that firms emit greenhouse gases that do not affect production directly. Furthermore, we regard utility function $u^i(x_1^i, x_2^i, E)$ as a composite function $v^i(x_1^i, x_2^i, \psi(E))$. Therefore, in this paper we do not distinguish between emissions E and pollution Q.

In the i-th country the household i is the owner of the initial endowment of materials $\bar{M}_i$ and labor $\bar{L}_i$.

Finally, we assume that there is no mobility of labor between countries and that the wages in two regions can be different. On the other hand, there are global markets for commodities 1,2, materials, and TEP.

2.1 Tradable Emission Permits Market

We assume that the legislatively given $\bar{e}_i (> 0)$ amount of TEP is distributed to the i-th country under an international treaty. Each government distributes the TEP to a firm and a household. In EU ETS, for example, they determine the quantitative overall emission target and issue a corresponding number of TEP. Finally each firms are given an allocation of TEP.

Let $\bar{e}_{fi}$ and $\bar{e}_{hi}$ be the amounts of TEP given to the firm i and the household i satisfying:

$$\bar{e}_i = \bar{e}_{fi} + \bar{e}_{hi}, i = 1, 2. \tag{1}$$

We call two triplets $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi}), i = 1, 2$ satisfying (1) an initial distribution of TEP. At first, we confine ourselves to the case $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$. That is to say, we assume that the government of the i-th country gives the firm all the TEP. This implies that *grandfathering* is institutionalized and that all the permits are allocated to the polluting firm according to their historical emissions. Many initial distributions of TEP other than $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ will be considered in Section 3.

Assume that a vector of prices $(p_1^*, p_2^*, p_m^*, p_e^*, w_1^*, w_2^*)$ is given, where $p_i^*(i = 1, 2)$, p_e^*, p_m^*, $w_i^*(i = 1, 2)$ are the price of goods i, the price of TEP, the price of materials and the wage in country i, respectively.

The i-th firm's choice $(y_i^*, L_i^*, m_i^*, e_{fi}^*)$ is a solution to the problem:

$$\begin{cases} \pi_i^* = \max_{(y_i, L_i, m_i, e_{fi})} p_i^* y_i - p_m^* m_i - w_i^* L_i - p_e^*(e_{fi} - \bar{e}_i) \\ \qquad\qquad \text{subject to} \;\; y_i = f^i(m_i, e_{fi}, L_i), \qquad i = 1, 2. \end{cases} \tag{2}$$

The problem (2) is a usual maximizing behavior of a firm, while it includes a decision regarding emissions. That is, if the firm i chooses $e_{fi}^* > \bar{e}_i$, he has to buy an amount of $e_{fi}^* - \bar{e}_i$ in the market, because he is given initially $\bar{e}_i$. On the other hand, if $e_{fi}^* < \bar{e}_i$, he can sell the residual and he is a supplier of TEP. Furthermore, there is no international migration. Then the labor markets are separated into two countries so that wages w_1 and w_2 of two countries can be different[5].

A household can control the quality of environment through purchasing TEP although she suffers from total emissions E. If she buys an amount of TEP, e_{hi}, $i = 1, 2$, then the amount of emissions E which she faces with is

$$\begin{aligned} E &= \text{sum of initial issued TEP} - \text{sum of two households' purchases of TEP} \\ &= \bar{e}_1 + \bar{e}_2 - (e_{hi} + e_{hj}). \end{aligned}$$

The household i, however, cannot know the amount of TEP the other household j buys. Expecting the amount of TEP of the other household's purchase is $\tilde{e}_{hj}$, she can calculate the total emissions from her purchase e_{hi} as:

$$E = \bar{e}_1 + \bar{e}_2 - (e_{hi} + \tilde{e}_{hj}).$$

Given $(\tilde{e}_{h1}, \tilde{e}_{h2})$, the household i's choice $(x_1^{i*}, x_2^{i*}, e_{hi}^*)$ is a solution to the problem:

$$\begin{cases} \max_{(x_1^i, x_2^i, e_{hi})} u^i(x_1^i, x_2^i, \bar{e}_1 + \bar{e}_2 - (e_{hi} + \tilde{e}_{hj})) \\ \text{subject to} \;\; p_1^* x_1^i + p_2^* x_2^i + p_e^* e_{hi} = w_i^* \bar{L}_i + p_m^* \bar{M}_i + \pi_i^*, \;\; e_{hi} \geqq 0, \;\; i = 1, 2, \;\; i \neq j, \end{cases} \tag{3}$$

where we assume the firm i is possessed by the household i. We require that a relation between $(\tilde{e}_{h1}, \tilde{e}_{h2})$ and (e_{h1}^*, e_{h2}^*) holds in equilibrium. That is:

$$(\tilde{e}_{h1}, \tilde{e}_{h2}) = (e_{h1}^*, e_{h2}^*). \tag{4}$$

The equality is a requirement of Nash equilibrium. Including this condition, we can restate (3) and (4) as follows. That is, the vector $(x_1^{i*}, x_2^{i*}, e_{hi}^*)$ is a solution to the problem:

$$\begin{cases} \max_{(x_1^i, x_2^i, e_{hi})} u^i(x_1^i, x_2^i, \bar{e}_1 + \bar{e}_2 - (e_{hi} + e_{hj}^*)) \\ \text{subject to} \;\; p_1^* x_1^i + p_2^* x_2^i + p_e^* e_{hi} = w_i^* \bar{L}_i + p_m^* \bar{M}_i + \pi_i^*, \;\; e_{hi} \geqq 0, \;\; i = 1, 2, \;\; i \neq j. \end{cases} \tag{5}$$

[5]The wages will be identical when global labor market prevails. It will be easy for us to handle this case.

Furthermore, the market equilibrium conditions are expressed as:

$$\begin{cases} x_i^{1*} + x_i^{2*} = y_i^*, & i = 1, 2 \\ m_1^* + m_2^* = \bar{M}_1 + \bar{M}_2 \\ L_i^* = \bar{L}_i, & i = 1, 2 \\ e_{f1}^* + e_{f2}^* + e_{h1}^* + e_{h2}^* = \bar{e}_1 + \bar{e}_2, \end{cases} \tag{6}$$

where the left hand side is total demand and the right hand side is total supply, respectively.

Definition 1 *A pair of price and allocation vectors:*

$$\big((p_1^*, p_2^*, p_m^*, p_e^*, w_1^*, w_2^*), ((x_1^{i*}, x_2^{i*}, e_{hi}^*), (y_i^*, L_i^*, m_i^*, e_{fi}^*, e_{fi}^* - \bar{e}_{fi}), i = 1, 2)\big)$$

satisfying the conditions (2), (5), and (6) *is called a market equilibrium with initial distribution* $(\bar{e}_i, \bar{e}_i, 0)$, $i = 1, 2$ *of TEP under grandfathering.*

Let $((x_1^i, x_2^i, e_{hi}), (y_i, L_i, m_i, e_{fi}, e_{fi} - \bar{e}_{fi}), i = 1, 2)$ be an allocation vector. We call an allocation vector without TEP variables $((x_1^i, x_2^i), (y_i, L_i, m_i, e_{fi}), i = 1, 2)$ a *real allocation vector.*

2.2 Efficiency

In this subsection we analyze the Pareto efficiency of resource allocations. We assume that the utility functions and the production functions are sufficiently smooth in this and next subsections.

We can obtain a Pareto efficient allocation through maximizing one household's utility while leaving the other household's utility constant. An efficient allocation is a solution to the problem:

$$\begin{cases} \max_{(x_1^1, x_2^1, x_1^2, x_2^2, e_{f1}, e_{f2}, m_1, m_2)} u^1(x_1^1, x_2^1, e_{f1} + e_{f2}) \\ \quad \text{subject to} \;\; x_1^1 + x_1^2 \leqq f^1(m_1, e_{f1}, \bar{L}_1) \\ \qquad\qquad\qquad x_2^1 + x_2^2 \leqq f^2(m_2, e_{f2}, \bar{L}_2) \\ \qquad\qquad\qquad m_1 + m_2 \leqq \bar{M}_1 + \bar{M}_2 \\ \qquad\qquad\qquad \bar{u}^2 \leqq u^2(x_1^2, x_2^2, e_{f1} + e_{f2}), \end{cases} \tag{7}$$

where $\bar{u}^2$ is a constant. The corresponding Lagrangian is

$$\begin{aligned} u^1(x_1^1, x_2^1, e_{f1} + e_{f2}) &+ \lambda_1(f^1(m_1, e_{f1}, \bar{L}_1) - x_1^1 - x_1^2) + \lambda_2(f^2(m_2, e_{f2}, \bar{L}_2) - x_2^1 - x_2^2) \\ &+ \lambda_3(\bar{M}_1 + \bar{M}_2 - m_1 - m_2) + \lambda_4(u^2(x_1^2, x_2^2, e_{f1} + e_{f2}) - \bar{u}^2). \end{aligned}$$

Assuming the problem has an interior solution, we get the following the necessary conditions:

$$u_1^1 - \lambda_1 = 0 \tag{8}$$
$$u_2^1 - \lambda_2 = 0 \tag{9}$$
$$\lambda_4 u_1^2 - \lambda_1 = 0 \tag{10}$$
$$\lambda_4 u_2^2 - \lambda_2 = 0 \tag{11}$$
$$u_e^1 + \lambda_1 f_e^1 + \lambda_4 u_e^2 = 0 \tag{12}$$
$$u_e^1 + \lambda_2 f_e^2 + \lambda_4 u_e^2 = 0 \tag{13}$$
$$\lambda_1 f_m^1 - \lambda_3 = 0 \tag{14}$$
$$\lambda_2 f_m^2 - \lambda_3 = 0, \tag{15}$$

where

$$u_e^i = \frac{\partial u^i}{\partial E} < 0, \quad u_j^i = \frac{\partial u^i}{\partial x_j^i} > 0, \ j = 1, 2, \ i = 1, 2,$$
$$f_m^i = \frac{\partial f^i}{\partial m_i} > 0, \quad f_e^i = \frac{\partial f^i}{\partial e_{fi}} > 0, \ i = 1, 2.$$

The equalities (12) and (13) describe the optimality on emissions. Cancelling the Lagrangian multipliers λ_1, λ_2, λ_4 by using (10) and (11), we obtain from (12) that:

$$\left(-\frac{u_e^1}{u_1^1}\right) + \left(-\frac{u_e^2}{u_1^2}\right) = f_e^1. \tag{16}$$

The left hand side of (16) is the summation of "the first household's marginal evaluation of a decrease in total emissions in terms of the commodity one" and that of the second household. The right hand side is "the marginal decrease in the production of commodity one caused by decreasing emissions." In other words, this is the equality between sum of two marginal rates of substitution of emissions to commodity one and marginal cost of emissions, that is, a kind of Samuelson condition for the efficiency of public goods.

In the same way, we have

$$\left(-\frac{u_e^1}{u_2^1}\right) + \left(-\frac{u_e^2}{u_2^2}\right) = f_e^2. \tag{17}$$

2.3 Excessiveness of the total emissions

In this subsection, we examine whether the quantity of emissions in the equilibrium defined in Definition 1 is excessive or not. First, from the marginal condition of the maximization problems (5) and (2), we can have:

$$-\frac{u_e^1}{u_1^1} = -\frac{u_e^2}{u_1^2} = \frac{p_e}{p_1}, \quad f_e^1 = \frac{p_e}{p_1}, \quad -\frac{u_e^1}{u_2^1} = -\frac{u_e^2}{u_2^2} = \frac{p_e}{p_2}, \quad f_e^2 = \frac{p_e}{p_2}. \tag{18}$$

Then we have for emissions and commodity i

$$\text{sum of marginal rates of substitution} = 2 \times \frac{p_e}{p_i} > \frac{p_e}{p_i} = \text{marginal cost}, \;\; i = 1, 2.$$

This implies that the demand price is higher than the supply price for TEP. This leads us to a possibility that the total emissions may be excessive in comparison with the optimal level of emissions.

Now, let us show rigorously that the total amount of emissions in equilibrium is excessive. Let $((p_1^*, p_2^*, p_m^*, p_e^*, w_1^*, w_2^*), ((x_1^{i*}, x_2^{i*}, e_{hi}^*), (y_i^*, L_i^*, m_i^*, e_{fi}^*, e_{fi}^* - \bar{e}_{fi}), i = 1, 2))$ be an equilibrium with $(\bar{e}_i, \bar{e}_i, 0)$, $i = 1, 2$ under grandfathering. By the first order conditions for utility maximization of households' we have:

$$\frac{p_e^*}{p_i^*} = -\frac{u_e^j(x_1^{j*}, x_2^{j*}, e_{f1}^* + e_{f2}^*)}{u_i^j(x_1^{j*}, x_2^{j*}, e_{f1}^* + e_{f2}^*)} = f_e^i(m_i^*, e_{fi}^*, L_i^*), \;\; i = 1, 2, \;\; j = 1, 2. \tag{19}$$

Furthermore, the allocation satisfies market equilibrium conditions such as

$$\begin{cases} x_i^{1*} + x_i^{2*} = y_i^*, & i = 1, 2 \\ m_1^* + m_2^* = \bar{M}_1 + \bar{M}_2, & \\ L_i^* = \bar{L}_i, & i = 1, 2 \\ e_{f1}^* + e_{f2}^* + e_{h1}^* + e_{h2}^* = \bar{e}_1 + \bar{e}_2. & \end{cases} \tag{20}$$

Suppose that the amount of emissions is reduced in the country 1 by $\Delta e (> 0)$ units while other inputs remains intact. Then the production of commodity 1 will decrease by $\Delta y_1 (\fallingdotseq f_e^1 \Delta e)$.[6] Let the consumption of good 1 of each household decrease equally by $\Delta y_1/2 (\fallingdotseq f_e^1 \Delta e/2)$. Then their utilities will decrease by $\Delta \ell^i (\fallingdotseq u_1^i \Delta y_1/2)$, $i = 1, 2$. On the other hand, their utilities will increase by $\Delta b^i (\fallingdotseq -u_e^i \Delta e)$, $i = 1, 2$ because the total emissions are reduced by Δe. By (19), the net benefit of household i is approximately:

$$\begin{aligned} \text{net benefit of household } i &= b^i - \ell^i \\ &\fallingdotseq -u_e^i \Delta e - u_1^i \frac{f_e^1 \Delta e}{2} \\ &= u_1^i \Delta e \frac{p_e^*}{2 p_1^*} > 0, i = 1, 2. \end{aligned}$$

On the other hand, the real allocation generated by this procedure is

$$((x_1^{1*} - \Delta y_1/2, x_2^{1*}), (x_1^{2*} - \Delta y_1/2, x_2^{2*}), (y_1^* - \Delta y_1, L_1^*, m_1^*, e_{f1}^* - \Delta e), (y_2^*, L_2^*, m_2^*, e_{f2}^*)).$$

By (20), we can see that the real allocation is feasible.

We have shown that all the households can be made better off if emissions are reduced by a small amount. Then we can summarize our result as follows:

Theorem 1 (**Excessiveness of Total Emissions**): *The amount of emissions attained in equilibrium is excessive from the Paretean viewpoint.*

[6] The symbol $\fallingdotseq$ implies that the left hand side is approximately equal to the right hand side.

3 Coase property

We will establish in this section that the Coase property broadly holds in our economy, i.e., that a change in the initial distribution of TEP has no effects on the equilibrium real allocation. We do not assume in this section that utility functions and production functions are smooth.

3.1 Coase property under grandfathering

Let $((p_1^*, p_2^*, p_m^*, p_e^*, w_1^*, w_2^*), ((x_1^{i*}, x_2^{i*}, e_{hi}^*), (y_i^*, L_i^*, m_i^*, e_{fi}^*, e_{fi}^* - \bar{e}_{fi}), i = 1, 2))$ be an equilibrium satisfying Definition 1 with initial distribution $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ of TEP under grandfathering. The real allocation of this equilibrium is $((x_1^{i*}, x_2^{i*}), (y_i^*, L_i^*, m_i^*, e_{fi}^*), i = 1, 2)$.

Step 1 Suppose that the initial TEP distribution is modified from $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ to $(\bar{e}'_i, \bar{e}'_i, 0)$, $i = 1, 2$. Here, note that this alteration includes a possibility, $\bar{e}_1 + \bar{e}_2 \neq \bar{e}'_1 + \bar{e}'_2$. That is to say, the total amount of TEP can be different after the alteration. We assume that

Assumption 1 *Each component in the equilibrium allocation vector* $((x_1^{i*}, x_2^{i*}, e_{hi}^*), (y_i^*, L_i^*, m_i^*, e_{fi}^*, e_{fi}^* - \bar{e}_{fi}), i = 1, 2)$ *constitutes an interior solution to the corresponding maximizing problem. And the new initial distribution of TEP lies in a range*:

$$e_{hi}^* > \bar{e}_i - \bar{e}'_i, \quad i = 1, 2. \tag{21}$$

This implies that the change in initial distribution of TEP in country i should not be decreased largely. The inequality (21) allows a case that the new amount $\bar{e}'_i$ of the TEP distributed to country i exceeds the old amount $\bar{e}_i$.

Furthermore, we assume the following:

Assumption 2 *The equilibrium under grandfathering exists uniquely for each economy with initial distribution of TEP which is* $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ *or* $(\bar{e}'_i, \bar{e}'_i, 0), i = 1, 2$.

In what follows, we will show the price vector $(p_1^*, p_2^*, p_m^*, p_e^*, w_1^*, w_2^*)$ is the equilibrium price vector in the economy with new initial distribution $(\bar{e}'_i, \bar{e}'_i, 0), i = 1, 2$ of TEP.

Step 2 The producer's behavior under new distribution of TEP is given by:[7]

$$\pi'_i \stackrel{\text{def}}{=} \max_{(y_i, L_i, m_i, e_{fi})} p_i^* y_i - p_m^* m_i - w_i^* L_i - p_e^*(e_{fi} - \bar{e}'_i)$$
$$\text{subject to} \quad y_i = f_i(m_i, e_{fi}, L_i), i = 1, 2.$$

The solution to this problem is identical with that to (2) since $p_e^* \bar{e}'_i$ and $p_e^* \bar{e}_i$ are constant. This implies that the vector (m_i^*, e_{fi}^*, L_i^*) is the solution. Let π^* be the profit of firm i in the equilibrium with distribution $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ of TEP and we have a relation between previous profit π^* and new profit π'_i as follows:

$$\pi'_i = \pi_i^* + p_e^*(\bar{e}'_i - \bar{e}_i), \quad i = 1, 2.$$

[7]The symbol "$\stackrel{\text{def}}{=}$" indicates that the left hand side is define by the right hand side.

Step 3 Letting $i \neq j$, we represent the i-th household's behavior under new initial distribution $(\bar{e}'_k, \bar{e}'_k, 0), k = 1, 2$ of TEP as follows:

$$\begin{cases} \max_{(x_1^i, x_2^i, e_{hi})} u^i(x_1^i, x_2^i, \bar{e}'_1 + \bar{e}'_2 - (e_{hi} + (e^*_{hj} + \bar{e}'_j - \bar{e}_j))) \\ \quad \text{subject to} \ \ p_1^* x_1^i + p_2^* x_2^i + p_e^* e_{hi} = I'_i, i = 1, 2, \end{cases} \tag{22}$$

where I'_i is the income of household i under $(\bar{e}'_k, \bar{e}'_k, 0), k = 1, 2$ and is defined as:

$$I'_i = w_i^* \bar{L}_i + p_m^* \bar{M}_i + \pi'_i, \ \ i = 1, 2.$$

Moreover, in (22) note that the household i assumes the amount of other household j's purchase of TEP is $(e^*_{hj} + \bar{e}'_j - \bar{e}_j)$. The reason for this will be clear later. Let a solution to (22) be $(x_1^{i*\prime}, x_2^{i*\prime}, e^{*\prime}_{hi})$.

On the other hand, the vector $(x_1^{i*}, x_2^{i*}, e^*_{hi})$ is the solution to the problem:

$$\begin{cases} \max_{(x_1^i, x_2^i, e_{hi})} u^i(x_1^i, x_2^i, \bar{e}_1 + \bar{e}_2 - (e_{hi} + e^*_{hj})) \\ \quad \text{subject to} \ \ p_1^* x_1^i + p_2^* x_2^i + p_e^* e_{hi} = I_i^*, \end{cases} \tag{23}$$

where I_i^* is the income of household i in the economy with initial distribution $(\bar{e}_k, \bar{e}_k, 0), k = 1, 2$ of TEP and is defined as $w_i^* \bar{L}_i + p_m^* \bar{M}_i + \pi_i^*$. The relation of two incomes I'_i and I_i^* is:

$$I'_i = I_i^* + p_e^*(\bar{e}'_i - \bar{e}_i), \quad i = 1, 2.$$

Step 4 Defining $E'_{hi} = \bar{e}'_1 + \bar{e}'_2 - (e_{hi} + e^*_{hj} + (\bar{e}'_j - \bar{e}_j))$, we can rewrite the utility function in (22) as:

$$u^i\left(x_1^i, x_2^i, E'_{hi}\right).$$

We regard E'_{hi} as a variable. Rewriting the budget constraint in (22), we have

$$\begin{aligned} p_1^* x_1^i + p_2^* x_2^i + p_e^* e_{hi} &= w_i^* \bar{L}_i + p_m^* \bar{M}_i + \pi'_i \\ p_1^* x_1^i + p_2^* x_2^i - p_e^* E'_{hi} &= w_i^* \bar{L}_i + p_m^* \bar{M}_i + \pi_i^* + p_e^*(\bar{e}'_i - \bar{e}_i) \\ &\quad - p_e^*\left(\bar{e}'_1 + \bar{e}'_2 - e^*_{hj} - (\bar{e}'_j - \bar{e}_j)\right) \\ &= w_i^* \bar{L}_i + p_m^* \bar{M}_i + \pi_i^* - p_e^*(\bar{e}_1 + \bar{e}_2 - e^*_{hj}), \end{aligned}$$

where $i \neq j, i = 1, 2, j = 1, 2$. Therefore, the problem (22) is equivalently rewritten as:

$$\begin{cases} \max_{(x_1^i, x_2^i, E'_{hi})} u^i\left(x_1^i, x_2^i, E'_{hi}\right) \\ \quad \text{subject to} \ \ p_1^* x_1^i + p_2^* x_2^i - p_e^* E'_{hi} = w_i^* \bar{L}_i + p_m^* \bar{M}_i + \pi_i^* - p_e^*\left(\bar{e}_1 + \bar{e}_2 - e^*_{hj}\right) \\ \qquad\quad \text{and} \quad E'_{hi} \leqq \bar{e}'_1 + \bar{e}'_2 - (e^*_{hj} + (\bar{e}'_j - \bar{e}_j)), i = 1, 2. \end{cases} \tag{24}$$

The second constraint in (24) implies that $e_{hi} \geqq 0$.

Step 5 Let us rewrite the problem (23) in the same manner as in Step 4. Putting $E_{hi} \stackrel{\text{def}}{=} \bar{e}_1 + \bar{e}_2 - (e_{hi} + e^*_{hj})$, we can represent the maximization problem (23) equivalently as

$$\begin{cases} \max_{(x^i_1, x^i_2, E_{hi})} u^i\left(x^i_1, x^i_2, E_{hi}\right) \\ \quad \text{subject to } \ p^*_1 x^i_1 + p^*_2 x^i_2 - p^*_e E_{hi} = w^*_i \bar{L}_i + p^*_m \bar{M}_i + \pi^*_i - p^*_e\left(\bar{e}_1 + \bar{e}_2 - e^*_{hj}\right) \\ \qquad\qquad \text{and } \ E_{hi} \leqq \bar{e}_1 + \bar{e}_2 - e^*_{hj}. \end{cases} \tag{25}$$

Step 6 Comparing the maximization problem (24) with (25), we can find that two problems are identical except the second constraints. Furthermore, the vector $(x^{i*}_1, x^{i*}_2, \bar{e}_1 + \bar{e}_2 - (e^*_{hi} + e^*_{hj}))$ is the interior solution to (25) since it satisfies the second constraint of (25) in strictly inequality by (21). We can see the vector $(x^{i*}_1, x^{i*}_2, \bar{e}_1 + \bar{e}_2 - (e^*_{hi} + e^*_{hj}))$ satisfies the second constraint of (24) in strict inequality. In fact, from the assumption (21), we have:

$$\begin{aligned} \bar{e}_1 + \bar{e}_2 - (e^*_{hi} + e^*_{hj}) &= \bar{e}'_1 + \bar{e}'_2 - (e^*_{hi} + (\bar{e}'_i - \bar{e}_i)) - (e^*_{hj} + (\bar{e}'_j - \bar{e}_j)) \\ &< \bar{e}'_1 + \bar{e}'_2 - (e^*_{hj} + (\bar{e}'_j - \bar{e}_j)). \end{aligned}$$

Therefore, a triplet

$$\left(x^{i*}_1, x^{i*}_2, \bar{e}_1 + \bar{e}_2 - (e^*_{hi} + e^*_{hj})\right)$$

is a solution to (24). By definition, this implies:

$$e'^*_{hi} = e^*_{hi} + \bar{e}'_i - \bar{e}_i.$$

Step 7 The steps so far developed show that the equilibrium with new distribution $(\bar{e}'_i, \bar{e}'_i, 0), i = 1, 2$ of TEP under grandfathering is

$$\left((p^*_1, p^*_2, p^*_m, p^*_e, w^*_1, w^*_2),\ ((x^{i*}_1, x^{i*}_2, e^*_{hi} + \bar{e}'_i - \bar{e}_i), (y^*_i, L^*_i, m^*_i, e^*_{fi}, e^*_{fi} - \bar{e}'_{fi}), i = 1, 2)\right).$$

In addition to this, the real allocation is

$$((x^{i*}_1, x^{i*}_2), (y^*_i, L^*_i, m^*_i, e^*_{fi}), i = 1, 2).$$

We can observe that a pair of price and the real allocation vectors obtained is identical with that of the equilibrium with $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ under grandfathering. We must stress that the total emissions are identical in two equilibria.

This result can be summarized in the following theorem.

Theorem 2 [Coase Property under grandfathering] *Suppose that there are two economies which are different only in the given initial TEP distributions which are $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ and $(\bar{e}'_i, \bar{e}'_i, 0), i = 1, 2$ respectively. Suppose that these economies satisfy Assumptions 1 and 2 then the equilibrium price and the equilibrium real allocation are identical in two economies.*

Assumption 1 is essential for Theorem 2. On the other hand, it is straightforward that we can establish a similar proposition without Assumption 2. That is:

Corollary 1 *Suppose that there are two economies which are different only in the given initial TEP distributions which are* $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ *and* $(\bar{e}'_i, \bar{e}'_i, 0), i = 1, 2$ *respectively. Suppose that these economies satisfy Assumption 1 then for any equilibrium in the economy with* $(\bar{e}_i, \bar{e}_i, 0)$, $i = 1, 2$ *there exists an equilibrium in the economy with* $(\bar{e}'_i, \bar{e}'_i, 0)$, $i = 1, 2$ *whose pair of price and real allocation is identical with that of the equilibrium with* $(\bar{e}_i, \bar{e}_i, 0)$, $i = 1, 2$.

It is noteworthy that Assumption 1 is not necessarily a local constraint. In fact, any $\{\bar{e}'_1, \bar{e}'_2\}$ such that $\bar{e}_i < \bar{e}'_i, i = 1, 2$ satisfies (21). This implies an international agreement to increase the total TEP has no effects on real allocation. Furthermore, even if a policy to decrease total TEP is agreed by governments it will have no effects on real allocation as far as Assumption 1 hold. The policy of decreasing total TEP will have effects on total emissions when at least one household stops buying TEP.

3.2 A graphical exposition

Let us illustrate the essence of the Theorem 2 graphically. First, noting that

$$\begin{aligned} E^*_{hi} &= \bar{e}_1 + \bar{e}_2 - (e^*_{h1} + e^*_{h2}) \\ &= \bar{e}'_1 + \bar{e}'_2 - ((e^*_{h1} + \bar{e}'_1 - \bar{e}_1) + \left(e^*_{h2} + \bar{e}'_2 - \bar{e}_2)\right), \end{aligned}$$

and defining

$$Y_i \stackrel{\text{def}}{=} w^*_i \bar{L}_i + p^*_m \bar{M}_i + \pi^*_i - p^*_e \left(\bar{e}_1 + \bar{e}_2 - e^*_{hj}\right),$$

then we know that the first constraints in (24) and (25) are identical, that is,

$$p^*_1 x^i_1 + p^*_2 x^i_2 - p^*_e E_{hi} = Y_i.$$

This is the budget line in Figure 1. The budget line is touching an indifference curve at a point (E^*_{hi}, x^{i*}) in Figure 1. Rearranging the second constraints in (24) and (25), we have:

$$E'_{hi} \leqq \bar{E}'_{hi} = \bar{e}'_1 + \bar{e}'_2 - (e^*_{hj} + (\bar{e}'_j - \bar{e}_j)), \quad E_{hi} \leqq \bar{E}_{hi} = \bar{e}_1 + \bar{e}_2 - e^*_{hj}.$$

Then a change in the initial distribution of TEP from $(\bar{e}_1, \bar{e}_2)$ to $(\bar{e}'_1, \bar{e}'_2)$ makes the boundary in the second constraint vary from $\bar{E}_{hi}$ to $\bar{E}'_{hi}$. The touching point is intact when the initial distribution of TEP varies. And thus, if E^*_{hi} is an interior solution and if the modification is sufficiently small, $\bar{E}'_{hi}$ can be located at the right of E^*_{hi} as illustrated in Figure 1. This situation make the pair (E^*_{hi}, x^{i*}) be the best choice for household i even when the boundary of the second constraint is $\bar{E}'_{hi}$.

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Figure 1 is around here.

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3.3 Comprehensive Coase property

In the previous subsections, we have considered the case where the method of distributing initial TEP is subject to grandfathering. In this subsection, we consider the more general case that the government of country i gives not only the firm but also the household the initial distribution of TEP by $\bar{e}_{fi}$ and $\bar{e}_{hi}$, respectively.

The new equilibrium concept in the economy is given as follows:

Definition 2 *A pair of price and allocation vectors*

$$\big((p_1^*, p_2^*, p_m^*, p_e^*, w_1^*, w_2^*), ((x_1^{i*}, x_2^{i*}, e_{hi}^* - \bar{e}_{hi}), (y_i^*, L_i^*, m_i^*, e_{fi}^*, e_{fi}^* - \bar{e}_{fi}), i = 1,2)\big).$$

is an equilibrium with initial distribution $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi})$, $i = 1,2$ *of TEP satisfying* (1) *when*

(i) $(y_i^*, L_i^*, m_i^*, e_{fi}^*)$ *is a solution to the problem:*

$$\begin{cases} \pi_i^* = \max_{(y_i, L_i, m_i, e_{fi})} p_i^* y_i - p_m^* m_i - w_i^* L_i - p_e^*(e_{fi} - \bar{e}_{fi}) \\ \qquad\qquad \text{subject to} \;\; y_i = f^i(m_i, e_{fi}, L_i), \end{cases} \tag{26}$$

$i = 1,2.$

(ii) $(x_1^{i*}, x_2^{i*}, e_{hi}^*)$ *is a solution to the problem:*

$$\begin{cases} \max_{(x_1^i, x_2^i, e_{hi})} u^i(x_1^i, x_2^i, \bar{e}_1 + \bar{e}_2 - (e_{hi} + e_{hj}^*)) \\ \;\; \text{subject to} \;\; p_1^* x_1^i + p_2^* x_2^i + p_e^* e_{hi} = w_i^* \bar{L}_i + p_m^* \bar{M}_i + p_e^* \bar{e}_{hi} + \pi_i^*, \;\; e_{hi} \geqq 0, \end{cases} \tag{27}$$

$i = 1,2, \; i \neq j.$

(iii) *The demand-supply equalities corresponding to* (6) *must hold.*

The vector $((x_1^{i*}, x_2^{i*}), (y_i^*, L_i^*, m_i^*, e_{fi}^*), i = 1,2)$ associated with the equilibrium is a real allocation vector in this economy.

We assume instead of Assumption 2 that:

Assumption 3 *There exists a unique equilibrium in the economy with the initial distribution of TEP which is* $(\bar{e}_i, \bar{e}_i, 0), i = 1,2$ *or* $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi}), i = 1,2$.

We can show that the total emissions is *excessive* from the Paretean viewpoint in the equilibrium described by Definition 2. In fact, the marginal conditions obtained from problems (26) and (27) are identical with those in (18) when functions are smooth. Therefore, we can apply the same discussion as in Section 2.2.

We can describe many economies through choosing a suitable initial distribution $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi})$, $i = 1,2$ of TEP. As we have seen, we can depict grandfathering by the equilibrium with initial distribution $(\bar{e}_i, \bar{e}_i, 0), i = 1,2$ of TEP. Another interesting economy is that with initial distribution $(\bar{e}_i, 0, \bar{e}_i)$, $i = 1,2$ of TEP. That is, all the TEP are given to households in this economy.

We can interpret this economy as that where each government introduces auctioning for allocating TEP because of the following reasons. First, the government revenues from auctioning are returned to households through reducing income taxes or through transfer payments. Second, a household's income under auctioning is identical with the income that she will have when she is given all the TEP as her initial holding.

Now, let us suppose that the following pair of price and allocation vectors be an equilibrium with $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ under grandfathering:

$$((p_1', p_2', p_m', p_e', w_1', w_2'), ((x_1^{i\prime}, x_2^{i\prime}, e_{hi}'), (y_i', L_i', m_i', e_{fi}', e_{fi}' - \bar{e}_{fi}), i = 1, 2)).$$

Let π_i' be the profit of firm i associated with the equilibrium, $i = 1, 2$. Furthermore, suppose that an initial distribution of TEP is altered from $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ to $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi})$, $i = 1, 2$ of TEP satisfying (1). We will show that the price $(p_1', p_2', p_m', p_e', w_1', w_2')$ is the equilibrium price vector in the economy with the initial distribution $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi})$, $i = 1, 2$ of TEP. Then from (26) and (27), we can observe following three facts:

(a) Given the new initial distribution $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi})$, $i = 1, 2$ of TEP, we can see that there are no changes in the production, inputs and final emissions of firms. Therefore, the solution to (26) is $(y_i', L_i', m_i', e_{fi}')$, $i = 1, 2$. Furthermore, the profit of firm i is $\pi_i' + p_e'(\bar{e}_{fi} - \bar{e}_i)$.

(b) The new total income of individual i is:

$$w_i'\bar{L}_i + p_m'\bar{M}_i + \pi_i' + p_e'(\bar{e}_{fi} - \bar{e}_i) + p_e'\bar{e}_{hi} = w_i'\bar{L}_i + p_m'\bar{M}_i + \pi_i'.$$

The right hand side coincides with the income of household i in the equilibrium with initial distribution $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ of TEP. This implies the change in the initial distribution of TEP does not cause that in consumer choice. This implies the consumer i chooses:

$$(x_1^{i\prime}, x_2^{i\prime}, e_{hi}').$$

(c) The demand-supply equalities hold since the allocation $((x_1^{i\prime}, x_2^{i\prime}, e_{hi}'), (y_i', L_i', m_i', e_{fi}', e_{fi}' - \bar{e}_{fi}), i = 1, 2)$ satisfies the conditions in (iii) of Definition 2.

Above discussions lead us to the conclusion that the pair of price and allocation vectors :

$$((p_1', p_2', p_m', p_e', w_1', w_2'), ((x_1^{i\prime}, x_2^{i\prime}, e_{hi}'), (y_i', L_i', m_i', e_{fi}', e_{fi}' - \bar{e}_{fi}), i = 1, 2))$$

is an equilibrium with initial distribution $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi})$, $i = 1, 2$ of TEP. And finally the real allocation associated with an economy with TEP distribution $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi})$, $i = 1, 2$ is :

$$(x_1^{i\prime}, x_2^{i\prime}), (y_i', L_i', m_i', e_{fi}'), i = 1, 2$$

We can summarize the above result in the following theorem:

Theorem 3 [The independency of the redistribution of initial TEP] *Suppose Assumption 3 holds. The real allocation of the equilibrium with initial distribution* $(\bar{e}_i, \bar{e}_i, 0)$*,* $i = 1, 2$ *of TEP is identical with that of the equilibrium allocation with* $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi})$*,* $i = 1, 2$ *satisfying* (1).

Theorems 2 and 3 lead us to a striking conclusion. Let us pick two economies. One is an economy with initial distribution $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi}), i = 1, 2$ of TEP. We call this economy simply the economy with $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi}), i = 1, 2$. The other is an economy with $(\bar{e}'_i, \bar{e}'_{fi}, \bar{e}'_{hi}), i = 1, 2$. By Theorem 3, the real allocation associated with the equilibrium in the economy $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi}), i = 1, 2$ is identical with that in the economy with $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$. The same discussion applies to the economy with $(\bar{e}'_i, \bar{e}'_{fi}, \bar{e}'_{hi}), i = 1, 2$ and that with $(\bar{e}'_i, \bar{e}'_i, 0), i = 1, 2$. Suppose that $\bar{e}_1, \bar{e}_2, \bar{e}'_1, \bar{e}'_2$, and the equilibrium allocation in the economy with $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ satisfy Assumptions 1. Then by Theorem 2, the real allocation attained in the economy with $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ is identical to that with $(\bar{e}'_i, \bar{e}'_i, 0), i = 1, 2$. Finally we obtain that the real allocation in the economy with $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi}), i = 1, 2$ is identical with that with $(\bar{e}'_i, \bar{e}'_{fi}, \bar{e}'_{hi}), i = 1, 2$.

This discussion can be explained as in a following format:

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Figure 2 is around here.

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An thus we can summarize the environmental policy implications of Theorems 1, 2, and 3 as follows:

> **Environmental Policy Implications**: The real allocation attained in equilibrium remains intact even when the governments alter the initial distribution from $(\bar{e}_i, \bar{e}_i, 0), i = 1, 2$ of TEP to $(\bar{e}_i, \bar{e}_{fi}, \bar{e}_{hi}), i = 1, 2$ of TEP. Especially, total amount of emissions in one economy is identical with that in the other economy. If governments agree to decrease total emissions then they must restrict total TEP to the extent that it violates Assumption 1. For example, governments must decrease the total amount of TEP so small that at least one household, i.e., one nation cannot purchase the TEP.

4 Concluding Remarks

In this paper, we have constructed a general equilibrium model with tradable emission permits which is expected to be an effective policy instrument widely. Our result on inefficiency states that the amount of emissions will be excessive from the view point of Pareto efficiency. It is, however, obvious that the inefficiency does not occur in the economy where the number of households is unity. Furthermore, It DOES NOT necessarily imply that an environmental policy using TEP makes firms emit an excessive amount of greenhouse gases from the environmental viewpoint. It is fairly possible that the amount of emissions obtained in our model is compatible with the level of emissions for us to keep the earth from climate change.

We must stress that our Coase Property is NOT necessarily a negative result. It states that the total amount of emissions will be invariant even when the governments' decision on the total amount of TEP varies. This implies that the governments can issue TEP in a rough expectation. The institutional distinction between grandfathering and auctioning does not cause changes in the level of emissions. The amount of greenhouse gases that peoples want is determined in the TEP market. This will make costs administrating TEP system fairly small.

Three kinds of important problems remain unanswered in this paper. One is the dynamic aspect of the TEP system. Another is the comparative statics analysis of the model. The other is to generalize the setting of our model.

Our present model is perfectly static. Therefore, we cannot fully analyze the effects of accumulation of greenhouse gases. For example, decreasing one percent of greenhouse gases in the air will be much more difficult and costly than decreasing emissions in greenhouse gases by the same percent. Therefore, it will be very important for economists to develop a growth model to treat the accumulation of greenhouse gases. In this event, introducing capital into the model would enrich the results.

The Coase property established in our paper is very comprehensive. Then the next problem we must ask is what factors are effective in decreasing the total amount of emissions. For example, does the technical progress which makes production technology more material-saving decrease the total emissions? Does newly discovered carbon energy make the amount of emissions decrease? Trying to answer these questions will be challenging and interesting. The comparative static analysis could help us to answer the questions.

Furthermore, our model is a simple general equilibrium model of match-box size. It contains only two countries, two households, two goods, three production factors. We have a question whether our results remain true in more complicated models or not. Especially, it will be important to analyze the robustness of the Coase Property.

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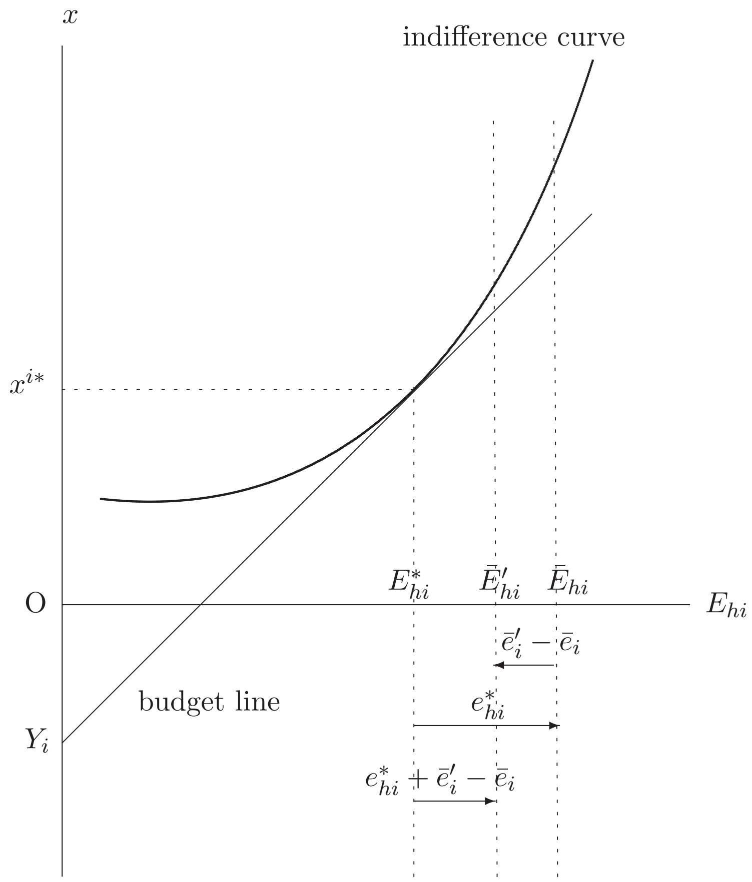
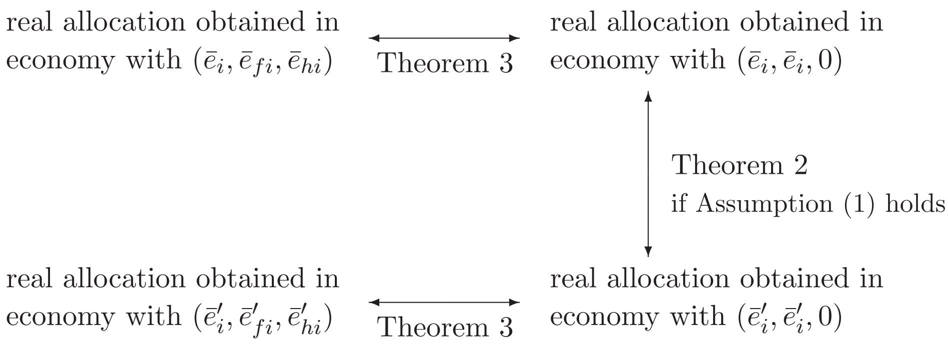


Figure 1: Graphical Exposition



The symbol "↔" indicates two real allocations are identical.

Figure 2: Flow of arguments