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Dual Opportunity Cost of Childcare and the Low Fertility Trap^{*}

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Abstract

This paper develops a model wherein labor income is affected by work experience; further, it analyzes how the fertility rate is determined. In the case where the wage rate is affected through on-the-job training (OJT) or work experience, the fertility rate converges to a low level or continues to increase. Moreover, this paper demonstrates the effects of two childcare support policies. A childcare allowance for children (direct child payment policy) always boosts the fertility rate but decreases the labor participation rate. On the contrary, an infrastructure improvement for childcare, such as building a nursery school, increases not only the fertility rate but also the labor participation rate.

Keywords: childcare support, fertility rate, labor participation

JEL classification: H24, J13, J24

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1 Introduction

In 2005, the total number of new born babies in Japan was less than 1.1 million-the lowest number of births ever. Further, Japan also accounted for the lowest birthrate at 1.26, which is lower than that in the West. Fig.1 illustrates the fertility rate in developed countries.

[Insert Fig.1 around here.]

There are some reasons for the tendency toward fewer children: (1) the increase in late marriages and late childbirths, (2) the decrease in the number of children per household, and (3) the increase in the number of unmarried persons. It is said that there is a strong relationship between these reasons and the opportunity cost for childcare. For example, households must quit employment in order to have children because of the shortage of nursery schools in Japan.

The Cabinet Office (2003) estimates a standard Japanese woman's opportunity cost for childcare at 85 million yen.¹ This high opportunity cost for birth and childcare is believed to be one of the reasons for the decrease in the fertility rate. Iguchi and Nishimura (2002) estimate income loss (opportunity cost) arising due to childcare in many countries; at 4.18 million yen in U.S., 2.63 million yen in France, and 63.61 million yen in Japan, Japan has the highest income loss among the U.S. and the European countries.

A woman who quits her job after childbirth for the purpose of childcare forfeits not only her current income but also the opportunity to accumulate work experience. If a higher accumulation of experience leads to higher labor productivity, quitting brings about current and future income losses. For instance, even when a woman reenters the job market after a period of retirement, she has to start working at low wage rate because she is not considered to have accumulated sufficient experience. Thus, it can be said that childcare entails dual opportunity costs.

This paper analyzes the manner in which the fertility rate is endogenously determined based on dynamic general equilibrium by introducing the mechanism whereby work experience en-

¹The Cabinet Office assumes that the standard woman graduates from university at the age of 22, has her first child at 28, retires temporarily for childcare purposes, and returns to her job at the age of 34. The opportunity cost is calculated by comparing the lifetime income of the standard woman describes above with that of a woman who does not have children and continues to work. If a woman retires to have children and later returns to a part-time job, the income loss is estimated at 240 million yen.

hances labor productivity. Moreover, we consider two childcare support policies and demonstrate the validity of these policies based on the criterion of the fertility rate. Finally, this paper mentions the relationship between the fertility rate and the labor participation rate, which have exhibited a positive correlation in OECD countries in recent years.

Some papers have analyzed the endogenous fertility rate in the model considering opportunity cost and female labor participation. Galor and Weil (1996) demonstrate the relationship between a decrease in the fertility rate and an increase in the rate of women employees; they show that the wage rate of women relative to that of men increases with a decrease in the fertility rate.

However, Sleebos (2003) finds that in OECD countries, the data describing the relationship between the fertility rate and female labor participation has recently changed from negative to positive. Yamaguchi (2005) also finds a positive relationship in OECD countries. Apps and Rees (2001) analyze that the effectiveness of the government policy to raise the fertility rate in high female labor participation rate. Further, Apps and Rees (2004) incorporate childcare support service into Galor and Weil's (1996) model and show that a rising wage rate among women does not always decrease the fertility rate because of both the substitution and income effects. Da Rocha and Fuster (2006) present a model with friction in the labor market and describe this positive relationship.

Some studies such as Galor and Weil (1996) and Apps and Rees (2001, 2004) assume that having children requires a certain amount of time, which implies a reduction in job tenure. However, they assume that the wage rate earned by a household is constant despite the accumulation of experience through continuing with the job. In other words, no study has analyzed the endogenous fertility rate using a model where the wage rate is enhanced through an accumulation of experience and an increase in labor productivity, namely, on-the-job training (OJT). The purpose of this paper is to develop a model that considers labor productivity by accumulating experience, in addition to showing how fertility rate is determined endogenously.

In our model, multiple equilibria-with high and low levels of fertility rate in the steady state-are generated. Since the steady state with low fertility rate is stable, the fertility rate converges

to a low level. This situation is known as the “low fertility trap.”² In this paper, the low fertility trap is explained as follows: since the per capita income is low, households must work hard to maintain some standard of living; therefore, they are unable raise children. In Japan, one of the reasons for the low fertility rate is the issue of income.³

In addition, this paper considers two types of childcare support policies and reveals whether or not they are effective. A childcare allowance (direct child payment) always boosts the fertility rate but diminishes the labor participation. On the contrary, an improvement in the infrastructure for childcare, which decreases the opportunity cost for childcare, can raise both the fertility rate and the labor participation. This result is consistent with recent empirical papers that describe the positive relationship between the fertility rate and female labor participation.

This paper is structured as follows. Section 2 explains our model setting, and Section 3 provides the dynamic equilibrium solution. Section 4 analyzes the effect of childcare support policies, and the final section contains some concluding remarks.

2 The Model

The model economy in this paper is constructed in terms of a two-period overlapping generations model. The economy comprises two types of agents: households and firms.

2.1 Households

First, we consider the behavior of the households. The households comprise two generations: the young and the old. Each household supplies its young members as labor to the firms and earns labor income. Households have one unit of time, which is assumed to be allocated to labor and childcare. Moreover, we assume that an instantaneous wage rate depends on labor time. Since wage rate in a modern labor market is determined based on experience, this assumption

²Lutz and Skirbekk (2005) found three mechanisms that cause the low fertility trap; the demographic, economic, and social norms mechanisms. They explain the low fertility trap in terms of the economic mechanism as follows. “Lower fertility leads to faster population aging and thus to deeper cuts in the welfare state, less job creation, and an expectation of lower economic growth in the future; at the same time, aspirations for personal consumption are still on the rise owing to parental wealth and fewer siblings; further, the match of high aspirations and pessimism about the economic future will result in even lower fertility. This assumed economic mechanism has the potential to create a continuing downward spiral toward lower fertility.”

³The 13th fundamental survey of birth trend.

is realistic. The total labor income each individual can earn by inputting labor l_t is $\frac{\epsilon w_t l_t^2}{2}$, where $\epsilon w_t l_t$ is instantaneous wage rate, w_t is wage rate per effective labor $\frac{\epsilon w_t l_t^2}{2}$, $\epsilon(>0)$ is a nonnegative parameter and t represents a period.⁴ Each household distributes its income into consumption, childcare, and saving. Thus, we get the following budget constraint:

$$zn_t + c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = \frac{\epsilon w_t(1-\phi n_t)^2}{2}, \quad 0 < \phi < 1, \quad (1)$$

where c_{1t} and c_{2t+1} denote consumption by the young and old generations, respectively; r_{t+1} is the interest rate; n_t states the number of children; and it takes ϕ units of time to raise one child. In addition, the parents need to buy some childcare goods z for their child.

This paper would like to stress the relationship between career mobility (rising wage rate by continuing with the job or undergoing OJT) and the number of children. It is natural to consider that a household with many children faces a slow increase in wage rate because of the lack of experience accumulation due to the short job tenure.

The utility of each young individual is described as

$$U = \alpha \ln c_{1t} + \beta \ln c_{2t+1} + (1 - \alpha - \beta) \ln n_t, \quad \alpha, \beta > 0, \quad 1 - \alpha - \beta > 0. \quad (2)$$

This utility function is similar to that in Galor and Weil (1996). Each young household maximizes its utility (2) under the budget constraint (1). We get the following optimum:

$$\begin{aligned} c_{1t} &= \frac{\alpha \epsilon}{2} (1 - \phi^2 n_t^2) w_t, \\ c_{2t+1} &= \frac{(1 + r_{t+1}) \beta \epsilon}{2} (1 - \phi^2 n_t^2) w_t, \\ n_t &= \frac{\epsilon(1 - \alpha - \beta)}{2(z + \epsilon \phi(1 - \phi n_t) w_t)} (1 - \phi^2 n_t^2) w_t. \end{aligned}$$

Under a given w_t , each household uniquely determines the number of its children as follows:⁵

$$n_t = \frac{(\epsilon \phi w_t + z) - ((\epsilon \phi w_t + z)^2 - (1 - (\alpha + \beta)^2) \epsilon^2 \phi^2 w_t^2)^{\frac{1}{2}}}{(1 + \alpha + \beta) \epsilon \phi^2 w_t}. \quad (3)$$

s_t is defined as household saving in period t , $s_t = \frac{\beta}{1 - \alpha - \beta} (z + \epsilon \phi(1 - \phi n_t) w_t) n_t$.

⁴See Appendix for detail.

⁵See Appendix for the uniqueness of the solution.

2.2 Firms

Next, we consider the behavior of the firms. The firms produce final goods in the competitive market. Similar with Grossman and Yanagawa (1993), the following constant-return-to-scale production function is assumed:

$$Y_t = F(K_t, BL_t), \quad B = \frac{K_t}{L_t} \frac{1}{b}, \quad b > 0,$$

where Y_t denotes the output, K_t is the capital stock and L_t is effective labor in period t , which is defined as $L_t \equiv \frac{\epsilon(1-\phi n_t)^2 N_t}{2}$. N_t is the population size of young generation in period t .

In the perfectly competitive market, wage and capital rent are equal to the marginal product of each factor input.

$$1 + r_t = f'(b), \tag{4}$$

$$w_t = \frac{2\omega}{\epsilon b} \frac{k_t}{(1 - \phi n_t)^2}, \tag{5}$$

where $k_t \equiv \frac{K_t}{N_t}$ and $\frac{Y_t}{BL_t} \equiv f(b)$. $\omega \equiv f(b) - f'(b)b$ is constant. We assume that the capital stock fully depreciates within a period.

3 The Equilibrium

Having considered the behavior of the agents, we proceed to the analysis of the equilibrium. The equilibrium of this economy depends on the capital per capita k_t . Using the capital market clearing condition $K_{t+1} = N_t s_t$, we obtain the following difference equation for k_t :

$$k_{t+1} = \frac{\beta}{1 - \alpha - \beta} \left(z + \frac{2\phi\omega}{b} \frac{1}{1 - \phi n_t} k_t \right).$$

There are some phases of k_t . If z is high and the slope of k_{t+1} is somewhat steep, the phase of k_t is depicted as shown in Fig.4(a). If z is low and the slope of k_{t+1} is somewhat gentle, the phase of k_t is depicted as shown in Fig.4(b). Further, if z and the slope of k_{t+1} is midway between the two above cases, the phase of k_t is depicted as shown in Fig.4(c).

[Insert Fig.4 around here.]

We define the steady state as a situation in which the capital per capita is constant or $k_{t+1} = k_t = k^*$. Then, the capital per capita in the steady state is written as

$$k^* = \frac{\beta}{1 - \alpha - \beta} \left(z + \frac{2\phi\omega}{b} \frac{1}{1 - \phi n^*} k^* \right). \quad (6)$$

The fertility rate in steady state n^* increases with the capital per capita in steady state k^* .⁶

The capital per capita in the steady state is not always unique. The case of Fig.4(a) has no steady state equilibrium. The case of Fig.4(b) has one steady state equilibrium with low income and fertility. The case of Fig.4(c) has two steady state equilibria, one with low income and low fertility and other with high income and high fertility.⁷ We define low and high capital per capita in the steady state as k^{low} , k^{high} , respectively. k^{low} is stable and k^{high} is unstable. In the case of Fig.4(b), given any k_0 , k_t converges to k^{low} . In the case of both Fig.4(b) and Fig.4(c), given any k_0 that is less than k^{high} , k_t converges to k^{low} . Then, the following proposition is provided.

Proposition 1 In the economy where the wage rate depends on experience, the economy may experience multiple equilibria. One is the state with high capital per capita and high fertility rate, and the other is that with low capital per capita and low fertility rate. Moreover, the equilibrium with low income and fertility is stable.

k^{low} is the equilibrium in which households must work hard because of the low wage rate. As a result, it is difficult for them to raise children due to the shortage of income. We can think two ways of exiting the low fertility trap; one is through the Big Push (the government boosts k_0 to the extent that it exceeds k^{high}), and the other is the childcare support policy, which is examined in the next section.

We consider the cases where ϵ (individual ability) and ω (the rising wage rate) change. If capital-labor ratio k_t is constant, an increasing ϵ boosts the fertility rate. However, considering the dynamics, we find that the fertility rate given by (3) and (5) is determined irrespective of

⁶We can show $\frac{dn^*}{dk^*} = \frac{zb(1-\phi n^*)^2}{2\phi^2\omega k^{*2}} > 0$.

⁷See the Appendix for this proof.

the value of ϵ . Therefore, a change in ϵ does not influence the fertility rate and the dynamic path. On the contrary, an increase in ω increases the fertility rate and k_{t+1} for any k_t . If the value of ω is somewhat large, there are two equilibria with low and high fertility rate. In the case of small ω , there is only the equilibrium with a low fertility rate as illustrated in Fig.6.

[Insert Fig.6 around here.]

4 Analysis of Government Policy

Having studied the steady state in the economy, we consider the effects of policies to boost the fertility rate. This paper refers to such policies as childcare support policies. The government levies income tax τ on wage income to finance the support policy. In this paper, we consider two types of childcare support policies: one is the childcare allowance and the other is infrastructure improvement for childcare, that is, the policy aiming at compatibility between pursuing a job and childcare or at hastening the return to job after retiring for childcare.

We assume that the government grants a subsidy q per child (direct child payment) or provides l to lower ϕ that is, $\frac{d\phi(l)}{dl} (\equiv \phi') < 0$. l is considered as the childcare service per household or the resources for the implementation of laws that enable mothers to simultaneously hold jobs and engage in childcare, among other things. Then, we can rearrange the budget constraint (1) as follows:

$$(z - q)n_t + c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = \frac{\epsilon(1 - \tau)}{2} (1 - \phi(l)n_t)^2 w_t.$$

Under the above constraint, the fertility rate n_t is determined as

$$n_t = \frac{\epsilon(1 - \alpha - \beta)(1 - \tau)}{2(z - q + (1 - \tau)\epsilon\phi(l)(1 - \phi(l)n_t)w_t)} (1 - \phi(l)^2 n_t^2) w_t. \quad (7)$$

Given w_t , n_t is determined uniquely. The government manages the balance of budget as follows.

$$qn_t + l = \frac{\tau}{2} (1 - \phi(l)n_t)^2 w_t \quad (8)$$

We can now analyze the effects of the government policies. First, we examine the effect of the childcare allowance. For simplicity, we assume that the wage rate w_t is fixed. Substituting

(8) into (7) and totally differentiating (7) with respect to τ , q , n , k at approximation of $\tau = 0$ (and therefore, $q = 0$, $l = 0$), we obtain

$$\frac{dn_t}{d\tau} = \frac{(\alpha + \beta)w_t(1 - \phi n_t)(1 + \phi n_t)}{2(z + \epsilon\phi w_t(1 - (\alpha + \beta)\phi n_t))} > 0. \quad (9)$$

The sign of $\frac{dn_t}{d\tau}$ is always positive. A childcare allowance can boost the fertility rate. On the contrary, an allowance policy causes a decrease in the rate of labor participation $1 - \phi n$.

Second, we analyze the effect of an infrastructure improvement for childcare. Totally differentiating τ , l , n , k at the approximation of $\tau = 0$ (therefore, $q = 0$, $l = 0$) leads the following equation:

$$\frac{dn_t}{d\tau} = \frac{\frac{\epsilon w_t}{2}(1 - \phi n_t)((1 + \alpha + \beta)\phi n_t - (1 - \alpha - \beta)) - \epsilon w_t \phi' n_t(1 - \phi n_t)(1 - (1 + \alpha + \beta)\phi n_t)}{z + \epsilon\phi(1 - \phi n_t)w_t + (1 - \alpha - \beta)\epsilon\phi^2 w_t n_t}. \quad (10)$$

The first term in the numerator depicts the income effect through the tax burden, which is always negative. The second term in the numerator states the effects of boosting the fertility rate through the decrease in the time spent on childcare per child, which is always positive. If the effect of the boost is dominant, the fertility rate can be raised through this policy. The policy can also increase the labor participation $1 - \phi n_t$ if a decrease in ϕ is large and an increase in n_t is small.

Moreover, if n_t is inelastic to ϵ and w_t , a high ϵ or w_t raises the possibility of satisfying $\frac{dn_t}{d\tau} > 0$. Then, the following proposition is arrived at.

Proposition 2 Both childcare allowances and an improvement in the infrastructure for childcare (the policy for decreasing an opportunity cost of having a child) boost the fertility rate. However, their effects on the labor participation rate are ambiguous. The allowance policy always decreases the labor participation rate when the fertility rate increases, whereas an infrastructure improvement policy may increase the labor participation rate.

The relationship between the fertility rate and the female labor participation rate was observed to have been negative in the 1980s. However, this correlation became positive in the 2000s. Yamaguchi (2005) explains that the correlation changes upon creating circumstances

wherein women can simultaneously hold jobs and raise their children. Apps and Rees (2001, 2004) theoretically arrive at this positive correlation. This paper demonstrates that an improvement in the infrastructure for childcare boosts both the fertility rate and the labor participation rate in the case where income loss caused by a rising tax rate is small or that where the time spent for childcare decreases largely due to childcare support policies.

5 Concluding Remarks

In this paper, we develop a model where work experience determines the wage rate; we consider the dynamics of the fertility rate and analyze the effects of the childcare support policies on the fertility and labor participation rates. In our model, the economy has a unique equilibrium or multiple equilibria. In the case of multiple equilibria, the economy converges to a low income per capita with low fertility rate or continues to increase both the income and the fertility rate.

We summarize the effects of childcare support policies on the fertility rate in the economy with constant capital per capita. A childcare allowance increases the fertility rate. However, this policy decreases the labor participation rate. On the contrary, infrastructure improvement increases both the fertility and labor participation rates. This result is consistent with the relationship recently observed in some OECD countries.

Japan faces an aging society with the households exhibiting a tendency toward fewer children, and it expects to suffer from a shortage of labor in the future. In order to solve these problems, the government should employ policies that increase the fertility and labor participation rates.

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Appendix

The labor income in the case where the wage rate is determined by experience

We assume that an instantaneous wage rate is expressed by $\epsilon l_t w_t$, that is, the wage rate per unit of the tenure increases with the tenure because of experience, among other things. Then, when households work for a unit of time, the total labor income is $\frac{\epsilon w_t}{2}$. On the other hand, the tenure is less than one unit due to the time spent in having children, and the total labor income is $\frac{\epsilon w_t}{2} - ABCD - CDEF$ as shown Fig.2(a). AB and $1 - AB$ denote the time spent on childcare and the tenure, respectively. $ABCD$ is the direct opportunity cost generated as a result of quitting the job. $CDEF$ is the indirect opportunity cost, which the cost is incurred by a decrease in the wage rate as a result of quitting the job. Fig.2(a) can be described as Fig.2(b).

[Insert Fig.2 around here.]

The proof that an allocation of n_t is unique

(5) can be expressed as follows.

$$(1 + \alpha + \beta)\epsilon\phi^2 w_t n_t^2 - 2(z + \epsilon\phi w_t)n_t + (1 - \alpha - \beta)\epsilon w_t = 0. \quad (11)$$

The above equation can be illustrated as follows:

[Insert Fig.3 around here]

This equation has two solutions, n_t^0 and n_t^1 . However, since $1 - \phi n_t$ must be nonnegative, n_t^1 is excluded.⁸ Therefore, n_t is determined uniquely.

The dynamics of capital per capita k_t

The phases of k_t have three patterns. The slope of k_{t+1} determines how the phase of k_t is depicted. The slope of k_{t+1} is $\frac{\beta}{1-\alpha-\beta} \frac{2\phi\omega}{b} \frac{1}{1-\phi n_t} \cdot \frac{dn_t}{dw_t}$ is given as follows.

$$\frac{dn_t}{dw_t} = \frac{zn_t}{w_t} \frac{1}{z + \epsilon\phi w_t(1 - (1 - \alpha - \beta)\phi n_t)}.$$

⁸Since $1 - \phi n_t$ is nonnegative, n_t^1 must hold. When $n_t = \frac{1}{\phi}$, the left-hand side of (11) is negative, and hence, $n_t^0 < \frac{1}{\phi} < n_t^1$.

The upper limit of n_t is expressed as follows.⁹

$$\lim_{w_t \rightarrow \infty} n_t = \frac{1 - \alpha - \beta}{1 + \alpha + \beta} \frac{1}{\phi}.$$

Hence, the sign of $\frac{dn_t}{dw_t}$ is positive due to $0 < n_t < \frac{1 - \alpha - \beta}{1 + \alpha + \beta} \frac{1}{\phi}$.

Next, we show that the sign of $\frac{dn_t}{dk_t}$ is positive. n_t satisfies the following budget constraint.

$$\frac{\epsilon}{2} (1 - \phi n_t)^2 w_t = z n_t + \left(\frac{\alpha + \beta}{1 - \alpha - \beta} \right) (z n_t + \epsilon \phi (1 - \phi n_t) w_t n_t),$$

where $c_{1t} = \frac{\alpha}{1 - \alpha - \beta} (z n_t + \epsilon \phi (1 - \phi n_t) w_t n_t)$ and $c_{2t+1} = \frac{\beta}{1 - \alpha - \beta} (z n_t + \epsilon \phi (1 - \phi n_t) w_t n_t)$ are substituted into this budget constraint. This equation can be presented as follows.

$$z n_t = \frac{\omega}{b} \frac{k_t}{1 - \phi n_t} ((1 - \alpha - \beta) - (1 + \alpha + \beta) \phi n_t).$$

Defining $L \equiv z n_t$ and $R \equiv \frac{\omega}{b} \frac{k_t}{1 - \phi n_t} ((1 - \alpha - \beta) - (1 + \alpha + \beta) \phi n_t)$, we obtain the following figure.

[Insert Fig.5 around here.]

An increase in k_t shifts the R line upwards and increases n_t , thus $\frac{dn_t}{dk_t} > 0$.¹⁰

If the slope of k_{t+1} in an infinite period is less than one, that is, $n < \frac{1}{\phi} - \frac{\beta}{1 - \alpha - \beta} \frac{2\omega}{b}$ (that is, $\phi < \frac{1 - \alpha - \beta}{1 + \alpha + \beta} \frac{\alpha + \beta}{\beta} \frac{b}{\omega}$), then the phase of k_t shows the case of Fig.4(b). On the one hand, if $n > \frac{1}{\phi} - \frac{\beta}{1 - \alpha - \beta} \frac{2\omega}{b}$ (that is, $\phi > \frac{1 - \alpha - \beta}{1 + \alpha + \beta} \frac{\alpha + \beta}{\beta} \frac{b}{\omega}$), then the phase of k_t shows the case of Fig.4(a) or Fig.4(c).

Dynamics of k_t in the wage system wherein the wage rate does not depend on the tenure

The budget constraint is given as follows:

$$z n_t + c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = \epsilon (1 - \phi n_t) w_t.$$

⁹We define $g(w) \equiv (\epsilon \phi w + z) - ((\epsilon \phi w + z)^2 - (1 - (\alpha + \beta)^2) \epsilon^2 \phi^2 w^2)^{\frac{1}{2}}$ and $f(w) \equiv (1 + \alpha + \beta) \epsilon \phi^2 w$, (then, $n = \frac{g(w)}{f(w)}$). Since $\lim_{w \rightarrow \infty} \frac{g(w)}{f(w)} = \frac{\infty}{\infty}$, then using l'Hospital theorem, we can show that $\lim_{w \rightarrow \infty} \frac{g(w)}{f(w)} = \lim_{w \rightarrow \infty} \frac{g'(w)}{f'(w)} = \frac{1 - \alpha - \beta}{1 + \alpha + \beta} \frac{1}{\phi}$.

¹⁰ $0 < n_t < \frac{1 - \alpha - \beta}{1 + \alpha + \beta} \frac{1}{\phi}$ (at $0 < k_t < \infty$) can be depicted in this figure (When $n_t = 0$ at $k_t = 0$, $n_t = \frac{1 - \alpha - \beta}{1 + \alpha + \beta} \frac{1}{\phi}$).

In this case, we find $n_t = \frac{1-\alpha-\beta}{z+\epsilon\phi w_t} \epsilon w_t$ and $s_t = \beta \epsilon w_t$. Considering the market clearing condition, it is expressed as $k_{t+1} = \frac{\beta}{1-\alpha-\beta} (z + \epsilon \phi w_t)$. With $w_t = \frac{\omega}{b} \frac{1}{\epsilon(1-\phi n_t)} k_t$, the following equation is obtained:

$$k_{t+1} = \frac{\beta}{1-\alpha-\beta} \left(z + \frac{\phi \omega}{b} \frac{1}{1-\phi n_t} \right). \quad (12)$$

This dynamic equation is nearly same as the wage system wherein the wage rate depends on the tenure. The upper limit of n_t is $\lim_{w_t \rightarrow \infty} n_t = \frac{1-\alpha-\beta}{\phi}$; this, $0 < n_t < \frac{1-\alpha-\beta}{\phi}$ at $0 < k_t < \infty$. In this case, if $\phi > (1-\alpha-\beta) \frac{\alpha+\beta}{\beta} \frac{b}{\omega}$, the dynamic are as depicted in Fig.4(c).

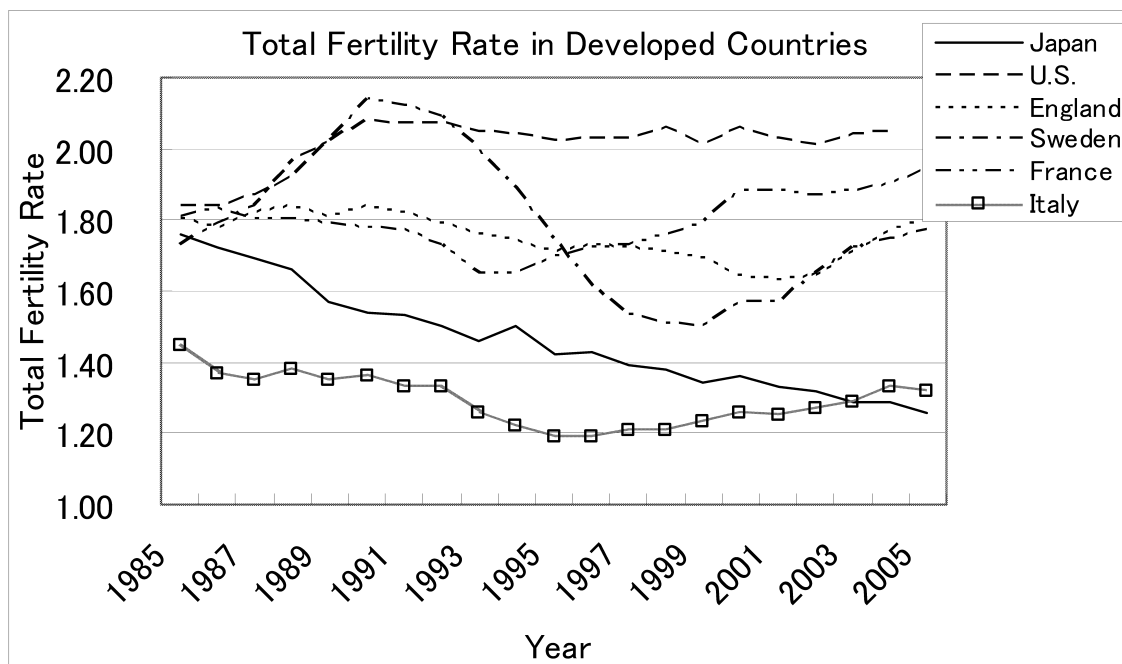


Fig.1: Total fertility rate in developed countries (Data: The Population Statistic 2006
(National Institute of Population and Social Security Research))

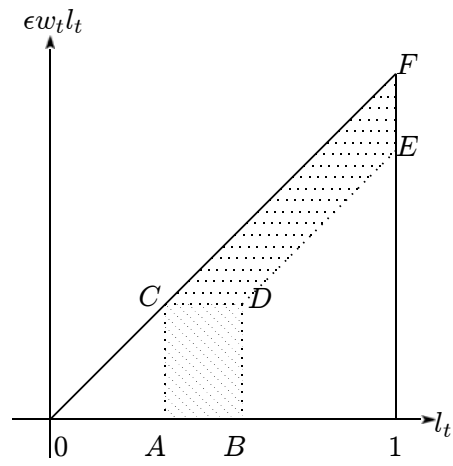


Fig.2(a): Labor income under dual opportunity cost for child care

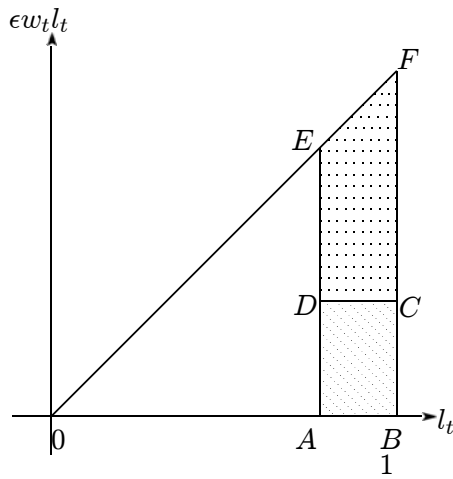


Fig.2(b): Labor income under dual opportunity cost for child care

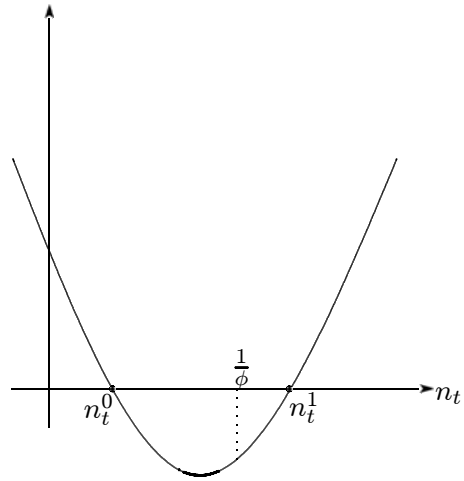


Fig.3: Allocations of n_t

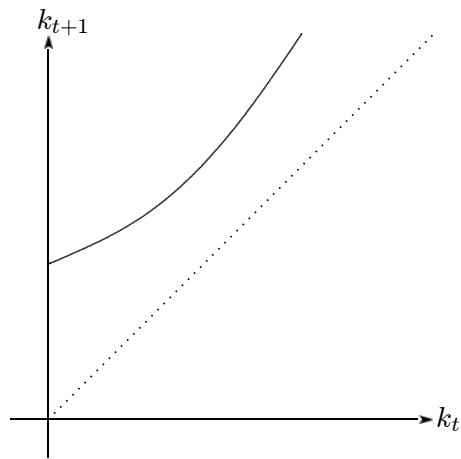


Fig.4(a): No equilibrium

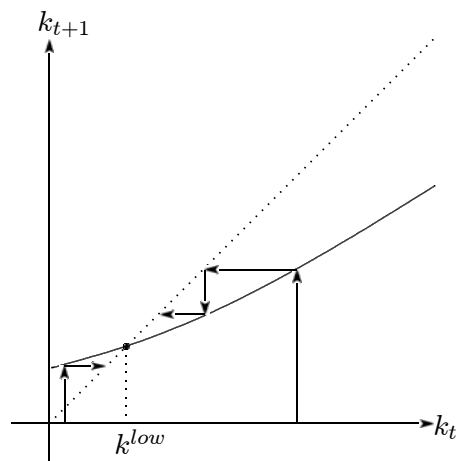


Fig.4(b): A unique equilibria with low k

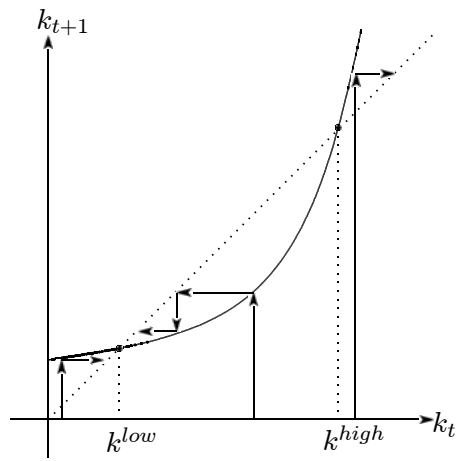


Fig.4(c): Multiple equilibrium with both high and low k

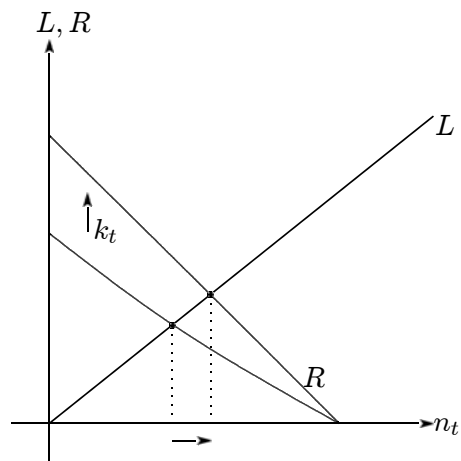


Fig.5: An increase in k_t and determination of n_t

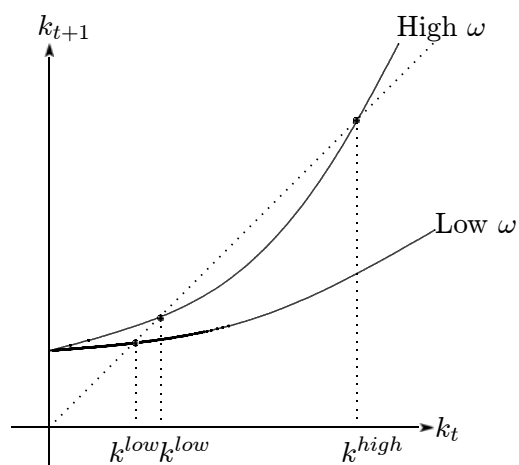


Fig.6: Dynamics of k_t and ω