



Demographic Changes, Fiscal Policy and Economic Growth: Theory and Evidence

Haruyama, Tetsugen

Ashiya, Masahiro

Hamori, Shigeyuki

(Citation)

神戸大学経済学研究科 Discussion Paper, 712

(Issue Date)

2007-09

(Resource Type)

technical report

(Version)

Version of Record

(URL)

<https://hdl.handle.net/20.500.14094/80200054>



Demographic Changes, Fiscal Policy and Economic Growth: Theory and Evidence*

Tetsugen HARUYAMA[†] Masahiro ASHIYA[‡]

Shigeyuki HAMORI[§]

Abstract

Demographic changes have two faces: (i) a falling birth rate, and (ii) a longer life expectancy. This paper analyzes the effect that those two phenomena have on fiscal policy and economic growth in a politico-economic model of endogenous growth. In particular, the model is framed in continuous time to cope with problems that existing studies (OLG models) have in common. In our model, income taxes are used to finance public investment, which in turn drives economic growth. Consumers who live for a finite time length have their own ideal tax rate, given a trade-off between current and future consumption. An equilibrium tax rate is determined by majority voting, i.e. median voters, whose ideal tax rate depends on model parameters, including the birth rate and life expectancy. There are two key results: (1) the birth rate and taxes (and growth) have an inverted U relationship, and (2) a longer life expectancy leads to higher tax rate (and growth). Empirical analysis confirm that data of the G7 countries support well theoretical predictions of the model.

Key words: demographic changes, taxes, economic growth, median voter

JEL classification number: E62, H50, J10, O40, O50

*This research is partly supported by the Grant-in-Aid for Scientific Research (No. 17330042) from the Ministry of Education, Culture, Sports, and Technology, Japan.

[†]Graduate School of Economics, Kobe University, 2-1 Rokkodai, Nada, Kobe 657-8501, Japan. (Email) haruyama@econ.kobe-u.ac.jp

[‡](Corresponding author) Graduate School of Economics, Kobe University, 2-1 Rokkodai, Nada, Kobe 657-8501, Japan. (Email) ashiya@econ.kobe-u.ac.jp

[§]Graduate School of Economics, Kobe University, 2-1 Rokkodai, Nada, Kobe 657-8501, Japan. (Email) hamori@econ.kobe-u.ac.jp

1. Introduction

According to United Nations (2007), the 5-year average of the crude birth rate (expressed as the number of births per 1000 persons) in developed economies was 22.4 in the period of 1950-55, and halved to 11.1 in 2000-05. A significant and steady drop in the birth rate is one face of demographic changes taking place in advanced countries.¹ Another important aspect of the demographic changes is that people live longer. Life expectancy in developed economies was 66.1 years in 1950-55 and rose to 82.4 years in 2000-05. These two faces of demographic changes increase old people in number relative to young people. This means a rise in the median age of population, and more importantly, an increase in the age of *median voters* than decades ago. For example, the age of Japanese median voters was 38.4 in 1950 and rose to 51.3 in 2005. To the extent that median voters are important in policy determination in democracy, on-going demographic changes are bound to cause policy shifts in developed countries. Then, the following questions arise. Does an economy adopt lower or higher tax rates, as demographic structure changes? How do demographic changes affect economic growth? Is economic growth slowdown an inevitable outcome?

Motivated by these observations, the present paper examines the impact of demographic changes on fiscal policy and economic growth in a politico-economic model with emphasis on conflicts of interests among different generations in policy determination. More specifically, our model is based on three traits. First, public capital accumulation, financed by tax revenue, drives economic growth. Second, consumers who live for a finite period have their own ideal tax rate, depending on their remaining life time. Third, the median voter determines a tax rate in equilibrium. In this framework, comparative statistics related to demographic changes, e.g. a fall in the birth rate, are considered, and theoretical predictions are empirically examined, using data of G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States). Before explaining results, it is instructive to highlight key differences between the present paper and existing studies.

Many studies explore the issue of demographic changes in models similar to ours.² However, given the over-lapping generations (OLG) framework adopted in those studies, they have either or both of the following features, which we argue are problematic in politico-economic analysis of demographic changes.

- (1) As long as population grows in OLG models where consumers live for two periods, the median voter always belongs to the "young" generation. This means that population growth slowdown does not change the "age" of the median voter. Existing studies deal with the problem by introducing heterogeneity among consumers in, e.g. skills. However, policy shifts caused under this assumption reflect changes in skills rather than the age of the median voter. We argue that this

¹The crude birth rate has also been steadily falling in less developed countries. It was 44.4 in 1950-55, and fell to 25.4 in 2000-05 according to United Nations (2007). Those numbers do not significantly change even if China is excluded.

²For example, see Beauchemin (1998), Boldrin (2005), Gradstein and Kaganovich (2004), Holtz-Eakin et al. (2004), Kemnitz (1999, 2000), Levy (2005), Pecchenino and Pollard (2002), Pecchenino and Utendorf (1999), Razin et al. (2002), Tosun (2003), and Zhang et al. (2003).

feature is most problematic, since the primary problem of aging is related to the concept of time, whereas the skill problem is secondary in the sense that incentives to enhance skills depend on time, i.e. workers' age and remaining life time. Furthermore, empirical studies show that productivity and age of workers are not necessarily related in a monotonic way.³

- (2) OLG models where consumers live for three periods also suffer the same problem. The age of median voters in industrialized countries typically falls in the range of 35-45. This implies that median voters' age is crucially dependent on whether a greater number of consumers are in their 30s or 40s.⁴ However, in OLG models with three-periods-lived consumers, voters in their 30s or 40s are all in the "middle-age" generation group, if life is plausibly assumed to last for about 75 years in developed economies.⁵

The present paper deals with the above two problems by developing a continuous-time model where consumers live for a fixed time interval.⁶ Especially, the assumption of continuous time allows us to establish results which are purely based on the "time effect" of aging rather than something else.

In our model, consumers pay taxes, which are used to finance public investment. Since public capital accumulation drives economic growth,⁷ consumers have their own ideal tax rate chosen on the basis of a trade-off between current and future consumption. An equilibrium tax rate is determined through majority voting, i.e. by the median voters. In turn, long-run growth of income depends on an equilibrium tax rate selected by the median voter. Since her ideal tax rate depends on model parameters (e.g. birth rate and life expectancy), our model allows us to derive theoretical predictions about the effect of demographic changes on fiscal policy and growth.

The first key result is that a rise in life expectancy increases an equilibrium tax rate. This leads to higher economic growth, given that the two variables are positively related. This result can be easily explained. Note that younger voters prefer higher tax

³For example, see Medoff and Abraham (1980) and references in Skirbekk (2004).

⁴For example, if there are more voters in their 30s than 40s, median voters' age is closer to 35.

⁵Another aspect of the problem is that analysis of the impacts of baby boom cohort aging on fiscal policy is bound to be unsatisfactory in OLG models with three-periods-lived voters.

⁶A deterministic life length can be easily made stochastic. But this extension is suppressed since little additional insight is gained.

⁷Previous research has produced conflicting empirical results on the correlation of the government size and economic growth. Barro (1996) and Alesina et al. (2005) obtain a negative correlation between total government spending (as a percentage of GDP) and growth. Levine and Renelt (1992) and Sala-i-Martin (1997a, b) find no stable correlation between them. However, Caselli et al. (1996) address the issues of correlation among country-specific effects and endogeneity of explanatory variables, and obtain a positive correlation between government spending and growth. As for a correlation between government investment expenditure (as a percentage of GDP) and growth, Kelly (1997) and Sala-i-Martin (1997b) demonstrate a significant positive correlation, whereas Levine and Renelt (1992) do not obtain robust results.

rates than older voters, because the former have a longer remaining life time during which they can benefit from higher economic growth. For the same reason, people come to prefer higher tax rates as they live longer (Proposition 1). Since the remaining life time of the median voter is increasing in life expectancy of the population (Corollary 1), the equilibrium tax rate is also increasing in the life expectancy (Proposition 3).

The second main result concerns a decline in the birth rate. This increases the number of older people relative to younger consumers (with life expectancy unchanged). That is, the median voter becomes older and prefers a lower tax rate. On the other hand, public capital is assumed to be subject to congestion, following many studies on growth and public capital.⁸ As population growth slows down, congestion is eased and the marginal benefit of a higher tax increases. This makes the median voters choose a higher tax rate. These two opposing effects give rise to an inverted U relationship between the birth rate and taxes (and growth). To put it differently, if the initial birth rate is sufficiently low, a falling birth rate will lead to lower equilibrium tax rates. In industrialized countries with a very low birth rate, therefore, a lower birth rate is expected to bring about lower economic growth.

Note that population aging (a longer life expectancy) and a lower rate of child birth (a greater number of the old relative the young) are different aspects of demographic changes. Our analysis shows that they have contrasting impacts on fiscal policy and economic growth. In industrialized countries, therefore, the relative strength of these phenomena will determine the directions of future changes in taxes and economic growth.

Given these results, we use data on the G7 countries for the period of 1980-2004 to empirically analyze whether the above predictions are observed. First, predicted relationships between tax rates and other variables (median voter's age, birth rate, and life expectancy) are well supported by data. Second, data also support predictions regarding the relationship between growth on one hand and the birth rate and life expectancy on the other. An inverted U-shaped relationship between the birth rate and economic growth is observed. A positive correlation between a longer life expectancy and growth is also shown to hold in data. To check robustness of the results, the sample period is extended to cover the period of 1970-2004. Overall, results turn out to be convincing.

There is another important contribution of the present paper. There are many empirical studies on the impact of changes in the demographic structure of an economy on growth.⁹ To the best of our knowledge, this paper is the first attempt to empirically analyze the effects of a falling birth rate and aging population on tax rates and economic growth in the politico-economic framework.

The structure of the present paper is as follows. Section 2 develops a politico-economic model of growth in a continuous-time framework. Three key propositions are derived. In the next section, empirical analysis is conducted to examine

⁸For example, see Gramlich (1994) on congestion of infrastructure investment. The congestion effect of public capital is stressed by many studies, including Barro and Sala-i-Martin (1992).

⁹See Acemoglu and Johnson (2006), Ahituv (2001), An and Jeon (2006), Durlauf et al. (2005), Feyrer (2007), Kelley and Schmidt (2005), Li and Zhang (2007), and Zhang and Zhang (2005) and the references therein.

whether or not data support theoretical predictions of the model. Section 4 gives concluding remarks.

2. The Model

2.1. Technology

Time is continuous. Final output Y (a numeraire) is produced according to the following production function:

$$Y(t) = \tilde{G}(t)L(t). \quad (1)$$

L denotes the number of workers. \tilde{G} represents the effect of the stock of public physical capital, which is captured by the following assumption:¹⁰

$$\tilde{G}(t) = \frac{G(t)}{L(t)}. \quad (2)$$

The presence of L in the denominator means that public capital is subject to congestion.¹¹ The absence of private physical capital and the assumptions (1) and (2), albeit simple, allow us to develop our argument in a tractable framework.

To finance public investment, the government taxes final output at a given rate $1 > \tau > 0$. Given this government policy, profits of final output producers are maximized when the following condition holds:

$$w(t) = (1 - \tau) \frac{G(t)}{L(t)}. \quad (3)$$

Public capital G accumulates according to

$$G(t) = A \int_{-\infty}^t \tau Y(s) ds. \quad (4)$$

The integral on the right-hand side is the sum of tax revenues collected up to time t . The parameter A captures the extent to which tax revenues are efficiently used to produce public capital. In other words, A is interpreted to be dependent on the quality of institutions in the economy. Then, using (1) and (4), we can derive the growth rate of aggregate output

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{G}(t)}{G(t)} = A\tau. \quad (5)$$

It shows that growth accelerates with a tax rate. As shown shortly, τ will be endogenously determined through a political process.

2.2. Consumers

2.2.1. Demographic Structure

The total number of consumers-cum-workers at time t is L . At every moment, $\beta e^{\beta t}$

¹⁰A minor generalization of (2) is to assume $\tilde{G}(t) = G(t)/L(t)^\phi$, $\phi > 0$. But, this modification does not change key results in any fundamental way.

¹¹This assumption follows many studies on public infrastructure (see footnote 8). One consequence of this assumption is the removal of so-called scale effects (i.e. a positive relation between a growth rate and the size of an economy), which is not supported by data (e.g. see Jones (2005)).

workers are born where $\beta > 0$ is a constant birth rate. They live for a finite time period $0 < T < \infty$. To track the cohort of consumers in a demographic structure, let $a \in [0, T]$ denote their age. Given these assumptions, the total population L is given by

$$L(t) = \int_0^T \beta e^{\beta(t-a)} da = e^{\beta t} (1 - e^{-\beta T}). \quad (6)$$

The total population is increasing in β and T . Since β and T are constant, population grows at a rate of

$$\frac{\dot{L}(t)}{L(t)} = \beta \quad (7)$$

2.2.2 Voting

Consumers are assumed to be risk-neutral. The intertemporal utility function of a consumer of age a is

$$U_a(t) = \int_t^{t+T-a} e^{-\rho(s-t)} c(s) ds \quad (8)$$

where ρ is the subjective rate of time preference and $c(s)$ denotes consumption of final output. We assume that consumers spend all their wage income in each instant. Therefore,

$$c(s) = (1 - \tau) \frac{G(t)}{L(t)} e^{(A\tau - \beta)(s-t)} \quad \text{for } s \geq t \quad (9)$$

using (3). $(1 - \tau)$ on the right-hand side of (9) represents the *static* negative effect of the tax. This arises because the tax lowers the marginal product of labor, generating a distortionary effect on wage. On the other hand, $e^{(A\tau - \beta)s}$ captures the *dynamic* positive effect of the tax. A higher tax rate accelerates the accumulation of public capital, so that wage grows at a faster rate. Substituting (9) into (8) gives

$$U_a(t) = \frac{G(t)}{L(t)} \underbrace{(1 - \tau)}_{\text{Static effect}} \cdot \overbrace{F(\tau; a)}^{\text{Dynamic effect}} \quad (10)$$

where $F(\tau, a) = \int_t^{t+T-a} e^{-(\rho + \beta - A\tau)(s-t)} ds$ and $\rho + \beta - A\tau > 0$ is assumed. An ideal tax rate of consumers of age a can be derived by maximizing U_a by choosing τ . A trade-off is between the *static* negative effect of a higher tax, captured by $(1 - \tau)$ in (10) and the *dynamic* positive effect, represented by $F(\tau; a)$. Note that the dynamic effect depends on the age of consumers. This is the channel through which the demographic structure of population affects growth in a politico-economic equilibrium.

2.2.3 Ideal Tax Rate

The ideal tax rate of consumers of age a can be derived by maximizing (10) with respect to τ . The F.O.C. is

$$\overbrace{(1 - \tau) F_\tau(\tau; a)}^{MB(\tau; a)} = \overbrace{F(\tau; a)}^{MC(\tau; a)} \quad (11)$$

$MB(\tau; a)$ of (11) is the marginal benefit of a higher tax rate, which is realized

through a higher growth rate (dynamic effect). On the other hand, $MC(\tau; a)$ is the marginal cost of a higher τ due to the distortionary tax (static effect). Appendix A shows that utility maximization requires $A > \rho + \beta$, which means $\tau \in (0, (\rho + \beta)/A)$ in equilibrium. It also establishes that the second-order condition is satisfied and a unique ideal tax rate exists, given age of consumers.¹² In addition, the appendix demonstrates that both of $MB(\tau; a)$ and $MC(\tau; a)$ are increasing in τ and that the MB curve is less steep than the MC curve at equilibrium, as shown in Figure 1.

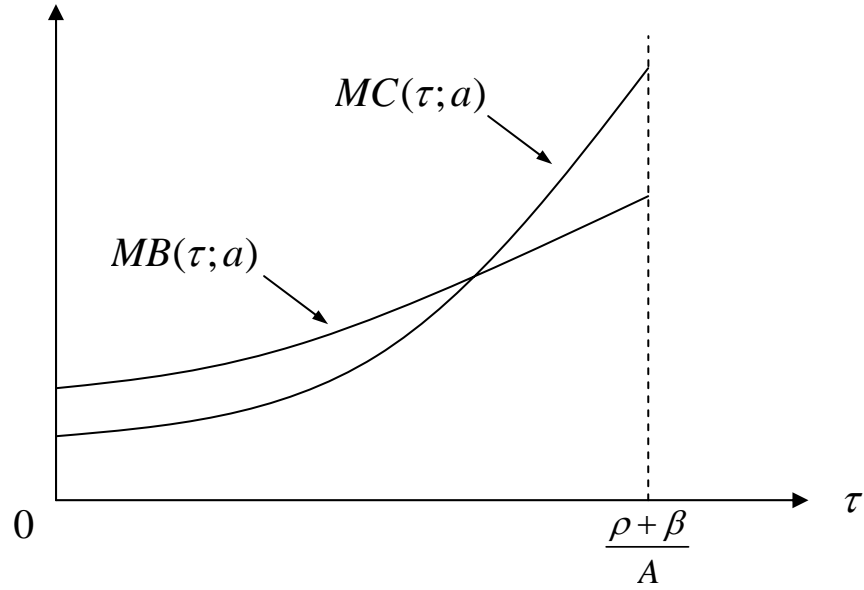


Figure 1: The determination of the ideal tax rate of consumers of age a .

Note that $T - a$ is the remaining life time of consumers of age a . Given the above discussion and using (11), the ideal tax rate of consumers who will live for $T - a$ is implicitly defined by

$$\tau = f(T - a; \beta, \rho, A) \quad (12)$$

Then, the next proposition can be established:

Proposition 1: *The ideal tax rate of consumers, whose remaining life time is $T - a$, has the following properties.*

$$(a) \frac{\partial \tau}{\partial (T - a)} > 0, \quad (b) \frac{\partial \tau}{\partial \beta} < 0, \quad (c) \frac{\partial \tau}{\partial \rho} < 0, \quad (d) \frac{\partial \tau}{\partial A} > 0. \quad (13)$$

Proof. See Appendix B.

We develop intuitive explanations for Proposition 1, which are important for later

¹²An equilibrium does not need to be interior for all consumers for our key results, as long as the median voter's ideal tax rate is between 0 and $(\rho + \beta)/A$.

analysis. First, consider the effects of the remaining life time of consumers $T - a$. The marginal benefit of a higher tax rate is realized through a higher growth rate. Such a dynamic positive effect is larger with the remaining life time of consumers. That is, the marginal benefit is larger with T and smaller with a . Turning to the effect of β (the birth rate), the acceleration of population growth aggravates congestion of using public capital, which results in a lower growth rate of wage, and hence, consumption per capita. This reduces the marginal benefit of the public investment, leading to a lower ideal tax rate. When ρ (the subjective rate of time preference) is large, consumers care less about the future benefit of the public investment and hence prefers lower tax rate. Finally, remember that A shows how efficiently tax revenues are used for public investment by the government. As the institutional quality rises, consumers increase their ideal tax rate.

2.2.4. Median Voter

We assume that the median voter determines the equilibrium tax rate. This is a simple, but widely used modeling approach of a political process of democratic decisions. Denote the age of the median voter by a_m . It is implicitly determined by

$$\frac{L_t}{2} = \int_0^{a_m} \beta e^{\beta(t-a)} da. \quad (14)$$

The right-hand side is the number of population of age from zero to a_m . Using (6), one can rewrite (14) as

$$a_m = \ln \left(\frac{2}{1 + e^{-\beta T}} \right)^{\frac{1}{\beta}} \equiv a_m(\beta, T). \quad (15)$$

Given (15), the following proposition can be established.

Proposition 2: *The age of the median voter, a_m , has the following properties.*

$$(a) \ 1 > \frac{\partial a_m}{\partial T} > 0, \quad (b) \ \frac{\partial a_m}{\partial \beta} < 0. \quad (16)$$

Proof: See Appendix C.

To give intuitive explanations for Proposition 2, consider the effect of T first. A higher T means an increase in life expectancy of consumers. As they live longer, the number of older people increases, and hence the median voter gets older. It should be easy to understand that the age of the median voter does not increase as much as life expectancy. Turning to the effect of β , recall that $\beta e^{\beta t}$ of workers are born at time t . Therefore, given a constant life-span T , the relative number of younger people increases in β . This makes the median voter younger.

Now, the remaining life time of the median voter is

$$T - a_m(\beta, T) \quad (17)$$

Given Proposition 2, the following results are obvious.

Corollary 1: *The remaining life time of the median voter has the following properties*

$$(a) \frac{\partial(T - a_m(\beta, T))}{\partial T} > 0, \quad (b) \frac{\partial(T - a_m(\beta, T))}{\partial \beta} > 0. \quad (18)$$

Proof: Omitted.

We will exploit this result to establish our key results below.

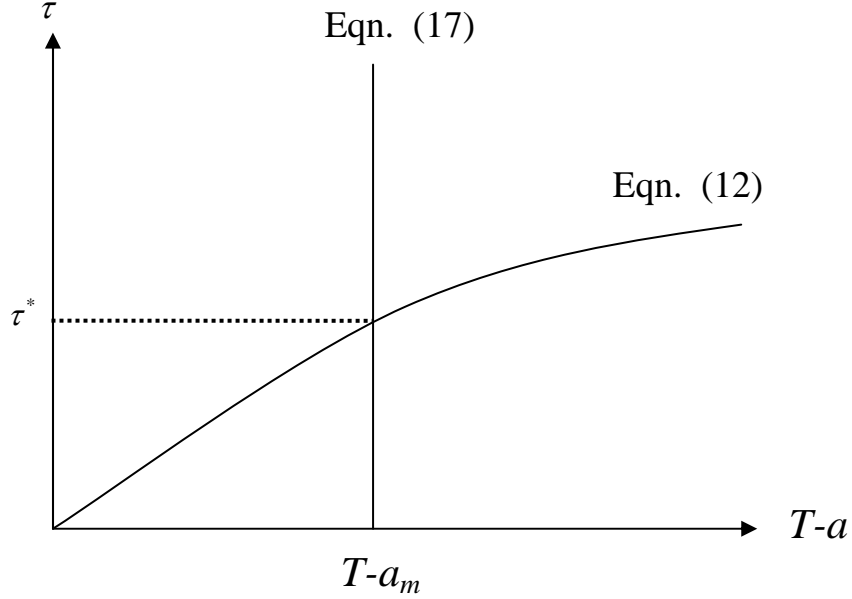


Figure 2: Equilibrium tax rate.

2.3. Political Equilibrium

The equilibrium conditions are given by (12) and (17) with two unknowns τ and $T - a$, and they are depicted in Figure 2. (12) shows a positive relation between a tax rate and the remaining life time. As explained above, younger voters prefer a higher tax rate, since they gain more from public investment than old consumers. On the other hand, (17) defines the remainder of the median voter's life time. Using these conditions, we can establish the following proposition:

Proposition 3: *The equilibrium tax rate τ^* has the following properties.*

$$(a) \frac{\partial \tau^*}{\partial T} > 0, \quad (b) \frac{\partial \tau^*}{\partial \beta} \geq \text{ or } \leq 0, \quad (c) \frac{\partial \tau^*}{\partial \rho} < 0, \quad (d) \frac{\partial \tau^*}{\partial A} > 0. \quad (19)$$

Proof : See Appendix D.

Consider result (a) first. Recall that the median voter lives longer as life expectancy increases (Corollary 1). Thus, the pivotal voter with a longer remaining life time gains more from future consumption and votes for a higher tax rate. Therefore, the equilibrium tax rate τ^* rises as people live longer. This result is represented by a rightward shift of the (17) line in Figure 2. The result also implies that economic growth accelerates due to public investment as peoples' life expectancy rises.

Behind result (a) is that the median voter prefers a higher tax rate as she lives longer. In fact, another reason for a longer remaining life time of the median voter can be introduced into the model. Suppose that consumers whose age is \tilde{a} or over only can vote. \tilde{a} is the minimum voting age and assume $0 < \tilde{a} < T$. Then, the remaining life time of the median voter is given by

$$T - a_m(\beta, T, \tilde{a}) \quad \text{where } a_m(\beta, T, \tilde{a}) = \ln \left(\frac{2}{2e^{-\beta\tilde{a}} - 1 + e^{-\beta T}} \right)^{1/\beta}. \quad (20)$$

One can confirm that $T - a_m$ is decreasing in \tilde{a} . If the minimum voting age is reduced, the median voter gets younger and lives longer. A consequence of this extension is obvious – an equilibrium tax rate is higher with a higher growth rate. Indeed, the issue of whether or not the minimum voting age should be reduced has been hotly debated in Japan and the UK in recent years. Our paper offers an additional benefit of the reduction of the voting age which is not considered by those governments.¹³

Next, consider result (b) of Proposition 3. There are two opposing effects. First, as explained above, a higher β makes the median voters younger, making their remaining life time longer. We call this the *rejuvenating effect*. This is represented by a rightward shift of the (17) curve in Figure 2, tending to increase τ^* . Second, a higher β aggravates congestion of public capital and reduces the ideal tax rate of all consumers, including the median voter. Because of this *congestion effect*, the (12) curve shifts down in Figure 2, tending to decrease τ^* . In general, the direction of changes in τ is ambiguous due to those two opposing effects. However, we can demonstrate the following result.

Let $\tau^* = \tau(\beta)$ denote a function relating an equilibrium tax rate to the birth rate. Appendix D demonstrates that (i) $\tau'(0) > 0$ under a certain condition and (ii) there exists $\tilde{\beta} > 0$ such that $0 = \tau(\tilde{\beta})$. Result (i) means that an equilibrium tax rate increases as the birth rate rises when the latter is initially small enough. On the other hand, result (ii) implies that an equilibrium tax rate goes to zero if the birth rate is too high. These results suggest that an equilibrium tax rate and the birth rate are related in a non-monotonic way, indicating the possibility of an inverted U relationship. Indeed, extensive simulation analysis suggests that an inverted U relationship with a single peak is a robust prediction of the model with plausible parameter values. Figure 3 shows an example where life expectancy is 83 years old and the long-run real interest rate is 3%. The value of A is chosen such that tax rates are broadly consistent with a real economy. In the figure, when β is low, the rejuvenating effect of a higher birth rate dominates the congestion effect, so that τ^* rises. The dominance of the first over the second effects is reversed when β is high.

¹³See Williams and White (2007) for reasons in favor of the plan in the UK. In Japan, a legislation to reduce the minimum voting age for national referendum to 18 from 20 was enacted in May 2007 as part of an effort to amend the constitution. This move is followed by the creation of a governmental panel to consider a plan to reduce the minimum voting age in public election to 18 from 20 (as of writing the paper).

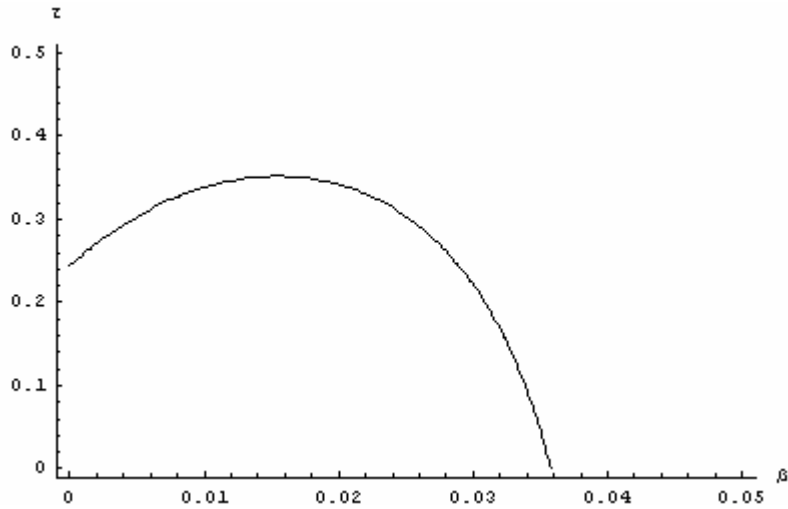


Figure 3: The effect of β on the tax rate ($A=0.07$, $\rho=0.03$, $T=83$).

There are two points worth mention regarding result (b). First, in developed economies, rising social security payments are expected to absorb a greater proportion of public expenditure due to demographic changes. It is creating concerns about sustainability of public deficits with a mounting pressure for higher tax rates. Our analysis, on the other hand, indicates that population growth slowdown leads to higher tax rates even in the absence of social security payments (as long as the birth rate is initially high). The reason for higher tax rates is not related to government budget issues, but voters' preferences. Second, an inverted U relationship suggests that an economy experiencing population growth slowdown may initially face an increasing tax rate, followed by a falling tax rate. Our model also suggests that the rate of economic growth may also follow a similar pattern. Indeed, we will confirm these results in empirical analysis later.

3. Empirical Analysis

3.1 Data

We use the annual data of G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States). The real growth rate (g), the tax burden ratio (τ), the age of the median voter (a_m), the birth rate (β), and life expectancy (T) are obtained from OECD (Organization for Economic Co-operation and Development) and other sources. Details of data construction are explained in the Appendix E. Since we have only annual data with small sample size for each country, we employ a panel data analysis to cope with this problem. The implication of this approach is that a combination of data for multiple countries can be used to perform powerful tests, when only annual data is available for individual countries.

In order to check the robustness of our empirical results, we use two sample periods, i.e., sample A: 1970-2004 and sample B: 1980-2004. This enables us to check whether our empirical results depend on a particular sample period.

3.2 Empirical Techniques

We perform empirical analyses based on (12) and (15) of the theoretical model. For

this end, we linearly specify the model as follows.

$$\tau_t = \alpha_1 + \alpha_2 \cdot a_{m,t} + \alpha_3 \cdot \beta_t + \alpha_4 \cdot T_t + u_{1t} \quad (\alpha_2 < 0, \alpha_3 < 0, \text{ and } \alpha_4 > 0) \quad (21)$$

$$a_{m,t} = \alpha_5 + \alpha_6 \cdot \beta_t + \alpha_7 \cdot T_t + u_{2t} \quad (\alpha_6 < 0 \text{ and } \alpha_7 > 0) \quad (22)$$

where α_i ($i = 1, 2, \dots, 7$) are unknown parameters, and u_{it} ($i=1,2$) are error terms with $E[u_{i,t} | I_{t-1}] = 0$ and I_{t-1} is the information set available at time $(t-1)$. Here, it is assumed that $I_{t-1} \subset I_t$. Equations (21) and (22) correspond to (12) and (15) of the theoretical model, respectively. We expect $\alpha_2 < 0$, $\alpha_3 < 0$, and $\alpha_4 > 0$ from Proposition 1, and $\alpha_6 < 0$ and $\alpha_7 > 0$ from Proposition 2.

We estimate the fixed effect model using the generalized method of moments (GMM). GMM has several characteristics (Hansen, 1982). The use of GMM enables us to take into consideration the endogeneity of explained variables in the system. An HAC (heteroskedasticity autocorrelation consistent) covariance matrix makes it possible to deal with heteroscedasticity and serial correlation in our analysis, and we are able to test the over-identifying restrictions of our empirical model by using Hansen's J-test. To enhance the efficiency of our estimates, we use iterated GMM to obtain estimates and their standard error.

3.3 Empirical results

Tables 1 and 2 show the estimates obtained using (21) for 1970-2004 and 1980-2004, respectively. In this case, we use instrumental variables as follows¹⁴:

$$z = (1, \tau_{t-1}, \tau_{t-2}, a_{m,t-1}, a_{m,t-2}, \beta_{t-1}, \beta_{t-2}, T_{t-1}, T_{t-2}).$$

In Table 1, every coefficient estimate meets the theoretical sign conditions specified in Propositions 1 and 2, and is statistically significant at the 1% level. Additionally, with a low J-statistic of 29.9901 for 1970-2004 and 23.1258 for 1980-2004, the over-identifying restriction of the model is empirically accepted. The coefficient of the median voter age (α_2) is estimated to be -0.7269 for 1970-2004 and -0.4945 for 1980-2004, which indicate that one-year increase in the age of the median voter results in 0.73%pts (0.49%pts) lower tax rates using the data of 1970-2004 (1980-2004).¹⁵ The birth rate coefficient (α_3) is estimated to be -1.0527 for 1970-2004 and -0.4760 for 1980-2004. Therefore, holding the age of the median voter constant, one-point increase in the birth rate results in an decrease of 1.05%pts (0.48%pts) in tax rates using the data of 1970-2004 (1980-2004).^{16,17} Lastly, the coefficient of the average life expectancy

¹⁴We obtain similar empirical results when we use the following instruments:

$$z = (1, \tau_{t-1}, a_{m,t-1}, \beta_{t-1}, T_{t-1})$$

¹⁵The difference between the peak and the bottom ages of the median voter in this period is 11.7 years for Japan, 6.0 years for Canada, 4.7 years for the US, 2.1 years for the UK, and between 3.0 and 3.6 years for other countries.

¹⁶This result does *not* necessarily imply that the equilibrium tax rate is decreasing in the birth rate, since the age of the median voter decreases as the birth rate increases.

¹⁷The decrease in the birth rate from 1970 to 1980 is 5.6 for Italy, 5.0 for Japan, 3.3 for Germany, and between 2.4 and 3.0 for other countries. The decrease in the birth rate from 1980 to 2004 is 4.8 for Japan, 4.5 for Canada, and between 1.4 and 2.1 for other countries.

(α_4) is estimated to be 0.6241 for 1970-2004 and 0.6723 for 1980-2004. Holding the age of the median voter constant, one-year increase in the average life expectancy results in more than 0.6%pts higher tax rates.¹⁸

Table 2 shows the estimates obtained using (22) for 1970-2004 and 1980-2004. In this case, we use the following instrumental variables:¹⁹

$$z = (1, a_{m,t-1}, a_{m,t-2}, \beta_{t-1}, \beta_{t-2}, T_{t-1}, T_{t-2}).$$

Every coefficient estimate meets the theoretical sign condition and is statistically significant at the 1% level. In addition, with a low J-statistic of 29.9337 for 1970-2004 and 22.5087 for 1980-2004, the over-identifying restriction of the model is empirically accepted. The birth rate coefficient (α_6) is estimated to be -0.5276 for 1970-2004 and -0.8024 for 1980-2004: the age of the median voter decreases as the birth rate increases. The average life expectancy coefficient (α_7) is estimated to be 0.1754 for 1970-2004 and 0.3985 for 1980-2004: the age of the median voter increases as the average life expectancy increases.

In summary, the individual coefficient estimates meet the theoretical sign conditions and are statistically significant. The J-statistic indicates that the over-identifying restriction of the model is empirically accepted. These results are robust to the sample periods. We conclude that our politico-economic model of the equilibrium tax rate is empirically supported by the data of G7 countries.

Next we estimate the following empirical model:

$$g_t = \alpha_8 + \alpha_9 \beta_t + \alpha_{10} \beta_t^2 + \alpha_{11} T_t \quad (\alpha_9 > 0, \alpha_{10} < 0, \alpha_{11} > 0).$$

Equation (5) shows that the growth rate is increasing with the tax rate, and Proposition 3 shows that the tax rate has an inverted U relationship with the birth rate. Hence we expect $\alpha_9 > 0$ and $\alpha_{10} < 0$. Moreover, Proposition 3 shows that an increase in the average life expectancy results in higher tax rate. Hence we expect $\alpha_{11} > 0$. In the empirical analysis, we use the following instrumental variables:

$$z = (1, g_{t-1}, g_{t-2}, \beta_{t-1}, \beta_{t-2}, T_{t-1}, T_{t-2}).$$

Results are summarized in Table 3. As for the birth rate, both coefficients are statistically significant at the 5% level and meet the theoretical sign conditions. α_9 is estimated to be 7.1147 for 1970-2004 and 17.7922 for 1980-2004, and α_{10} is estimated to be -0.2536 for 1970-2004 and -0.7239 for 1980-2004. An inverted U-shape relationship between the birth rate and the growth rate is consistent with the finding of Kelley and Schmidt (1995), An and Jeon (2006), and Li and Zhang (2007). The estimation result for 1970-2004 (1980-2004) indicates that the growth rate is maximized when the birth rate is 14.027 (12.289), and that the growth rate becomes 4.11%pts (3.79%pts) lower than the maximum if the birth rate is decreased to 10.000.²⁰

¹⁸The increase in the average life expectancy from 1970 to 1980 is 4.1 years for Japan, 1.7 years for the UK, and between 2.1 and 2.9 for other countries. The increase in the average life expectancy from 1980 to 2004 is 4.2 years for the US, 4.9 years for Canada, and between 5.2 years and 6.0 years for other countries.

¹⁹We obtained similar empirical results when we use the following instruments:

$$z = (1, a_{m,t-1}, \beta_{t-1}, T_{t-1})$$

²⁰The birth rate for 1980/2004 is 15.4/10.9 for Canada, 14.8/12.7 for France, 10.1/8.6

The coefficient of the average life expectancy (α_{11}) is estimated to be 0.0140 for 1970-2004 and 0.3412 for 1980-2004. Although the coefficient is not statistically significant for 1970-2004, it is significant for 1980-2004. These results indicate that the growth rate is an increasing function of the average life expectancy, which is consistent with Proposition 3.²¹

4. Some Concluding Remarks

In this paper, we analyzed the effect that demographic changes have on fiscal policy and growth in a politico-economic model where an equilibrium tax rate is determined by majority voting. The model is distinguished from existing studies in the assumption regarding the time domain. The continuous-time framework adopted in the present paper allows us to cope with problematic features of the previous studies that originate in the use of the OLG framework. We believe that our modeling approach contributes to a deeper understanding of demographic issues facing developed economies.

In our analysis, we distinguish between two closely related aspects of demographic changes: (i) a longer life time of consumers, and (ii) a lower birth rate, both of which increases the old in number relative to the young. Although these two aspects are captured by the expression "population aging", we show that they have different effects on fiscal policy and growth. Key results are two-fold. First, when life expectancy rises, both of taxes and growth increase. Second, there exists an inverted U relationship between the birth rate on one hand and taxes and growth on the other.

Furthermore, we examine whether or not data support theoretical predictions of our model. For this purpose, we use data from G7 countries. Our intension is to consider as many developed economies as data allows in order to show validity of our approach. Empirical analysis clearly demonstrates that the model's predictions are observed in data. In addition, robustness of the results is largely confirmed when the sample period is extended by 10 years. Note that our empirical analysis represents the first attempt in the literature to show the link between demographic variables, taxes and the median voter.

References

- Acemoglu, Daron, and Johnson, Simon (2006) "Disease and Development: the Effect of Life Expectancy on Economic Growth." NBER Working Paper 12269, June, 2006.
- Ahituv, Avner (2001) "Be Fruitful or Multiply: On the Interplay between Fertility and Economic Development." *Journal of Population Economics*, May 14(1), 51-71.
- Alesina, Alberto, Spolaore, Enrico, and Wacziarg, Romain (2005) "Trade, Growth, and the Size of Countries." in Philippe Aghion and Steven N. Durlauf eds., *Handbook*

for Germany, 11.8/9.7 for Italy, 13.5/8.7 for Japan, 13.4/12.0 for the UK, and 15.9/14.0 for the US.

²¹ The increase in the average life expectancy from 1980 to 2004 is 4.2 years for the US, 4.9 years for Canada, and between 5.2 and 6.0 years for other countries.

of Economic Growth, North-Holland.

- An, Chong-Bum, and Jeon, Seung-Hoon (2006) "Demographic Changes and Economic Growth: An Inverted-U Shape Relationship." *Economics Letters*, September 92(3), 447-454.
- Barro, Robert J. (1996) "Democracy and Growth." *Journal of Economic Growth*, March 1(1), 1-27.
- Barro, Robert J. and Sala-I-Martin, Xavier (1992) "Public Finance in Models of Economic Growth." *Review of Economic Studies*, Vol.59, No.4, pp. 645-661.
- Beauchemin, Kenneth R. (1998) "Intergenerational Politics, Fiscal Policy and Productivity," *Review of Economic Dynamics*, October 1(4), 835-858.
- Boldrin, Michele (2005) "Public Education and Capital Accumulation." *Research in Economics*, June 59(2), 85-109
- Caselli, Francesco, Esquivel, Gerardo, and Lefort, Fernando (1996) "Reopening the Convergence Debate: A New Look at Cross-Country Growth Empirics." *Journal of Economic Growth*, September 1(3), 363-389.
- Durlauf, Steven N., Johnson, Paul A., and Temple, Jonathan R. W. (2005) "Growth Econometrics." in Philippe Aghion and Steven N. Durlauf eds., *Handbook of Economic Growth*, North-Holland.
- Feyrer, James (2007) "Demographics and Productivity." *Review of Economics and Statistics*, February 89(1): 100-109.
- Gradstein, Mark, and Kaganovich, Michael (2004) "Aging Population and Education Finance." *Journal of Public Economics*, December 88(12), 2469-2485.
- Gramlich, Edward M. (1994) "Infrastructure Investment: A Review Essay." *Journal of Economic Literature*, Vol.32, No.3, pp. 1176-1196.
- Levy, Gilat (2005) "The Politics of Public Provision of Education." *Quarterly Journal of Economics*, November 120(4), 1507-1534.
- Hansen, Lars Peter (1982) "Large Sample Properties of Generalized Method of Moments Estimators." *Econometrica*, July 50(4), 1029-1054.
- Holtz-Eakin, Douglas, Lovely, Mary E., and Tosun, Mehmet S. (2004) "Generational Conflict, Fiscal Policy, and Economic Growth." *Journal of Macroeconomics*, March 26(1), 1-23.
- Jones, Charles I. (2005) "Growth and Ideas." In *Handbook of Economic Growth*, (eds) Philippe Aghion and Steven N. Durlauf, Vol.1B, pp.1063-1114, Amsterdam: Elsevier.

- Kelley, Allen C., and Schmidt, Robert M. (1995) "Aggregate Population and Economic Growth Correlations: The Role of the Components of Demographic Change." *Demography*, November 32(4), 543-555.
- Kelly, Trish (1997) "Public Expenditures and Growth," *Journal of Development Studies*, October 34(1), 60-84.
- Kelley, Allen C., and Schmidt, Robert M. (2005) "Evolution of Recent Economic-Demographic Modeling: A Synthesis." *Journal of Population Economics*, June 18(2), 275-300.
- Kemnitz, Alexander (1999) "Demographic Structure and the Political Economy of Education Subsidies." *Public Choice*, December 101(3-4), 235-249.
- Kemnitz, Alexander (2000) "Social Security, Public Education, and Growth in a Representative Democracy." *Journal of Population Economics*, August 13(3), 443-462.
- Levine, Ross, and Renelt, David (1992) "A Sensitivity Analysis of Cross-Country Growth Regressions." *American Economic Review*, September 82(4), 942-963.
- Li, Hongbin, and Zhang, Junsen (2007) "Do High Birth Rates Hamper Economic Growth?" *Review of Economics and Statistics*, February, 89(1), 110-117
- Medoff, James. L. and Abraham, Katherine. G. (1980). "Experience, Performance, and Earnings.", *Quarterly Journal of Economics*, Vol. 95, No. 4, pp. 703-736.
- Pecchenino, Rowena A., and Pollard, Patricia S. (2002) "Dependent Children and Aged Parents: Funding Education and Social Security in an Aging Economy." *Journal of Macroeconomics*, June 24(2), 145-169.
- Pecchenino, Rowena A., and Utendorf, Kelvin R. (1999) "Social Security, Social Welfare and the Aging Population." *Journal of Population Economics*, November 12(4), 607-623.
- Razin, Assaf, Sadka, Efraim, and Swagel, Phillip (2002) "The Aging Population and the Size of the Welfare State." *Journal of Political Economy*, August 110(4), 900-918.
- Sala-i-Martin, Xavier X. (1997a) "I Just Ran 4 Million Regressions." National Bureau of Economic Research Working Paper no. 6252.
- Sala-i-Martin, Xavier X. (1997b) "I Just Ran 2 Million Regressions." *American Economic Review, Papers and Proceedings*, May 87(2), 178-183.
- Skirbekk, Vegard (2004) "Age and Individual Productivity: A Literature Survey" in Gustav Feichtinger (ed.) *Vienna Yearbook of Population Research 2004*,

pp.133-53.

Tosun, Mehmet Serkan (2003) "Population Aging and Economic Growth: Political Economy and Open Economy Effects." *Economics Letters*, December 81(3), 291-296.

United Nations (2007) *World Population Prospects: The 2006 Revision*, Department of Economic and Social Affairs, United Nations.

Williams, Kevin and White, Isobel (2007) "Reduction in Voting Age." Standard Note: SN/PC/1747, *House of Commons Library*, UK (available at <http://www.parliament.uk/commons/lib/research/notes/snpc-01747.pdf>, accessed on 29 Aug 2007).

Zhang, Jie, and Zhang, Junsen (2005) "The Effect of Life Expectancy on Fertility, Saving, Schooling and Economic Growth: Theory and Evidence." *Scandinavian Journal of Economics*, March 107(1), 45-66.

Zhang, Jie, Zhang, Junsen, and Lee, Ronald (2003) "Rising Longevity, Education, Savings, and Growth." *Journal of Development Economics*, February 70(1), 83-101.

Table 1 Empirical Results

$$(21) \quad \tau_t = \alpha_1 + \alpha_2 a_{m,t} + \alpha_3 \beta_t + \alpha_4 T_t + u_{1t}, \quad \alpha_2 < 0, \alpha_3 < 0, \alpha_4 > 0$$

Coefficient	Estimate	SE	p-value	Estimate	SE	p-value
	1970-2004			1980-2004		
α_2	-0.7269	0.0825	0.0000	-0.4945	0.0884	0.0000
α_3	-1.0527	0.0783	0.0000	-0.4760	0.1497	0.0018
α_4	0.6241	0.0631	0.0000	0.6723	0.0744	0.0000
J statistic	29.9901			23.1298		

Note:

Instrumental variables: $z = (1, \tau_{t-1}, \tau_{t-2}, a_{m,t-1}, a_{m,t-2}, \beta_{t-1}, \beta_{t-2}, T_{t-1}, T_{t-2})$

Table 2 Empirical Results

$$(22) \quad a_{m,t} = \alpha_5 + \alpha_6 \beta_t + \alpha_7 T_t + u_{2t}, \quad \alpha_6 < 0, \alpha_7 > 0$$

Coefficient	Estimate	SE	<i>p</i> -value	Estimate	SE	<i>p</i> -value
	1970-2004			1980-2004		
α_6	-0.5276	0.0610	0.0000	-0.8024	0.0659	0.0000
α_7	0.1754	0.0273	0.0000	0.3985	0.0622	0.0000
J statistic	29.9337			22.5087		

Note:

Instrumental variables: $z = (1, a_{m,t-1}, a_{m,t-2}, \beta_{t-1}, \beta_{t-2}, T_{t-1}, T_{t-2})$

Table 3 Empirical Results

$$g_t = \alpha_8 + \alpha_9 \beta_t + \alpha_{10} \beta_t^2 + \alpha_{11} T_t, \quad \alpha_9 > 0, \alpha_{10} < 0, \alpha_{11} > 0$$

Coefficient	Estimate	SE	<i>p</i> -value	Estimate	SE	<i>p</i> -value
	1970-2004			1980-2004		
α_9	7.1147	3.3817	0.0365	17.7922	4.0209	0.0000
α_{10}	-0.2536	0.1231	0.0404	-0.7239	0.1677	0.0000
α_{11}	0.0140	0.1446	0.9229	0.3412	0.1410	0.0167
J statistic	99.7853			85.4835		

Note:

Instrumental variables: $z = (1, g_{t-1}, g_{t-2}, \beta_{t-1}, \beta_{t-2}, T_{t-1}, T_{t-2})$

Appendix A: Derivation of Ideal Tax Rate of Consumers of Age a

A.1 Preliminary

Consider the following incomplete gamma function

$$\gamma(n, x) \equiv \int_0^x m^{n-1} e^{-m} dm = (n-1)! \left(1 - e^{-x} \sum_{k=0}^{n-1} \frac{x^k}{k!} \right) \quad (23)$$

where n is an integer. (23) has the following properties

$$\gamma(1, x) > \gamma(2, x) > \frac{\gamma(3, x)}{2}, \quad \gamma(2, x) - \gamma(3, x) = e^{-x} (x^2 + x + 1 - e^x) > 0. \quad (24)$$

Using those equations and defining $x = (\rho + \beta - A\tau)(T - a)$, one can write

$$\begin{aligned} F(\tau; a) &= \frac{T-a}{x} \gamma(1, x), & F_\tau(\tau; a) &= A \left(\frac{T-a}{x} \right)^2 \gamma(2, x), \\ F_{\tau\tau}(\tau; a) &= A^2 \left(\frac{T-a}{x} \right)^3 \gamma(3, x). \end{aligned} \quad (25)$$

A.2 First-order and Second-order Conditions

Making use of (25), the FOC (11) can be rewritten as

$$\frac{A(1-\tau^*)}{\underbrace{\rho + \beta - A\tau^*}_{\Phi(\tau^*)}} = \frac{\gamma(1, x^*)}{\gamma(2, x^*)} > 1 \quad (26)$$

where $x^* = (\rho + \beta - A\tau^*)(T - a)$ and the inequality is due to (24). To ensure the existence of an interior equilibrium, assume $\Phi(0) = A/(\rho + \beta) > 1$, due to which one can easily confirm $\Phi'(\tau) > 0$ with

$$\Phi(\tau) \in \left(\frac{A}{\rho + \beta}, \infty \right) \quad \text{for } \tau \in \left(0, \frac{\rho + \beta}{A} \right). \quad (27)$$

The second-order condition is

$$\begin{aligned} \left. \frac{\partial^2 U_a}{\partial \tau^2} \right|_{\tau=\tau^*} &= \frac{G(t)}{L(t)} \left[(1-\tau^*) F_{\tau\tau}(\tau^*) - 2F_\tau(\tau^*) \right] \\ &= \frac{G(t)}{L(t)} A \left(\frac{T-a}{x^*} \right)^2 \left[\Phi(\tau^*) \gamma(3, x^*) - 2\gamma(2, x^*) \right] \\ &= \frac{G(t)}{L(t)} A \left(\frac{T-a}{x^*} \right)^2 \left[\frac{\gamma(1, x^*)}{\gamma(2, x^*)} \gamma(3, x^*) - 2\gamma(2, x^*) \right] \\ &= -\frac{G(t)}{L(t)} \left(\frac{T-a}{x^*} \right)^2 \frac{2A}{\gamma(2, x^*)} \left[\gamma(2, x^*) (1 - \gamma(1, x^*)) + e^{-x^*} \left(\frac{x^*}{2} \right)^2 \gamma(1, x^*) \right] < 0 \end{aligned} \quad (28)$$

where the second equality uses (25), the third equality uses (26) and the last equality uses

$$\gamma(3, x) = 2\gamma(2, x) - e^{-x} x^2. \quad (29)$$

A.3 Existence of τ^*

From the FOC (11),

$$\frac{\partial MC(\tau)}{\partial \tau} = F_\tau(\tau) = A \left(\frac{T-a}{x} \right)^2 \gamma(2, x) > 0, \quad (30)$$

$$\begin{aligned} \frac{\partial MB(\tau)}{\partial \tau} &= (1-\tau)F_{\tau\tau}(\tau) - F_\tau(\tau) = \left(\frac{T-a}{x} \right)^2 A [\Phi(\tau)\gamma(3, x) - \gamma(2, x)] \\ &= - \left(\frac{T-a}{x} \right)^2 \frac{A}{\gamma(2, x)} [\gamma(1, x)\gamma(3, x) - \gamma(2, x)^2] \\ &= \left(\frac{T-a}{x} \right)^2 \frac{Ae^{-x}}{\gamma(2, x)} [\gamma(2, x) + x^2\gamma(1, x)] > 0 \end{aligned} \quad (31)$$

where the first line of (31) uses (25), and the second line is due to (26) and the last line can be obtained after rearrangement. Now, (28) can be rewritten as

$$\frac{\partial^2 U_a}{\partial \tau^2} \Big|_{\tau=\tau^*} = \frac{G(t)}{L(t)} \left(\frac{\partial MB(\tau)}{\partial \tau} - \frac{\partial MC(\tau)}{\partial \tau} \right) \Big|_{\tau=\tau^*} < 0, \quad (32)$$

which means that the *MB* curve is less steep than the *MC* curve at the ideal tax rate of consumers of age a , as depicted in Figure 1. Given these results, an odd number of intersection points between the *MB* and *MC* curves exist if the following conditions hold:

$$\begin{aligned} \frac{A\gamma(2, x_0)}{(\rho + \beta)^2} \equiv MB(0) &> MC(0) \equiv \frac{\gamma(1, x_0)}{\rho + \beta}, \\ \frac{[A - (\rho + \beta)](T-a)^2}{2} \equiv MB\left(\frac{\rho + \beta}{A}\right) &< MC\left(\frac{\rho + \beta}{A}\right) \equiv T-a, \end{aligned} \quad (33)$$

where $x_0 = (\rho + \beta)(T-a)$. An interior solution requires the following parameter restriction, which is derived from (33):

$$(\rho + \beta) \left(1 + \frac{2}{x_0} \right) > A > (\rho + \beta) \frac{\gamma(1, x_0)}{\gamma(2, x_0)}. \quad (34)$$

This condition is assumed to hold in the main text.

A.4 Uniqueness of τ^*

Using the definition of x , the FOC (11) can be rewritten as

$$[A - (\rho + \beta)](T-a) = x \left(\frac{\gamma(1, x)}{\gamma(2, x)} - 1 \right). \quad (35)$$

Moreover,

$$\frac{d}{dx} \left(x \left(\frac{\gamma(1, x)}{\gamma(2, x)} - 1 \right) \right) = - \frac{e^{-x}x}{\gamma(2, x)} \left(x \frac{\gamma(1, x)}{\gamma(2, x)} - 2 \right) < 0. \quad (36)$$

To confirm the sign, note that

$$\frac{d}{dx} \left(x \frac{\gamma(1, x)}{\gamma(2, x)} \right) = \frac{\Delta(x)}{\gamma(2, x)^2} > 0, \quad \lim_{x \rightarrow 0} x \frac{\gamma(1, x)}{\gamma(2, x)} = 2 \quad (37)$$

where

$$\Delta(x) \equiv \left(\gamma(1, x) + e^{-\frac{x}{2}} \right) \left(\gamma(1, x) - e^{-\frac{x}{2}} \right) > 0 \quad (38)$$

and $\gamma(1, x) - e^{-\frac{x}{2}}$ is positive because it is monotonically increasing and equal to zero at $x = 0$. Therefore, (36) implies that a unique x exists, hence the ideal tax rate of consumers of age a is unique.

Appendix B: Proof of Proposition 1

Result (a)

Totally differentiating (11) gives

$$\frac{d\tau}{d(T-a)} = -\frac{[(1-\tau)A(T-a)-1]e^{-x}}{(1-\tau)F_{\tau\tau}(\tau) - 2F_{\tau}(\tau)} > 0 \quad (39)$$

where the denominator is negative due to (28). To show that the numerator is positive, rewrite it as

$$[(1-\tau)A(T-a)-1]e^{-x} = \left(\frac{F(\tau)}{F_{\tau}(\tau)} A(T-a) - 1 \right) e^{-x} = [x - (1 - e^{-x})] \frac{e^{-x}}{\gamma(2, x)} > 0 \quad (40)$$

where the first equality is due to (11) and the second equality makes use of (25).

Results (b) and (c)

Define $\sigma = \rho + \beta$. Note that we can write

$$\frac{dF(\tau)}{d\sigma} = -\frac{F_{\tau}(\tau)}{A}, \quad \frac{dF_{\tau}(\tau)}{d\sigma} = -\frac{F_{\tau\tau}(\tau)}{A}. \quad (41)$$

Using those equations, totally differentiate (11) to obtain

$$\frac{\partial\tau}{\partial\sigma} = -\frac{(1-\tau)F_{\tau\tau}(\tau) - F_{\tau}(\tau)}{A[(1-\tau)F_{\tau\tau}(\tau) - 2F_{\tau}(\tau)]} < 0. \quad (42)$$

The denominator is negative due to (28) and the numerator is negative since

$$(1-\tau)F_{\tau\tau}(\tau) - F_{\tau}(\tau) = -\left(\frac{T-a}{x}\right)^2 \frac{\gamma(1, x)\gamma(3, x) - \gamma(2, x)^2}{\gamma(2, x)} = -\left(\frac{T-a}{x}\right)^2 \frac{\Delta(x)}{\gamma(2, x)} < 0 \quad (43)$$

where the first equality uses (25) and the last equality is due to (38).

Result (d)

Note that

$$\frac{dF(\tau)}{dA} = -\frac{\tau}{A} F_{\tau}(\tau), \quad \frac{dF_{\tau}(\tau)}{dA} = -\frac{\tau}{A} F_{\tau\tau}(\tau). \quad (44)$$

Then, totally differentiating (11) yields

$$\frac{d\tau}{d(T-a)} = \frac{\tau[(1-\tau)F_{\tau\tau}(\tau) - F_{\tau}(\tau)]}{A[(1-\tau)F_{\tau\tau}(\tau) - 2F_{\tau}(\tau)]} > 0 \quad (45)$$

where the denominator is negative due to (28) and the numerator is also negative because of (43).

Appendix C: Proof of Proposition 2

Result (a)

Differentiating (15) w.r.t. T gives

$$\frac{\partial a_m}{\partial T} = \frac{e^{-\beta T}}{1+e^{-\beta T}} > 0. \quad (46)$$

Result (b)

Differentiating w.r.t. β yields

$$\frac{\partial a_m}{\partial \beta} = -\beta^{-2} \ln \frac{2}{1+e^{-\beta T}} - \beta^{-1} \frac{T e^{-\beta T}}{1+e^{-\beta T}} < 0. \quad (47)$$

Appendix D: Proof of Proposition 3

D.1 Results (a), (c), and (d)

Graphical analysis is sufficient to establish the results. Result (a) holds due to (a) of Corollary 1, which is represented by a rightward shift of the (17) curve. Regarding ρ and A , they do not affect the (17) curve, and we only need to consider the impact of those parameters on the (12) curve. Results (c) and (d) of Proposition 1 imply that the (12) curve shifts downward and upward, respectively.

D.2 Result (b)

It is easier to consider τ^* which is defined by (11) and (15), which gives

$$(1-\tau^*)F_\tau(\tau^*, a_m(\tau^*)) = F(\tau^*, a_m(\tau^*)). \quad (48)$$

This equation implicitly defines $\tau^* = \tau(\beta)$. Totally differentiating this equation yields

$$\tau'(\beta) \equiv \frac{d\tau^*}{d\beta} = -\frac{(1-\tau^*)\frac{\partial F_\tau(\tau^*)}{\partial \beta} - \frac{\partial F(\tau^*)}{\partial \beta}}{(1-\tau^*)F_{\tau\tau}(\tau^*) - 2F_\tau(\tau^*)} \quad (49)$$

where $a_m(\tau^*)$ in $F(\tau^*, a_m(\tau^*))$ and $F_\tau(\tau^*, a_m(\tau^*))$ is suppressed for simplicity.

Defining $x^* = (\rho + \beta - A\tau^*)(T - a_m)$, rewrite two terms in the numerator as

$$\frac{\partial F(\tau^*)}{\partial \beta} = -\frac{F_\tau(\tau^*)}{A} - e^{-x^*} \frac{\partial a_m}{\partial \beta}, \quad \frac{\partial F_\tau(\tau^*)}{\partial \beta} = -\frac{F_{\tau\tau}(\tau^*)}{A} - A(T - a_m)e^{-x^*} \frac{\partial a_m}{\partial \beta}. \quad (50)$$

Then, using (25) and (50), we can write

$$\begin{aligned} (1-\tau^*)\frac{\partial F_\tau(\tau^*)}{\partial \beta} - \frac{\partial F(\tau^*)}{\partial \beta} &= -\frac{(1-\tau^*)F_{\tau\tau}(\tau^*) - F_\tau(\tau^*)}{A} \left[A \frac{T - a_m}{x^*} \gamma(3, x^*) - \gamma(2, x^*) \right] \\ &\quad - \left[\frac{(1-\tau^*)A}{\rho + \beta - A\tau^* x^*} - 1 \right] e^{-x^*} \frac{\partial a_m}{\partial \beta} \\ &= -\left(\frac{T - a_m}{x^*} \right)^2 \frac{\Delta(x^*)}{\gamma(2, x^*)} - \frac{\Upsilon(x^*)}{\gamma(2, x^*)} e^{-x^*} \frac{\partial a_m}{\partial \beta} \geq \text{or} \leq 0. \end{aligned} \quad (51)$$

where

$$\Upsilon(x^*) = x^* \gamma(1, x^*) - \gamma(2, x^*) = x^* - (1 - e^{-x^*}) > 0. \quad (52)$$

Since $\partial a_m / \partial \beta < 0$, the sign of (51) is ambiguous, suggesting the possibility of a non-monotonic relationship between τ and β (i.e. the existence of a stationary point

at $\partial\tau/\partial\beta=0$). To explore this possibility, consider the following points.

D.2.a Slope at Vertical Intercept

We evaluate (49) at $\beta=0$, i.e. $\tau'(0)$. Define

$$x_\beta \equiv (\rho - A\tau^*)(T - a_m) = \frac{(\rho - A\tau)T}{2} \quad (53)$$

at $\beta=0$. Then, the numerator of (49) at $\beta=0$ is

$$(1-\tau) \frac{\partial F_\tau}{\partial \beta} - \frac{\partial F}{\partial \beta} \Big|_{\beta=0} = \frac{(T/2x_\beta)^2}{\gamma(3, x_\beta)} \left[-\frac{\gamma(1, x_\beta)}{\gamma(2, x_\beta)} + \frac{1}{2} + x_\beta \frac{\gamma(1, x_\beta)}{\gamma(2, x_\beta)} \left(\frac{\gamma(2, x_\beta)}{\gamma(3, x_\beta)} - \frac{1}{2} \right) \right]. \quad (54)$$

To evaluate the sign of this expression, note that

$$\frac{\gamma(1, x_\beta)}{\gamma(2, x_\beta)} = \frac{1}{x_\beta} \left(2 - \frac{\gamma(3, x_\beta)}{\gamma(2, x_\beta)} \right) + 1, \quad (55)$$

which allows us to rewrite (54) as

$$(1-\tau) \frac{\partial F_\tau}{\partial \beta} - \frac{\partial F}{\partial \beta} \Big|_{\beta=0} = \frac{(T/2)^2}{x_\beta^2 \gamma(3, x_\beta)} (\alpha_1 x_\beta^2 + \alpha_2 x_\beta + \alpha_3) \quad (56)$$

where

$$\alpha_1 \equiv \eta, \quad \alpha_2 \equiv -(2 - \eta - \eta^2), \quad \alpha_3 \equiv -2(2 - \eta)\eta, \quad \eta \equiv 2 - \frac{\gamma(3, x_\beta)}{\gamma(2, x_\beta)}. \quad (57)$$

Note that $2 > \eta > 0$ for $0 < x < \infty$. Moreover, rearrangement of (35) at $\beta=0$ gives

$$\frac{\gamma(3, x_\beta)}{\gamma(2, x_\beta)} = 2 \left(1 - (A - \rho) \frac{T}{4} \right), \quad (58)$$

which allows us to rewrite η as

$$\eta \equiv (A - \rho) \frac{T}{2}. \quad (59)$$

Therefore, (56) is zero when

$$\bar{x}_\beta(\eta) = \frac{-\alpha_2 + \sqrt{\alpha_2^2 - 4\alpha_1\alpha_3}}{2\alpha_1} > 0 \quad \text{for } 2 > \eta > 0. \quad (60)$$

Therefore, (56) is positive for $x_\beta > \bar{x}_\beta(\eta)$. Given that the denominator of (49) is negative, we have $d\tau^*/d\beta > 0$ for $x_\beta > \bar{x}_\beta(\eta)$.

D.2.b Horizontal Intercept

Here we show the existence of $\tilde{\beta} > 0$ such that $0 = \tau(\tilde{\beta})$. Define

$\tilde{x}_0 \equiv (\rho + \tilde{\beta})(T - a_m(\tilde{\beta}))$, so that (48) is rewritten as

$$\underbrace{A(T - a_m(\tilde{\beta}))}_{\text{LHS}} = \tilde{x}_0 \underbrace{\frac{\gamma(1, \tilde{x}_0)}{\gamma(2, \tilde{x}_0)}}_{\text{RHS}} \quad (61)$$

at $\tau=0$. Vary T from zero to infinity, holding $\tilde{\beta}$ constant. Then, note that

$$T \in (0, \infty) \Rightarrow T - a_m \in (0, \infty) \Rightarrow x_0 \in (0, \infty). \quad (62)$$

Now, *LHS* is linear in $T - a_m$ and starts from the origin in Figure 4. *RHS* is monotonically increasing in $T - a_m$ and is greater than 2 (see (37)). Taking a difference between the slopes of *LHS* and *RHS* gives

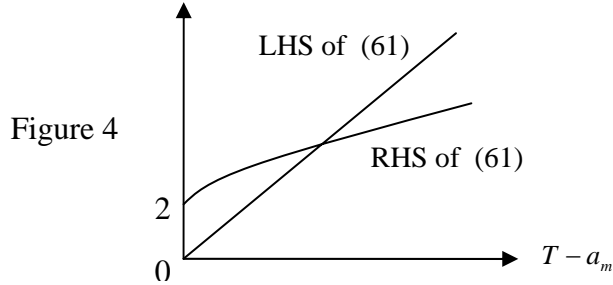
$$\begin{aligned} \frac{dLHS}{d(T - a_m)} - \frac{dRHS}{d(T - a_m)} &= A - \left[(\rho + \tilde{\beta}) \frac{\gamma(1, \tilde{x}_0)}{\gamma(2, \tilde{x}_0)} + \tilde{x}_0 \frac{d}{d\tilde{x}_0} \left(\frac{\gamma(1, \tilde{x}_0)}{\gamma(2, \tilde{x}_0)} \right) \frac{d\tilde{x}_0}{d(T - a_m)} \right] \\ &= -\tilde{x}_0 \frac{d}{d\tilde{x}_0} \left(\frac{\gamma(1, \tilde{x}_0)}{\gamma(2, \tilde{x}_0)} \right) \underbrace{\frac{d\tilde{x}_0}{d(T - a_m)}}_{(+)} \end{aligned} \quad (63)$$

where the second line uses $\frac{A}{\rho + \tilde{\beta}} = \frac{\gamma(1, \tilde{x}_0)}{\gamma(2, \tilde{x}_0)}$, which must hold for utility maximization

(see (26)). To sign (63), calculate the following derivative:

$$\frac{d}{d\tilde{x}_0} \left(\frac{\gamma(1, \tilde{x}_0)}{\gamma(2, \tilde{x}_0)} \right) = -e^{-\tilde{x}_0} \frac{\tilde{x}_0 - (1 - e^{-\tilde{x}_0})}{\gamma(2, \tilde{x}_0)^2} < 0. \quad (64)$$

Given that (63) is positive, there must be a unique intersection point between *LHS* and *RHS*, which defines a unique $T - a_m$. Moreover, given that $T - a_m$ and $\tilde{\beta}$ have a one-to-one relationship, there must be a unique $\tilde{\beta} > 0$.



Appendix E: Data

Data on the real GDP growth rate is taken from “SourceOECD OECD Economic Outlook No 80.” The tax burden ratio, defined as the ratio of total tax revenue to nominal GDP, is obtained from “SourceOECD Revenue Statistics of OECD Member Countries.” The age of the median voter is calculated using “Demographic Yearbook Historical supplement” published by the United Nations (for Canada, France, Italy, and the UK), the US Census Bureau website (for Canada, France, and the US), Federal Statistical Office GENESIS-Online Statistical Information System (for Germany), National Institute of Statistics website (for Italy), “Population Census” and “Annual Report on Current Population Estimates” published by Ministry of Public Management, Home Affairs, Posts and Telecommunications (for Japan), and National Statistics Online (for the UK). The birth rate is taken from “SourceOECD OECD Health Data 2006.” The life expectancies data is from “SourceOECD OECD Health Data 2006” (for Canada, France, Germany, Italy, Japan, and the US), “Demographic Yearbook Historical supplement” published by the United Nations (for Canada and Italy), and National Statistics Online (for the UK).