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GRADUATE SCHOOL OF BUSINESS ADMINISTRATION

KOBE UNIVERSITY

ROKKO KOBE JAPAN

2005・21

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Discussion Paper Series

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July 11, 2005

Abstract

We investigate a mixed market in which a state-owned, welfare-maximizing public firm competes against profit-maximizing n domestic private firms and m foreign private firms. A circular city model with quantity-setting competition is employed. We find that the equilibrium location pattern depends on m. All private firms agglomerate in the unique equilibrium if m is zero or one. Two foreign firms induce differentiation between domestic and foreign private firms. More than two foreign firms yield differentiation among foreign firms. Regardless of n and m, the agglomeration of all domestic private firms appears in equilibrium. We provide several conditions in which eliminating the public firm from the market enhances social welfare.

JEL classification numbers: H42, L13, R32

Key words: spatial agglomeration, shipping model, foreign firms, herd behavior

* We are grateful to Masahiro Ashiya, Shingo Ishiguro and participants in the seminars at Institute of Statistical Research, University of Tokyo, and Japanese Economic Association Annual Meeting 2003 for their helpful comments and suggestions. We are also indebted to the Editor Richard Arnott and anonymous referees for their valuable and constructive suggestions. Needless to say, we are responsible for any remaining errors. Financial supports of the Grant-in-Aid from Zengin Foundation for Studies on Economics and Finance and from the Japanese Ministry of Education, Science and Culture, and of ISS Comparative Regionalism Project (CREP) are greatly appreciated.

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1 Introduction

Studies of mixed markets, in which state-owned welfare-maximizing public firms compete against profit-maximizing private firms, have become increasingly popular in recent years.[1] Mixed oligopolies are common in developed, developing, and former communist transitional economies.[2] In Japan, in particular, competition between private and public firms exists in many oligopolistic markets, such as those for banking services, housing loans, life insurance, broadcasting services, and overnight deliveries.[3]

In many of these mixed markets, it is often the case that private firms adopt very similar strategies, exhibiting "herd behavior" that differs from that of public firms. The herd behavior exhibited by Japanese city banks is a typical example. In this market, private banks compete domestically against strong public banks, such as the Postal Bank and the Public House Loan Corporation. Accordingly, many of these private banks rush into the international financial markets to avoid domestic competition.[4]

Most existing works on mixed oligopoly, as well as our earlier work, investigate the competition between public and domestic private firms. In real world economies, however, competitors of public firms are not limited to domestic private firms. For example, the New Zealand government set up a state-owned public bank to compete against private foreign banks. Similarly, when the government of Brazil bargained with the Swiss medical company Roche, it used a public medical institution as a potential competitor in the domestic market. Électricité de France and Gas de France also compete against foreign private firms in the EU energy markets. Recently, many foreign private financial institutions rushed into the Japanese financial markets, which are typical mixed markets, as discussed above. Airline, telecommunication, natural gas,

[1] For pioneering work on mixed oligopolies, see Merrill and Schneider (1966). See Bös (1986, 1991), Vickers and Yarrow (1988), and Nett (1993) for excellent surveys.

[2] The interest in mixed oligopolies is due to their importance to the economies of Europe, Canada, and Japan more than to that of the US. However, there are examples of mixed oligopolies in the US, such as the packaging and overnight-delivery industries.

[3] See, e.g., Ide and Hayashi (1992).

[4] Several examples of herd behavior are described in Matsushima and Matsumura (2003).

electric power, automobile, and steel industries in many developed and developing countries are also typical examples. Recently, literature on mixed oligopoly with foreign competitors has begun to appear, including Fjell and Pal (1996), Pal and White (1998), and Matsumura (2003a). All of these studies indicate that the existence of foreign competitors (even a single one) drastically changes the equilibrium outcomes.

In this paper, we also consider foreign competitors explicitly and investigate how the presence of foreign competitors affects the "herd behavior" in mixed oligopolies. We again use a location model with a circular city in which firms deliver goods (shipping model).[5] We find that the number of foreign firms substantially affects the equilibrium location patterns. If the number of foreign competitors is zero or one, the equilibrium location pattern is unique, and all private firms (both domestic and foreign) agglomerate at the side of the circle opposite the location of the public firm. In other words, a single foreign firm does not affect the equilibrium locational choices of private firms. However, if the number of foreign firms is two, multiple equilibria appear. In every equilibrium, each domestic private firm inevitably changes its location, while it is possible that two foreign private firms still locate at the side of the circle opposite the location of the public firm. If the number of foreign private firms is more than two, the agglomeration of foreign firms never appears in equilibrium. In other words, more than two foreign firms yield differentiation among foreign firms. Regardless of the number of foreign private firms and that of domestic private firms, it is possible that all domestic private firms agglomerate at one point, although the point of agglomeration depends on the number of foreign firms. These results then indicate that when the number of foreign firms is relatively small, the effects on the locational choices by domestic firms are limited. An increase in the number of foreign firms causes a change of locational choice by domestic private firms, and a further increase yields diversification among foreign private firms, while it is possible for diversification among domestic private firms to be limited (herd behavior).

[5] For discussions on mixed oligopoly with spatial competition, see Cremer, Marchand, and Thisse (1991), Matsumura and Matsushima (2003, 2004), and Nilssen and Sørgard (2002). For applications of circular-city shipping Cournot models see, for example, Matsushima (2001) and Matsumura (2003b).

In this paper, we use spatial price discrimination models with Cournot competition. Hamilton, Thisse, and Weskamp (1989) and Anderson and Neven (1991) have carried out pioneering work on location models with quantity competition.[6] In a spatial price discrimination model, we can interpret "space" as product variety and each firm's location as its most efficient sector. We can also interpret distant locations from a firm as inefficient sectors of the firm. For example, in the automobile industry, "space" represents car size, and a firm's location indicates that the firm produces small cars efficiently but produces large cars inefficiently. This interpretation is similar to those of Eaton and Schmitt (1994) and Norman and Thisse (1999). To explain flexible manufacturing systems (FMS), they use spatial price discrimination models.

Following this interpretation, our model is applicable to the analysis of mixed markets, where multi-product firms face Cournot competition. The European automobile industry is a typical example of such mixed markets. Most automobile enterprises are multi-product firms. Several automobile manufacturers are state ownership companies. Renault is a partially state-owned company, and Volkswagen is also owned by the government of Lower Saxony, which owns a 20% stake in the firm. Most economists describe the competition in the automobile industry using Cournot models. The airline industry is another typical example. In this industry, there were (and still are) state-owned airline companies, such as Air France. Airline companies are also multi-product (multi-market) firms, and there are papers treating airline companies as multi-product firms (e.g., Borenstein (1991) and Gimeno (1999)). The market structure of airline industry is reasonably consistent with a Cournot model, where firms commit to quantities and then prices adjust along the reaction curves. The Cournot assumption is common to most empirical studies on the airline industry (e.g., Reiss and Spiller (1989) and Richard (2003)).

The remainder of this paper is organized as follows. In Section 2, we present the basic model. In Section 3, we investigate the equilibrium outcomes of the model. Section 4 discusses welfare implications. Section 5 extends the basic model and investigate three issues that are

[6] Greenhut and Greenhut (1975) and Norman (1981) have already examined Cournot competition in spatial models, but they discussed the equilibrium price pattern rather than the equilibrium pattern of location. Recently, the literature on location-quantity models has become richer and more diverse. For example, Chamorro-Rivas (2000) and Pal and Sarkar (2002) consider spatial Cournot competition among multi-plant firms.

ignored in the basic analysis. Section 6 concludes the paper.

2 The model

We formulate an oligopoly model in a mixed market, in which a welfare-maximizing public firm competes against profit-maximizing domestic private firms and foreign private firms. Firm 0 is the public firm, and there exist n domestic private firms (firm 1, firm 2,..., firm n) and m foreign private firms.[7] Let $D \equiv \{1, 2, ..., n\}$ denote the set of domestic private firms and $F \equiv \{n+1, n+2, ..., n+m\}$ denote the set of foreign private firms.

We now present a two-stage location-quantity game. The basic structure of the model is from Pal (1998a). Let x_i ($i \in \{0, 1, \ldots, n+m\}$) be the locations of firm i. x_i is the point on the circle located at a distance from 0 (measured clockwise).

In the first stage, firm 0 locates at a point on the circle. Without loss of generality, we assume that firm 0 locates at $x_0 = 0$. Later, each private firm i ($i \in \{1, \ldots, n+m\}$) simultaneously chooses its location x_i. Let $q_i(x)$ denote the firm i's output offered at each point $x \in [0, 1]$. x is the point on the circle located at a distance from 0 (measured clockwise). In the second stage, each firm i ($i \in \{0, \ldots, n+m\}$) observes its competitors' locations and simultaneously chooses $q_i(x) \in [0, \infty)$ for $x \in [0, 1]$. Let $p(x)$ denote the price of the product at x and

$$q(x) \equiv \sum_{i=0}^{n+m} q_i(x)$$

denote the total quantity supplied at x. We assume that the demand function at each point x is linear and is given by:

$$p(x) = a - bq(x),$$

where a and b are positive constants. Let $d(x, x_i)$ denote the distance between x and x_i.

[7] In this paper, the government is not permitted to nationalize more than one firm. As pointed out by Merrill and Schneider (1966), the most efficient outcome is achieved by the nationalization of all firms, if nationalization does not change the costs of firms (i.e., no X-inefficiency in the public firm exists). The need for the analysis of a mixed oligopoly lies in the fact that it is impossible or undesirable, for political or economic reasons, to nationalize an entire sector. For example, without competitors, public firms may lose the incentive to improve their costs, resulting in a loss of social welfare. Thus, we do not consider the possibility of nationalizing all firms.

This signifies the shorter distance of the two possible ways to transfer the goods along the perimeter. To ship a unit of the product from its own location to a consumer at point x, each firm i ($i \in \{0, \ldots, n+m\}$) pays a transport cost $td(x, x_i)$, where t is a constant value. Firms are able to discriminate among consumers since they control transportation. Consumer arbitrage is assumed to be prohibitively costly.[8] Each of $(n + m + 1)$ firms has identical technology and constant marginal cost of production, which is normalized to zero. These assumptions are standard and also made in many other location–quantity models.

3 Equilibrium

In this section, we discuss the equilibrium in the model formulated above. We use subgame perfection as the equilibrium concept. The game is solved by backward induction. First, we discuss the equilibrium outcomes in the second-stage subgames given the location of each firm.

3.1 Quantity choice

We follow the Cournot assumption that firms compete in quantities at each point in the market. Since marginal production costs are constant, quantities set at different points by the same firm are strategically independent. Cournot equilibria can be characterized by a set of independent Cournot equilibria, one for each point x. Let $\pi_i(x)$ denote firm i's ($i \in \{1, \ldots, n + m\}$) profit at x, given the locations of all firms;

$$\pi_i(x) = (a - bq(x) - t(d(x, x_i)))\, q_i(x). \tag{1}$$

Let $w(x)$ denote the domestic social surplus (consumer surplus plus profits of all domestic firms) at x.

$$w(x) = \int_0^{q(x)} (a - bm)dm - q(x)(a - bq(x)) + \sum_{i=0}^{n} (a - bq(x) - td(x, x_i))q_i(x). \tag{2}$$

[8] This assumption is not essential. Unless transportation costs for consumers are strictly smaller than those of firms, consumer arbitrage plays no role in our model. For this discussion, see Hamilton, Thisse, and Weskamp (1989).

The first-order condition of firm 0 and firm i ($i \in \{1, \ldots, n+m\}$) is given, respectively, by

$$a - td(x, x_0) - bq_0(x) - b\sum_{i=1}^{n} q_i(x) = 0, \tag{3}$$

$$a - td(x, x_i) - bq_i(x) - b\sum_{i=0}^{n+m} q_i(x) = 0. \tag{4}$$

In this paper, we assume that the whole market will always be served by firm 0. This assumption is satisfied if $2a \geq t(n+1)$.[9]

We first show that the output level of all domestic firms (both public and private) does not depend on the locations of domestic private firms.

Lemma 1 *(i) $q_0 + \sum_{i \in D} q_i$ does not depend on $x_i \in D$. (ii) For any $i \in D \cup F$, $q_i(x) = 0$ if $d(x, x_0) \leq d(x, x_i)$.*

Proof: See Appendix.

We explain Lemma 1(i) intuitively. Let $R_0(q_1, q_2, ..., q_{n+m+1})$ denote the reaction function of firm 0 in the second-stage game. From (3), we have $\partial R_0/\partial q_i = -1$ for all $i \in D$. In other words, one unit reduction of firm i's output ($i \in D$) increases the best output of firm 0 by one unit. x_i affects the marginal cost of firm i for each market x and may therefore affect the equilibrium $q_i(x)$. However, this effect is offset by the behavior of firm 0, so the output level of all domestic firms does not depend on x_i ($i \in D$). Since the total output level of all firms other than firm j ($j \in F$) does not depend on x_i ($i \in D$), x_i never affects the output of each foreign firm.

On the other hand, the locations of foreign firms affect the total output, so they also affect the profits of all firms. Let $\tilde{F}(x)$ and $\tilde{m}(x)$ denote the set of foreign firms supplying at market x and the number of such firms, respectively, The total quantity supplied and the price are:

$$q(x) \quad = \quad \frac{(\tilde{m}(x)+1)a - td(x, x_0) - \sum_{j \in \tilde{F}(x)} td(x, x_j)}{(\tilde{m}(x)+1)b}, \tag{5}$$

[9] A similar assumption (sufficiently large a) is also made in many studies of mixed oligopoly and quantity-setting spatial models. See, among others, Anderson and Neven (1991) and Pal (1998a).

$$p(x) = \frac{td(x,x_0) + \sum_{j\in\tilde{F}(x)} td(x,x_j)}{(\tilde{m}(x)+1)}. \tag{6}$$

The output quantity and profit of firm i supplying for market x are given by

$$q_i(x) = \frac{td(x,x_0) + \sum_{j\in\tilde{F}(x)} td(x,x_j) - (\tilde{m}(x)+1)td(x,x_i)}{(\tilde{m}(x)+1)b}, \tag{7}$$

$$\pi_i(x) = \frac{\left[td(x,x_0) + \sum_{j\in\tilde{F}(x)} td(x,x_j) - (\tilde{m}(x)+1)td(x,x_i)\right]^2}{(\tilde{m}(x)+1)^2 b}. \tag{8}$$

If firm i does not supply for market x (i.e., (7) is non-positive), its profit from market x is zero. From (8), we obtain the following Lemma.

Lemma 2 *The profits of firm $i \in D \cup F$ do not depend on x_j if $j \neq i$ and $j \in D$.*

A foreign firm j supplies for market x only if $d(x,x_0) \geq d(x,x_j)$. $p(x)$ in (6) is smaller than or equal to $td(x,x_0)$ (the price in which foreign firms do not enter). Therefore, we have the following Lemma.

Lemma 3 *Suppose that foreign firms supply at x. The price at x, $p(x)$, is smaller than that in which foreign firms do not enter. In other word, consumer surplus at x is higher than that in which foreign firms do not enter.*

3.2 Location choice

In this subsection, the equilibrium locations are discussed. First, we present a result describing the equilibrium location without foreign private firms as a benchmark.

Result (Matsushima and Matsumura (2003)) *If $m = 0$, in the unique equilibrium, all private firms agglomerate at 1/2 (the side of the circle opposite the location of the public firm).*

The intuition behind this result is as follows. If $m = 0$, the price at market x is $td(x,x_0)$ (see (6)), that is, the price at each local market is equal to the unit transportation cost of the public firm. The further away the public firm is from a market, the higher the public firm's transport cost at the market will be. Each private firm faces tough competition from the public firm in the market nearer the location of the public firm. For each private firm, markets near the location of the public firm are thin, and those far away from that location are thick. To minimize the transportation costs at the largest market for private firms, each firm prefers the location that is farthest from the public firm. This produces an agglomeration of private firms.

We now discuss the equilibrium location patterns with foreign private firms. Lemma 2 states that the location of each domestic private firm does not affect the profits of other firms at all. This implies that none of the firms needs to worry about where the domestic private firms locate. Thus, each domestic private firm can choose its location without considering its strategic effect. Under these conditions, if one point is the best location for a domestic private firm, then it is also the best location for all other domestic firms. Thus, the agglomeration of all domestic private firms can always appear in equilibrium.

Proposition 1 *At least one equilibrium exists in which all domestic private firms agglomerate at one point.*

Proof: See Appendix.

However, this property does not hold true for foreign firms. The location of one foreign firm does affect the output choice of all other foreign firms; thus, the location choices made by a foreign firm have a strategic effect. Owing to this strategic interaction, the optimal location of one foreign firm depends on the locations of other foreign firms.

Proposition 2 *Suppose that $m = 1$. In the unique equilibrium $x_i = 1/2$ for all $i \in D \cup F$.*

Proof: See Appendix.

Proposition 2 indicates that agglomeration of all private firms still appears even after one foreign firm enters the market. This result, however, does not hold true if there are two or more foreign firms. Proposition 3 indicates that two foreign firms yield a differentiation between foreign and domestic private firms, although it is possible that two foreign firms still locate at the side of the circle opposite the location of the public firm. Proposition 4 states that more than two foreign firms yield differentiation among foreign firms.

Proposition 3 *Suppose that* $m = 2$. *(i) The following location choices constitute an equilibrium: each domestic private firm* i *chooses either* $x_i = \frac{5-\sqrt{3}}{11} \sim 0.297$ *or* $x_i = \frac{6+\sqrt{3}}{11} \sim 0.703$*, and each foreign firm* j *chooses* $x_j = 1/2$. *(ii) The following location choices also constitute an equilibrium: each domestic private firm* i *chooses either* $x_i = \frac{18-\sqrt{66}}{30} \sim 0.329$ *or* $x_i = \frac{12+\sqrt{66}}{30} \sim 0.671$*, one foreign firm* j *chooses* $x_j = 13/30$ *and the other foreign firm* k *chooses* $x_k = 17/30$.

Proof: See Appendix.

Proposition 4 *Suppose that* $m > 2$. *The agglomeration of all foreign firms never appears in equilibrium.*

Proof: See Appendix.

We explain the intuition behind Propositions 2–4. Suppose that all private firms locate at the point 1/2. We consider whether or not each private firm has an incentive for deviating this location strategy, given the other firms' location.

From (8), we can assume that the markets near 0 (the location of the public firm) are not profitable, since $td(x, x_0)$ is small. In other words, the markets near 0 are not profitable for each private firm; thus, there is a strong incentive to avoid this severe competition against the public firm. As a result, a private firm chooses the furthest location from the public firm. This mechanism is in common with Matsushima and Matsumura (2003). This is called the "public firm effect." At the same time, the markets near 1/2 (the location of the foreign firms) are not profitable, since $\tilde{m} = m$ for the markets near point 1/2 (we have defined $\tilde{m}$ as the number of supplying foreign firms). Thus, each firm has an incentive to be far away from point 1/2. This is called the "foreign firm effect." An increase in m accelerates the competition and reduces the prices at the markets near 1/2. Therefore, the foreign firm effect depends on m.

Then, given the locations of all other firms, one firm (firm k) deviates and slightly reduces x_k from 1/2. This reduces the distance from the location of the public firm and increases the distance from the locations of foreign firms. If the foreign firm effect dominates the public firm effect, it increases the profits of firm k. The foreign firm effect is increasing in the number of foreign firms other than firm k; thus, there naturally exists a threshold value dominating the public firm effect. In our model, this threshold value is two. If the number of other foreign firms is two or more, firm k has the above-mentioned deviation incentive.

The number of foreign firms other than itself locating at $1/2$ is m for each domestic private firm and $m-1$ for each foreign firm. Suppose that $m = 1$. The number of foreign firms other than itself locating at $1/2$ is 0 or 1 for all private firms and the public firm effect dominates the foreign firm effect. This is the reason why all private firms agglomerate at $1/2$ when $m = 1$ (Proposition 2). Suppose that $m = 2$. The number of foreign firms other than itself locating at $1/2$ is 1 for foreign firms and 2 for domestic firms. Thus, the public firm effect dominates the foreign firm effect for each foreign firm, but the foreign firm effect dominates the public firm effect for each domestic private firm. This is the reason why domestic private firms do not chooses the location $1/2$, while two foreign firms choose it (Proposition 3(i)). Suppose that $m \geq 3$. The number of foreign firms other than itself locating at $1/2$ is 2 or more for all private firms, and the foreign firm effect dominates the public firm effect. Thus, there is no equilibrium where m foreign firms locate at $1/2$ (Proposition 4).

We now mention the difference between the location patterns of Proposition 3(i) and 3(ii). Consider the two-foreign-firm case. We denote one foreign firm as "firm a" and the other as "firm b." Proposition 3(i) states that firm a's optimal location is $1/2$ when $x_b = 1/2$, and Proposition 3(ii) states that it is $13/30$ when $x_b = 17/30$. This implies that an increase in the distance between firm b's location and the public firm's enlarges firm a's optimal distance from the public firm's location. We explain the reason behind this strategic complementarity. Suppose that $x_b = 17/30$. Firm a chooses its location which balances the public firm and the foreign firm effects, and it is $13/30$. Suppose that firm b moves from $17/30$ to $1/2$. The move enhances the public firm effect at market $13/30$ because the move reduces firm b's cost for market $13/30$ and induces the larger output of the public firm. Since the move strengthens the public firm effect, firm a has a higher incentive for moving away from the public firm. As a result, firm a's optimal location becomes 1/2. In short, a longer distance between firm b and the public firm yields the longer optimal distance between firm a and the public firm. This strategic complementarity yields multiple equilibria.

Proposition 4 presents a property of equilibrium location but does not fully describe the equilibrium location pattern when $m \geq 3$. Since foreign firms never agglomerate at 1/2 in equilibrium, the asymmetries between foreign firms inevitably arise. For example, the distance between each foreign firm and the public firm never becomes the same across all foreign firms. Thus, as opposed to the case without foreign firms, it is impossible to solve the m-foreign firm case systematically. Although we can solve each of the problems in the case where $m = 3, m = 4, m = 5, ..$, in the interests of brevity we only present the results of a 3-foreign-firm case.[10]

Proposition 5 *Suppose that $m = 3$. The following location choices constitute an equilibrium: each domestic private firm i chooses either $x_i = \frac{9216\sqrt{2}-10391}{5750} \sim 0.460$ or $x_i = \frac{16141-9216\sqrt{2}}{5750} \sim 0.540$, and the foreign firms choose $x_a = \frac{3-\sqrt{2}}{4} \sim 0.396$, $x_b = 1/2$, and $x_c = \frac{1+\sqrt{2}}{4} \sim 0.604$, respectively.*

4 Welfare implication

Given that n domestic private firms exist, we compare domestic welfare among four cases: (1) no foreign firm exists (SW_0); (2) one foreign firm exists (SW_1); (3) two foreign firms exist, and they locate at $x = 1/2$ (SW_{2a}); and (4) two foreign firms exist, and each of them locates at $x = 13/30$ and $x = 17/30$ (SW_{2b}).

4.1 No foreign firm

We first consider the case in which no foreign firm exists. In this case, the public firm's profit is zero. Social welfare (SW_0) is the consumers' surplus (CS_0) plus the sum of each private firm's

[10] The proof is available from the authors on request.

profit (Π_0).

$$\begin{aligned} CS_0 &= 2\int_0^{\frac{1}{2}} \frac{b}{2}\left(\frac{a-p(x)}{b}\right)^2 dx = 2\int_0^{\frac{1}{2}} \frac{b}{2}\left(\frac{a-tx}{b}\right)^2 dx = \frac{12a^2-6at+t^2}{24b}, \\ \Pi_0 &= 2n\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{(tx-t(1/2-x))^2}{b}dx = \frac{nt^2}{24b}, \\ SW_0 &= \frac{12a^2-6at+(n+1)t^2}{24b}. \end{aligned}$$

4.2 One foreign firm

We next consider the case in which one foreign firm exists. In this instance, the profit of the public firm is negative. Social welfare (SW_1) is the consumers' surplus (CS_1) plus the profit of the public firm (π_{01}) plus the sum of each private firm's profit (Π_1).

In this case, the foreign firm locates at $x = 1/2$ (see Proposition 2). From the proof of Proposition 2, we find that the foreign firm supplies at $x \in [1/4, 3/4]$ (see the last paragraph before equation (21)).

First, we calculate CS_1. The consumers' surplus at x is $bq(x)^2/2$. From (5) and (6), we find that this is equal to $(a - p(x))^2/2b$. From (6), the price at x is (by symmetry, we only consider the range $[0, 1/2]$):

$$p(x) = \begin{cases} tx & \text{if } x \in [0, 1/4], \\ t/4 & \text{if } x \in [1/4, 1/2]. \end{cases} \tag{9}$$

$$CS_1 = 2\int_0^{\frac{1}{2}} \frac{(a-p(x))^2}{2b}dx = \frac{24a^2-9at+t^2}{48b}.$$

Next, we calculate π_{01}. From (3), we obtain

$$q_0(x) = \frac{a-td(x,x_0)}{b} - \sum_{i\in D} q_i(x). \tag{10}$$

From (7) and proof of Proposition 2 (see the first paragraph after (26)), we have

$$q_i(x) = \begin{cases} 0 & \text{if } x \in [0, 1/4], \\ \frac{t(2x-1/2)}{2b} & \text{if } x \in [1/4, 1/2]. \end{cases} \tag{11}$$

$$\pi_{01} = 2\int_0^{\frac{1}{2}} (p(x) - td(x,x_0))q_0(x)dx = -\frac{t(12a-(5+2n)t)}{192b}.$$

Finally, we calculate Π_1. Π_1 is n times of each domestic private firm's profit. From (27), we obtain the profit of each domestic firm and the total profit:

$$\Pi_1 = \frac{n(16(1/2)^3 - 24(1/2)^2 + 12(1/2) - 1)t^2}{96b} = \frac{nt^2}{96b}.$$

We have social welfare:

$$SW_1 = CS_1 + \pi_{01} + \Pi_1 = \frac{96a^2 - 48at + (9+4n)t^2}{192b}.$$

4.3 Two foreign firms

Third, we consider the case in which one foreign firm exists. In this case, the profit of the public firm is negative. Social welfare (SW_2) is the consumers' surplus (CS_2) plus the profit of the public firm (π_{02}) plus the sum of each private firm's profit (Π_2).

In this instance, two equilibrium outcomes exist: (a) foreign firms locate at $x = 1/2$, and each domestic firm locates at $z_a \equiv (5-\sqrt{3})/11$ (or $(6+\sqrt{3})/11$); (b) foreign firms locate at $x = 13/30$ and $x = 17/30$ respectively, and each domestic firm locates at $z_b \equiv (18-\sqrt{66})/30$ (or $(12+\sqrt{66})/30$).

Case (a): From the proof of Proposition 3(i), we find that both foreign firms supply at $x \in [1/2, 3/4]$ (see the last paragraph before Equation (34)).

First we calculate CS_{2a}. As mentioned earlier, consumers' surplus at x is $(a-p(x))^2/2b$. From (6), we have

$$p(x) = \begin{cases} tx & \text{if } x \in [0, 1/4], \\ t(1-x)/3 & \text{if } x \in (1/4, 1/2]. \end{cases} \tag{12}$$

$$CS_{2a} = 2\int_0^{\frac{1}{2}} \frac{(a-p(x))^2}{2b} dx = \frac{216a^2 - 72at + 7t^2}{432b}.$$

Next, we calculate π_{02a}. From (7) and the proof of Proposition 3(i) (see the last paragraph before Equation (39)), we have

$$q_i(x) = \begin{cases} 0 & \text{if } x \in [0, z_a/2], \\ \frac{t(2x-z_a)}{b} & \text{if } x \in [z_a/2, 1/4], \\ \frac{t(1+2x-3z_a)}{3b} & \text{if } x \in [1/4, z_a], \\ \frac{t(1-4x+3z_a)}{3b} & \text{if } x \in [z_a, (1+3z_a)/4], \\ 0 & \text{if } x \in [(1+3z_a)/4, 1/2] \cup [1/2, 1], \end{cases}$$

$$\pi_{02a} = \int_0^1 (p(x) - td(x, x_0))q_0(x)dx = -\frac{t(15972a - (6655 + (1035 - 240\sqrt{3})n)t)}{191664b}.$$

Finally, we calculate Π_{2a}. Π_{2a} is n times each domestic private firm's profit. From (39), we obtain the profit of each domestic firm and the total profit:

$$\Pi_{2a} = \frac{n(22z_a^3 - 30z_a^2 + 12z_a - 1)t^2}{72b} = \frac{n(13 + 4\sqrt{3})t^2}{2904b}.$$

$$SW_{2a} = CS_{2a} + \pi_{02a} + \Pi_{2a} = \frac{287496a^2 - 143748at + (29282 + (5679 + 72\sqrt{3})n)t^2}{574992b}.$$

Case (b): From the proof of Proposition 3(ii), we find that the foreign firm located at $x = 13/30$ supplies at $x \in [13/60, 13/20]$ and that located at $x = 17/30$ supplies at $x \in [7/20, 47/60]$ (see item "(4) $x_2 \in (17/45, 1/2]$" in the proof of Proposition 3(ii)).

First, we calculate CS_{2b}. As mentioned earlier, consumers' surplus at x is $(a - p(x))^2/2b$. From (6), we have

$$p(x) = \begin{cases} tx & \text{if } x \in [0, 13/60], \\ 13t/60 & \text{if } x \in [13/60, 7/20], \\ t(1-x)/3 & \text{if } x \in [7/20, 13/30], \\ t(15x+2)/45 & \text{if } x \in [13/30, 1/2]. \end{cases} \tag{13}$$

$$CS_{2b} = 2\int_0^{\frac{1}{2}} \frac{(a - p(x))^2}{2b} dx = \frac{48600a^2 - 16056at + 1531t^2}{97200b}.$$

Next, we calculate π_{02b}. From (7) and the proof of Proposition 3(ii) (see item "(5) $x_1 \in (13/45, 7/20]$" in the proof of Proposition 3(ii)), we have

$$q_i(x) = \begin{cases} 0 & \text{if } x \in [0, z_b/2], \\ \frac{t(2x - z_b)}{b} & \text{if } x \in [z_b/2, 13/60], \\ \frac{t(13/30 + 2x - 2z_b)}{2b} & \text{if } x \in [13/60, z_b], \\ \frac{t(13/30 - 2x + 2z_b)}{2b} & \text{if } x \in [z_b, 7/20], \\ \frac{t(1 - 4x + 3z_b)}{3b} & \text{if } x \in [7/20, 13/30], \\ \frac{t(2/15 - 2x + 3z_b)}{3b} & \text{if } x \in [13/30, 1/2], \\ \frac{t(17/15 - 4x + 3z_b)}{3b} & \text{if } x \in [1/2, (17 + 45z_b)/60], \\ 0 & \text{if } x \in [(17 + 45z_b)/60, 1], \end{cases}$$

$$\pi_{02b} = \int_0^1 (p(x) - td(x, x_0))q_0(x)dx = -\frac{t(494640a - (201300 - (103655 - 16389\sqrt{66})n)t)}{5832000b}.$$

Finally, we calculate Π_{2b}. Π_{2b} is n times each domestic private firm's profit. From (60), we obtain the profit of each domestic firm and the total profit:

$$\Pi_{2b} = \frac{n(243000z_b^3 - 437400z_b^2 + 208980z_b - 12269)t^2}{2916000b} = \frac{n(8143 + 1188\sqrt{66})t^2}{2916000b}.$$

We summarize them:

$$SW_{2b} = CS_{2b} + \pi_{02b} + \Pi_{2b} = \frac{972000a^2 - 486000at + (97720 - (29123 - 6255\sqrt{66})n)t^2}{1944000b}.$$

4.4 Comparison

Given that n domestic private firms exist, we compare four cases: (1) no foreign firm exists (SW_0), (2) one foreign firm exists (SW_1), (3) two foreign firms exist, and they locate at $x = 1/2$ (SW_{2a}), (4) two foreign firms exist, and they locate at $x = 13/30$ and $x = 17/30$ (SW_{2a}).

First, we compare SW_{2b} with SW_{2a}. We obtain

$$SW_{2b} - SW_{2a} = \frac{(-1703680 + (8325405\sqrt{66} - 324000\sqrt{3} - 64318213)n)t^2}{2587464000b},$$

and it is positive for any $n \geq 1$. This implies the following proposition.

Proposition 6 *(i) $SW_{2b} < SW_{2a}$ if $n = 0$, and (ii) $SW_{2b} > SW_{2a}$ for any $n \geq 1$.*

The differentiation between two foreign firms yields larger welfare than the agglomeration of them if domestic firms exist. On the other hand, if no domestic private firm exists, agglomeration of foreign firms is better for domestic welfare.

Suppose that there is no domestic private firm. The cost of the public firm, which is the sole domestic supplier, is highest at the market 1/2. Thus, the increase of the supply for the market 1/2 improves welfare most efficiently. Suppose that a foreign domestic firm a locates at point $y < 1/2$ and the other foreign firm b locates at point $1 - y$. Suppose that both firms relocate, one at point $y'(y < y' \leq 1/2)$ and the other at point $1 - y'$. We can show that the total output for market x ($y' \leq x \leq 1 - y'$) increases, while that for the other market remains unchanged as long as both firms provide a supply. Since the relocation increases the total output and, thus, consumer surplus for market 1/2, it improves domestic welfare. Thus, the agglomeration of foreign firms improves domestic welfare.

Suppose that domestic private firms exist. As mentioned above, the agglomeration of foreign firms yields a higher total output of foreign firms, resulting in the profit transfer from domestic firms to foreign firms. It reduces the profits of domestic private firms and total social domestic surplus.

Next, we compare SW_0, SW_1, and SW_{2b}. We obtain

$$SW_0 - SW_1 = \frac{(-1+4n)t^2}{192b}, \quad SW_0 - SW_{2b} = \frac{(-16720 + (110123 - 6255\sqrt{66})n)t^2}{1944000b}.$$

These two differences are positive for any $n \geq 1$. We obtain

$$SW_1 - SW_{2b} = \frac{(-6595 + (69623 - 6255\sqrt{66})n)t^2}{1944000b},$$

which is positive for any $n \geq 2$. These equations imply the following proposition.

Proposition 7 *(i) $SW_0 < SW_1 < SW_{2b}$ for $n = 0$, (ii) $SW_0 \geq \max\{SW_1, SW_{2b}\}$ for any $n \geq 1$, and (iii) $SW_1 > SW_{2b}$ for any $n \geq 2$.*

Proposition 7 states that an increase in the number of foreign firms improves welfare if no domestic firm exists. Conversely, if domestic private firms exist, eliminating foreign firms improves welfare. Eliminating foreign firms increases the profits of domestic firms when domestic private firms exist; thus, it improves domestic welfare. No such effect exists in the case without domestic private firms, so eliminating foreign firms does not improve welfare.[11]

To check the efficiency of the locations, we consider the following solution. Suppose that the social planner cannot control the output of each private firm but can control the locations of domestic private firms. The following lemma shows that when the number of foreign firms is zero or one, the location equilibrium is efficient from the viewpoint of social welfare.[12]

[11]For a discussion on private duopoly, see Ono (1990).
[12]The proof is available upon a request. The mathematica file is available.

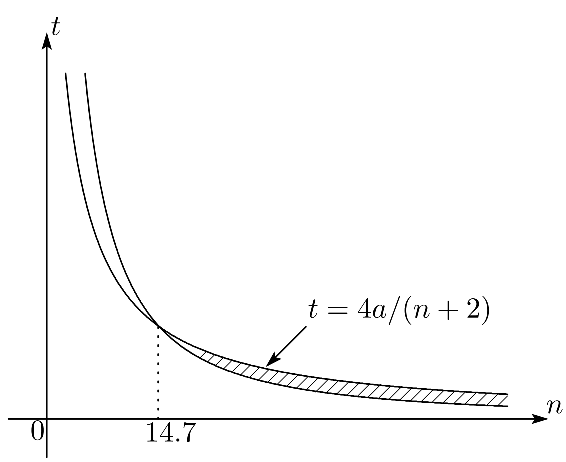
Lemma 4 *Suppose that the social planner cannot control the output of each private firm but can control the locations of the domestic private firms. In that case, the social planner chooses the locations* $x_1 = x_2 = \ldots = x_n = 1/2$.

The result is similar to that of Matsushima and Matsumura (2003). In our model, the public firm is inferior at the market near 1/2 and superior at the market near 0. Thus, additional production by a private firm greatly improves social welfare at the market near 1/2 but not at the market near 0. If each private firm locates at 1/2, the additional output is supplied most intensively at the market where the additional supply has the most value.

As shown in Proposition 3, when there are two foreign firms, domestic private firms do not agglomerate at 1/2. The location pattern in which two foreign firms exist is inefficient. We can show that eliminating the public firm may enhance social welfare when there are two foreign firms.

Proposition 8 *(i) Suppose that there are one public firms, one or zero foreign firm, and* n *domestic private firms. Eliminating the public firm reduces social welfare. (ii) Suppose that there are one public firm, two foreign firms, and* n *domestic private firms. Eliminating the public firm enhances social welfare, if* t *satisfies the following inequalities:*

$$\frac{900\left(1350-(n+3)\sqrt{6((69623-6255\sqrt{66})n-36970)}\right)a}{303750-(n+3)^2((69623-6255\sqrt{66})n-36970))} < t < \frac{4a}{n+2}. \tag{14}$$



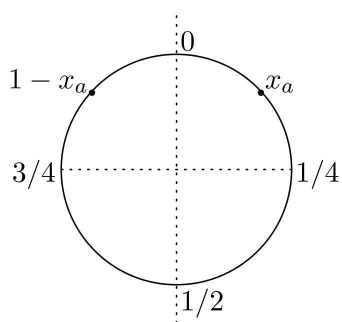
When there is only one or zero foreign firms, the locations of domestic private firms are efficient from the viewpoint of social welfare. On the contrary, when there are two foreign firms, the locations of domestic private firms are inefficient from the viewpoint of social welfare. As the transport cost per length increases, the welfare loss induced by the inefficient locations increases. As the number of domestic private firms increases, the significance of the welfare loss increases. Therefore, when the above inequalities are satisfied, the public firm harms social welfare.

5 Extensions

In this section, we investigate three problems that have been disregarded in the previous sections.

5.1 Multiple public firms

In the previous sections, we assume that the number of public firms is one. In this section, we investigate a model with two public firms. We consider the following three stage games: first, one public firm (firm 0) chooses its location; second, the other public firm (firm 00) chooses its location after observing firm 0's location; third, observing the locations of the public firms, each private firm chooses its location simultaneously; fourth, the firms set the quantities supplied at each point $x \in [0, 1]$. Without loss of generality, we assume that the public firms locate at x_a and $1 - x_a$, respectively ($x_a \in [0, 1/4]$). In other words, firm 00 chooses the distance between two public firms, $2x_a$.



5.1.1 No foreign firm

First, we consider the second stage location choices of domestic private firms, given the locations of the public firms. For the same reason discussed in the previous sections, a private firm's profit is not affected by the locations of other domestic private firms, and each domestic private firm's location choice is strategically independent. Thus, it is sufficient to consider the location of a domestic private firm.

When a domestic private firm locates at $x_i \in [0, x_a]$, from (8), its profit is:

$$\begin{aligned}\pi &= \int_0^{x_i} \frac{(t(x_a - m) - t(x_i - m))^2}{b} dm + \int_{x_i}^{\frac{x_i + x_a}{2}} \frac{(t(x_a - m) - t(m - x_i))^2}{b} dm \\ &+ \int_{\frac{1+x_i+1-x_a}{2}}^{1} \frac{(t(m-(1-x_a)) - t(1+x_i - m))^2}{b} dm \;=\; \frac{t^2(x_a - x_i)^2(x_a + 2x_i)}{3b}.\end{aligned}$$

When $x_i = 0$, π is maximized and then the profit is $x_a^3 t^2/3b$.

When a domestic private firm locates at $x_i \in [x_a, 1/2]$, from (8), the profit of it is:

$$\begin{aligned}\pi &= \int_{\frac{x_i + x_a}{2}}^{x_i} \frac{(t(m - x_a) - t(x_i - m))^2}{b} dm + \int_{x_i}^{\frac{1}{2}} \frac{(t(m - x_a) - t(m - x_i))^2}{b} dm \\ &+ \int_{\frac{1}{2}}^{\frac{1-x_a+x_i}{2}} \frac{(t(1 - x_a - m) - t(m - x_i))^2}{b} dm \;=\; \frac{t^2(x_a - x_i)^2(3 - 2x_a - 4x_i)}{6b}.\end{aligned}$$

When $x_i = 1/2$, π is maximized and then the profit is $(1/2 - x_a)^3 t^2/3b$. This profit is larger than that in which the private firm locates at $x_i = 0$.

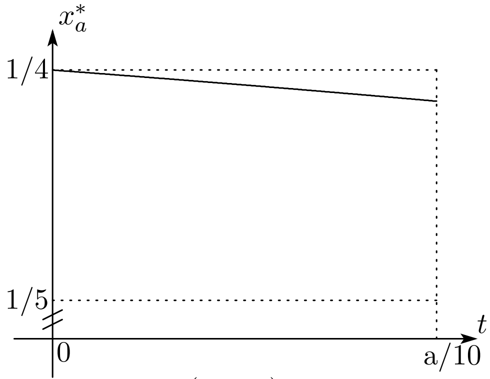
Next, we consider the location choices of the public firms in the first and second stages. Taking subsequent locations of private firms into account, social welfare (consumers' surplus plus the sum of the firms' profits) is given by

$$\begin{aligned}SW &= 2\left(\int_0^{x_a} \frac{(a - t(x_a - m))^2}{2b} dm + \int_{x_a}^{\frac{1}{2}} \frac{(a - t(m - x_a))^2}{2b} dm\right) + n \times \frac{(1/2 - x_a)^3 t^2}{3b} \\ &= \frac{12a^2 - 6(8x_a^2 - 4x_a + 1)at + (12x_a^2 - 6x_a + 1 + n(1 - 2x_a)^3)t^2}{24b}.\end{aligned}$$

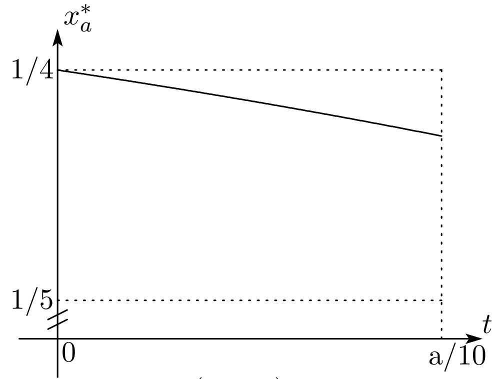
This is maximized:

$$x_a^* = \frac{-(4a - (n+1)t) + \sqrt{(4a - t)(4a - (n+1)t)}}{2nt}. \tag{15}$$

From the following figures, we can see that $x_a^* \leq 1/4$, and strict inequality holds if $n > 0$.



$(n = 4)$



$(n = 8)$

We have the following proposition:[13]

Proposition 9 *Suppose that there are two public firms and that there is no foreign firm. In equilibrium the public firms locate at the point x_a^* and at the point $1 - x_a^*$ respectively, where x_a^* is (15). In the equilibrium all domestic private firm locate at the point $x = 1/2$.*

Substituting x_a^* into SW, we have:

$$\begin{aligned} SW &= \frac{2(4a-t)(4a-(n+1)t)\sqrt{(4a-t)(4a-(n+1)t)}}{24n^2bt} \\ &\quad - \frac{128a^3 - 12(8+4n+n^2)a^2t + 6(n+2)^2at^2 - (n+1)(n+2)t^3}{24n^2bt}. \end{aligned}$$

If $n = 0$, the public firms choose the maximal distance between them because it minimizes the total transport costs. We explain why the public firms choose the non-maximal distance when $n > 0$. By increasing the distance between the public firms, they deprive market shares from private firms locating at $1/2$. Since the private firms' transport costs are lower than those of the public firms for the markets served by both the private and the public firms, the above production substitution from the private to the public firms reduces welfare. Although the non-maximal distance is inefficient from the viewpoint of the minimization concerning the public firms' transport costs, the non-maximal distance is selected to avoid the welfare-reducing production substitution. The production substitution effect becomes stronger when n is larger. This is the reason that the distance between the public firms decreases in n.

[13] Taking subsequent locations of private firms into account, social welfare depends only on the distance between two public firms. Thus, optimal location choice $(x_0, x_{00}) = (x_a^* 1 - x_a^*)$ is achieved in equilibrium.

5.1.2 One foreign firm

First, we discuss the location of the foreign firm. From (8), we conclude that the profit of the foreign firm is the quarter of a domestic private firm's profit function in the former subsection. Therefore, the location of the foreign firm is identical to that of the domestic firm discussed in the former subsection. That is, the foreign firm locates at $x = 1/2$.

Second, we discuss the locations of domestic private firms. As discussed in the former subsection, on $[0, x_a]$, the optimal location of a domestic private firm is $x = 0$ and then the profit is $x_a^3 t^2/3b$. On $[x_a, 1-x_a]$, the foreign firm supplies at $x \in [(1/2+x_a)/2, (1-x_a+1/2)/2]$ (see, $q_i(x)$ in (7)). Given the location of a domestic private firm $x_i (\in [x_a, 1/2])$, we now present the range in which the domestic private firm supplies a positive amount of goods. From (7), at $m \in [1/2, 1-x_a]$, if the following inequality is satisfied, the quantity supplied by the domestic private firm is positive:

$$\frac{t(1-x_a-m)+t(m-1/2)-2t(m-x)}{2b} > 0 \ \Leftrightarrow \ m < \frac{1-2x_a+4x}{4} \equiv H(x_a, x) \ \left(< \frac{1-x_a+1/2}{2} \right).$$

$H(x_a, x)$ is larger than $1/2$, if and only if $x > (1+2x_a)/4$. We have to consider two cases: (i) $x \in [x_a, (1+2x_a)/4]$, (ii) $x \in [(1+2x_a)/4, 1/2]$.

(i) $x \in [x_a, (1+2x_a)/4]$: On $[(x_a+x)/2, x]$, the foreign firm does not supply; on $[x, (1/2+x_a)/2]$, the foreign firm does not supply; on $[(1/2+x_a)/2, (1-2x_a+4x)/4]$, the foreign firm supplies. From (8), the profit of the domestic firm is presented by

$$\int_{\frac{x+x_a}{2}}^{x} \frac{(t(m-x_a)-t(x-m))^2}{b} dm + \int_{x}^{\frac{1/2+x_a}{2}} \frac{(t(m-x_a)-t(m-x))^2}{b} dm$$
$$+ \int_{\frac{1/2+x_a}{2}}^{\frac{1-2x_a+4x}{4}} \frac{(t(m-x_a)+t(1/2-m)-2t(m-x))^2}{4b} dm \ = \ \frac{t^2(x_a-x)^2(1-2x)}{4b}.$$

This is maximized at $x = (1+x_a)/3 (> (1+2x_a)/4)$. That is, the optimal location is the boundary, $x = (1+2x_a)/4$.

(ii) $x \in [(1+2x_a)/4, 1/2]$: On $[(x_a+x)/2, (1/2+x_a)/2]$, the foreign firm does not supply; on $[(1/2+x_a)/2, x]$, the foreign firm supplies; on $[x, (1-2x_a+4x)/4]$, the foreign firm supplies.

From (8), the profit of the domestic firm is presented by

$$\begin{aligned}
&\int_{\frac{x+x_a}{2}}^{\frac{1/2+x_a}{2}} \frac{(t(m-x_a)-t(x-m))^2}{b}dm + \int_{\frac{1/2+x_a}{2}}^{x} \frac{(t(m-x_a)+t(1/2-x_a)-2t(x-m))^2}{4b}dm \\
&+ \int_{x}^{\frac{1}{2}} \frac{(t(m-x_a)+t(1/2-m)-2t(m-x))^2}{4b}dm \\
&+ \int_{\frac{1}{2}}^{\frac{1-2x_a+4x}{4}} \frac{(t(1-x_a-m)+t(m-1/2)-2t(m-x))^2}{4b}dm \\
= \;& \frac{t^2((1-2x_a)^3-2(1-2x)^3)}{96b}.
\end{aligned}$$

This is maximized at $x = 1/2$ and then the profit is $(1-2x_a)^3t^2/96b$. We find that on $[x_a, 1/2]$, the optimal location of the domestic private firm is $x = 1/2$.

We have shown that either $x = 0$ and $x = 1/2$ is the best for domestic firms. We then compare the two locations; $x = 0$ and $x = 1/2$. The difference between the profits in which the private firm locates at $x = 0$ and $x = 1/2$ is:

$$\frac{(1-2x_a)^3t^2}{96b} - \frac{x_a^3t^2}{3b} = \frac{(1-6x_a+12x_a^2-40x_a^3)t^2}{96b}.$$

This is positive if and only if $x_a < \bar{x}_a \simeq 0.193$, where $\bar{x}_a$ satisfies $1-6\bar{x}_a+12\bar{x}_a^2-40\bar{x}_a^3 = 0$.

Third, we derive the optimal locations of the public firms. Consumers surplus is given by

$$\begin{aligned}
CS &= 2\left(\frac{b}{2}\int_0^{x_a}\left(\frac{a-t(x_a-m)}{b}\right)^2 dm + \frac{b}{2}\int_{x_a}^{\frac{1/2+x_a}{2}}\left(\frac{a-t(m-x_a)}{b}\right)^2 dm \right.\\
&\quad \left. +\frac{b}{2}\int_{\frac{1/2+x_a}{2}}^{\frac{1}{2}}\left(\frac{2a-t(m-x_a)-t(1/2-m)}{2b}\right)^2 dm\right) \\
&= \frac{24a^2-3(3-12x_a+28x_a^2)at+(1-6x_a+12x_a^2+8x_a^3)t^2}{48b}.
\end{aligned}$$

When $x_a > \bar{x}_a$, each domestic firm locates at $x = 0$ and the foreign firm locates at $x = 1/2$. The total profits of the public firms are

$$\begin{aligned}
\pi_0 &= 2\int_{\frac{1/2+x_a}{2}}^{\frac{1}{2}}\left(\frac{t(m-x_a)+t(1/2-m)}{2} - t(m-x_a)\right) \times \frac{a-t(m-x_a)}{b}dm \\
&= -\frac{(1-2x_a)^2t(12a-5(1-2x_a)t)}{192b}.
\end{aligned}$$

Social welfare (consumers surplus plus the sum of the domestic firms) is

$$SW = CS + \pi_0 + n\pi_i$$

$$= \frac{96a^2 - 48(1-4x_a+8x_a^2)at + (9-54x_a+108x_a^2+8(8n-1)x_a^3)t^2}{192b}.$$

The first-order condition lead to

$$x_a = \frac{32a - 9t - 2\sqrt{2(128a^2 - 4(8n+17)at + 9(n+1)t^2)}}{2(8n-1)t} \ (> \frac{1}{4}).$$

On $[\bar{x}_a, 1/4]$, the optimal locations of the public firms are $x_a = 1/4$ and $1 - x_a = 3/4$ and then SW is

$$SW = \frac{768a^2 - 192at + (17+8n)t^2}{1536b}. \tag{16}$$

When $x_a \leq \bar{x}_a$, each domestic firm locates at $x = 1/2$ and the foreign firm locates at $x = 1/2$. The total profits of the public firms are

$$\begin{aligned}\pi_0 &= 2\int_{\frac{1/2+x_a}{2}}^{\frac{1}{2}} \left(\frac{t(m-x_a)+t(1/2-m)}{2} - t(m-x_a)\right) \\ &\quad \times \left(\frac{a-t(m-x_a)}{b}dm - n \times \frac{t(m-x_a)+t(1/2-m)-2t(1/2-m)}{2b}\right) \\ &= -\frac{(1-2x_a)^2 t(12a - 5(1-2x_a)t - 2n(1-2x_a)t)}{192b}.\end{aligned}$$

Social welfare (consumers surplus plus the sum of the domestic firms) is

$$\begin{aligned}SW &= CS + \pi_0 + n\pi_i \\ &= \frac{96a^2 - 48(1-4x_a+8x_a^2)at + (9+4n-6(9+4n)x_a+12(9+4n)x_a^2 - 8(1+4n)x_a^3)t^2}{192b}.\end{aligned}$$

The first-order condition lead to

$$x_a = \frac{-32a + (9+4n)t + 2\sqrt{2(128a^2 - 4(4n+17)at + (4n+9)t^2)}}{2(4n+1)t}.$$

When this is the interior solution, social surplus is

$$\begin{aligned}SW &= \frac{4(4a-t)(32a-(4n+9)t)\sqrt{2(4a-t)(32a-(4n+9)t)}}{24b(4n+1)^2t} \\ &\quad - \frac{8192a^3 - 12(545+136n+16n^2)a^2t + 6(17+4n)^2at^2 - (153+104n+16n^2)t^3}{24b(4n+1)^2t}.\end{aligned}$$

After tedious calculus, we find that SW in the above equation is smaller than that in which $x_a = 1/4$. We have the following proposition:

Proposition 10 *Suppose that there are two public firms and a foreign firm. In equilibrium, the public firms locate at point* $1/4$ *and at point* $3/4$*, respectively. In equilibrium, all private firms locate at point* 0 *and the foreign firm locates at point* $1/2$*.*

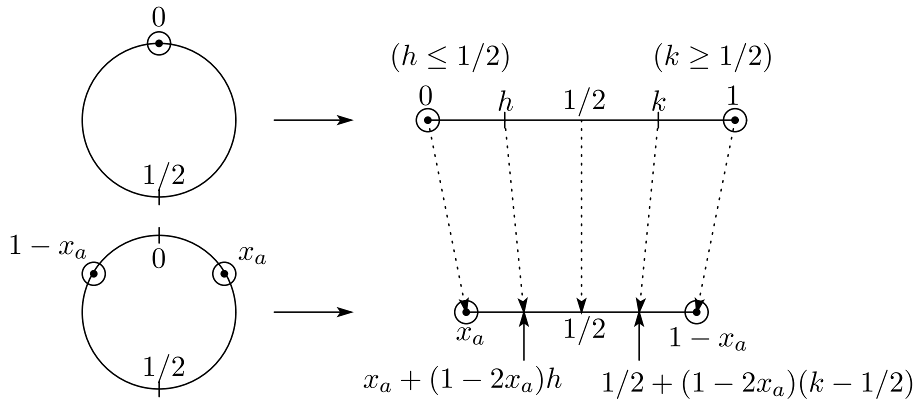
Propositions 9 and 10 indicate that the equilibrium locations of the public firms depend on the number of foreign firms. If no foreign firm exists, the public firms choose the non-maximal distance between them (Proposition 9). We explain why the public firms choose the maximal location when a foreign firm exists. As noted above, increasing the distance between the public firms induces production substitution from the private to the public firms. Since the substitution reduces the foreign firm's output, it increases the market share of the domestic firms (public and private) and increases domestic social surplus. This is the reason that the public firms choose the maximal distance.

5.1.3 Two foreign firms

We investigate the case where two foreign firms exist. We consider two cases: (i) a foreign firm locates at $x \in [0, x_a] \cup [1 - x_a, 1]$ and another foreign firm locates at $x \in [x_a, 1 - x_a]$; (ii) each foreign firm locates on the range $[x_a, 1 - x_a]$, (note that, we can exclude the case in which each foreign firm locates on the range $[0, x_a] \cup [1 - x_a, 1]$, because this is inferior to the second case).

In the first case, a foreign firm locates at $x = 0$ and another foreign firm locates at $x = 1/2$. The reason has already been explained in the former subsection.

In the second case, the market structure is similar to that in which a public firm exists on the circle with length $1 - 2x_a$.



Therefore, we can use the result (Proposition 3) in section 3: When two foreign firms exist, each domestic private firm i chooses either $x_i = \frac{5-\sqrt{3}}{11} \sim 0.297$ or $x_i = \frac{6+\sqrt{3}}{11} \sim 0.703$, and each foreign firm j chooses $x_j = 1/2$. The following location choices also constitute an equilibrium: each domestic private firm i chooses either $x_i = \frac{18-\sqrt{66}}{30} \sim 0.329$ or $x_i = \frac{12+\sqrt{66}}{30} \sim 0.671$, one foreign firm j chooses $x_j = 13/30$ and the other foreign firm k chooses $x_k = 17/30$. We translate these location patterns into those which are suitable to the situation considered here.

Lemma 5 *Suppose that two foreign firms exist and that each of them locates on the range* $[x_a, 1-x_a]$. *(i) Each domestic private firm i chooses either* $x_i = \frac{5-\sqrt{3}+(1+2\sqrt{3})x_a}{11}$ *or* $x_i = \frac{6+\sqrt{3}-(1+2\sqrt{3})x_a}{11}$, *and each foreign firm j chooses $x_j = 1/2$. (ii) The following location choices also constitute an equilibrium: each domestic private firm i chooses either* $x_i = \frac{18-\sqrt{66}+2(\sqrt{66}-3)x_a}{30}$ *or* $x_i = \frac{12+\sqrt{66}-2(\sqrt{66}-3)x_a}{30}$, *one foreign firm j chooses $x_j = (13+4x_a)/30$ and the other foreign firm k chooses $x_k = (17-4x_a)/30$.*

In this case, profits of firms are equal to "those in Section 3" times "$(1-2x_a)^3$. For instance, if a firm's profit is 2 in Section 3, the profit in this case is equal to $2 \times (1-2x_a)^3$. Like a case with single public firm discussed in Section 3, there are two equilibrium location patterns. We restrict our attention to the second outcome because the profits of foreign firms are larger than those in the first outcome.

When the location pattern is the second one in Lemma (one foreign firm j chooses $x_j = (13+4x_a)/30$ and the other foreign firm k chooses $x_k = (17-4x_a)/30$.), the profit of a foreign firm is $1369(1-2x_a)^3t^2/243000b$. Given the location pattern, if a foreign firm locates at $x=0$, then it's profit is $(2x_a)^3/96b$. If the former profit is larger than the latter one, this is an equilibrium outcome. After tedious calculus, we find that if $x_a < 0.2245\ldots$, the second outcome in Lemma is an equilibrium, but if not, a foreign firm locates at $x=0$ and another foreign firm locates at $x=1/2$ in equilibrium. The public firms take the location strategies of foreign firms above into account and set their locations. Messy calculus, we have the following proposition:

Proposition 11 *Suppose that there are two public firms and two foreign firms. Then the equilibrium locations of the public firms are x_a^{**} and $1-x_a^{**}$, where x_a^{**} is given by*

$$x_a^{**} = \frac{\sqrt{(32a-9t)(32a-(4n+9)t)} - (32a-(4n+9)t)}{8nt}. \tag{17}$$

The equilibrium locations of the foreign firms are $x=0$ and $x=1/2$. The equilibrium location of each domestic private firm is $x=1/2$.

Proposition 11 indicates that the public firms again choose a non-maximal distance between them. As is shown above, one foreign firm locates at 0 and the other foreign firm locates at 1/2. Increasing the distance between the public firms has three production substitution effects. It induces production substitution from all domestic private firms and one foreign firm locating at 1/2 to the public firms. At the same time, the increase in x_a induces production substitution from the public firms to the other foreign firm locating at 0. The last effect reduces domestic welfare. On the other hand, when only one foreign firm exists, this effect does not exist. Thus, the public firms have smaller incentives for increasing their distance, which yields the non-maximal equilibrium distance.

5.1.4 Welfare implication

When two public firms exist, each one firm locates at a different point. From the equilibrium locations of public firms, the two yield a larger welfare than one public firm does. The social planner can locate the public firms at the same point, which yields the same equilibrium welfare as that in the case of one public firm. Therefore, the additional public firm never reduces social welfare.

As we discuss in Proposition 8, it is possible that no public firm is better than another from the normative viewpoint. The question then naturally arises of whether no public firm or two public firms yields a larger welfare. After tedious calculus, we can show that social welfare is larger in the latter case. However, this result depends on the assumption that public firms are as efficient as private ones. If we consider the cost differences between public and private firms, which are discussed in the next subsection, it is possible that this result does not hold.

5.2 Inefficient public firm

In previous sections, we assume that public firms are as efficient as private firms. In this subsection, we consider a case in which a public firm is less efficient than private firms. We assume that the marginal cost of the public firm is $c > 0$, while those of private firms are normalized to zero.[14] We restrict our attention to the case in which no foreign firms exist and one public firm exists. We also assume that $t < 2(a-(n+1)c)/(n+1)$, which ensures a positive quantity supplied by the public firm at each point on the circular city.

Without loss of generality, we can assume that $x_0 = 0$. From (8), the profit of a private firm i locating at x_i is given by the following function (note that, at each point, the marginal cost of the public firm is $td(x, x_0) + c$):

$$\begin{aligned}\pi_i &= \int_{\frac{x_i}{2}}^{x_i} \frac{(tm + c - t(x_i - m))^2}{b} dm + \int_{x_i}^{\frac{1}{2}} \frac{(tm + c - t(m - x_i))^2}{b} dm \\ &\quad + \int_{\frac{1}{2}}^{\frac{1+x_i}{2}} \frac{(t(1-m) + c - t(m - x_i))^2}{b} dm \\ &= \frac{3c^2 + 6ctx_i + 3t(t-2c)x_i^2 - 4t^2 x_i^3}{6b}.\end{aligned}$$

Differentiating the function with respect to x_i, we have:

$$\frac{\partial \pi_i}{\partial x_i} = \frac{t(1-2x_i)(c + tx_i)}{b}.$$

It is positive for any $x_i \in [0, 1/2)$, so the optimal location of each private firm is still 1/2. This yields the following proposition.

[14] In this paper, we assume that these costs are given exogenously. For a discussion of endogenous cost differences, see Corneo and Rob (2003), Ishibashi and Matsumura (2005), Matsumura and Matsushima (2004), and Nett (1993).

Proposition 12 *Suppose that there are one public firm, m domestic private firms and no foreign firm. Suppose that $t < 2(a-(n+1)c)/(n+1)$. The equilibrium location pattern does not depend on c.*

This proposition implies that the cost difference between public and private firms does not affect the equilibrium location patterns. However, introducing a cost difference between public and private firms yields quite an important welfare implication.

We compare the equilibrium welfare of a mixed oligopoly with that of a pure oligopoly, in which a public firm is eliminated from the market.

In the mixed oligopoly, the equilibrium profit of each private firm is

$$\pi_i = \frac{t^2 + 6ct + 12c^2}{24b}. \tag{18}$$

Consumers' surplus is ($td(x, x_0)$ in (5) is replaced by $td(x, x_0) + c$)

$$CS = 2 \times \frac{b}{2} \int_0^{\frac{1}{2}} \left(\frac{a-c-tm}{b}\right)^2 dm = \frac{12(a-c)^2 - 6(a-c)t + t^2}{24b}.$$

The profit of the public firm is zero. Social welfare is equal to $CS + n\pi_i$, that is,

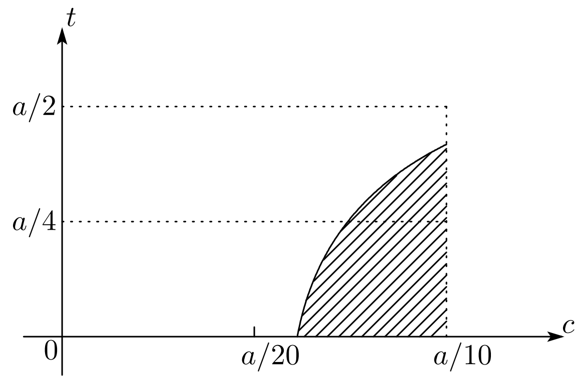
$$SW_m = \frac{12(a^2 - 2ac + (n+1)c^2) - 6(a-(n+1)c)t + (n+1)t^2}{24b}$$

Suppose that the public firm is removed from the market and then the private firms relocate their locations. In this case, an equidistance location pattern is an equilibrium outcome (see, Pal (1998)). Social welfare is:[15]

$$SW_p = \begin{cases} \dfrac{n(48(n+2)a^2 - 24(n+2)at + (2n^2+7n+8)t^2)}{96(n+1)^2 b}, & \text{if } n \text{ is even,} \\[2ex] \dfrac{48(n+2)n^3a^2 - 24n^3(n+2)at + (n^3(2n^2+7n+8) - (2n+3))t^2}{96(n+1)^2 n^2 b}, & \text{if } n \text{ is odd.} \end{cases}$$

The following figure presents the area where removing public firm improves welfare.

[15] The result is described in Matsushima (2001).



$(n = 2)$

From this figure, we can see that removing a public firm never improves welfare as long as the public firm is as efficient as the private firm (i.e., $c = 0$). By removing the public firm, each private firm produces more. In other words, production substitutions from the public firm to the private firm take place. When c is large, these production substitutions save the production cost, resulting in the improvement of welfare.

From this figure, we derive an interesting implication. Removing the public firm is more likely to improve welfare when t is smaller. We explain the intuition behind this feature. When $t = 0$, for any point, the public firm is less efficient than the private firms. Thus, the production substitution discussed above significantly improves welfare. An increase in t decreases the relative inefficiency of the public firm to the private firm for the markets close to the public firm's location and increases that for the markets close to the private firms' location. Since the public firm's outputs are larger for the former markets than for the latter markets, an increase in t reduces the relative inefficiency of the public firm, resulting in an increase in the value of the public firm. Thus, the removal of the public firm is less likely to improve the welfare when t is large.

5.3 Entries by domestic private firms

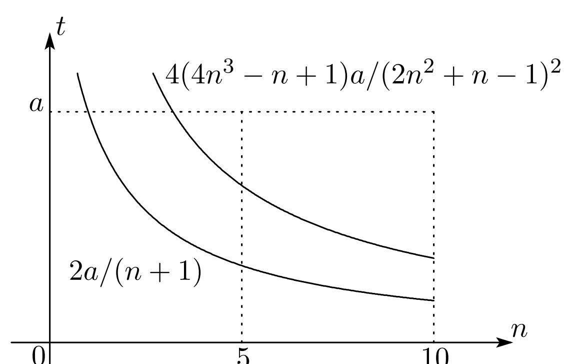
In other sections, we assume that the number of firms is given exogenously. In this subsection, we consider entries of domestic private firms. As opposed to the other sections, we assume that a sufficiently large number of potential entrants exist and the number of entering firms is determined by a zero-profit condition. We consider the case in which there is one public firm and no foreign firm. Let $F(>)0$ be the entry cost of each private firm. Let $\pi(n)$ be each domestic private firm's gross profit. The zero profit condition, $\pi(n) = F$, yields the equilibrium number of entering firms.[16] We then discuss $\pi(n)$.

As long as the public firm supplies its product for all of the points (or equivalently $(n+1)t \leq 2a$), $\pi(n)$ does not depend on n. This is because the equilibrium price at each market is equal to the marginal cost of the public firm regardless of n and each firm's output does not depend on n (see the discussion in Section 3.1).

We then derive $\pi(n)$ when $(n+1)t > 2a$. We assume that the public firm locates at $x = 0$ and the domestic private firms locate at $x = 1/2$. If this location pattern is an equilibrium outcome, we have

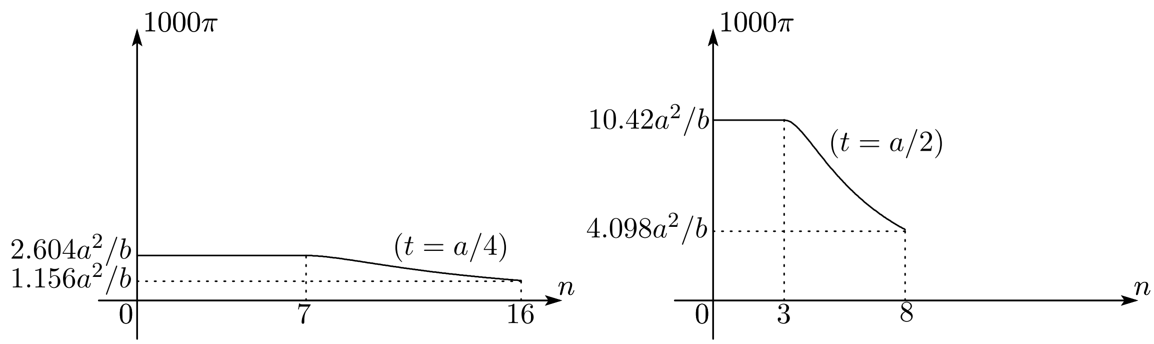
$$\begin{aligned} \pi(n) &= \int_{\frac{1}{4}}^{\frac{2a+tn}{2(2n+1)t}} \frac{(tm - t(1/2-m))^2)}{b} dm + \int_{\frac{2a+tn}{2(2n+1)t}}^{\frac{1}{2}} \frac{(a - t(1/2-m))^2)}{(n+1)^2 b} dm \\ &= \frac{-32(4n+3)a^3 + 96(n+1)^2a^2t - 24(n+1)^2at^2 + 2(n+1)^2t^3}{48b(n+1)^2(2n+1)^2t}. \end{aligned} \tag{19}$$

We can show that if $t \leq 4(4n^3 - n + 1)a/(2n^2 + n - 1)^2$, this location pattern in fact becomes an equilibrium one (see Appendix). From Figure 6, we can see that the condition $(n+1)t \leq 2a$ is stricter than the condition $t \leq 4(4n^3 - n + 1)a/(2n^2 + n - 1)^2$, so there exists n such that both $(n+1)t > 2a$ and $t \leq 4(4n^3 - n + 1)a/(2n^2 + n - 1)^2$ are satisfied.



[16] For discussions on free entry equilibrium in mixed oligopoly, see Anderson et al. (1997) and Matsumura and Kanda (2005).

We then discuss examples of $\pi(n)$. We consider two cases ($t = a/4$ and $t = a/2$). Figure 7 depicts $\pi(n)$ in the two cases.



Consider the first case ($t = a/4$). If $F > 2.604a^2/1000b$, no domestic private firm enters the market. If $F = 2.604a^2/1000b$, the number of firms is indeterminate (from 0 to 7). If $1.156a^2/1000b < F < 2.604a^2/1000b$, the number of private firms is between 7 and 15 (and all of them agglomerate at one point). If $F < 1.156a^2/1000b$, the number of firms exceeds 15, and private firms do not agglomerate at one point (we failed to derive the location pattern explicitly in the low fixed-cost case).

6 Concluding remarks

This study investigates a location–quantity model of mixed markets. The equilibrium location pattern depends on the number of foreign private firms and not on the number of domestic private firms. Spatial agglomeration of all private firms appears in a unique equilibrium if the number of foreign firms is zero or one. If the number of foreign firms is two, domestic and foreign private firms agglomerate at different points. The presence of more than two foreign firms yields differentiation among foreign firms (i.e., there is no equilibrium in which all foreign firms agglomerate at one point). Regardless of the number of firms, agglomeration of all domestic private firms appears in equilibrium.

In this paper, we assume that the public firm is a welfare maximizer. Although this assumption is very popular among the models of mixed oligopoly,[17] other approaches also exist.[18] Deviation from this welfare-maximizing assumption and the application of other approaches to this problem remain topics for future research.

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[17] See, among others, Anderson, de Palma, and Thisse (1997), De Fraja and Delbono (1989), Fjell and Pal (1996), Pal (1998b), and Matsumura (1998).

[18] See, e.g., Fershtman (1990) and Futagami (1999).

APPENDIX

Proof of Lemma 1 From (3), we obtain that $b(q_0 + \sum_{i \in D} q_i) = a - td(x, x_0)$. This yields Lemma 1(i). From (3), we obtain that

$$td(x, x_0) = a - b\left(q_0 + \sum_{i \in D} q_i\right) \geq p(x) \equiv a - b\left(q_0 + \sum_{i \in D \cup F} q_i\right). \tag{20}$$

Thus, if $d(x, x_0) \leq d(x, x_i)$, $p(x) \leq td(x, x_0) \leq td(x, x_i)$. That is, firm i's cost is never lower than the price, so it does not supply for the market, x. This yields Lemma 1(ii). **Q.E.D.**

Proof of Proposition 1 Proposition 1 obviously holds true if $n = 1$. Suppose that $n \geq 2$. We assume that there is no equilibrium where $x_1 = x_2$ and derive a contradiction. Suppose that firm 2 deviates from the equilibrium strategy and chooses the same location as firm 1. From Lemma 1, we have that the locations of all other firms are still optimal. Because of the symmetries between all domestic private firms this location is best for firm 2 since it is best for firm 1. Thus, this location pattern constitutes an equilibrium, a contradiction. **Q.E.D.**

Proof of Proposition 2 We have already shown that the location of private firms does not affects the profits of each private firm. Thus, if Proposition 2 holds true when $n = 1$, it also holds true for any $n \geq 1$. Thus, we prove Proposition 2 in the case of $n = m = 1$. Suppose that firm 1 (firm 2) is a domestic (foreign) private firm.

First, we show that the optimal location of firm 2 is 1/2 given that $x_0 = 0$ and $x_1 = 1/2$. By symmetry, without loss of generality, we assume that $x_2 \in [0, 1/2]$. Since no other foreign firm exists, $q_2(x)$ in (7) is positive if and only if $td(x, x_0) > td(x, x_2)$. Therefore, firm 2 supplies at $x \in (x_2/2, (1 + x_2)/2)$. For $x \in (x_2/2, (1 + x_2)/2)$, the quantity supplied by firm 2 is

$$q_2(x) = \frac{td(x, x_0) - td(x, x_2)}{2b}. \tag{21}$$

From (8), firm 2's profit is:

$$\begin{aligned} \pi_2 \quad = \quad & \int_{\frac{x_2}{2}}^{x_2} \frac{(tx + t(x_2 - x) - 2t(x_2 - x))^2}{4b} dx + \int_{x_2}^{\frac{1}{2}} \frac{(tx + t(x - x_2) - 2t(x - x_2))^2}{4b} dx \\ & + \int_{\frac{1}{2}}^{\frac{1+x_2}{2}} \frac{(t(1 - x) + t(x - x_2) - 2t(x - x_2))^2}{4b} dx = \frac{t^2 x_2^2 (3 - 4x_2)}{24b}. \end{aligned} \tag{22}$$

Differentiating π_2 with respect to x_2, we have $\partial\pi_2/\partial x_2 = (t^2 x_2(1-2x_2))/4b \geq 0$. This implies that the best reply of firm 2 is $x_2 = 1/2$.

Next, we show that firm 1's best location is $x_1 = 1/2$ given the others' locations. By symmetry, without loss of generality, we assume that $x_1 \in [0, 1/2]$. We consider the following two segments: (1) $x_1 \in [0, 1/4]$ and (2) $x_1 \in (1/4, 1/2]$.

(1) $x_1 \in [0, 1/4]$: From Lemma 1(ii), firm 1 does not supply at $x \in [0, x_1/2]$, so its profit from market $x \in [0, x_1/2]$ is zero. From Lemma 1(ii), firm 2 does not supply at $x \in [0, 1/4]$. From (8), firm 1's profit at $x \in (x_1/2, 1/4]$ is

$$\pi_1(x) = \frac{(tx - td(x_1, x))^2}{b}. \tag{23}$$

Consider the market $x > 1/4$. From Lemma 1(ii), firm 1 does not supply for the market $x \geq 3/4 (\geq (x_1+1)/2)$. In the former part of this proof, we have already shown that firm 2 supplies at $x \in (1/4, 3/4)$ (since $x_2 = 1/2$). From (7) and (21), firm 1 supplies only for the market $x < (1+4x_1)/4$. For the market $x \in (1/4, (1+4x_1)/4)$, the profit of firm 1 is given by

$$\pi_1(x) = \frac{(tx + td(x_2, x) - 2td(x_1, x))^2}{4b}. \tag{24}$$

Thus, the total profit of the domestic firm is:

$$\begin{aligned}\pi_1 &= \int_{\frac{x_1}{2}}^{x_1} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{x_1}^{\frac{1}{4}} \frac{(tx - t(x - x_1))^2}{b} dx \\ &\quad + \int_{\frac{1}{4}}^{\frac{1+4x_1}{4}} \frac{(tx + t(\frac{1}{2} - x) - 2t(x - x_1))^2}{4b} dx \;=\; \frac{t^2 x_1^2 (1 - 2x_1)}{4b}.\end{aligned} \tag{25}$$

Differentiating π_1 with respect to x_1, we have:

$$\frac{\partial \pi_1}{\partial x_1} = \frac{t^2 x_1 (1 - 3x_1)}{2b} > 0. \tag{26}$$

(2) $x_1 \in (1/4, 1/2]$: From Lemma 1(ii), firm 1 does not supply at $x \in [0, x_1/2]$, so its profit from market $x \in [0, x_1/2]$ is zero. From Lemma 1(ii), firm 2 does not supply at $x \in [0, 1/4]$. From (8), firm 1's profit at $x \in (x_1/2, 1/4]$ is given by (23).

Consider the market $x > 1/4$. From Lemma 1(ii) firm 1 does not supply for the market $x \geq 3/4 (\geq (x_1+1)/2)$. In the former part of this proof, we have already shown that firm 2 supplies at $x \in (1/4, 3/4)$ (since $x_2 = 1/2$). From (7) and (21), firm 1 supplies only for the market $x < (1+4x_1)/4$. For the market $x \in (1/4, (1+4x_1)/4)$, the profit of firm 1 is given by (24). Noting that $(1+4x_1)/4 \geq 1/2$, we have that the resulting profit of firm 1 is:

$$\begin{aligned}
\pi_1 &= \int_{\frac{x_1}{2}}^{\frac{1}{4}} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{\frac{1}{4}}^{x_1} \frac{(tx + t(1/2 - x) - 2t(x_1 - x))^2}{4b} dx \\
&\quad + \int_{x_1}^{\frac{1}{2}} \frac{(tx + t(1/2 - x) - 2t(x - x_1))^2}{4b} dx + \int_{\frac{1}{2}}^{\frac{1+4x_1}{4}} \frac{(t(1-x) + t(x - 1/2) - 2t(x - x_1))^2}{4b} dx \\
&= \frac{t^2(16x_1^3 - 24x_1^2 + 12x_1 - 1)}{96b}. \qquad (27)
\end{aligned}$$

Differentiating π_1 with respect to x_1, we have:

$$\frac{\partial \pi_1}{\partial x_1} = \frac{t^2(1-2x_1)^2}{8b} \geq 0. \qquad (28)$$

(26) and (28) imply that the optimal location of firm 1 is $x_1 = 1/2$. **Q.E.D.**

Proof of Proposition 3 For the same reason in Proof of Proposition 2, we consider the case in which $n = 1$ and $m = 2$. Suppose that firm 1 is domestic and firms 2 and 3 are foreign.

Proof of Proposition 3(i) First, we show that the location pattern, $x_2 = x_3 = 1/2$, is an equilibrium outcome. By symmetry, without loss of generality, we assume that the best response of firm 2 is $x_2 = 1/2$ given that $x_0 = 0$ and $x_3 = 1/2$. We consider 3 segments: (1) $x_2 \in [0, 1/4]$, (2) $x_2 \in (1/4, 1/3]$, and (3) $x_2 \in (1/3, 1/2]$.

(1) $x_2 \in [0, 1/4]$ From Lemma 1(ii) and (7), on $[0, 1/4]$, firm 3 does not supply but firm 2 supplies at $x \in [x_2/2, 1/4]$. On $[1/4, (x_2 + 1/2)/2]$, the quantity supplied by firm 2 is positive. On this range the quantity supplied by firm 3 is

$$q_3(x) = \max\left\{\frac{td(x, x_0) + td(x, x_2) - 2td(x, x_3)}{3b}, 0\right\} = \max\left\{\frac{tx + t(x - x_2) - 2t(1/2 - x)}{3b}, 0\right\}. \qquad (29)$$

For any $x \in ((1+x_2)/4, (x_2+1/2)/2]$, $tx + t(x - x_2) - 2t(1/2 - x)$ is positive, and it is negative for $x \leq (1+x_2)/4$. This implies that firm 3 also supplies for $x \in ((1+x_2)/4, (x_2+1/2)/2]$ but does not supply for $x \leq (1+x_2)/4$. On $((x_2+1/2)/2, 1/2]$, the quantity supplied by firm 3 is positive, and that supplied by firm 2 is

$$q_2(x) = \max\left\{\frac{td(x, x_0) + td(x, x_3) - 2td(x, x_2)}{3b}, 0\right\} = \max\left\{\frac{tx + t(1/2 - x) - 2t(x - x_2)}{3b}, 0\right\}. \qquad (30)$$

For any $x \in [(x_2+1/2)/2, (1+4x_2)/4)$, $tx+t(1/2-x)-2t(x-x_2)$ is positive, and it is non-positive for $x \in [(1+4x_2)/4, 1/2]$. This implies that firm 2 supplies for $x \in [(x_2+1/2)/2, (1+4x_2)/4)$ but not for $x \in [(1+4x_2)/4, 1/2]$. From Lemma 1(ii) firm 2 does not supply for $x \in [1/2, 1]$.

We summarize the discussion as follows: (a) firm 2 does not supply at $x \in [0, x_2/2]$ and $x \in [(1+4x_2)/4, 1]$; (b) firm 2 and firm 0 supply at $x \in [x_2/2, (1+x_2)/4]$; (c) firm 2, firm 3, and firm 0 supply at $x \in [(1+x_2)/4, (1+4x_2)/4]$. The resulting profit of firm 2 is:

$$\begin{aligned}
\pi_2 &= \int_{\frac{x_2}{2}}^{x_2} \frac{(tx-t(x_2-x))^2}{4b}dx + \int_{x_2}^{\frac{1+x_2}{4}} \frac{(tx-t(x-x_2))^2}{4b}dx \\
&+ \int_{\frac{1+x_2}{4}}^{\frac{1+4x_2}{4}} \frac{(tx+t(1/2-x)-2t(x-x_2))^2}{9b}dx \\
&= \frac{t^2x_2^2(3-4x_2)}{48b}. \qquad (31)
\end{aligned}$$

For any $x_2 \in [0, 1/4]$, π_2 is non-decreasing, and it is increasing for $x_2 \in (0, 1/4]$.

(2) $x_2 \in (1/4, 1/3]$ From the same discussions in segment (1), we obtain the following supply pattern, which is exactly the same as that in segment (1). (a) firm 2 does not supply at $x \in [0, x_2/2]$ and $x \in [(1+4x_2)/4, 1]$; (b) firm 2 and firm 0 supply at $x \in [x_2/2, (1+x_2)/4]$; (c) firm 2, firm 3, and firm 0 supply at $x \in [(1+x_2)/4, (1+4x_2)/4]$. The profit of firm 2 is:

$$\begin{aligned}
\pi_2 &= \int_{\frac{x_2}{2}}^{x_2} \frac{(tx-t(x_2-x))^2}{4b}dx + \int_{x_2}^{\frac{1+x_2}{4}} \frac{(tx-t(x-x_2))^2}{4b}dx \\
&+ \int_{\frac{1+x_2}{4}}^{\frac{1}{2}} \frac{(tx+t(\frac{1}{2}-x)-2t(x-x_2))^2}{9b}dx + \int_{\frac{1}{2}}^{\frac{1+4x_2}{4}} \frac{(t(1-x)+t(x-\frac{1}{2})-2t(x-x_2))^2}{9b}dx \\
&= \frac{t^2x_2^2(3-4x_2)}{48b}. \qquad (32)
\end{aligned}$$

For any $x_2 \in (1/4, 1/3]$, π_2 is increasing.

(3) $x_2 \in (1/3, 1/2]$ The range for which firm 2 supplies is exactly the same as that in segments (1) and (2). We focus the different points only. On $[1/4, x_2]$, the quantity supplied by firm 3 is

$$q_3(x) = \max\Big\{\frac{td(x,x_0)+td(x,x_2)-2td(x,x_3)}{3b}, 0\Big\} = \max\Big\{\frac{tx+t(x_2-x)-2t(1/2-x)}{3b}, 0\Big\}. \qquad (33)$$

For this range, $tx+t(x_2-x)-2t(1/2-x) > 0$, if and only if $x \in ((1-x_2)/2, x_2]$. Thus, $q_3(x)$

is positive if and only if $x \in ((1-x_2)/2, x_2]$. Note that threshold value is $(1-x_2)/2$ in segment (3), while it is $(1+x_2)/4$ in segments (1) and (2).

The following is the supply pattern in this case. (a) firm 2 does not supply at $x \in [0, x_2/2]$ and $x \in [(1+4x_2)/4, 1]$; (b) firm 2 and firm 0 supply at $x \in [x_2/2, (1-x_2)/2]$; (c) firm 2, firm 3, and firm 0 supply at $x \in ((1-x_2)/2, (1+4x_2)/4)$. Thus, the profit of firm 2 is:

$$\begin{aligned}
\pi_2 &= \int_{\frac{x_2}{2}}^{\frac{1-x_2}{2}} \frac{(tx - t(x_2 - x))^2}{4b} + \int_{\frac{1-x_2}{2}}^{x_2} \frac{(tx + t(1/2 - x) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{x_2}^{\frac{1}{2}} \frac{(tx + t(1/2 - x) - 2t(x - x_2))^2}{9b} dx \\
&+ \int_{\frac{1}{2}}^{\frac{1+4x_2}{4}} \frac{(t(1-x) + t(x - 1/2) - 2t(x - x_2))^2}{9b} dx \\
&= \frac{t^2(-7 + 54x_2 - 108x_2^2 + 72x_2^3)}{432b}. \qquad (34)
\end{aligned}$$

Differentiating it, we have $\partial \pi_2 / \partial x_2 = (t^2(1-2x_2)^2)/8b \geq 0$. Since π_2 is non-decreasing in $x_2 \in [0, 1/2]$ and increasing in $x_2 \in (0, 1/2)$, we have that π_2 is maximized at $x_2 = 1/2$. The profit is

$$\pi_2 = \frac{t^2}{216b}. \qquad (35)$$

Next, we show that, given the above locations of firms 2 and 3 (the foreign firms), firm 1's (the domestic firm's) best reply is $x_1 = \frac{5-\sqrt{3}}{11}$. By symmetry, without loss of generality, we assume that $x_1 \in [0, 1/2]$. We consider the following 2 segments: (1) $x_1 \in [0, 1/3]$ and (2) $x_1 \in (1/3, 1/2]$.

(1) $x_1 \in [0, 1/3]$ From the former part of the proof, we have that the quantity of each foreign firm is positive if and only if $x \in (1/4, 3/4)$. Firm 1 supplies for the market $x \in (x_1/2, 1/4]$, and it is given by

$$q_1(x) = \frac{(tx - td(x_1, x))}{b}. \qquad (36)$$

Consider the market $x \in (1/4, 1/2]$. As is shown in the former part of this proof, the quantity supplied by firm 2 (and that by firm 3) is

$$q_2(x) = \frac{td(x, x_0) + td(x, x_3) - 2td(x, x_2)}{3b} = \frac{tx - t(1/2 - x)}{3b}. \qquad (37)$$

From (7), on $(1/4, 1/2]$, the quantity supplied by firm 1 is

$$q_1(x) = \max\Big\{\frac{tx + 2t(1/2 - x) - 3t(x - x_1)}{3b}, 0\Big\}. \tag{38}$$

$tx + 2t(1/2 - x) - 3t(x - x_1)$ is positive if and only if $x \in (1/4, (1 + 3x_1)/4)$. Thus, firm 1 supplies for the market x if and only if $x \in (1/4, (1 + 3x_1)/4)$. From a similar discussion, we have that firm 1 does not supply for the market $x \in [(1 + 3x_1)/4, 1]$.

We summarize the discussion as follows: (a) firm 1 does not supply at $x \in [0, x_1/2]$ and $x \in [(1 + 3x_1)/4, 1]$; (b) firm 1 and firm 0 supply at $x \in [x_1/2, 1/4]$; (c) firm 1, firm 2, firm 3, and firm 0 supply at $x \in [1/4, (1 + 3x_1)/4]$. The profit of firm 1 is:

$$\begin{aligned}
\pi_1 &= \int_{\frac{x_1}{2}}^{\frac{1}{4}} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{\frac{1}{4}}^{x_1} \frac{(tx + 2t(1/2 - x) - 3t(x_1 - x))^2}{9b} dx \\
&\quad + \int_{x_1}^{\frac{1+3x_1}{4}} \frac{(tx + 2t(1/2 - x) - 3t(x - x_1))^2}{9b} \\
&= \frac{t^2(22x_1^3 - 30x_1^2 + 12x_1 - 1)}{72b}.
\end{aligned} \tag{39}$$

The first-order condition of optimality is:

$$\frac{\partial \pi_1}{\partial x_1} = 0 \quad \Leftrightarrow \quad \frac{t^2(2 - 10x_1 + 11x_1^2)}{12b} = 0. \tag{40}$$

This yields

$$x_1 = \frac{5 - \sqrt{3}}{11}. \tag{41}$$

(2) $x_1 \in (1/3, 1/2]$ From the former part of the proof, we have that the quantity of each foreign firm is positive if and only if $x \in (1/4, 3/4)$. Firm 1 supplies for the market $x \in (x_1/2, 1/4]$, and it is given by (36). Consider the market $x \in (1/4, 1/2]$. The quantity supplied by firm 2 (and that by firm 3) is given by (37). From (7), on $(1/4, 1/2]$, the quantity supplied by firm 1 is

$$q_1(x) = \max\Big\{\frac{tx + 2t(1/2 - x) - 3t(x - x_1)}{3b}, 0\Big\}. \tag{42}$$

$tx + 2t(1/2 - x) - 3t(x - x_1)$ is positive for all $x \in (1/4, 1/2]$ because $x_1 > 1/3$. Thus, firm 1 supplies for these markets. On $[1/2, 3/4]$, the quantity supplied by firm 1 is

$$q_1(x) = \max\Big\{\frac{t(1 - x) + 2t(x - 1/2) - 3t(x - x_1)}{3b}, 0\Big\}. \tag{43}$$

$t(1-x)+2t(x-1/2)-3t(x-x_1)$ is positive if and only if $x \in (1/2, 3x_1/2)$. Thus, on this range ($x \in (1/2, 3/4]$), firm 1 supplies for the market x if and only if $x \in (1/2, 3x_1/2)$. On $[3/4, 1]$, firm 1 does not supply (Lemma 1(ii)).

We summarize the discussion as follows: (a) firm 1 does not supply at $x \in [0, x_1/2]$ and $x \in [3x_1/2, 1]$; (b) firm 1 and firm 0 supply at $x \in (x_1/2, 1/4]$; (c) firm 1, firm 2, firm 3, and firm 0 supply at $x \in (1/4, 3x_1/2]$. The profit of firm 1 is:

$$\begin{aligned}
\pi_1 &= \int_{\frac{x_1}{2}}^{\frac{1}{4}} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{\frac{1}{4}}^{x_1} \frac{(tx + 2t(1/2 - x) - 3t(x_1 - x))^2}{9b} dx \\
&+ \int_{x_1}^{\frac{1}{2}} \frac{(tx + 2t(1/2 - x) - 3t(x - x_1))^2}{9b} dx \\
&+ \int_{\frac{1}{2}}^{\frac{3x_1}{2}} \frac{(t(1-x) + 2t(x - 1/2) - 3t(x - x_1))^2}{9b} dx \\
&= \frac{t^2(120x_1^3 - 144x_1^2 + 54x_1 - 5)}{216b}. \qquad (44)
\end{aligned}$$

Differentiating π_1 with respect to x_1, we have:

$$\frac{\partial \pi_1}{\partial x_1} = \frac{t^2(1-2x_1)(3-10x_1)}{12b} < 0. \qquad (45)$$

This implies that no $x \in (1/3, 1/2]$ maximizes π_1. **Q.E.D.**

Proof of Proposition 3(ii) First, we show that the location pattern, $x_2 = 13/30$ and $x_3 = 17/30$ constitutes an equilibrium. By symmetry, it is sufficient to show that the best response of firm 2 is $x_2 = 13/30$ given that $x_0 = 0$ and $x_3 = 17/30$. To show the best response, we consider the following 8 segments: (1) $x_2 \in [0, 13/60]$, (2) $x_2 \in (13/60, 7/20]$, (3) $x_2 \in (7/20, 17/45]$, (4) $x_2 \in (17/45, 1/2]$, (5) $x_2 \in (1/2, 17/30]$, (6) $x_2 \in (17/30, 32/45]$, (7) $x_2 \in (32/45, 47/60]$, and (8) $x_2 \in (47/60, 1]$.

(1) $x_2 \in [0, 13/60]$

$$\begin{aligned}
\pi_2 &= \int_{\frac{x_2}{2}}^{x_2} \frac{(tx - t(x_2 - x))^2}{4b} dx + \int_{x_2}^{\frac{17+15x_2}{60}} \frac{(tx - t(x - x_2))^2}{4b} dx \\
&+ \int_{\frac{17+15x_2}{60}}^{\frac{17+60x_2}{60}} \frac{(tx + t(17/30 - x) - 2t(x - x_2))^2}{9b} dx \\
&= \frac{t^2 x_2^2 (17 - 20x_2)}{240b}. \qquad (46)
\end{aligned}$$

For any $x_2 \in [0, 13/60]$, π_2 is non-decreasing and it is increasing for $x_2 \in (0, 13/60]$.

(2) $x_2 \in (13/60, 7/20]$

$$\begin{aligned}
\pi_2 &= \int_{\frac{x_2}{2}}^{x_2} \frac{(tx - t(x_2 - x))^2}{4b} dx + \int_{x_2}^{\frac{17+15x_2}{60}} \frac{(tx - t(x - x_2))^2}{4b} dx \\
&+ \int_{\frac{17+15x_2}{60}}^{\frac{1}{2}} \frac{(tx + t(17/30 - x) - 2t(x - x_2))^2}{9b} dx \\
&+ \int_{\frac{1}{2}}^{\frac{47+60x_2}{120}} \frac{(t(1-x) + t(17/30 - x) - 2t(x - x_2))^2}{9b} dx \\
&= \frac{t^2(2197 - 30420x_2 + 346950x_2^2 - 459000x_2^3)}{2916000b}. \qquad (47)
\end{aligned}$$

For any $x_2 \in (13/60, 7/20]$, π_2 is increasing.

(3) $x_2 \in (7/20, 17/45]$

$$\begin{aligned}
\pi_2 &= \int_{\frac{x_2}{2}}^{x_2} \frac{(tx - t(x_2 - x))^2}{4b} dx + \int_{x_2}^{\frac{17+15x_2}{60}} \frac{(tx - t(x - x_2))^2}{4b} dx \\
&+ \int_{\frac{17+15x_2}{60}}^{\frac{1}{2}} \frac{(tx + t(17/30 - x) - 2t(x - x_2))^2}{9b} dx \\
&+ \int_{\frac{1}{2}}^{\frac{17}{30}} \frac{(t(1-x) + t(17/30 - x) - 2t(x - x_2))^2}{9b} dx \\
&+ \int_{\frac{17}{30}}^{\frac{13+60x_2}{60}} \frac{(t(1-x) + t(x - 17/30) - 2t(x - x_2))^2}{9b} dx \\
&= \frac{t^2(-3532 + 24480x_2 + 60075x_2^2 - 121500x_2^3)}{1458000b}. \qquad (48)
\end{aligned}$$

For any $x_2 \in (7/20, 17/45]$, π_2 is increasing.

(4) $x_2 \in (17/45, 1/2]$

$$\begin{aligned}
\pi_2 &= \int_{\frac{x_2}{2}}^{\frac{17-15x_2}{30}} \frac{(tx - t(x_2 - x))^2}{4b} dx + \int_{\frac{17-15x_2}{30}}^{x_2} \frac{(tx + t(\frac{17}{30} - x) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{x_2}^{\frac{1}{2}} \frac{(tx + t(\frac{17}{30} - x) - 2t(x - x_2))^2}{9b} dx \\
&+ \int_{\frac{1}{2}}^{\frac{17}{30}} \frac{(t(1-x) + t(\frac{17}{30} - x) - 2t(x - x_2))^2}{9b} dx \\
&+ \int_{\frac{17}{30}}^{\frac{13+60x_2}{60}} \frac{(t(1-x) + t(x - \frac{17}{30}) - 2t(x - x_2))^2}{9b} dx \\
&= \frac{t^2(-12641 + 86190x_2 - 152100x_2^2 + 81000x_2^3)}{486000b}. \qquad (49)
\end{aligned}$$

The first-order condition for the optimality is:

$$\frac{\partial \pi_2}{\partial x_2} = 0 \Longleftrightarrow \frac{t^2(13-30x_2)(221-270x_2)}{16200b} = 0. \tag{50}$$

The second-order condition is satisfied. From this condition, we find that π_2 is maximized at $x_2 = 13/30$. The profit is

$$\pi_2 = \frac{1369t^2}{243000b}. \tag{51}$$

(5) $x_2 \in (1/2, 17/30]$

$$\begin{aligned}
\pi_2 &= \int_{\frac{x_2}{2}}^{\frac{17-15x_2}{30}} \frac{(tx - t(x_2 - x))^2}{4b} dx + \int_{\frac{17-15x_2}{30}}^{\frac{1}{2}} \frac{(tx + t(\frac{17}{30} - x) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{\frac{1}{2}}^{x_2} \frac{(t(1-x) + t(\frac{17}{30} - x) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{x_2}^{\frac{17}{30}} \frac{(t(1-x) + t(\frac{17}{30} - x) - 2t(x - x_2))^2}{9b} dx \\
&+ \int_{\frac{17}{30}}^{\frac{13+60x_2}{60}} \frac{(t(1-x) + t(x - \frac{17}{30}) - 2t(x - x_2))^2}{9b} dx \\
&= \frac{t^2(-30641 + 194190x_2 - 368100x_2^2 + 225000x_2^3)}{486000b}.
\end{aligned} \tag{52}$$

For any $x_2 \in (1/2, 17/30]$, π_2 is decreasing.

(6) $x_2 \in (17/30, 32/45]$

$$\begin{aligned}
\pi_2 &= \int_{\frac{-17+60x_2}{60}}^{\frac{1}{2}} \frac{(tx + t(17/30 - x) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{\frac{1}{2}}^{\frac{17}{30}} \frac{(t(1-x) + t(17/30 - x) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{\frac{17}{30}}^{x_2} \frac{(t(1-x) + t(x - 17/30) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{x_2}^{\frac{32-15x_2}{30}} \frac{(t(1-x) + t(x - 17/30) - 2t(x - x_2))^2}{9b} dx \\
&+ \int_{\frac{32-15x_2}{30}}^{\frac{1+x_2}{2}} \frac{(t(1-x) - t(x - x_2))^2}{4b} dx \\
&= \frac{t^2(75119 - 301770x_2 + 456300x_2^2 - 243000x_2^3)}{1458000b}.
\end{aligned} \tag{53}$$

For any $x_2 \in (17/30, 32/45]$, π_2 is decreasing.

(7) $x_2 \in (32/45, 47/60]$

$$\begin{aligned}
\pi_2 &= \int_{\frac{-17+60x_2}{60}}^{\frac{1}{2}} \frac{(tx + t(17/30 - x) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{\frac{1}{2}}^{\frac{17}{30}} \frac{(t(1-x) + t(17/30 - x) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{\frac{17}{30}}^{\frac{32+15x_2}{60}} \frac{(t(1-x) + t(x - 17/30) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{\frac{32+15x_2}{60}}^{x_2} \frac{(t(1-x) - t(x_2 - x))^2}{4b} dx \\
&+ \int_{x_2}^{\frac{1+x_2}{2}} \frac{(t(1-x) - t(x - x_2))^2}{4b} dx \\
&= \frac{t^2(-5339 + 46290x_2 - 80775x_2^2 + 40500x_2^3)}{486000b}. \qquad (54)
\end{aligned}$$

For any $x_2 \in (32/45, 47/60]$, π_2 is decreasing.

(8) $x_2 \in (47/60, 1]$

$$\begin{aligned}
\pi_2 &= \int_{x_2 - \frac{13}{60}}^{\frac{32+15x_2}{60}} \frac{(t(1-x) + t(x - 17/30) - 2t(x_2 - x))^2}{9b} dx \\
&+ \int_{\frac{32+15x_2}{60}}^{x_2} \frac{(t(1-x) - t(x_2 - x))^2}{4b} dx + \int_{x_2}^{\frac{1+x_2}{2}} \frac{(t(1-x) - t(x - x_2))^2}{4b} dx \\
&= \frac{t^2(1-x_2)^2(20x_2 - 7)}{240b}. \qquad (55)
\end{aligned}$$

For any $x_2 \in (47/60, 1]$, π_2 is non-increasing.

Next, we derive the location pattern in which the foreign firms locate at $x_2 = 13/30$ and $x_3 = 17/30$. By symmetry, without loss of generality, we assume that $x_1 \in [0, 1/2]$. To show the best response, we have to consider 8 segments: (1) $x_1 \in [0, 2/15]$, (2) $x_1 \in (2/15, 13/60]$, (3) $x_1 \in (13/60, 11/45]$, (4) $x_1 \in (11/45, 13/45]$, (5) $x_1 \in (13/45, 7/20]$, (6) $x_1 \in (7/20, 17/45]$, (7) $x_1 \in (17/45, 13/30]$, and (8) $x_1 \in (13/30, 1/2]$.

(1) $x_1 \in [0, 2/15]$

$$\begin{aligned}
\pi_1 &= \int_{\frac{x_1}{2}}^{x_1} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{x_1}^{\frac{13}{60}} \frac{(tx - t(x - x_1))^2}{b} dx \\
&+ \int_{\frac{13}{60}}^{\frac{13+60x_1}{60}} \frac{(tx + t(13/30 - x) - 2t(x - x_1))^2}{4b} dx
\end{aligned}$$

$$= \frac{t^2 x_1^2(13-30x_1)}{60b}. \tag{56}$$

For any $x_1 \in [0, 2/15]$, π_1 is non-decreasing.

(2) $x_1 \in (2/15, 13/60]$

$$\begin{aligned}
\pi_1 &= \int_{\frac{x_1}{2}}^{x_1} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{x_1}^{\frac{13}{60}} \frac{(tx - t(x - x_1))^2}{b} dx \\
&+ \int_{\frac{13}{60}}^{\frac{7}{20}} \frac{(tx + t(13/30 - x) - 2t(x - x_1))^2}{4b} dx \\
&+ \int_{\frac{7}{20}}^{\frac{1+3x_1}{4}} \frac{(tx + t(13/30 - x) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
&= \frac{t^2(-23625x_1^3 + 10125x_1^2 - 180x_1 + 8)}{40500b}.
\end{aligned} \tag{57}$$

For any $x_1 \in (2/15, 13/60]$, π_1 is increasing.

(3) $x_1 \in (13/60, 11/45]$

$$\begin{aligned}
\pi_1 &= \int_{\frac{x_1}{2}}^{\frac{13}{60}} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{\frac{13}{60}}^{x_1} \frac{(tx + t(13/30 - x) - 2t(x_1 - x))^2}{4b} dx \\
&+ \int_{x_1}^{\frac{7}{20}} \frac{(tx + t(13/30 - x) - 2t(x - x_1))^2}{4b} dx \\
&+ \int_{\frac{7}{20}}^{\frac{1+3x_1}{4}} \frac{(tx + t(13/30 - x) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
&= \frac{t^2(3000x_1^3 - 6600x_1^2 + 3220x_1 - 237)}{36000b}.
\end{aligned} \tag{58}$$

For any $x_1 \in (13/60, 11/45]$, π_1 is increasing.

(4) $x_1 \in (11/45, 13/45]$

$$\begin{aligned}
\pi_1 &= \int_{\frac{x_1}{2}}^{\frac{13}{60}} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{\frac{13}{60}}^{x_1} \frac{(tx + t(13/30 - x) - 2t(x_1 - x))^2}{4b} dx \\
&+ \int_{x_1}^{\frac{7}{20}} \frac{(tx + t(13/30 - x) - 2t(x - x_1))^2}{4b} dx \\
&+ \int_{\frac{7}{20}}^{\frac{13}{30}} \frac{(tx + t(13/30 - x) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
&+ \int_{\frac{13}{30}}^{\frac{2+45x_1}{30}} \frac{(tx + t(x - 13/30) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
&= \frac{t^2(194400x_1^3 - 213840x_1^2 + 78300x_1 - 5969)}{583200b}.
\end{aligned} \tag{59}$$

For any $x_1 \in (11/45, 13/45]$, π_1 is increasing.

(5) $x_1 \in (13/45, 7/20]$

$$\begin{aligned}
\pi_1 \;=\; & \int_{\frac{x_1}{2}}^{\frac{13}{60}} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{\frac{13}{60}}^{x_1} \frac{(tx + t(13/30 - x) - 2t(x_1 - x))^2}{4b} dx \\
& + \int_{x_1}^{\frac{7}{20}} \frac{(tx + t(13/30 - x) - 2t(x - x_1))^2}{4b} dx \\
& + \int_{\frac{7}{20}}^{\frac{13}{30}} \frac{(tx + t(13/30 - x) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
& + \int_{\frac{13}{30}}^{\frac{1}{2}} \frac{(tx + t(x - 13/30) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
& + \int_{\frac{1}{2}}^{\frac{17+45x_1}{60}} \frac{(t(1 - x) + t(x - 13/30) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
= \;& \frac{t^2(243000x_1^3 - 437400x_1^2 + 208980x_1 - 12269)}{2916000b}. \qquad (60)
\end{aligned}$$

Differentiating π_1 with respect to x_1, we have:

$$\frac{\partial \pi_1}{\partial x_1} = \frac{t^2(150x_1^2 - 180x_1 + 43)}{600b}. \qquad (61)$$

From the first-order condition of the optimality, we have

$$x_1 = \frac{18 - \sqrt{66}}{30} \sim 0.329. \qquad (62)$$

(6) $x_1 \in (7/20, 17/45]$

$$\begin{aligned}
\pi_1 \;=\; & \int_{\frac{x_1}{2}}^{\frac{13}{60}} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{\frac{13}{60}}^{\frac{7}{20}} \frac{(tx + t(13/30 - x) - 2t(x_1 - x))^2}{4b} dx \\
& + \int_{\frac{7}{20}}^{x_1} \frac{(tx + t(13/30 - x) + t(17/30 - x) - 3t(x_1 - x))^2}{9b} dx \\
& + \int_{x_1}^{\frac{13}{30}} \frac{(tx + t(13/30 - x) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
& + \int_{\frac{13}{30}}^{\frac{1}{2}} \frac{(tx + t(x - 13/30) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
& + \int_{\frac{1}{2}}^{\frac{17+45x_1}{60}} \frac{(t(1 - x) + t(x - 13/30) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
= \;& \frac{t^2(222750x_1^3 - 279450x_1^2 + 111780x_1 - 10013)}{729000b}. \qquad (63)
\end{aligned}$$

For any $x_1 \in (7/20, 17/45]$, π_1 is decreasing.

(7) $x_1 \in (17/45, 13/30]$

$$
\begin{aligned}
\pi_1 \;=\;& \int_{\frac{x_1}{2}}^{\frac{13}{60}} \frac{(tx - t(x_1 - x))^2}{b} dx + \int_{\frac{13}{60}}^{\frac{7}{20}} \frac{(tx + t(13/30 - x) - 2t(x_1 - x))^2}{4b} dx \\
&+ \int_{\frac{7}{20}}^{x_1} \frac{(tx + t(13/30 - x) + t(17/30 - x) - 3t(x_1 - x))^2}{9b} dx \\
&+ \int_{x_1}^{\frac{13}{30}} \frac{(tx + t(13/30 - x) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
&+ \int_{\frac{13}{30}}^{\frac{1}{2}} \frac{(tx + t(x - 13/30) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
&+ \int_{\frac{1}{2}}^{\frac{17}{30}} \frac{(t(1 - x) + t(x - 13/30) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
&+ \int_{\frac{17}{30}}^{\frac{3x_1}{2}} \frac{(t(1 - x) + t(x - 13/30) + t(x - 17/30) - 3t(x - x_1))^2}{9b} dx \\
=\;& \frac{t^2(135000x_1^3 - 162000x_1^2 + 63270x_1 - 6613)}{243000b}. \qquad (64)
\end{aligned}
$$

For any $x_1 \in (17/45, 13/30]$, π_1 is decreasing.

(8) $x_1 \in (13/30, 1/2]$

$$
\begin{aligned}
\pi_1 \;=\;& \int_{\frac{-13+60x_1}{60}}^{\frac{7}{20}} \frac{(tx + t(13/30 - x) - 2t(x_1 - x))^2}{4b} dx \\
&+ \int_{\frac{7}{20}}^{\frac{13}{30}} \frac{(tx + t(13/30 - x) + t(17/30 - x) - 3t(x_1 - x))^2}{9b} dx \\
&+ \int_{\frac{13}{30}}^{x_1} \frac{(tx + t(x - 13/30) + t(17/30 - x) - 3t(x_1 - x))^2}{9b} dx \\
&+ \int_{x_1}^{\frac{1}{2}} \frac{(tx + t(x - 13/30) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
&+ \int_{\frac{1}{2}}^{\frac{17}{30}} \frac{(t(1 - x) + t(x - 13/30) + t(17/30 - x) - 3t(x - x_1))^2}{9b} dx \\
&+ \int_{\frac{17}{30}}^{\frac{13}{20}} \frac{(t(1 - x) + t(x - 13/30) + t(x - 17/30) - 3t(x - x_1))^2}{9b} dx \\
&+ \int_{\frac{13}{20}}^{\frac{13+60x_1}{60}} \frac{(t(1 - x) + t(x - 17/30) - 2t(x - x_1))^2}{4b} dx \\
=\;& \frac{t^2(487 - 2400x_1 + 4650x_1^2 - 3000x_1^3)}{13500b}. \qquad (65)
\end{aligned}
$$

For any $x_1 \in (13/30, 1/2]$, π_1 is decreasing. **Q.E.D.**

Proof of Proposition 4 We assume that all foreign firms agglomerate at h in an equilibrium and derive a contradiction. Without loss of generality, we assume that $h \leq 1/2$.

First, we assume that $h < 1/2$ and show that, a foreign firm can enhance its profit if it deviates from the equilibrium strategy and relocates at $h + \varepsilon$, where ε is small and positive constant. The profit of each foreign firm in the equilibrium is:

$$\begin{aligned}
\pi &= \int_{\frac{h}{2}}^{h} \frac{[tx + mt(h-x) - (m+1)t(h-x)]^2}{(m+1)^2 b} dx \\
&\quad + \int_{h}^{\frac{1}{2}} \frac{[tx + mt(x-h) - (m+1)t(x-h)]^2}{(m+1)^2 b} dx \\
&\quad + \int_{\frac{1}{2}}^{\frac{1+h}{2}} \frac{[t(1-x) + mt(x-h) - (m+1)t(x-h)]^2}{(m+1)^2 b} dx \\
&= \frac{h^2(3-4h)t^2}{6b(m+1)^2}. \qquad (66)
\end{aligned}$$

When a foreign firm relocates from h to $h + \varepsilon$, the firm's profit becomes:

$$\begin{aligned}
\pi &= \int_{\frac{h+m\varepsilon}{2}}^{h} \frac{[tx + (m-1)t(h-x) + t(h+\varepsilon-x) - (m+1)t(h+\varepsilon-x)]^2}{(m+1)^2 b} dx \\
&\quad + \int_{h}^{h+\varepsilon} \frac{[tx + (m-1)t(x-h) + t(h+\varepsilon-x) - (m+1)t(h+\varepsilon-x)]^2}{(m+1)^2 b} dx \\
&\quad + \int_{h+\varepsilon}^{\frac{1}{2}} \frac{[tx + (m-1)t(x-h) + t(x-h-\varepsilon) - (m+1)t(x-h-\varepsilon)]^2}{(m+1)^2 b} dx \\
&\quad + \int_{\frac{1}{2}}^{\frac{1+h-\varepsilon}{2}} \frac{[t(1-x) + (m-1)t(x-h) + t(x-h-\varepsilon) - (m+1)t(x-h-\varepsilon)]^2}{(m+1)^2 b} dx \\
&\quad + \int_{\frac{1+h-\varepsilon}{2}}^{\frac{1+h+\varepsilon}{2}} \frac{[t(1-x) - t(x-h-\varepsilon)]^2}{4b} dx \\
&= \frac{[h^2(3-4h) + 6(1-2h)hm\varepsilon + 3(m-4h)m\varepsilon^2 + (1+m-5m^2-m^3)\varepsilon^3]t^2}{6b(m+1)^2}. \qquad (67)
\end{aligned}$$

The difference between π in (67) and π in (66) is:

$$\frac{[6(1-2h)hm + 3(m-4h)m\varepsilon + (1+m-5m^2-m^3)\varepsilon^2]\varepsilon t^2}{6b(m+1)^2}. \qquad (68)$$

For any $h < 1/2$ and sufficiently small ε, this is positive, a contradiction.

Next, we assume that $h = 1/2$. We show that a foreign firm is able to enhance its profit if it relocates from $1/2$ to $y < 1/2$, where y is sufficiently close to $1/2$.

From (66), the profit of each foreign firm in the equilibrium is

$$\pi = \frac{t^2}{24b(m+1)^2}. \tag{69}$$

If a foreign firm relocates at $y < 1/2$, the profit of the firm is:

$$\begin{aligned}
\pi \;=\; & \int_{\frac{y}{2}}^{\frac{1-y}{2}} \frac{(tx - t(y-x))^2}{4b} dx + \int_{\frac{1-y}{2}}^{y} \frac{[tx + (m-1)t(\frac{1}{2} - x) + t(h-x) - (m+1)t(y-x)]^2}{(m+1)^2 b} dx \\
& + \int_{y}^{\frac{1}{2}} \frac{[tx + (m-1)t(\frac{1}{2} - x) + t(x-y) - (m+1)t(x-y)]^2}{(m+1)^2 b} dx \\
& + \int_{\frac{1}{2}}^{\frac{3-m+2ym}{4}} \frac{[t(1-x) + (m-1)t(x - \frac{1}{2}) + t(h-y) - (m+1)t(x-y)]^2}{(m+1)^2 b} dx \\
=\; & \frac{[8(m^3 + 5m^2 - 9m - 1)y^3 - 12(m^3 + 3m^2 - 5m - 1)y^2]t^2}{48b(m+1)^2} \\
& + \frac{[6(m-1)(m+1)^2 y - (m-1)(m^2+3)]t^2}{48b(m+1)^2}.
\end{aligned} \tag{70}$$

The difference between π in (70) and π in (69) is:

$$\frac{(1-2y)^2(1 - 3m + m^2 - m^3 - 2(1 + 9m - 5m^2 - m^3)y)t^2}{48b(m+1)^2}. \tag{71}$$

For any y which is close to $1/2$, this is positive since $\lim_{y \to 1/2} (1 - 3m + m^2 - m^3 - 2(1 + 9m - 5m^2 - m^3)y) = 6m(m-2) > 0$, a contradiction. **Q.E.D.**

Proof of the result in Section 5.3 Suppose that there are one public firm, n domestic private firms and no foreign firm. Without loss of generality, we assume that $x_0 = 0$. We then prove that the domestic private firms locate at $x = 1/2$ in equilibrium if $t \leq 4(4n^3 - n + 1)a/(2n^2 + n - 1)^2$. We show that, given the locations of the public firm and the other private firms, each private firm cannot increase its profit by relocating from the point $1/2$.

We first consider the conditions that the public firm does not supply to a range on the circular city. We consider three ranges: (i) $m \in [0, x]$ (x is the location of the relocating firm), (ii) $m \in [x, 1/2]$, (iii) $m \in [1/2, 1]$.

When the public firm does not supply at $m \in [0, x]$, the following equation holds:

$$(n-1)\frac{tm - t(1/2 - m)}{b} + \frac{tm - t(x-m)}{b} > \frac{a - tm}{b}, \quad m > \frac{2a + (n-1)t + 2tx}{2(2n+1)t} \equiv m_1. \tag{72}$$

$m_1 \leq x$, if and only if

$$\frac{2a+(n-1)t}{4nt} < x \leq 1/2. \tag{73}$$

When the public firm does not supply at $m \in [x, 1/2]$, the following equation holds:

$$(n-1)\frac{tm - t(1/2-m)}{b} + \frac{tm - t(m-x)}{b} > \frac{a - tm}{b}, \quad m > \frac{2a+(n-1)t-2tx}{2(2n-1)t} \equiv m_2. \tag{74}$$

$x \leq m_2 \leq 1/2$, if and only if

$$\frac{2a-nt}{2t} \leq x \leq \frac{2a+(n-1)t}{4nt}. \tag{75}$$

When the public firm does not supply at $m \in [1/2, 1]$, the following equation holds:

$$(n-1)\frac{t(1-m) - t(m-1/2)}{b} + \frac{t(1-m) - t(m-x)}{b} > \frac{a - t(1-m)}{b}, \tag{76}$$

$$m < \frac{-2a+(3n+1)t+2tx}{2(2n+1)t} \equiv m_3. \tag{77}$$

$1/2 \leq m_3$, if and only if

$$\frac{2a-nt}{2t} < x \leq 1/2. \tag{78}$$

From the discussions above, we can see that we have to consider following three cases: (i) $x \in [(2a+(n-1)t)/4nt, 1/2]$ (the public firm does not supply at $m \in [m_1, m_3]$), (ii) $x \in [(2a-nt)/2t, (2a+(n-1)t)/4nt]$ (the public firm does not supply at $m \in [m_2, m_3]$), (iii) $x \in [0, (2a-nt)/2t]$ (the public firm supplies at all points on the circular city).

(i) $x \in [(2a+(n-1)t)/4nt, 1/2]$. In the case, the public firm does not supply at $m \in [m_1, m_3]$. The profit of the private firm locating at $x \in [(2a+(n-1)t)/4nt, 1/2]$ is:

$$\begin{aligned}
\pi_i &= \int_{\frac{x}{2}}^{m_1} \frac{(tm - t(x-m))^2}{b} dm + \int_{m_1}^{x} \frac{(a + (n-1)t(1/2-m) - nt(x-m))^2}{(n+1)^2 b} dm \\
&\quad + \int_{x}^{\frac{1}{2}} \frac{(a + (n-1)t(1/2-m) - nt(m-x))^2}{(n+1)^2 b} dm \\
&\quad + \int_{\frac{1}{2}}^{m_3} \frac{(a + (n-1)t(m-1/2) - nt(m-x))^2}{(n+1)^2 b} dm + \int_{m_3}^{\frac{1+x}{2}} \frac{(t(1-m) - t(m-x))^2}{b} dm \\
&= \frac{8(n-1)n(2n+1)^2 t^3 x^3}{12b(n+1)^2(2n+1)^2 t}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{6t^2(4(4n^3-n+1)a+(4n^4-4n^3-3n^2-1)t)x^2+6(4n^3-n+1)(4a-t)t^2x}{12b(n+1)^2(2n+1)^2t}\\
&+\frac{-8a^3(4n+3)+24a^2(n+1)^2t-6a(4n^3+n^2+n+2)t^2+(2n^4+6n^3-n^2-n+2)t^3}{12b(n+1)^2(2n+1)^2t}.
\end{aligned}
$$

Differentiating it with respect to x, we have:

$$
\frac{\partial \pi_i}{\partial x}=\frac{t(1-2x)((4a-t)(4n^3-n+1)+2nt(1+3n-4n^3)x)}{2b(n+1)^2(2n+1)^2}. \tag{79}
$$

If $t \leq 4(4n^3-n+1)a/(2n^2+n-1)^2$, this is non-negative for any $x \leq 1/2$.

(ii) $x \in [(2a-nt)/2t, (2a+(n-1)t)/4nt]$. In this case, the public firm does not supply at $m \in [m_2, m_3]$. The profit of the private firm locating at $x \in [(2a-nt)/2t, (2a+(n-1)t)/4nt]$ is:

$$
\begin{aligned}
\pi_i =& \int_{\frac{x}{2}}^{x} \frac{(tm-t(x-m))^2}{b}dm+\int_{x}^{m_2}\frac{(tm-t(m-x))^2}{b}dm\\
&+\int_{m_2}^{\frac{1}{2}}\frac{(a+(n-1)t(1/2-m)-nt(m-x))^2}{(n+1)^2b}dm\\
&+\int_{\frac{1}{2}}^{m_3}\frac{(a+(n-1)t(m-1/2)-nt(m-x))^2}{(n+1)^2b}dm+\int_{m_3}^{\frac{1+x}{2}}\frac{(t(1-m)-t(m-x))^2}{b}dm\\
=& -\frac{8n(8n^4+20n^3+26n^2+11n-1)t^3x^3}{12b(n+1)^2(2n+1)^2(2n-1)t}\\
&+\frac{6(4(8n^3+6n^2+n+1)a+(8n^5+4n^4+2n^3-3n^2-6n-1)t)t^2x^2}{12b(n+1)^2(2n+1)^2(2n-1)t}\\
&-\frac{6(4n^3-n+1)(2a-nt)^2tx+(6n^2+3n-1)(2a-nt)^3}{12b(n+1)^2(2n+1)^2(2n-1)t}.
\end{aligned}
$$

Differentiating it with respect to x, we have:

$$
\begin{aligned}
h(x) \equiv& -\frac{24n(8n^4+20n^3+26n^2+11n-1)t^3x^2}{12b(n+1)^2(2n+1)^2(2n-1)t}\\
&+\frac{12(4(8n^3+6n^2+n+1)a+(8n^5+4n^4+2n^3-3n^2-6n-1)t)t^2x}{12b(n+1)^2(2n+1)^2(2n-1)t}\\
&-\frac{6(4n^3-n+1)(2a-nt)^2t}{12b(n+1)^2(2n+1)^2(2n-1)t}.
\end{aligned}
$$

We now show that $h(x)$ is positive for any $x \in [(2a-nt)/2t, (2a+(n-1)t)/4nt]$. We can easily show that $h''(x) < 0$. That is, $h(x)$ is a concave function. On $[(2a-nt)/2t, (2a+(n-1)t)/4nt]$,

$h(x)$ is minimized at one of the boundaries, $x = (2a - nt)/2t$ or $x = (2a + (n-1)t)/4nt$. Substituting each of the boundaries into $h(x)$, we have:

$$\begin{aligned} h\left(\frac{2a-nt}{2t}\right) &= \frac{(n+1)^2(2n-1)(2n+1)^2(2a-nt)^2((3+2n)t-4a}{24b(n+1)^2(2n+1)^2(2n-1)t} > 0, \\ h\left(\frac{2a+(n-1)t}{4nt}\right) &= \frac{(2n-1)}{48bn^2(n+1)^2(2n+1)^2(2n-1)t} \times \\ & [(n+1)^2(4n^4+4n^3-n^2-n+2)t^3 - 3(n+1)^2(12n^3+8n^2-5n+5)at^2 \\ & \quad +12(n+1)(16n^3+8n^2-2n+3)a^2t - 4(52n^3+24n^2-3n+7)a^3]. \end{aligned}$$

We now define that $k(t) \equiv (n+1)^2(4n^4+4n^3-n^2-n+2)t^3 - 3(n+1)^2(12n^3+8n^2-5n+5)at^2+12(n+1)(16n^3+8n^2-2n+3)a^2t-4(52n^3+24n^2-3n+7)a^3$. Substituting $t = 2a/(n+1)$ (the lowest value of t in this section) into $k(t)$, we have

$$k\left(\frac{2a}{n+1}\right) = \frac{16n^2(2n-1)(2n+1)^2a^3}{(2n-1)(n+1)} > 0. \tag{80}$$

If $k'(t)$ is positive for any t, $k(t)$ is positive for any $t > 2a/(n+1)$, that is, $h((2a+(n-1)t)/4nt)$ is positive. $k'(t)$ is

$$\begin{aligned} k'(t) &= 3(n+1)^2(4n^4+4n^3-n^2-n+2)t^2 \\ & \quad -6(n+1)^2(12n^3+8n^2-5n+5)at + 12(n+1)(16n^3+8n^2-2n+3)a^2. \end{aligned}$$

This is a convex function and minimized at $t = (12n^3+8n^2-5n+5)a/(4n^4+4n^3-n^2-n+2)$. Substituting it into $k'(t)$, we have

$$k'\left(\frac{(12n^3+8n^2-5n+5)a}{4n^4+4n^3-n^2-n+2}\right) = \frac{3(n-1)(n+1)(2n+1)^4(7n^2-4n+1)a^2}{4n^4+4n^3-n^2-n+2} \geq 0.$$

We find that $k'(t)$ is non negative and $k(t)$ is non decreasing. $k(t)$ is positive for any $t > 2a/(n+1)$, that is, $h((2a+(n-1)t)/4nt)$ is positive. Therefore, $h(x)$ is positive for any $x \in [(2a-nt)/2t, (2a+(n-1)t)/4nt]$, that is, the optimal location of the private firm does not exist on $[(2a-nt)/2t, (2a+(n-1)t)/4nt]$.

(iii) $x \in [0, (2a-nt)/2t]$. In this case, the public firm supplies at all points on the circular city. The profit of the private firm locating at $x \in [0, (2a-nt)/2t]$ is:

$$\pi_i = \int_{\frac{x}{2}}^{x} \frac{(tm - t(x-m))^2}{b} dm + \int_{x}^{\frac{1}{2}} \frac{(tm - t(m-x))^2}{b} dm + \int_{\frac{1}{2}}^{\frac{1+x}{2}} \frac{(t(1-m) - t(m-x))^2}{b} dm$$

$$
\begin{aligned}
&= \frac{t^2x^2(3-4x)}{6b}, \\
\frac{\partial \pi_i}{\partial x} &= \frac{t^2x(1-2x)}{b}.
\end{aligned}
$$

π_i is increasing with respect to x. This implies that the optimal location of the private firm does not exist on $[0,(2a-nt)/2t]$.

Accordingly, we have that the optimal location of the private firm is $x = 1/2$ if $t \leq 4(4n^3 - n + 1)a/(2n^2 + n - 1)^2$. Q.E.D.

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