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Privatization and entries of foreign enterprises in a differentiated industry

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Abstract

We investigate whether or not privatization is beneficial from the viewpoint of social welfare in a monopolistic competition model. We discuss the relationship between the welfare effects of privatization and the degree of foreign direct investment in the private sector, which is an important problem in developing countries and in transition economies such as China and Central and Eastern European countries. We find that, in the long run, privatization of a public firm is more likely to improve welfare when the country depends on foreign capital in the private sector, whereas the opposite tendency exists in the short run.

JEL classification numbers: H42, L13, C72

Key words: mixed oligopoly, privatization, product differentiation, foreign firms, free entry

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1 Introduction

Since the 1980s, we have observed a worldwide wave of privatization. Nevertheless, public firms still exist, and many of them compete with private firms in private goods markets in developed, developing, and former communist countries. Competition between public and private firms existed, or still exists, in a range of industries, including the airline, rail, telecommunications, natural gas, electricity, steel, and overnight-delivery industries, as well as services including banking, home loans, health care, life insurance, hospitals, broadcasting, and education.[1]

Recently, studies of "mixed markets", involving both private and public enterprises, have become increasingly popular.[2] Most existing works have conducted short-run analyses, in which the number of firms is given exogenously. Anderson, de Palma, and Thisse (1997) is a pioneering work that provides a long-run analysis in mixed markets. They use a model with monopolistic competition. They show that, in the short run, the privatization of a public firm never improves welfare, whereas, in the long run, privatization may or may not improve welfare.[3]

Most existing works on mixed oligopoly, as well as that by Anderson, de Palma, and Thisse (1997), investigate the competition between public and domestic private firms. In real world economies, however, competitors of public firms are not limited to domestic private firms. For

[1] See, e.g., Matsushima and Matsumura (2003).

[2] This interest in mixed oligopolies is because of their importance to the economies of Europe, Canada, and Japan. Although they are less significant in the United States, there are some examples of mixed oligopolies such as the packaging and overnight-delivery industries. See Bös (1986, 1991) and Nett (1993) for excellent surveys of mixed oligopolies. The idea of mixed oligopoly dates to at least Merrill and Schneider (1966). Recently, the literature on mixed oligopoly has become richer and more diverse. For example, Ishibashi and Matsumura (2006) investigate R&D competition between public and private sectors. Bárcena-Ruiz and Garzón (2006) and Ohori (2006) analyze environmental policies. Mujumdar and Pal (1998) consider tax effects. Bárcena-Ruiz and Garzón (2003) discuss a merger problem. Pal (1998) and Matsumura (2003a) discuss endogenous role. Cremer, Marchand, and Thisse (1991), Li (2006), and Matsushima and Matsumura (2003) analyze endogenous product differentiation.

[3] For other papers on the long-run effect in mixed markets, see Futagami (1999) and Matsumura and Kanda (2005).

example, the New Zealand government set up a state-owned public bank to compete against private foreign banks. Similarly, when the government of Brazil bargained with the Swiss medical company Roche, it used a public medical institution as a potential competitor in the domestic market. Électricité de France and Gas de France also compete against foreign private firms in the EU energy markets. Especially in former communist transitional countries such as Eastern Europe countries and China, many national firms compete against both foreign and domestic private firms.[4]

In this paper, we consider foreign competitors explicitly and investigate how the presence of foreign competitors affects the efficiency of privatization. We extend the mixed oligopoly model of Anderson, de Palma, and Thisse (1997) to the case of foreign competitors. We find that the existence of foreign firms increases the welfare loss from the privatization of a public firm in the short run. In the long run, the existence of foreign firms causes the privatization to be more likely to improve welfare. If all private firms are foreign, the privatization of a public firm *always* improves welfare.

The result is noteworthy. When a public firm competes against foreign firms, the public firm lowers its price to enlarge the consumer surplus and to reduce the welfare loss, which stems from the profits of the foreign firms. In fact, in the short run, the social value of the public firm increases in the presence of foreign competitors. In the long run, however, this property does not hold. In equilibrium, social welfare is equal to consumer surplus plus the profit of a public firm because the profits of private firms are zero (as a result of free entry). The consumer surplus depends on product diversity. As discussed above, the public firm aggressively sets a lower price under foreign competition, resulting in a smaller number of entries. This loss increases in the presence of foreign competitors, and privatization is always beneficial when all competitors are foreign.

The relationship between the number of domestic and foreign firms and privatization is

[4] For competition between public and foreign private firms see Fjell and Pal (1996), Pal and White (1998), Matsumura (2003b), and Matsushima and Matsumura (2006).

related to the literature on foreign direct investment (FDI) into transition economies such as Central and Eastern European countries (CEECs). In our paper, in the long-run equilibrium, privatization induces the entry of foreign and domestic firms. That is, privatization has a positive impact on FDI into the host country. Using dynamic panel data methods, Carstensen and Toubal (2004) examine the determinants of FDI into CEECs. They show that both the level of privatization measured by private market share and the actual method of privatization have considerable positive impacts on the decision to invest in CEECs. Our results are consistent with the empirical evidence of the positive impact on the FDI decision.[5]

The remainder of this paper is organized as follows. In Section 2, we explain the basic setting of mixed oligopoly. Section 3 conducts a short-run analysis. It demonstrates that privatization never improves welfare and that welfare loss from privatization is larger when the competitors are foreign. Section 4 conducts a long-run analysis. It shows that privatization always improves welfare when all private firms are foreign. Section 5 discusses how cost differences between public and private firms affect our results. Section 6 is the conclusion.

2 The model

We first describe a market equilibrium in which all firms are privately owned. There are $n+1$ variants of a differentiated product. Market demand and consumer benefits are determined by a representative consumer's utility function:

$$U(x_1, \ldots, x_{n+1}; x_0) = \frac{1}{\rho} \ln \left(\sum_{i=1}^{n+1} x_i^{\rho} \right) + x_0, \tag{1}$$

where x_i denotes the quantity of variant i, x_0, the quantity of the numeraire, and $\rho \in (0, 1)$, the parameterized consumer's preference for variety. The restriction on ρ ensures that products are substitutes, with a high ρ corresponding to a low preference for variety; when $\rho \to 1$, all goods are perfect substitutes, while preferences have the Cobb–Douglas form for $\rho \to 0$. The

[5] Carstensen and Toubal (2004) also point out that the South Eastern European Countries, Bulgaria and Romania, were unsuccessful in attracting FDI during the first half of 1990s, and then began to attract investors only after they changed to foreign-oriented privatization policies in the late 1990s.

budget constraint of the consumer is:

$$\sum_{i=1}^{n+1} p_i x_i + x_0 = Y, \tag{2}$$

where p_i is the price of variant i, and Y, the consumer's income. Maximizing Eq. (1) subject to Eq. (2) yields the demand functions for the $(n+1)$ variants:

$$\bar{x}_i = \frac{p_i^{-\lambda-1}}{\sum_{j=1}^{n+1} p_j^{-\lambda}}, \quad i = 1, \ldots, n+1, \tag{3}$$

where $\lambda \equiv \rho/(1-\rho)$. Hence, λ is an inverse measure of consumer preference for variety: the smaller the former, the stronger the latter. When $\lambda = 0$, the variants are considered to be fully differentiated; when $\lambda \to \infty$, all the variants are regarded as perfect substitutes, and the consumer(s) buys the variant with the lowest price. Because total expenditure on the differentiated product is 1, we assume throughout that $Y > 1$ in order to have an interior solution to the representative consumer's maximization problem.

We now consider the supply side. Firm i produces variant i at a constant marginal cost c and bears a fixed cost F, which is sunk in the short run. These costs are assumed to be the same across firms so that all firms will be equally efficient, independent of their ownership. In other words, we ignore the cost difference between public and private firms, as did Anderson, de Palma, and Thisse (1997). Many (but not all) empirical works show that a public enterprise performs less efficiently than a private enterprise. In Section 5, we discuss how cost differences between public and private firms affect the results.[6] Hence, firm i's profits are:

$$\pi_i = (p_i - c)\bar{x}_i - F, \quad i = 1, \ldots, n+1. \tag{4}$$

In a case in which all firms are private, it is well known that there exists a unique Nash equilibrium when the $(n+1)$ firms compete in prices. The equilibrium is symmetric and is given by:

$$p^* = c\left[1 + \frac{(n+1)(1-\rho)}{n\rho}\right] = c\left(1 + \frac{n+1}{n\lambda}\right). \tag{5}$$

[6] For discussions of endogenous cost differences, see Bös and Peters (1995), Corneo and Rob (2003), Ishibashi and Matsumura (2006), Matsumura and Matsushima (2004), and Nett (1993, 1994).

The equilibrium outputs are the same for all firms, and the output per firm x^* is equal to:

$$x^* = \frac{n}{n+1} \frac{\rho}{(n+1-\rho)c}. \tag{6}$$

The total surplus of the variants is:

$$\Pi^* = \sum_{i=1}^{n+1} \pi_i^* = (n+1)\left(\frac{1-\rho}{n+1-\rho} - F\right).$$

To remove the case of natural monopolies, we assume that:

$$F < \frac{1-\rho}{2-\rho}.$$

In equilibrium, consumer surplus is given by the indirect utility function:

$$V^* = \frac{1}{\rho} \ln[(n+1)^{(1-\rho)}(p^*)^{-\rho}] + Y - 1, \tag{7}$$

where p^* is given by (5). Finally, the social surplus evaluated at equilibrium is:

$$W^* = V^* + \sum_{i=1}^{n+1} \alpha_i \pi_i^*, \tag{8}$$

where $\alpha_i \in [0,1]$ is the ownership share of domestic investors for firm i.

Now, we suppose that there is free entry and exit in the industry. This can be viewed as the long-run equilibrium whereby no costs are sunk. The equilibrium number of firms is obtained from the zero-profit condition:

$$n^e = \frac{1-\rho}{F} + \rho, \tag{9}$$

while the corresponding equilibrium price is:

$$p^e = \frac{c}{\rho(1-F)}. \tag{10}$$

3 The effect of privatization in the short run

To consider the effects of privatization, we characterize the equilibrium in which one firm is public and n firms are private. We let firm $n+1$ be the public firm and look for a price equilibrium at which the public firm maximizes social surplus. The government holds all shares in firm $n+1$, so $\alpha_{n+1} = 1$. For simplicity, we assume that $\alpha_i = \alpha$ for all $i \in \{1, 2, ..., n\}$.. The first-order conditions for the private firms are identical so that all private firms charge the same price $\hat{p}$, which is different from the price $\tilde{p}$ elected by the public firm. The objective function of the public firm is written as follows:

$$W = V(\hat{p}, \tilde{p}) + \alpha n \hat{\pi}(\hat{p}, \tilde{p}) + \tilde{\pi}(\hat{p}, \tilde{p}), \tag{11}$$

where $V(\hat{p}, \tilde{p})$ is indirect utility, $\hat{\pi}(\hat{p}, \tilde{p}) = (\hat{p} - c)\hat{x} - F$ is the profit of a private firm, and $\tilde{\pi} = (\tilde{p} - c)\tilde{x} - F$ is the profit made by the public firm evaluated at prices $(\hat{p}, \tilde{p})$.

To derive the equilibrium strategies of the private firms, we define that $\hat{X} \equiv \hat{P}/\hat{p}$ and $\tilde{X} \equiv \tilde{P}/\tilde{p}$ with:

$$\hat{P} \equiv \frac{\hat{p}^{-\lambda}}{n\hat{p}^{-\lambda} + \tilde{p}^{-\lambda}}, \quad \tilde{P} \equiv \frac{\tilde{p}^{-\lambda}}{n\hat{p}^{-\lambda} + \tilde{p}^{-\lambda}}. \tag{12}$$

Let us suppose that supplier i charges p_i, the private firms, $\hat{p}$, and the public firm, $\tilde{p}$. The demand function for firm i can then be rewritten as $\hat{X}_i = P_i/p_i$ with P_i defined by:

$$P_i \equiv \frac{p_i^{-\lambda}}{p_i^{-\lambda} + (n-1)\hat{p}^{-\lambda} + \tilde{p}^{-\lambda}}, \quad i = 1, \ldots, n. \tag{13}$$

Differentiating firm i's profit with respect to p_i, setting p_i equal to $\hat{p}$ and equating to zero yields the following implicit equation for $\hat{p}$:

$$\hat{p} = c\left[1 + \frac{1}{\lambda(1 - \hat{P})}\right], \tag{14}$$

where $\hat{P}$ is Eq. (13) evaluated at $p_i = \hat{p}$.

The first-order condition of the public firm is:

$$\begin{aligned}
\frac{\partial W}{\partial \tilde{p}} &= \frac{\partial V(\hat{p}, \tilde{p})}{\partial \tilde{p}} + \alpha n \frac{\partial \hat{\pi}(\hat{p}, \tilde{p})}{\partial \tilde{p}} + \frac{\partial \tilde{\pi}(\hat{p}, \tilde{p})}{\partial \tilde{p}} \\
&= -\tilde{x} + \tilde{x} + \alpha n(\hat{p} - c)\frac{\partial \hat{x}}{\partial \tilde{p}} + (\tilde{p} - c)\frac{\partial \tilde{x}}{\partial \tilde{p}} \quad (\because \text{Roy's identity}) \\
&= \alpha n(\hat{p} - c)\frac{\partial \hat{x}}{\partial \tilde{p}} + (\tilde{p} - c)\frac{\partial \tilde{x}}{\partial \tilde{p}} = 0. \qquad (15)
\end{aligned}$$

From the first-order condition and Eq. (12), we have the following lemma:

Lemma 1 *The public firm sets its price, which is lower than that of the private firms.*

Proof: From Eq. (12), we have:

$$\begin{aligned}\frac{\partial \hat{x}}{\partial \tilde{p}} &= \frac{\lambda \hat{p}^{-\lambda-1}\tilde{p}^{-\lambda-1}}{(n\hat{p}^{-\lambda}+\tilde{p}^{-\lambda})^2} = \lambda \frac{\hat{P}\tilde{P}}{\hat{p}\tilde{p}},\\ \frac{\partial \tilde{x}}{\partial \tilde{p}} &= \frac{-\tilde{p}^{-2\lambda} - n(\lambda+1)\tilde{p}^{-\lambda}\hat{p}^{-\lambda}}{\tilde{p}^2(n\hat{p}^{-\lambda}+\tilde{p}^{-\lambda})^2} = \frac{-\tilde{P}(n\lambda\hat{P}+1)}{\tilde{p}^2}.\end{aligned}$$

Substituting them into Eq. (15), we have:

$$\frac{\hat{p}-c}{\hat{p}} = \frac{\tilde{p}-c}{\tilde{p}}\left(\frac{1+n\lambda\hat{P}}{\alpha n\lambda\hat{P}}\right). \tag{16}$$

This equation implies Lemma 1. Q.E.D.

As mentioned in Anderson, de Palma, and Thisse (1997), the public firm takes into account consumer surplus, while the private firms do not. The public firm lowers its price to increase consumer surplus.

We now compare the price configuration above to the equilibrium price in Eq. (5) in the fully private oligopoly. From Lemma 1, we have the following lemma:

Lemma 2 *The equilibrium price of the private firm is lower in mixed oligopoly than that in private oligopoly where all firms are private.*

Proof: See Appendix.

We now discuss an effect of privatization of the public firm. Welfare after privatization is given by: (8) by substituting $\alpha_i = \alpha$ for $i \in \{1, 2, ..., n\}$ and $\alpha_{n+1} = 1$.[7] Lemmas 1 and 2 imply $p^* > \hat{p} > \tilde{p} \geq c$. That is, when there is a public firm in a differentiated oligopoly, all firms charge a lower price than they do in the pure market. As mentioned in Anderson, de Palma, and Thisse (1997), the allocation of consumers across variants is no longer optimal because the price of a public firm is lower than that of private firms. An answer is given in the following proposition.

[7] If foreign investors rather than domestic investors purchase the privatized firm, all of our results hold true as long as the privatized firm is correctly priced.

Proposition 1 *Social surplus falls in the short run if the public firm is privatized.*

Proof: See Appendix.

An implication of the proposition is that the existence of a public firm is always socially beneficial in a differentiated oligopoly with a fixed number of firms. A similar result is presented by Anderson, de Palma, and Thisse (1997) in the case where $\alpha = 1$.. Our Proposition 1 shows that their result holds regardless of α.

We then take a closer look at the relationship between α and the effect of privatization of the public firm. From Eqs. (14) and (16), we have the following lemma:

Lemma 3 *As α increases, $\hat{p}$ and $\tilde{p}$ increase.*

Proof: See Appendix.

The value of α represents the relative importance of private firms' profits from the viewpoint of social welfare. For instance, if the value is zero, the objective of the public firm is to maximize consumer surplus plus its own profit and then it sets its price equal to marginal cost. As the public firm takes into account the profits of the private firms (the value of α increases), the price of the public firm increases.

From Proposition 1 and Lemma 3, we find that, as the value of α increases, the prices of the private and public firms increase, and the profits of the firms increase. The former increment is a negative effect from the viewpoint of social welfare. In the latter increment, the increase in the domestic firms' profits is larger than that in the mixed oligopoly case because each firm's profit is larger than that in the mixed oligopoly ($\tilde{\pi} < \pi^*$, see Lemmas 1 and 2). The discussion leads to the following proposition:

Proposition 2 *As α increases, the welfare loss from privatization decreases.*

Proof: See Appendix.

The public firm sets its price at a lower level than the private firms because it takes consumer surplus into account. As the share of the foreign firms $(1 - \alpha)$ increases, the consumer surplus becomes more important than the producer surplus, so the public firm cuts its price. Through the strategic interaction between public and private firms, the prices of private firms fall. In other words, the public firm's pricing becomes tougher when $(1-\alpha)$ is large, and it effectively restricts the surplus belonging to the foreigner. Therefore, the difference between the two market configurations spreads, and we derive Proposition 2. This result states that the social value of the public firm increases in $1-\alpha$ (the share of foreign ownership). So in the short run, foreign investments in the private sector make the privatization of the public firm less beneficial for domestic welfare. As we show in the next section, however, this does not hold in the long run. In the long run, foreign investments in the private sector increase the benefit of privatization of the public firm.

4 The effect of privatization in the long run

In this section, we consider the effect of privatization in the long run. Suppose that private firms enter until they earn zero profits. Let $\tilde{n}$ be the number of private firms at the free-entry equilibrium involving one public firm. At this equilibrium, all private firms charge the same price $\hat{p}^e$, while the public firm sets a price $\tilde{p}^e$. Moreover, we have:

Lemma 4 *At the free-entry equilibrium, the private firms charge the same price regardless of the presence of a public firm:* $\hat{p}^e = p^e$.

Proof: Assume that there is a public firm in the market. For each private firm i, the first-order condition is:

$$\rho - 1 + (p_i - c)\left(\frac{1}{p_i} - \rho \hat{x}_i\right) = 0.$$

Substituting $\hat{x}_i$ into the zero-profit condition, we have the same price as Eq. (10). Q.E.D.

Note that the numbers of firms are not the same in the two market configurations. When n^e firms set p^e, the private firms make zero profits. From Lemmas 2 and 4, $\tilde{n}+1 < n^e$, that is, fewer firms are in business when one of them is a public firm.

The private firms' profits being zero by the free-entry condition, the social surplus at the free-entry equilibrium with a public firm can be rewritten as:

$$\tilde{W}^e = V(\hat{p}^e, \tilde{p}^e) + \tilde{\pi}^e(\hat{p}^e, \tilde{p}^e).$$

Because the social surplus at the free-entry equilibrium with private firms is given only by $W^e = V(p^e)$, we have:

$$\tilde{W}^e - W^e = \left(\frac{1}{\rho}\ln[\tilde{n}(\hat{x}^e)^\rho + (\tilde{x}^e)^\rho] + \tilde{\pi}^e(\hat{p}^e, \tilde{p}^e)\right) - \left(\frac{1}{\rho}\ln[n^e(x^e)^\rho]\right), \tag{17}$$

where $\hat{x}^e$ and $\tilde{x}^e$ denote a private or a public firm's demand at the preprivatization free-entry equilibrium, while x^e is a private firm's demand at the postprivatization case.

Proposition 3 *(i) At the free-entry equilibrium, the social gain from privatization is equal to the loss made by the public firm before it is privatized, and (ii) privatization improves welfare if α is sufficiently close to 0.*

Proof: Because $\hat{p}^e = p^e$ by Lemma 4, the zero-profit condition implies that each private firm sells the same quantity: $\hat{x}^e = x^e$. Accordingly, it follows from Eq. (3) that:

$$\frac{\hat{p}^{-\lambda-1}}{n^e\hat{p}^{-\lambda}} = \frac{\hat{p}^{-\lambda-1}}{\tilde{n}\hat{p}^{-\lambda} + \tilde{p}^{-\lambda}} \quad \Rightarrow \quad n^e = \tilde{n} + \left(\frac{\tilde{p}^e}{\hat{p}^e}\right)^{-\lambda}. \tag{18}$$

Using Eq. (3), we have:

$$\left(\frac{\tilde{x}^e}{\hat{x}^e}\right)^\rho = \left(\frac{\tilde{p}^{-\lambda-1}}{\hat{p}^{-\lambda-1}}\right)^\rho = \left(\frac{\tilde{p}^e}{\hat{p}^e}\right)^{-\lambda}.$$

Substituting it into Eq. (17), we have:

$$\begin{aligned}
\tilde{W}^e - W^e &= \left(\frac{1}{\rho}\ln\left[(\hat{x}^e)^\rho\left(\tilde{n} + \left(\frac{\tilde{p}^e}{\hat{p}^e}\right)^{-\lambda}\right)\right] + (\tilde{p}^e - c)\tilde{x}^e - F\right) - \left(\frac{1}{\rho}\ln[n^e(x^e)^\rho]\right) \\
&= \frac{1}{\rho}\ln\left[(\hat{x}^e)^\rho n^e\right] + (\tilde{p}^e - c)\tilde{x}^e - F - \frac{1}{\rho}\ln[n^e(x^e)^\rho] \\
&= (\tilde{p}^e - c)\tilde{x}^e - F, \quad (\because \text{Eq.(18) and } \hat{x}^e = x^e).
\end{aligned}$$

Therefore, Proposition 3(i) holds.

From (16), we have that $\tilde{p}$ is sufficiently close to c when the value of α is sufficiently close to 0. Because F is positive, the public firm has a deficit. From Proposition 3(i), we have

Proposition 3 (ii). Q.E.D.

The second result contains rich implications. As opposed to the short run, privatization improves welfare if the presence of foreign capital in the private sector is sufficiently large. At first glance, under the presence of foreign private firms, the existence of a public firm enhances welfare because the public firm lowers its price to enlarge consumer surplus and to reduce the welfare loss, which stems from the profits of the foreign firms. This effect in fact works in the short run (Proposition 2). In the long run, however, this is not true. In equilibrium, social welfare is equal to consumer surplus plus the profit of the public firm because the profits of private firms are zero (because of free entry). Consumer surplus depends on product diversity, that is, the number of firms is important for consumers. Because the public firm lowers its price, the number of entrants is lower than the case where the public firm privatizes. As the share held by foreigners increases, the loss of diversity induced by the smaller entry becomes significant because the price of a public firm becomes lower. This is why privatization improves welfare if α is close to zero.

5 Inefficiency of the public firm

In previous sections we ignore the cost difference between public and private firms. In reality, a public enterprise often performs less efficiently than a private enterprise. In this section, we assume that the public firm is less efficient. We assume that $\tilde{c} > c$, where $\tilde{c}$ is the marginal cost of the public firm.

5.1 Short run

Most of the calculus is similar to that in the former setting. We only show the difference between these settings.

We first derive the first-order condition of the public firm. This is related to Eq. (15) and as follows:

$$\frac{\partial W}{\partial \tilde{p}} = \alpha n(\hat{p} - c)\frac{\partial \hat{x}}{\partial \tilde{p}} + (\tilde{p} - \tilde{c})\frac{\partial \tilde{x}}{\partial \tilde{p}} = 0. \quad (19)$$

As in the proof of Lemma 1, we can rewrite Eq. (19) as follows:

$$\frac{\hat{p}-c}{\hat{p}} = \frac{\tilde{p}-\tilde{c}}{\tilde{p}}\left(\frac{1+n\lambda\hat{P}}{\alpha n\lambda\hat{P}}\right). \tag{20}$$

The first-order conditions of the private firms are presented in (14), that is,

$$\hat{p} = c\left[1+\frac{1}{\lambda(1-\hat{P})}\right]. \tag{14'}$$

From (14') and (20), we have the following Lemma:

Lemma 5 *The price of the public firm $\tilde{p}$ is equal to those of the private firms $\hat{p}$ if and only if:*

$$\tilde{c} = \left[1+\frac{(n+1)(n+1+(1-\alpha)n\lambda)}{n\lambda(n+1+n\lambda)}\right]c \equiv c^p. \tag{22}$$

$\tilde{p} > \hat{p} > p^*$, if and only if $\tilde{c} > c^p$. $\tilde{p} < \hat{p} < p^*$, if and only if $\tilde{c} < c^p$.

From (14') and (20), we also have the following Lemma:

Lemma 6 *Social welfare decreases as $\tilde{c}$ increases.*

From Lemmas 5 and 6, we have the following proposition:

Proposition 4 *Privatization of the public firm enhances social welfare if and only if $\tilde{c} > \tilde{c}^*$ $(c < \tilde{c}^* < c^p)$.*

Proof: From lemma 5, when $\tilde{c} = c^p$, $\tilde{p} = \hat{p} = p^*$. If the public firm is privatized, then prices do not change, but the efficiency of the public firm improves. We find that privatization improves welfare when $\tilde{c} = c^p$. As shown in Proposition 1, privatization reduces welfare when $\tilde{c} = c$. From lemma 6, as the value of $\tilde{c}$ increases, social welfare decreases. There exists the value of $\tilde{c} = \tilde{c}^*$ in which privatization is indifferent from the viewpoint of social welfare. Therefore, Proposition 4 holds. Q.E.D.

5.2 Long run

In this subsection, we consider the effect of privatization in the long run when the public firm is less efficient. Lemma 4 in the former section also holds in this case. Moreover, Proposition 3 also holds. The proof is made if we replace c with $\tilde{c}$ in the proof of Proposition 3. Obviously, given the costs of private firms, an increase in the cost of the public firm reduces its profits; thus, under Proposition 3 privatization of the public firm is more likely to improve welfare under cost difference between public and private firms.

6 Concluding remarks

In this paper, we extend the mixed oligopoly model of Anderson, de Palma, and Thisse (1997) to a case with foreign competitors. We find that the social value of the public firm increases under the presence of foreign competitors in the short run. On the contrary, in the long run, privatization of a public firm is more likely to improve welfare when the competitors of the public firm are foreign than when they are domestic. It is noteworthy that, when all private firms are owned by foreigners, privatization always improves welfare.

This result indicates that the time inconsistency problem becomes more serious when private competitors are foreign. The government has an incentive to privatize the public firm so as to stimulate new entry of foreign firms. After the entry, however, the government has an incentive to nationalize the privatized firm again. Expecting this renationalization, the entry of foreign firms is restricted. The market integration increases the possibility of international competition, and public firms are more likely to face competition against foreign firms. Under these conditions, it is more important to make a commitment not to renationalize the privatized firm.

In this paper, we assume that the public firm is a welfare maximizer. Although this assumption is very popular among the models of mixed oligopoly,[8] other approaches also exist.[9]

[8] See, among others, De Fraja and Delbono (1989), Fjell and Pal (1996), Pal (1998), and Matsumura (1998).

[9] See, e.g., Fershtman (1990), Futagami (1999), and Sappington and Sidak (2003).

Deviation from this welfare-maximizing assumption and the application of other approaches to this problem remain topics for future research.

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Appendix

Proof of Lemma 2: From Lemma 1, $\tilde{p}$ is smaller than $\hat{p}$. We rearrange $\hat{P}$ in Eq. (12):

$$\hat{P} = \frac{1}{n + (\tilde{p}/\hat{p})^{-\lambda}} = \frac{1}{n + (\hat{p}/\tilde{p})^{\lambda}} < \frac{1}{n+1}.$$

Using the inequality, we rearrange $\hat{p}$ in Eq. (14):

$$\hat{p} = c\left[1 + \frac{1}{\lambda(1-\hat{P})}\right] < c\left[1 + \frac{1}{\lambda(1-1/(n+1))}\right] = c\left[1 + \frac{1+n}{\lambda n}\right] = p^*.$$

$\hat{p}$ in the implicit equation in Eq. (14) is smaller than p^*. Q.E.D.

Proof of Proposition 1: We now show that social welfare is decreasing with respect to $\hat{p}$. If this is true, from Lemma 2, the proposition holds. Differentiating W in Eq. (11) with respect to $\hat{p}$, we have:

$$\begin{aligned}
\frac{\partial W}{\partial \hat{p}} &= \frac{\partial V}{\partial \hat{p}} + \alpha n \frac{\partial \hat{\pi}}{\partial \hat{p}} + \frac{\partial \tilde{\pi}}{\partial \hat{p}} \\
&= -n\hat{x} + \alpha n \hat{x} + \alpha n(\hat{p}-c)\frac{\partial \hat{x}}{\partial \hat{p}} + (\tilde{p}-c)\frac{\partial \tilde{x}}{\partial \hat{p}} \\
&= -(1-\alpha)n\hat{x} + \alpha n(\hat{p}-c)\frac{\partial \hat{x}}{\partial \hat{p}} + (\tilde{p}-c)\frac{\partial \tilde{x}}{\partial \hat{p}}.
\end{aligned}$$

Because $\tilde{p}$ maximizes W in Eq. (11), Eq. (15) holds, so that:

$$\begin{aligned}
\frac{\partial W}{\partial \hat{p}} &= -(1-\alpha)n\hat{x} + \alpha n(\hat{p}-c)\frac{\partial \hat{x}}{\partial \hat{p}} - \alpha n(\hat{p}-c)\frac{\partial \hat{x}}{\partial \tilde{p}}\frac{\partial \tilde{x}}{\partial \hat{p}} \Big/ \frac{\partial \tilde{x}}{\partial \tilde{p}} \\
&< \frac{\alpha n(\hat{p}-c)}{\partial \tilde{x}/\partial \tilde{p}}\left[\frac{\partial \hat{x}}{\partial \hat{p}}\frac{\partial \tilde{x}}{\partial \tilde{p}} - \frac{\partial \hat{x}}{\partial \tilde{p}}\frac{\partial \tilde{x}}{\partial \hat{p}}\right] \\
&= \frac{\alpha n(\hat{p}-c)}{-(n(\lambda+1)\tilde{p}^{-\lambda-2}\hat{p}^{-\lambda} + \tilde{p}^{-2\lambda-2})/(np^{-\lambda} + \tilde{p}^{-\lambda})^2}\left[\frac{(\lambda+1)\hat{P}\tilde{P}}{\hat{p}^2\tilde{p}^2}\right] < 0.
\end{aligned}$$

Therefore, Proposition 1 holds. Q.E.D.

Proof of Lemma 3: We now show that $d\hat{p}/d\alpha > 0$ and $d\tilde{p}/d\alpha > 0$. $\hat{p}$ and $\tilde{p}$ are determined by Eqs. (14) and (16). When α changes, $\hat{p}$ and $\tilde{p}$ also change. To identify the changes, we differentiate Eqs. (14) and (16).

The total differentiation of Eq. (14) is:

$$\frac{\lambda\hat{P}\tilde{P}}{\tilde{p}}\frac{d\tilde{p}}{d\alpha} = \left[\frac{\lambda\hat{P}\tilde{P}}{\hat{p}} + \frac{\lambda(1-\hat{P})^2}{c}\right]\frac{d\hat{p}}{d\alpha}. \tag{23}$$

The coefficients of $d\tilde{p}/d\alpha$ and $d\hat{p}/d\alpha$ are positive. Their signs are the same.

The total differentiation of Eq. (16) is:

$$\frac{\lambda(\hat{p}-c)\tilde{P} - c(1+n\lambda\hat{P})}{\hat{p}^2(1+n\lambda\hat{P})}\frac{d\hat{p}}{d\alpha} + \frac{c(1+n\lambda\hat{P}) - \lambda(\tilde{p}-c)\tilde{P}}{\tilde{p}^2\alpha n\lambda\hat{P}}\frac{d\tilde{p}}{d\alpha} = \frac{\hat{p}-c}{\alpha\hat{p}}. \tag{24}$$

Substituting $d\tilde{p}/d\alpha$ in Eq. (23) into the left-hand side of Eq. (24), we have:

$$\begin{aligned}
&\left[\frac{\lambda(\hat{p}-c)\tilde{P} - c(1+n\lambda\hat{P})}{\hat{p}^2(1+n\lambda\hat{P})} + \frac{c(1+n\lambda\hat{P}) - \lambda(\tilde{p}-c)\tilde{P}}{\tilde{p}^2\alpha n\lambda\hat{P}}\cdot\frac{\tilde{p}}{\lambda\hat{P}\tilde{P}}\left(\frac{\lambda\hat{P}\tilde{P}}{\hat{p}} + \frac{\lambda(1-\hat{P})^2}{c}\right)\right]\frac{d\hat{p}}{d\alpha}\\
> &\left[-\frac{1}{\hat{p}^2(1+n\lambda\hat{P})} + \frac{1}{\tilde{p}\alpha n\lambda\hat{P}}\left(\frac{1}{\hat{p}} + \frac{\lambda(1-\hat{P})^2}{c\hat{P}\tilde{P}}\right)\right]\frac{d\hat{p}}{d\alpha}\\
&(\because\ c(1+n\lambda\hat{P}) - \lambda(\tilde{p}-c)\tilde{P} > c(1+n\lambda\hat{P}) - \lambda(\hat{p}-c)\tilde{P})\\
= &\frac{1}{1+n\lambda\hat{P}}\left[-\frac{1}{\hat{p}^2} + \frac{1}{\tilde{p}}\cdot\frac{1+n\lambda\hat{P}}{\alpha n\lambda\hat{P}}\left(\frac{1}{\hat{p}} + \frac{\lambda(1-\hat{P})^2}{c\hat{P}\tilde{P}}\right)\right]\frac{d\hat{p}}{d\alpha}\\
= &\frac{1}{1+n\lambda\hat{P}}\left[-\frac{1}{\hat{p}^2} + \frac{1}{\tilde{p}}\cdot\frac{\tilde{p}(\hat{p}-c)}{(\tilde{p}-c)\hat{p}}\left(\frac{1}{\hat{p}} + \frac{\lambda(1-\hat{P})^2}{c\hat{P}\tilde{P}}\right)\right]\frac{d\hat{p}}{d\alpha}\quad(\because\ \text{Eq. (16)})\\
= &\frac{1}{1+n\lambda\hat{P}}\left[-\frac{(\tilde{p}-c)}{(\tilde{p}-c)\hat{p}^2} + \frac{(\hat{p}-c)}{(\tilde{p}-c)\hat{p}^2} + \frac{(\hat{p}-c)}{(\tilde{p}-c)\hat{p}}\cdot\frac{\lambda(1-\hat{P})^2}{c\hat{P}\tilde{P}}\right]\frac{d\hat{p}}{d\alpha}\\
= &\frac{1}{1+n\lambda\hat{P}}\left[\frac{\hat{p}-\tilde{p}}{(\tilde{p}-c)\hat{p}^2} + \frac{(\hat{p}-c)}{(\tilde{p}-c)\hat{p}}\cdot\frac{\lambda(1-\hat{P})^2}{c\hat{P}\tilde{P}}\right]\frac{d\hat{p}}{d\alpha}.
\end{aligned}$$

The coefficient of $d\hat{p}/d\alpha$ is positive. The right-hand side of Eq. (24) is positive. Therefore, $d\hat{p}/d\alpha$ is positive. We also find that $d\tilde{p}/d\alpha$ is positive. Q.E.D.

Proof of Proposition 2: Let W^* and $\tilde{W}$ be the welfare after and before privatization, respectively.

$$\frac{d\tilde{W}}{d\alpha} = \frac{\partial\tilde{W}}{\partial\hat{p}}\frac{d\hat{p}}{d\alpha} + \frac{\partial\tilde{W}}{\partial\tilde{p}}\frac{d\tilde{p}}{d\alpha} + n\tilde{\pi} \tag{25}$$

$$\frac{dW^*}{d\alpha} = n\pi^*. \tag{26}$$

From Proposition 1 and Lemma 3, we have that the first and second terms in (25) are negative. Because competition is more aggressive before privatization (see Lemmas 1 and 2), it is obvious that $\tilde{\pi} < \pi^*$. Under these conditions, (25) minus (26) is negative. Q.E.D.

Proof of Lemma 5: When $\hat{p} = \tilde{p}$, $\hat{P} = 1/(n+1)$ (see Eq. (12)). Taking this property into account, we solve Eq. (20) and then have Eq. (22).

Before we show the proof of the rest of the Lemma, we provide several properties related to the proof. $\hat{P}$ in Eq. (12) is larger than $1/(n+1)$ if and only if $\tilde{p} > \hat{p}$. From Eqs. (5) and (14'), we find that if $\tilde{p} > \hat{p}$, then $\hat{p} > p^*$, otherwise $\tilde{p} < \hat{p} < p^*$.

We now show that if $\tilde{c} < c^p$, then $\tilde{p} < \hat{p} < p^*$. We employ the proof by refutation. Suppose that $\hat{p} > p^*$ when $\tilde{c} < c^p$. We arrange Eq. (20) as follows:

$$H \equiv \frac{\hat{p}-c}{\hat{p}} - \frac{\tilde{p}-\tilde{c}}{\tilde{p}}\left(\frac{1+n\lambda\hat{P}}{\alpha n\lambda\hat{P}}\right).$$

If H is negative, the public firm sets a lower price. When the public firm sets its price at $\tilde{p} = \hat{p}$, H is:

$$\begin{aligned} H &= \frac{\hat{p}-c}{\hat{p}} - \frac{\hat{p}-\tilde{c}}{\hat{p}}\left(\frac{1+n\lambda/(n+1)}{\alpha n\lambda/(n+1)}\right) \\ &< \frac{\hat{p}-c}{\hat{p}} - \frac{\hat{p}-c^p}{\hat{p}}\left(\frac{1+n\lambda/(n+1)}{\alpha n\lambda/(n+1)}\right) \quad (\because \tilde{c} < c^p) \\ &< \frac{p^*-c}{p^*} - \frac{p^*-c^p}{p^*}\left(\frac{1+n\lambda/(n+1)}{\alpha n\lambda/(n+1)}\right) \quad (\because p^* < \hat{p}) \\ &= 0. \end{aligned}$$

Because $H < 0$, the public firm sets its price at $\tilde{p} < \hat{p}$. This contradicts the fact that if $\tilde{p} < \hat{p}$, $\hat{p} < p^*$. Therefore, when $\tilde{c} < c^p$, $p^* > \hat{p} > \tilde{p}$. Q.E.D.

Proof of Lemma 6: We first show that as the value of $\tilde{c}$ increases, the values of $\hat{p}$ and $\tilde{p}$ increase. Total differentials of the first-order conditions of the public and the private firms are:[10]

$$\frac{1}{\hat{p}}\left(\frac{\alpha n\lambda\hat{P}c}{\hat{p}}-\frac{\lambda\tilde{P}(\tilde{p}-\tilde{c})}{\tilde{p}}\right)d\hat{p}+\underbrace{\frac{1}{\tilde{p}^2}\left(\lambda\tilde{P}(\tilde{p}-\tilde{c})-\tilde{c}(1+n\lambda\hat{P})\right)}_{\text{the S.O.C of the public firm}}d\tilde{p}+\frac{1+n\lambda\hat{P}}{\tilde{p}}d\tilde{c}=0,$$

$$\tilde{p}\left((1-\hat{P})^2\hat{p}+c\hat{P}\tilde{P}\right)d\hat{p}-c\hat{P}\tilde{P}\hat{p}\,d\tilde{p}=0.$$

Substituting the second equation into the first, we have the coefficient of $d\hat{p}$ in the first equation as follows:

$$\begin{aligned}
& c\hat{P}\tilde{P}\hat{p}\cdot\frac{1}{\hat{p}}\left(\frac{\alpha n\lambda\hat{P}c}{\hat{p}}-\frac{\lambda\tilde{P}(\tilde{p}-\tilde{c})}{\tilde{p}}\right)+\frac{(\lambda\tilde{P}(\tilde{p}-\tilde{c})-\tilde{c}(1+n\lambda\hat{P}))((1-\hat{P})^2\hat{p}+c\hat{P}\tilde{P})}{\tilde{p}} \\
= \; & c\hat{P}\tilde{P}\left(\frac{\alpha n\lambda\hat{P}c}{\hat{p}}-\frac{\tilde{c}(1+n\lambda\hat{P})}{\tilde{p}}\right)-\frac{[\tilde{c}(1+n\lambda\hat{P})-\lambda\tilde{P}(\tilde{p}-\tilde{c})](1-\hat{P})^2\hat{p}}{\tilde{p}} \\
= \; & -c\hat{P}\tilde{P}(1+(1-\alpha)n\lambda\hat{P})-\frac{[\tilde{c}(1+n\lambda\hat{P})-\lambda\tilde{P}(\tilde{p}-\tilde{c})](1-\hat{P})^2\hat{p}}{\tilde{p}}<0.
\end{aligned}$$

Because the coefficient of $d\tilde{c}$ is positive, $\tilde{p}$ and $\hat{p}$ increase as the value of $\tilde{c}$ increases.

We now show that social welfare decreases as $\tilde{c}$ increases.

First, we consider the relationship between social welfare, $\tilde{p}$, and $\tilde{c}$. We now suppose that $\tilde{c}$ increases from c_l to c_h and that $\tilde{p}_l$ (*resp.* $\tilde{p}_h$) is the optimal value when $\tilde{c}=c_l$ (*resp.* $\tilde{c}=c_h$). Denote social welfare as $W(\tilde{p},\tilde{c})$. We can easily derive the following inequalities:

$$W(\tilde{p}_l,c_l)>W(\tilde{p}_h,c_l)>W(\tilde{p}_h,c_h).$$

The first inequality holds because $\tilde{p}_l$ is the optimal price when $\tilde{c}=c_l$ and the second holds because of the efficiency of the public firm. Therefore, the increment in $\tilde{c}$ enhances $\tilde{p}$ and then reduces welfare.

[10]

$$\text{coefficient of } d\hat{p} \;=\; \frac{n\lambda\hat{P}}{\hat{p}}\left(\frac{\alpha c}{\hat{p}}-\lambda\tilde{P}\left(\alpha\frac{\hat{p}-c}{\hat{p}}-\frac{\tilde{p}-\tilde{c}}{\tilde{p}}\right)\right).$$

Using Eq. (20), we arrange the equation:

$$\begin{aligned}
\text{coefficient of } d\hat{p} &= \frac{n\lambda\hat{P}}{\hat{p}}\left(\frac{\alpha c}{\hat{p}}-\lambda\tilde{P}\left(\frac{1}{n\lambda\hat{P}}\frac{\tilde{p}-\tilde{c}}{\tilde{p}}\right)\right) \\
&= \frac{1}{\hat{p}}\left(\frac{\alpha n\lambda\hat{P}c}{\hat{p}}-\frac{\lambda\tilde{P}(\tilde{p}-\tilde{c})}{\tilde{p}}\right).
\end{aligned}$$

Second, we consider the relationship between social welfare, $\hat{p}$, and $\tilde{c}$. As shown in the proof of Proposition 1, social welfare decreases as $\hat{p}$ increases. As the value of $\tilde{c}$ increases, the value of $\hat{p}$ increases. Therefore, the increment in $\tilde{c}$ enhances $\hat{p}$ and then reduces welfare.

From the two effects related to the increment in $\tilde{c}$, we find that social welfare decreases as the value of $\tilde{c}$ increases. Q.E.D.

References

Anderson, S. P., de Palma, A., Thisse, J.-F., 1997. Privatization and efficiency in a differentiated industry. European Economic Review 41, 1635–1654.

Bárcena-Ruiz J. C., Garzón, M. B., 2003. Mixed duopoly, merger and multiproduct firms. Journal of Economics 80, 27–42.

Bárcena-Ruiz J. C., Garzón, M. B., 2006. Mixed oligopoly and environmental policy Spanish Economic Review 8(2), 139–160.

Bös, D., 1986. *Public Enterprise Economics*. Amsterdam, North-Holland.

Bös, D., 1991. *Privatization: A Theoretical Treatment*. Clarendon Press, Oxford.

Bös, D., Peters, W., 1995. Double inefficiency in optimally organized firms. Journal of Public Economics 56, 355–375.

Carstensen, K., Toubal, F., 2004. Foreign direct investment in Central and Eastern European countries: a dynamic panel analysis. Journal of Comparative Economics 32, 3–22.

Corneo, G., Rob, R., 2003. Working in public and private firms. Journal of Public Economics 87, 1335–1352.

Cremer, H., Marchand, M., Thisse, J.-F., 1991. Mixed oligopoly with differentiated products. International Journal of Industrial Organization 9, 43–53.

De Fraja, G., Delbono, F., 1989. Alternative strategies of a public enterprise in oligopoly. Oxford Economic Papers 41, 302–311.

Fershtman, C., 1990. The interdependence between ownership status and market structure: the case of privatization. Economica 57, 319–328.

Fjell, K., Pal, D. 1996. A mixed oligopoly in the presence of foreign private firms. Canadian Journal of Economics 29, 737–743.

Futagami, K., 1999. On the range of the public sector. Financial Review 52, 1–13.

Ishibashi, I., Matsumura, T. 2006. R&D competition between public and private sectors. European Economic Review 50, 1347–1366.

Li, C., 2006. Location choice in a mixed oligopoly. Economic Modelling 23, 131–141.

Matsumura, T., 1998. Partial privatization in mixed duopoly. Journal of Public Economics 70, 473–483.

Matsumura, T., 2003a. Endogenous role in mixed markets: a two production period model. Southern Economic Journal 70, 403–413.

Matsumura, T., 2003b. Stackelberg mixed duopoly with a foreign competitor. Bulletin of Economic Research 55, 275–288.

Matsumura, T., Kanda, O., 2005. Mixed oligopoly at free entry markets. Journal of Economics 84(1), 27–48.

Matsumura, T., Matsushima, N., 2004. Endogenous cost differentials between public and private enterprises: a mixed duopoly approach. Economica 71, 671–688.

Matsushima, N., Matsumura, T., 2003. Mixed oligopoly and spatial agglomeration. Canadian Journal of Economics 36, 62–87.

Matsushima, N., Matsumura, T., 2006. Mixed oligopoly, foreign firms, and location choice. Regional Science and Urban Economics 36, 753–772.

Merrill, W., Schneider, N., 1966. Government firms in oligopoly industries: a short-run analysis. Quarterly Journal of Economics 80, 400–412.

Mujumdar, S., Pal, D., 1998. Effects of indirect taxation in a mixed oligopoly. Economics Letters 58, 199–204.

Nett, L., 1993. Mixed oligopoly with homogeneous goods. Annals of Public and Cooperative Economics 64, 367–393.

Nett, L., 1994. Why private firms are more innovative than public firms? European Journal of Political Economy 10, 639–653.

Ohori, S., 2006. Optimal environmental tax and level of privatization in an international duopoly. Journal of Regulatory Economics 29, 225–233.

Pal, D., 1998. Endogenous timing in a mixed oligopoly. Economics Letters 61, 181–185.

Pal, D., White, M. D., 1998. Mixed oligopoly, privatization, and strategic trade policy. Southern Economic Journal 65, 264–281.

Sappington, D. E. M., Sidak, J. G., 2003. Incentives for anticompetitive behavior by public enterprises. Review of Industrial Organization 22, 183–206.