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(Citation)

神戸大学経営学研究科 Discussion paper, 2007・34

(Issue Date)

2007-07

(Resource Type)

technical report

(Version)

Version of Record

(URL)

<https://hdl.handle.net/20.500.14094/80500117>



GRADUATE SCHOOL OF BUSINESS ADMINISTRATION

KOBE UNIVERSITY

ROKKO KOBE JAPAN

Discussion Paper Series

Why do large firms tend to integrate vertically?

— asymmetric vertical integration reconsidered —

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July 23, 2007

Abstract

We provide a theoretical framework to discuss the relation between firm size and vertical structures. The framework is based on a Hotelling model with three downstream and three upstream firms. Each downstream firm procures its input from each upstream firm and the procurement problems affect the locations of the firms. We show that the downstream firm that has the largest market share is more likely to integrate vertically. In other words, integrated firms tend to have a large market share. We also show that vertical integration enhances the degree of product differentiation. As a result, vertical integration mitigates the competition among the downstream firms. We briefly discuss whether inefficient downstream firms tend to integrate vertically. We conclude that this is true because those downstream firms tend to be far away from those rival firms and vertical integration enables downstream firms to escape tough competition.

JEL classification: L22, L13, R32

Key words: vertical integration, asymmetry, product differentiation, location

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1 Introduction

In many industries, vertically integrated and separated firms coexist. For instance, in the apparel industry, some brands (The Gap, L.L. Bean, Eddie Bauer, etc.) are distributed through vertically integrated specialized retailers. Other apparel brands (Tommy Hilfiger, Calvin Klein, etc.) are distributed primarily on a nonexclusive basis through department stores and other nonintegrated retailers (Gertner and Stillman (2001)). In the assembly industries, companies in Western European countries are less integrated than those in the USA, but still far more integrated than those in Japan. There are also significant differences in vertical industry structure between individual European economies (Hemmert (1999)).

Along this line of enquiry, Buehler and Schmutzler (2005) examine how asymmetric vertical structures come about and why integrated firms tend to be large in many industries.¹ Using a reduced-form successive Cournot model (see Salinger (1988)), they show the following two main results: (i) *there may be asymmetric equilibria where only one of the symmetric firms vertically integrates*; (ii) efficient firms are more likely to integrate when downstream firms differ with respect to their initial efficiency levels.² The latter result suggests that *large firms are more likely to integrate vertically*.³

Although the results in Buehler and Schmutzler (2005) are interesting and important in the literature of industrial organization, they mention several limitations. One limitation is that in reality, high-cost integrated firms and low-cost separated firms coexist in the food retail distribution sectors of several European countries. In their paper, since a firm is referred to as *a large firm* because of its cost advantage, the model cannot explain such a coexistence pattern in the retail sector. If we derive results that are similar to the two main results in Buehler and Schmutzler (2005) without any cost asymmetry, the model could be

¹ Buehler and Schmutzler (2005) provide several examples related to asymmetric vertical structures (the U.K. beer industry, the U.S. cable television industry, etc.).

² Linnemer (2003) and Dufeu (2004) also discuss vertical integration under Cournot competition.

³ Several papers also derive those asymmetric equilibria related to vertical integration. See Ordovery *et al.* (1990), Gaudet and Long (1996), Abiru *et al.* (1998), Church and Gandal (2000), Choi and Yi (2000), Elberfeld (2002), Jensen (2003), Matsushima (2004, 2006).

a complement for the model of Buehler and Schmutzler (2005). Furthermore, if we derive a result that high-cost integrated firms and low-cost separated firms coexist, we could say that the model provides an additional contribution to the literature on vertical integration.

To derive complementary results to those of Buehler and Schmutzler (2005), we also provide a theoretical framework to discuss the relation between firm size and vertical structures. To discuss this topic, we extend the models of Matsushima (2004, 2006), which provide frameworks for considering the relation between vertical structures and product differentiation. In these models, each downstream firm procures its input from each upstream firm and the procurement problems affect the locations of the firms.⁴

We now explain the basic structure of this paper. There exist three downstream firms and three upstream firms. One of the downstream firms and one of the upstream firms locate at the center of a linear city à la Hotelling (we call these national firms). On each side, one downstream and one upstream firm exists and the locations of the firms are determined endogenously (we call these local firms). All downstream firms have the same marginal cost.⁵ The differences among the downstream firms stem from product positioning: one is at the center and the others are around the peripheral points. According to Matsushima (2004, 2006), local upstream firms are differentiated along a linear city corresponding to the downstream product space. That is, if a downstream firm's product is differentiated toward the left, there is an efficiency benefit to be had by specializing its input accordingly. This is formalized by the location of the input in its product space. We can interpret transportation costs as the loss of conversion from an upstream firm's product into a suitable input for a downstream firm. After purchasing input from an upstream firm, each downstream firm sets its retail price.⁶

The setting is suitable for the following market structures. Suppose that there are several

⁴ In our model, the marginal costs of downstream firms depend on their locations. Using different frameworks, Karlson (1985) and Aiura and Sato (2007) also discuss those situations.

⁵ We also discuss the case of asymmetric costs in Section 5.

⁶ The location structure is similar to DeGraba (1987). In his model, however, there are no upstream firms.

firms. Large (mainstream) firms compete with many firms including small ones because they mainly produce basic and standard products, which are often recognized as the original ones in the market. On the other hand, small (marginal) firms mainly compete only with large (mainstream) firms that produce standard products, but do not compete with other small firms, because they produce quite different products and the substitutability of their products is quite low. We can apply the model to competition between firms in rural and urban areas. In this case, we can call national firms (*resp.* local firms) urban area firms (*resp.* rural area firms).

The paper shows that a national firm vertically integrates to mitigate price competition under any condition. When the per-unit transport cost of upstream firms is large, one of the local firms integrates vertically to commit not to locate near the national downstream firm. When the per-unit transport cost of upstream firms is large enough, the other local firms integrate vertically to commit not to locate near the national downstream firm. The first result of Buehler and Schmutzler (2005) also appears in our model, and the second one, where large firms are more likely to integrate vertically, is derived from a different mechanism. That is, without cost asymmetry among the downstream firms, we derive the second result in Buehler and Schmutzler (2005). We think that our result is a complementary for those in Buehler and Schmutzler (2005).

We now mention the intuition behind the result. When the national downstream and upstream firms are separate ones, each local downstream firm tends to locate near the center to decrease its procurement cost. The shorter distances between the national and local firms enhance the elasticity of demand for the *national* downstream firm (the shorter distance means that products are less differentiated). Because of the high elasticity of demand, the increase in the input price for the national downstream firm significantly decreases the quantity supplied by the national downstream firm. Vertical integration is the best way to avoid a significant decrease in the quantity supplied by the national firm, because the integrated firm is able to produce without variable costs. Given that the national firms vertically integrate, as the per-unit procurement costs increases, each local firm moves toward

the center. Vertical integration by a pair of local downstream and upstream firms mitigates price competition because the integrated local firm does not have to take into account the potential supplier. This induces vertical integration. The nonintegrated firms, however, do not react with a counter-merger of their own unless the per-unit transport cost of upstream firms is large enough.

The remainder of this paper is organized as follows. Section 2 presents the basic model. Section 3 shows the results under several integration patterns. Section 4 shows the main result regarding the decisions on vertical integration. Section 5 presents a brief discussion of asymmetric product costs. Section 6 concludes.

2 The Model

There are three downstream firms, D_A , D_B , and D_0 , which produce the same physical product. There is a linear city of length 2 ($[-1, 1]$), which lies on the abscissa of a line, and consumers are uniformly distributed with density 1 along the interval. We call $[-1, 0]$ market A and $[0, 1]$ market B . Suppose that in market A (*resp.* market B), D_A (*resp.* D_B) is located at point l_A (*resp.* l_B), which is a positive distance from zero. Suppose that D_0 is located at zero. In market A (*resp.* market B), a consumer living at y , which is a positive distance from zero, incurs a transportation cost of $(l_A - y)^2$ (*resp.* $(l_B - y)^2$) when purchasing a product from D_A (*resp.* D_B). The consumers have unit demands, i.e., each consumes one or zero units of the product. Each consumer derives a surplus from consumption (gross of price and transportation costs) equal to s . We assume that s is so large that every consumer consumes one unit of the product. This downstream market structure is also discussed in DeGraba (1987); however, he does not consider the vertical relation, which is the main concern of our paper.

Three upstream firms, U_A , U_B , and U_0 , supply inputs to three downstream firms. Suppose that U_A (*resp.* U_B) is located at point h_A (*resp.* h_B), which is a positive distance from zero. Suppose that U_0 is located at zero. Upstream firms engage in price competition for the business of downstream firms. Each input of a downstream firm produced by an upstream

firm is a perfect substitute. To supply an x distant downstream firm, an upstream firm incurs a transport cost τx . We assume that $\tau < 1/2$.

We analyze a four-stage game. In the first stage, upstream firms (U_A and U_B) and downstream firms (D_A and D_B) simultaneously choose their locations. In the second stage, each upstream firm, U_i ($i = 0, A, B$), simultaneously chooses its wholesale prices, $w_{ij} \in [0, \infty$ ($j = 0, A, B$), where j is the index of the downstream firm, D_j ($j = 0, A, B$). For instance, w_{AB} is U_A 's wholesale price for D_B . Each upstream firm engages in price competition for the business of downstream firms. In the third stage, observing the wholesale prices, each downstream firm chooses its supplier among U_A , U_B , and U_0 and then sets its retail price $p_i \in [0, \infty)$ ($i = 0, A, B$) simultaneously. In the fourth stage, observing the retail prices, consumers select between the sellers D_A , D_B , and D_0 .

3 The results

In this section, we show the main result of the paper. To derive the results, we consider six cases: (1) no integration; (2) integration of the national firms; (3) integration of one pair of local firms; (4) integrations of the national firms and one pair of local firms; (5) integrations of two pairs of local firms; (6) full integration.

3.1 No integration

This case is the basic setting in our model. The cases presented below are variants of the basic setting. After we explain the case thoroughly, we simply mention the difference between the basic setting and the remaining cases.

The third and fourth stages For a consumer living at

$$x_i = \frac{l_i^2 - p_0 + p_i}{2l_i}, \quad (1)$$

the full price (transport cost plus price) is the same at either of the two firms in market i .⁷

The profit of each downstream firm is given by

$$\pi_{dA} \equiv (p_A - w_A) \left(1 - \frac{l_A^2 - p_0 + p_A}{2l_A} \right), \quad (2)$$

$$\pi_{dB} \equiv (p_B - w_B) \left(1 - \frac{l_B^2 - p_0 + p_B}{2l_B} \right), \quad (3)$$

$$\pi_{d0} \equiv (p_0 - w_0) \left(\frac{l_A^2 - p_0 + p_A}{2l_A} + \frac{l_B^2 - p_0 + p_B}{2l_B} \right). \quad (4)$$

In eqs. (2), (3), and (4), $w_A = \min\{w_{AA}, w_{BA}, w_{0A}\}$, $w_B = \min\{w_{AB}, w_{BB}, w_{0B}\}$, and $w_0 = \min\{w_{A0}, w_{B0}, w_{00}\}$. These mean that D_A (D_B or D_0) procures its input from the upstream firm that bids the lowest wholesale price. From the first-order conditions of downstream firms, we obtain equilibrium price $p_i^*(l_A, l_B, w_A, w_B, w_0)$ ($i = 0, A, B$) in the third stage.

The second stage For each downstream firm, each upstream firm sets its price at the more efficient rival firm's transport cost if its cost is lower than the rival's (Bertrand competition), otherwise it sets at its own transport cost. The prices set by U_i ($i = 0, A, B$) are as follows:

$$U_A : w_{AA} = \max\{\tau|h_A - l_A|, \min\{\tau l_A, \tau(h_B + l_A)\}\} \quad (5)$$

$$w_{AB} = \max\{\tau(h_A + l_B), \min\{\tau l_B, \tau|h_B - l_B|\}\} \quad (6)$$

$$w_{A0} = \max\{\tau h_A, \min\{0, \tau h_B\}\} \quad (7)$$

$$U_B : w_{BA} = \max\{\tau(h_B + l_A), \min\{\tau|h_A - l_A|, \tau l_A\}\} \quad (8)$$

$$w_{BB} = \max\{\tau|h_B - l_B|, \min\{\tau(h_A + l_B), \tau l_B\}\} \quad (9)$$

$$w_{B0} = \max\{\tau h_B, \min\{0, \tau h_A\}\} \quad (10)$$

$$U_0 : w_{0A} = \max\{\tau l_A, \min\{\tau|h_A - l_A|, \tau(h_B + l_A)\}\} \quad (11)$$

$$w_{0B} = \max\{\tau l_B, \min\{\tau(h_A + l_B), \tau|h_B - l_B|\}\} \quad (12)$$

$$w_{00} = \max\{0, \min\{\tau h_A, \tau h_B\}\} \quad (13)$$

⁷ Eq. (1) is derived from the following equation: $-p_i - (l_i - x_i)^2 = -p_0 - x_i^2$.

The first stage We discuss the location choices of U_A and U_B . In this case $|h_A - l_A| < l_A$ must be satisfied in equilibrium and then the wholesale price is $w_A = \tau l_A$.⁸ A similar argument can be applied to the location of U_B and we obtain $w_B = \tau l_B$. The upstream firm's profits are

$$\pi_{uA} = (\tau l_A - \tau |h_A - l_A|) \left(1 - \frac{l_A^2 - p_0^* + p_A^*}{2l_A} \right), \quad (14)$$

$$\pi_{uB} = (\tau l_B - \tau |h_B - l_B|) \left(1 - \frac{l_B^2 - p_0^* + p_B^*}{2l_B} \right). \quad (15)$$

U_0 supplies its product to D_0 at the price $w_0 = \min\{\tau h_A, \tau h_B\}$. The retail prices p_i^* ($i = 0, A, B$) depend on h_A and h_B through the wholesale price w_0 . We can derive the following lemma:⁹

Lemma 1 $h_i = l_i$ ($i = A, B$) in equilibrium.

We can show that the intuition that U_i chooses $h_i = l_i$ ($i = A, B$) in equilibrium. We now suppose that $h_i = l_i$ and $l_A \geq l_B$ ($i = A, B$). In this case, the wholesale prices of D_0 , D_A , and D_B are τh_B , τl_A , and τl_B , respectively. Moreover, $|h_i - l_i| < l_i$ ($i = A, B$) must be satisfied because each upstream firm earns a positive profit.

First, we consider the location choice of U_A . If U_A chooses $h_A < l_A$, the wholesale price of D_0 is τh_B (if $h_B \leq h_A$) or τh_A (if $h_B > h_A$) and the per-unit transport cost of U_A is $\tau(l_A - h_A)(> 0)$. Moreover, the wholesale price of D_A does not change. The first and second changes decrease the profitability of U_A and the third one is neutral. If U_A chooses $h_A > l_A$, the wholesale prices of D_0 and D_A do not change and the per-unit transport cost of U_A is $h_A - l_A(> 0)$. The second change decreases the profitability of U_A and the first one is neutral. Since the profit of U_A does not increase in either case, $h_A = l_A$ is the optimal location of U_A .

Second, we consider the location choice of U_B . When U_B chooses $h_B < l_B$ or $h_A < h_B$, the properties in the case of U_A also hold. When U_B chooses $l_B < h_B \leq h_A$, the wholesale

⁸ When U_A chooses $h_A = 0$, it cannot gain any profit by selling to downstream firms, because U_A 's wholesale price is equal to its own transport cost ($w_{AA} = \tau l_A$, $w_{AB} = \tau l_B$, and $w_{A0} = 0$). From eqs. (5), (6), and (7), if $|h_A - l_A| \geq l_A$ in equilibrium, then U_A cannot supply their product to any downstream firms.

⁹ We can apply the lemma to the remaining cases.

prices of D_0 increase and the per-unit transport cost of U_B is $h_B - l_B (> 0)$. The first effect is the indirect positive effect through the increase in the quantity supplied by D_B and U_B . The second is, however, the direct negative effect. In the setting, the latter effect dominates the former one. $h_B = l_B$ is the optimal location of U_B .

From eqs. (2), (3), and (4), and solutions in stages 2 and 3, the profits of the downstream firms are:

$$\pi_{dA} \equiv (p_A^* - \tau l_A) \left(1 - \frac{l_A^2 - p_0^* + p_A^*}{2l_A} \right), \quad (16)$$

$$\pi_{dB} \equiv (p_B^* - \tau l_B) \left(1 - \frac{l_B^2 - p_0^* + p_B^*}{2l_B} \right). \quad (17)$$

The first-order conditions lead to the following results:

$$\begin{aligned} h_A^* &= h_B^* = l_A^* = l_B^* = \frac{6 - 5\tau}{8}, \\ \pi_{uA}^* &= \pi_{uB}^* = -\frac{\tau(-6 + 5\tau)(26 + 5\tau)}{384}, \quad \pi_{u0}^* = \frac{\tau(-22 + 5\tau)(-6 + 4\tau)}{192}, \\ \pi_{dA}^* &= \pi_{dB}^* = -\frac{(-6 + 5\tau)(26 + 5\tau)^2}{9216}, \quad \pi_{d0}^* = -\frac{(-22 + 5\tau)^2(-6 + 5\tau)}{4608}. \end{aligned}$$

Those results is summarized in Table 1.¹⁰ As the value of τ (per unit transport cost of input)

τ	$h_A^* = l_A^*$	$h_B^* = l_B^*$	π_{uA}^*	π_{uB}^*	π_{u0}^*	π_{dA}^*	π_{dB}^*	π_{d0}^*
0.05	0.71875	0.71875	0.01965	0.01965	0.03257	0.42992	0.42992	0.59030
0.10	0.68750	0.68750	0.03796	0.03796	0.06159	0.41909	0.41909	0.55173
0.15	0.65625	0.65625	0.05486	0.05486	0.08716	0.40763	0.40763	0.51448
0.20	0.62500	0.62500	0.07031	0.07031	0.10938	0.39551	0.39551	0.47852
0.25	0.59375	0.59375	0.08427	0.08427	0.12834	0.38272	0.38272	0.44383
0.30	0.56250	0.56250	0.09668	0.09668	0.14414	0.36926	0.36926	0.41040
0.35	0.53125	0.53125	0.10750	0.10750	0.15688	0.35512	0.35512	0.37820
0.40	0.50000	0.50000	0.11667	0.11667	0.16667	0.34028	0.34028	0.34722
0.45	0.46875	0.46875	0.12415	0.12415	0.17358	0.32473	0.32473	0.31743
0.50	0.43750	0.43750	0.12988	0.12988	0.17773	0.30847	0.30847	0.28882

Table 1: Locations and profits (no integration)

¹⁰ When only one pair of local firms vertically integrate (there exist two cases), we cannot explicitly solve the equilibrium outcomes. In those cases, we use a numerical simulation to derive the results. To compare the results in each subsection, we show these tables in the subsections.

increases, the values of $l_i = h_i$ ($i = A, B$) decrease. To understand the benchmark case, we now show the intuition behind the result.¹¹ In this model, given the locations of the other firms, when a downstream firm moves further away from its rival, three effects occur. First, price competition between downstream firms is softened (the “price effect”). The price effect enhances the profit of the downstream firm (D_A). Second, the demand for the downstream firm falls (the “demand effect”). The demand effect diminishes the profit of the downstream firm (D_A). Third, the wholesale price of the downstream firm rises (the “input price effect”) because the distance between the downstream firm and the other supplier increases. (e.g., D_A and U_0).

The input price effect is affected by the value of τ . The larger τ is, the larger are the wholesale prices. On the other hand, the price and demand effects are unaffected by the value of τ because the cost of each downstream firm is equal to that of its rival in equilibrium. Therefore, when τ is large, the third effect (the input price effect) is important for downstream firms. To lower the input price, each downstream firm shortens the distance to the rival’s supplier.

The profits of each downstream firm decrease as the value of τ increases. On the other hand, given that the value of τ is small, the profits of each upstream firm increase as the value of τ increases. The increase in τ enables the upstream firms to set their prices at higher levels because the rivals of each upstream firm have to incur higher transport costs, which are positively related to the value of τ . The increase in τ , however, induces the distances between the national and local downstream firms to be shortened. This enhances competition among the upstream firms. When τ is large (small), the latter negative (the former positive) effect dominates another one.

We now briefly mention the reason that in this case the location strategy of each downstream firm accelerates the tendency to locate near the center. That is, there exists complementarity concerning the location strategies of downstream firms. Although DeGraba (1987) does not point out the property of complementarity, we believe that it also exists in his study.

¹¹ The intuition mentioned in the paper is also mentioned in Matsushima (2004, 2006).

We also believe that this point is important in the literature of industrial organization and regional science.

Given that both downstream and upstream firms choose the same value of l and h , when one of the downstream firms (now denoted as D_A) moves toward the center, the competition between D_A and the national firm intensifies. Because of the intense competition between them, the national firm does not take into account the price of the other downstream firm (now denoted as D_B) to any significant extent. That is, from the viewpoint of D_B , which have not yet moved, the demand for D_B becomes less elastic. The change in the national firm's pricing strategy softens the "price effect" and then induces D_B to be near the center. Therefore, the movement toward the center by one of the downstream firms induces another downstream firm to move near the center. We will discuss this characteristic in Section 3.3.

3.2 Integration of the national firms

The remaining cases are variants of the basic setting mentioned above. We briefly mention what is the difference between the rest of the cases and the basic setting.

The third and fourth stages Suppose that U_0 and D_0 merge. Let I_0 be an integrated firm 0 and p_0 be I_0 's retail price. The integrated upstream firm sells its product to the integrated downstream firm at a wholesale price that is equal to own transport cost. Hence, $w_{00} = 0$, since D_0 and U_0 are located at zero. Then, $w_0 = 0$. The profit of I_0 is

$$\pi_{I0} \equiv p_0 \left(\frac{l_A^2 - p_0 + p_A}{2l_A} + \frac{l_B^2 - p_0 + p_B}{2l_B} \right). \quad (18)$$

From the first-order conditions of the firms, we obtain equilibrium price $p_i^*(l_A, l_B, w_A, w_B, w_0)$ ($i = 0, A, B$) in the third stage.

The second stage The pricing strategies of U_A and U_B are similar to those in the former subsection. The only difference is that U_0 sets $w_{00} = 0$.

The first stage As mentioned in Section 3.1, we obtain $w_A = \tau l_A$ and $w_B = \tau l_B$. By the integration, $w_0 = 0$ and then $p_i^*(l_A, l_B, w_A, w_B, w_0)$ ($i = A, B, 0$) depends only on the downstream firm's location (l_i , $i = A, B$). U_i ($i = A, B$) is able to locate at $h_i = l_i$ without the changes in the quantities supplied (D_0 , D_A , and D_B). Therefore, $h_i = l_i$ ($i = A, B$). The first-order conditions lead to the following result:

$$\begin{aligned} h_A^* &= h_B^* = l_A^* = l_B^* = \frac{3(2-\tau)}{8}, \\ \pi_{uA}^* &= \pi_{uB}^* = \frac{\tau(-2+\tau)(-26+5\tau)}{128}, \\ \pi_{dA}^* &= \pi_{dB}^* = \frac{(2-\tau)(-26+5\tau)^2}{3072}, \quad \pi_{I0}^* = \frac{(2-\tau)(22+5\tau)^2}{1536}. \end{aligned}$$

Those results are summarized in Table 2. The integrated national firm is a tough (efficient)

τ	$h_A^* = l_A^*$	$h_B^* = l_B^*$	π_{uA}^*	π_{uB}^*	π_{dA}^*	π_{dB}^*	π_{I0}^*
0.05	0.73125	0.73125	0.01961	0.01961	0.42089	0.42089	0.62850
0.10	0.71250	0.71250	0.03785	0.03785	0.40217	0.40217	0.62622
0.15	0.69375	0.69375	0.05474	0.05474	0.38395	0.38395	0.62337
0.20	0.67500	0.67500	0.07031	0.07031	0.36621	0.36621	0.61992
0.25	0.65625	0.65625	0.08459	0.08459	0.34895	0.34895	0.61588
0.30	0.63750	0.63750	0.09762	0.09762	0.33217	0.33217	0.61121
0.35	0.61875	0.61875	0.10941	0.10941	0.31585	0.31585	0.60593
0.40	0.60000	0.60000	0.12000	0.12000	0.30000	0.30000	0.60000
0.45	0.58125	0.58125	0.12942	0.12942	0.28460	0.28460	0.59342
0.50	0.56250	0.56250	0.13770	0.13770	0.26965	0.26965	0.58618

Table 2: Locations and profits (integration of the national firm)

competitor. To avoid competition with the national firm, each local downstream firm is far away from it. Because the distance between the national firm and each local firm is longer than in the no integration case, each nonintegrated upstream firm is able to set its price at a higher level. Moreover, the greater distance is a commitment device to discourage competition among the downstream firms. When the value of τ is sufficiently large, the commitment device is functioning well.

3.3 Integration of one pair of local firms

Suppose that U_A and D_A merge (note that we can apply the same logic to a merger of U_B and D_B). Let I_A be a integrated firm A and p_A be I_A 's retail price. The discussion of the third and fourth stages is similar to that in the former subsections.

The second stage The way to determine the input price of the integrated firm is the only difference between this case and the basic setting. The input price of the integrated local firm does not depend on the location of the downstream firm relative to the two upstream units (its own and the central units). If its own upstream unit is closer, then this is clearly the cost incurred by the integrated firm. Even if the central upstream firm is closer so that the integrated firm procures input from that firm, this remains the relevant expression of the cost, since the upstream firm will charge the next best price facing the integrated firm, which is the cost incurred. Therefore, $w_A = \tau|l_A - h_A|$.¹²

The first stage Since the argument is the same as that of section 3.1, $w_B = \tau l_B$. Because $w_A = \tau|l_A - h_A|$, the optimal location of U_I is $h_A = l_A$. Therefore $w_A = 0$.

In this case, the local firms are asymmetric with respect to the procurement conditions. The asymmetry affects the locations of the firms and then $l_A > l_B$ in equilibrium. We now mention that the intuition $l_A > l_B$ holds. Suppose that $l_A = l_B$. When I_A increases l_A (which moves to the edge), the marginal cost does not increase. On the other hand, when D_B increases l_B (which moves to the edge), the marginal cost increases because $w_B = \tau l_B$. Hence, I_A has more incentive to move farther from zero than D_B , that is $l_A > l_B$. By numerical calculation, we can show that $l_A > l_B$ in equilibrium. Applying the same reasoning in the former subsection, we have $h_B = l_B$. Since it is difficult to show the result analytically, we use numerical expression to show the result summarized in Table 3. The integration mainly has two effects on the interaction between the integrated and the national firms. First, the

¹² Note that we implicitly exclude the case in which the integrated upstream firm locates at the furthest point from the integrated downstream firm because the location is obviously inefficient from the viewpoint of the integrated firm.

τ	$h_A^* = l_A^*$	$h_B^* = l_B^*$	π_{uB}^*	π_{u0}^*	π_{dB}^*	π_{d0}^*	π_{IA}^*
0.05	0.73417	0.72231	0.01974	0.03255	0.43155	0.59143	0.44777
0.10	0.71862	0.69485	0.03832	0.06157	0.42275	0.55479	0.45447
0.15	0.70336	0.66765	0.05574	0.08727	0.41371	0.52022	0.46016
0.20	0.68842	0.64072	0.07199	0.10984	0.40445	0.48765	0.46482
0.25	0.67383	0.61410	0.08706	0.12948	0.39498	0.45703	0.46843
0.30	0.65961	0.58783	0.10096	0.14637	0.38531	0.42827	0.47097
0.35	0.64581	0.56193	0.11368	0.16072	0.37547	0.40133	0.47244
0.40	0.63245	0.53644	0.12524	0.17272	0.36548	0.37612	0.47285
0.45	0.61957	0.51141	0.13565	0.18256	0.35535	0.35258	0.47221
0.50	0.60720	0.48687	0.14493	0.19041	0.34512	0.33064	0.47054

Table 3: Locations and profits (integration of one pair of local firms)

integrated firm does not take into account its procurement cost and does not need to locate at a closer point to the national firm. The greater distance between the integrated and national firms mitigates the competition between them. Second, the efficiency improvement of the integrated firm enhances competition. In this setting, the former effect dominates the latter one (see the downstream prices of those firms). The integrated and the national firms set those prices at higher levels.

As mention in Section 3.1, for the nonintegrated downstream firm, the increment in the distance between the national and the integrated firms are similar to the enhancement in the elasticity of demand, which is related to the price effect. The nonintegrated downstream firm is far away from the center. This is reflected in the value of l_B under the first and the third cases. The location strategy of the nonintegrated downstream firm affects the nonintegrated upstream firm. To meet the location decision by the nonintegrated downstream firm, the nonintegrated upstream firm is also far away from the center. The location of the upstream firm increases the procurement cost of the national firm, which mitigates the competition between the national and the integrated firms.

3.4 Integrations of the national firms and one pair of local firms

Suppose that U_A and D_A merge and U_0 and D_0 do likewise. Let I_A (*resp.* I_0) be an integrated firm A (*resp.* firm 0) and p_A (*resp.* p_0) be I_A 's (*resp.* I_0 's) retail price. Each

integrated upstream firm sells its product to its own integrated downstream firm at its wholesale price, which is equal to its transport cost. Hence, $w_{AA} = \tau|h_A - l_A|$ and $w_{00} = 0$. Then, $w_0 = 0$. The pricing strategies of upstream firms are similar to those in the former subsections. We obtain $w_A = \tau|h_A - l_A|$, $w_B = \tau l_B$, and $w_0 = 0$. As mentioned earlier, we can show that $h_A = l_A$ and $h_B = l_B$ in equilibrium. Since it is difficult to show the result analytically, we use numerical expression to show the result summarized in Table 4. This

τ	$h_A^* = l_A^*$	$h_B^* = l_B^*$	π_{uB}^*	π_{dB}^*	π_{IA}^*	π_{IO}^*
0.05	0.74653	0.73475	0.01970	0.42236	0.43862	0.62940
0.10	0.74303	0.71960	0.03818	0.40517	0.43704	0.62842
0.15	0.73950	0.70455	0.05549	0.38852	0.43539	0.62726
0.20	0.73595	0.68961	0.07167	0.37239	0.43365	0.62593
0.25	0.73237	0.67477	0.08674	0.35678	0.43183	0.62441
0.30	0.72877	0.66005	0.10074	0.34168	0.42993	0.62272
0.35	0.72515	0.64544	0.11371	0.32707	0.42795	0.62085
0.40	0.72150	0.63096	0.12569	0.31295	0.42588	0.61880
0.45	0.71785	0.61661	0.13670	0.29931	0.42374	0.61658
0.50	0.71417	0.60238	0.14678	0.28613	0.42151	0.61418

Table 4: Locations and profits (integrations of the national and one local firms)

case is the combination of the cases 2 and 3. The integration of a pair of local downstream and upstream firms increases the distances between the local and national firms, respectively ($l_A = h_A$ and $l_B = h_B$). It mitigates the competition among them. The integration by the national firms also enlarges the distances. Because it decreases the marginal cost of the national firm, it harms the regional firms.

3.5 Integrations of two pairs of local firms

Suppose that U_A and D_A , and U_B and D_B merge, respectively. Let I_A (*resp.* I_B) be an integrated firm A (*resp.* firm B) and p_A (*resp.* p_B) be I_A 's (*resp.* I_B 's) retail price. The pricing strategies of upstream firms are similar to those in the former subsections. We obtain $w_A = \tau|l_A - h_A|$, $w_B = \tau|l_B - h_B|$, $w_0 = \min\{\tau h_A, \tau h_B\}$. As mentioned earlier, we can show that $h_A = l_A$ and $h_B = l_B$ in equilibrium and we have $w_A = w_B = 0$. The first-order

conditions lead to the following result:

$$h_A^* = h_B^* = l_A^* = l_B^* = \frac{11 - 5\tau}{24}, \quad \pi_{IA}^* = \pi_{IB}^* = \frac{(3 - \tau)(13 + 5\tau)^2}{1152},$$

$$\pi_{u0}^* = \frac{(3 - \tau)\tau(11 - 5\tau)}{48}, \quad \pi_{d0}^* = \frac{(3 - \tau)(11 - 5\tau)^2}{576}.$$

Those results are summarized in Table 5. The integrated local firms do not have to locate

τ	$h_A^* = l_A^*$	$h_B^* = l_B^*$	π_{u0}^*	π_{d0}^*	π_{IA}^*	π_{IB}^*
0.05	0.73750	0.73750	0.03303	0.59186	0.44957	0.44957
0.10	0.72500	0.72500	0.06344	0.55508	0.45879	0.45879
0.15	0.71250	0.71250	0.09129	0.51984	0.46773	0.46773
0.20	0.70000	0.70000	0.11667	0.48611	0.47639	0.47639
0.25	0.68750	0.68750	0.13965	0.45386	0.48474	0.48474
0.30	0.67500	0.67500	0.16031	0.42305	0.49277	0.49277
0.35	0.66250	0.66250	0.17874	0.39365	0.50047	0.50047
0.40	0.65000	0.65000	0.19500	0.36563	0.50781	0.50781
0.45	0.63750	0.63750	0.20918	0.33895	0.51479	0.51479
0.50	0.62500	0.62500	0.22135	0.31359	0.52138	0.52138

Table 5: Locations and profits (integrations of the local firms)

near the center. The distances between the national and each local firm increases. The longer distances mitigate the competition among them, but increase the procurement costs of the national firm. When the value of τ is small enough, the former positive effect dominates the latter one. The additional integration enhances the profit of the national firm.

3.6 Full integration

Suppose that U_A and D_A , U_B and D_B , and U_0 and D_0 merge, respectively. Let I_A (*resp.* I_B and I_0) be an integrated firm A (*resp.* B and 0) and p_A (*resp.* p_B and p_0) be I_A 's (*resp.* I_B 's and I_0 's) retail price. The pricing strategies of upstream firms are similar to those in the former subsections. We obtain $w_A = \tau|l_A - h_A|$, $w_B = \tau|l_B - h_B|$, and $w_0 = 0$. As mentioned earlier, we can show that $h_A = l_A$ and $h_B = l_B$ in equilibrium and we have $w_A = w_B = 0$. The first-order conditions lead to the following result:

$$h_A^* = h_B^* = l_A^* = l_B^* = \frac{3}{4}, \quad \pi_{IA}^* = \pi_{IB}^* = \frac{169}{384} \approx 0.440104, \quad \pi_{I0}^* = \frac{121}{192} \approx 0.630208. \quad (19)$$

The above equilibrium outcomes do not depend on τ . Since all firms merge and $h_A^* = h_B^* = l_A^* = l_B^*$, all wholesale prices are zero. Then, transport cost $\tau|h_i - l_i|$ ($i = A, B$) is always zero. Therefore, τ has no effect on the equilibrium outcome.

4 Integration decisions

We shall discuss decisions regarding vertical integration. The integration decisions are represented by the following $2 \times 2 \times 2$ matrix. $\Pi_j^*(H_A H_0 H_B)$ is the joint profit of upstream and

		Firm 0 : No integration		Firm B	
		No integration		Integration	
Firm A	No integration	$\Pi_A^*(NNN), \Pi_B^*(NNN), \Pi_0^*(NNN)$		$\Pi_A^*(NNI), \Pi_B^*(NNI), \Pi_0^*(NNI)$	
	Integration	$\Pi_A^*(INN), \Pi_B^*(INN), \Pi_0^*(INN)$		$\Pi_A^*(INI), \Pi_B^*(INI), \Pi_0^*(INI)$	

		Firm 0 : Integration		Firm B	
		No integration		Integration	
Firm A	No integration	$\Pi_A^*(NIN), \Pi_B^*(NIN), \Pi_0^*(NIN)$		$\Pi_A^*(NII), \Pi_B^*(NII), \Pi_0^*(NII)$	
	Integration	$\Pi_A^*(IIN), \Pi_B^*(IIN), \Pi_0^*(IIN)$		$\Pi_A^*(III), \Pi_B^*(III), \Pi_0^*(III)$	

Table 6: Payoff matrix

downstream firms j ($j = A, 0, B$) and $H_j \in \{N, I\}$ is the index of firm j 's vertical structure (N : nonintegrated, I : integrated).¹³ After tedious calculations, we have the following result:

Result *When vertical integration is determined endogenously. For any τ , integration of the national firm occurs. Integration of one local firm occurs if and only if $0.30 \leq \tau \leq 0.36$.*

¹³ Given the values of $\tau = 0.01, 0.02, \dots, 0.50$, $\Pi_j^*(\dots)$ ($j = A, B, 0$) are derived by the following calculations:

1. $\Pi_A^*(NNN) = \Pi_B^*(NNN)$ is $\pi_{uA}^* + \pi_{dA}^*$ and $\Pi_0^*(NNN)$ is $\pi_{u0}^* + \pi_{d0}^*$ in Section 3.1.
2. $\Pi_A^*(NIN) = \Pi_B^*(NIN)$ is $\pi_{uA}^* + \pi_{dA}^*$ and $\Pi_0^*(NIN)$ is π_{I0}^* in Section 3.2.
3. $\Pi_A^*(INN) = \Pi_B^*(NNI)$ is π_{IA}^* ; $\Pi_B^*(INN) = \Pi_A^*(NNI)$ is $\pi_{uB}^* + \pi_{dB}^*$; and $\Pi_0^*(INN) = \Pi_0^*(NNI)$ is $\pi_{u0}^* + \pi_{d0}^*$ in Section 3.3.
4. $\Pi_A^*(IIN) = \Pi_B^*(NII)$ is π_{IA}^* ; $\Pi_B^*(IIN) = \Pi_A^*(NII)$ is $\pi_{uB}^* + \pi_{dB}^*$; and $\Pi_0^*(IIN) = \Pi_0^*(NII)$ is π_{I0}^* in Section 3.4.
5. $\Pi_A^*(INI) = \Pi_B^*(INI)$ is π_{IA}^* and $\Pi_0^*(INI)$ is $\pi_{u0}^* + \pi_{d0}^*$ in Section 3.5.
6. $\Pi_A^*(III) = \Pi_B^*(III)$ is π_{IA}^* and $\Pi_0^*(III)$ is π_{I0}^* in Section 3.6.

*Full integration occurs if and only if $0.37 \leq \tau$.*¹⁴

For any τ , the national downstream firm vertically integrates with the upstream firm U_0 . As mentioned in Section 3.1, when they are separate firms, the price effect is weak, in which case each local downstream firm tends to locate near the center. The shorter distances between the national and the local firms enhance the elasticity of demand for the *national* downstream firm. Because of the high elasticity of demand, the increase in the input price for the global downstream firm significantly decreases the quantity supplied by the global firm. To avoid the significant decrease in the quantity supplied by the global firm, vertical integration is the best way because the integrated firm is able to produce its product without variable costs.

Given that national firms vertically integrate, as the value of τ increases, each local firm moves toward the center. Vertical integration mitigates price competition because the integrated local firm does not have to access the potential supplier. This induces vertical integration.

The nonintegrated firms, however, do not react with a counter-merger of their own unless the value of τ is large enough.¹⁵ Because of the complementarity of location, the location of the nonintegrated firms is far away from the center because the vertical integration is a commitment device not to locate near to the center. The equilibrium locations mitigate the competition between the national and the *other* local firms. Both profits of the nonintegrated firms (π_{uB} and π_{dB}) increase. Therefore, the regional asymmetry concerning the vertical structure appears in equilibrium.

The profit of the nonintegrated upstream firm increases with the value of τ because the nonintegrated upstream firm can set its wholesale price at τl_i . On the other hand, the profit of the nonintegrated downstream firm decreases with the value of τ because it moves toward

¹⁴ To check the result, we present several payoff matrixes under some values of τ in Appendix.

¹⁵ Matsushima (2006) also shows this type of asymmetric vertical structure. The reason why the asymmetric vertical structure appears is different from that in this paper. In Matsushima (2006), the nonintegrated downstream firm locates near to the center and the central position is superior to the edge. In this paper, the complementarity of location is the driving force.

the center. Because of the complementarity of location, the nonintegrated downstream firm's movement to the center induces the integrated local firm to move toward the center even though the integrated firm need not take into account the procurement cost (see Sections 3.3 and 3.4). The movements of the local firms enhance competition among the local and national firms. To avoid this competition, vertical integration by another pair appears when the value of τ is large.

5 Cost asymmetry among the downstream firms

To consider the relation between product inefficiency and vertical mergers, we add asymmetries of marginal costs to the basic setting. We briefly discuss the following two cases: (i) the national downstream firm is less efficient than the local downstream firms; (ii) one of the local downstream firms is less efficient than the other two downstream firms.

In the first case, if the inefficiency is not significant, we can apply the result in the former section, that is, the inefficient national downstream firm integrates unless the other downstream firms do not integrate. Because of its product positioning, the national firm essentially tends to integrate and the asymmetry does not change the property when the inefficiency is not significant.

In the second case, the inefficient local firm tends to integrate vertically, that is, for an intermediate range of τ , the following integration pattern appears in equilibrium: the national and the inefficient downstream firms vertically integrate, respectively. We now explain the intuition behind the result. The inefficient firm tends to be far away from the national firm because of its inefficiency. In other words, the inefficient firm's incentive to locate near the center is weaker than that of the efficient local downstream firm. As the value of τ increases, each local downstream firm moves head for the center. The complementarity of their locations strengthens the effect of the increment in τ . Because of its inefficiency, the inefficient firm's incentive to avoid moving to the center is stronger than that for the efficient firm. Therefore, for an intermediate range of τ , the integration pattern mentioned above appears in equilibrium.

As pointed out by Buehler and Schmutzler (2005), in the food retail distribution sector of several European countries, high-cost integrated firms and low-cost separated firms coexist. The integration pattern in the discussion is consistent with this fact. We believe that our model could complement that of Buehler and Schmutzler (2005).

6 Concluding remarks

We provide a theoretical framework to discuss the relation between firm size and vertical structures. The framework is based on a Hotelling model with three downstream and three upstream firms. We show that the downstream firm that has the largest market share is more likely to integrate vertically. In other words, integrated firms tend to have a large market share. We briefly discuss whether inefficient downstream firms tend to integrate vertically. We conclude that this would be true because those downstream firms tend to be far away from those rival firms and vertical integration enables downstream firms to escape from tough competition. Those results could explain the fact that high-cost integrated firms and low-cost separated firms coexist in the food retail distribution sector of several European countries. Therefore, we think that our model could be a complement for the model of Buehler and Schmutzler (2005).

In our paper, we assume that a national firm sets a uniform price. As discussed in DeGraba (1987), it could employ discriminatory pricing under some market conditions. The pricing would change the location strategies of local firms and the vertical structures of firms. Whether discriminatory pricing affects the locations and the vertical structure is a consideration for future research.

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Appendix

$\tau = 0.10$

Firm 0: Nointegration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.45705,	0.45705,	0.61332	0.46107,	0.45447,	0.61636
	Integratin	0.45447,	0.46107,	0.61636	0.45879,	0.45879,	0.61852

Firm 0: Integration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.44002,	0.44002,	0.62622	0.44335,	0.43704,	0.62842
	Integratin	0.43704,	0.44335,	0.62842	0.4401,	0.4401,	0.63021

$\tau = 0.20$

Firm 0: Nointegration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.46582,	0.46582,	0.58789	0.47644,	0.46482,	0.59749
	Integratin	0.46482,	0.47644,	0.59749	0.47639,	0.47639,	0.60278

Firm 0: Integration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.43652,	0.43652,	0.61992	0.44406,	0.43365,	0.62593
	Integratin	0.43365,	0.44406,	0.62593	0.4401,	0.4401,	0.63021

$\tau = 0.29$

Firm 0: Nointegration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.46633,	0.46633,	0.55822	0.48553,	0.47054,	0.57708
	Integratin	0.47054,	0.48553,	0.57708	0.49119,	0.49119,	0.58546

Firm 0: Integration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.4306,	0.4306,	0.6122	0.44268,	0.43032,	0.62307
	Integratin	0.43032,	0.44268,	0.62307	0.4401,	0.4401,	0.63021

$\tau = 0.30$

Firm 0: Nointegration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.46594,	0.46594,	0.55454	0.48627,	0.47097,	0.57465
	Integratin	0.47097,	0.48627,	0.57465	0.49277,	0.49277,	0.58336

Firm 0: Integration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.42979,	0.42979,	0.61121	0.44242,	0.42993,	0.62272
	Integratin	0.42993,	0.44242,	0.62272	0.4401,	0.4401,	0.63021

$\tau = 0.36$

Firm 0: Nointegration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.46167,	0.46167,	0.53099	0.48957,	0.47261,	0.55945
	Integratin	0.47261,	0.48957,	0.55945	0.50197,	0.50197,	0.57009

Firm 0: Integration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.42427,	0.42427,	0.60479	0.44039,	0.42754,	0.62045
	Integratin	0.42754,	0.44039,	0.62045	0.4401,	0.4401,	0.63021

$\tau = 0.37$

Firm 0: Nointegration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.46063,	0.46063,	0.52681	0.48993,	0.47273,	0.55683
	Integratin	0.47273,	0.48993,	0.55683	0.50345,	0.50345,	0.56777

Firm 0: Integration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.42325,	0.42325,	0.60363	0.43998,	0.42713,	0.62005
	Integratin	0.42713,	0.43998,	0.62005	0.4401,	0.4401,	0.63021

$\tau = 0.40$

Firm 0: Nointegration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.45694,	0.45694,	0.51389	0.49072,	0.47285,	0.54884
	Integratin	0.47285,	0.49072,	0.54884	0.50781,	0.50781,	0.56063

Firm 0: Integration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.42,	0.42,	0.6	0.43864,	0.42588,	0.6188
	Integratin	0.42588,	0.43864,	0.6188	0.4401,	0.4401,	0.63021

$\tau = 0.50$

Firm 0: Nointegration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.43835,	0.43835,	0.46655	0.49005,	0.47054,	0.52106
	Integratin	0.47054,	0.49005,	0.52106	0.52138,	0.52138,	0.53494

Firm 0: Integration		Firm B					
		No integratin			Integratin		
Firm A	No integratin	0.40735,	0.40735,	0.58618	0.43291,	0.42151,	0.61418
	Integratin	0.42151,	0.43291,	0.61418	0.4401,	0.4401,	0.63021