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Leadership by Confidence in Teams*

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Abstract

We study endogenous signaling by analyzing a team production problem with endogenous timing. Each agent of the team is privately endowed with some level of confidence about team productivity. Each of them must then commit a level of effort in one of two periods. At the end of each period, each agent observes his partner's move in this period. Both agents are rewarded by a team output determined by team productivity and total invested effort. Each agent must personally incur the cost of effort that he invested. We show a set of sufficient conditions under which an agent chooses to become a leader or a follower depending on his confidence about reward from the team in a stable equilibrium. This means that a player endogenously becomes a signal sender or a signal receiver depending only on the cost-benefits from becoming a sender or a receiver.

JEL Classification: C72, D82

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1 Introduction

1.1 Overview

The study of endogenous timing games has advanced our understanding of many economic phenomena through explaining when an agent takes an action. It has successfully covered many fields such as oligopoly, bargaining, and investment decisions. Yet, one field has remained unexplored to date. Leadership in teams is a typical phenomenon in which the choice of action timing becomes a central issue. It has been widely recognized that leadership is a key to understand teams. Nevertheless, no successful explanation has been made about when leadership emerges and who takes leadership in teams.

This paper investigates leadership in teams by analyzing the model of team production with endogenous timing. Leadership emerges in a team if one member moves voluntarily prior to others and causes favorable changes in the effort choices of the other members. We show in this paper that under a set of sufficient conditions, leadership emerges as an equilibrium of the team production with endogenous timing.

The failure to address leadership in teams in economics is mainly due to the lack of an appropriate model of the leadership mechanism. The focus of the studies of endogenous timing games is on mechanisms by which players are sorted to move at different points of time. By the types of mechanisms, the existing studies of endogenous timing games are broadly classified into two groups. The first group mainly adopts private information and time preference as driving forces. This type of models explains sorting by the degree to which players face trade-off between the cost of waiting by itself and the benefit of learning the rival's private information by waiting (or signaling the player's own private information by waiting). These models have been applied to areas such as investment decisions (Hendricks and Kovenock (1989), Chamley and Gale (1994), and Gul and Lundholm (1995)), bargaining (Kambe (1999) and Abreu and Gul (2000)), and war of attrition (Fudenberg and Tirole (1986)).

The second group explains the sorting of the timings of moves by the advantage or disadvantage of commitment that is determined by strategic relations such as strategic complementarity or substitution. This type of models has been mainly applied to oligopolistic situations such as Cournot competition and Bertrand competition (Gal-Or (1987), Hamilton and Slutsky (1990), Mailath (1993), Normann (1997), van-Damme and Hurkens (1999), Normann (2002), van Damme and Hurkens (2004), Amir and Stepanova (2006), and so on).

Emergence of leadership in teams cannot be fully explained by those types of combinations of the forces. Either time preference or strategic complementarity or substitution is not necessarily present in team production.

Rather, team output and contribution cost do not usually depend on timing of investing efforts. Then, the team production problem is free from time discounting. Furthermore, as suggested by Holmstrom (1982), although there are possibly many types of team production functions, the natural specification of the function is the product of total efforts and the productivity parameter. This function is the prototype of the team production function, as has appeared in many applications (e.g., Hermalin (1998)). Observe that the function implies neither strategic complementarity nor substitution. Therefore, the above two existing groups are not sufficient to explain the emergence of leadership in teams.

We adopt a new combination of driving forces in order to explain emergence of leadership. The driving forces are *multi-sided private information* and *simple payoff externality*. Multi-sided private information means that every player has private information about team productivity. Simple payoff externality is that a player's payoff depends on the action choices of other players, but a player's optimal action does not depend on the action choices by other players. The production function that is the product of total efforts and the productivity parameter implies simple payoff externality.

The combination of multi-sided private information and simple payoff externality makes leadership emerge in teams as follows. Every agent holds his personal expectation about team productivity based on his private information. The expectation pushes an agent to voluntarily move before others do when he is more confident and it induces him to wait for action at a later timing when he is less confident. When an agent becomes a leader or a follower according to his private information, signaling from the leader to the follower is realized. Then, the sender enjoys a benefit due to the simple payoff externality from increased effort investment by the receiver. The receiver benefits from learning the sender's private information. The expectation of this signaling is the cause that sorts some agents into leaders and others into followers. We say that *endogenous signaling* is realized when (i) an agent chooses to be a sender or a receiver solely by comparing the benefit of sending a signal with the benefit of learning a signal; and (ii) a sender-receiver relation is endogenously realized by these choices.

We formalize the situation above into a simple team production game. There are two members and each agent is privately endowed with some level of confidence about team productivity. Each member must then commit a level of effort in one of two periods. At the end of each period, each agent observes his partner's move in this period. Both agents are rewarded by a team output determined by team productivity and total invested effort. Each agent must personally incur the cost of the effort that he invested.

In this model, leadership by confidence prevails when (i) an agent, who holds an optimistic belief about the reward from his effort investment for the team, moves voluntarily and commits a certain level of effort in the first period; and (ii) the other agent, seeing the first mover's action, is induced to invest levels of efforts in the second period higher than he would invest if the first move action did not take place. We show a sufficient condition under which leadership by confidence is supported as a stable outcome of sequential equilibria in the team production game. The sufficient condition consists of four assumptions on the environment. First, confidence is independent across agents. Second, agents are symmetric. Third, the increase in the conditional expectations of team productivity when an agent learns his partner is more confident is smaller when the agent himself is more confident than when he is less confident. Fourth, effort cost is subject to strong convexity.

The intuition is as follows. In order for leadership by confidence to emerge, two properties must hold: (i) when an agent is more confident about team productivity, he prefers moving first and signaling his confidence to moving second and learning his partner's confidence; and (ii) when an agent is less confident about team productivity, he is unwilling to mimic the behavior he would take if he were more confident.

The essential part of (i) is the trade-off between signaling and learning. If an agent chooses to move second, he can make a better decision about his effort by learning his partner's confidence about team productivity. If he moves first, he must give up the value of this information; however, he can enjoy the increase in his partner's effort. When agents are symmetric (the second assumption above), the size of one's own effort adjustment in moving second and the size of an increase in his partner's effort become comparable. When an agent is more confident about team productivity, the impact on utility is smaller for the former factor by the third assumption. Therefore, he prefers signaling to learning.

Property (ii) is a single crossing property about a choice between moving first and moving second. When an agent is more confident about team productivity, he expects a larger impact on team output from an increase in his partner's effort by the first assumption. He also expects a smaller loss of value of information by the third and fourth assumptions, because his expectation about team productivity is less influenced by his partner's information, and he foresees that effort adjustment in moving second is relatively negligible. Therefore, an agent is more willing to move first when he is more confident about team productivity.

1.2 The Literature

We model endogenous signaling under multi-sided private information as the driving force for leadership in teams. We relate our model to the literature of endogenous timing games in the light of signaling.

There are broadly two groups of studies of endogenous timing games, as mentioned above. The studies in the first group are exactly the ones that paid attention to private information and signaling in endogenous timing games. However, once we remove time discounting from the models, every player prefers to be a receiver and never desires to be a sender. The mere private information is not enough to explain endogenous timings of moves in teams.

On the other hand, the studies in the second group simply extend static games such as Cournot competition into dynamic complete information games by allowing players to choose the timings of moves. Signaling has no role to play in explaining endogenous timings of moves.

Some papers in the second group studied the endogenous timing mechanism of strategic substitution in the presence of private information. Mailath (1993) is the first attempt. He studied a Cournout market in which only one firm has private information and has a choice of timing for move in one of two periods while the other firms have no private information and must move in the second period. He showed that the informed firm endogenously becomes a Stackelberg leader in equilibrium. Normann (1997) and Normann (2002) extended Mailath (1993) to a model in which all the firms have choices of timing for move while keeping private information restricted to one firm. Signaling takes place in equilibrium in these models. However, since only one firm is privately informed, the potential leader is determined a priori and strategic substitution continues to play a crucial role for endogenous timings of moves.

There is one previous study that explores endogenous choice of timing solely by signaling and without assuming time discounting or strategic relations such as strategic substitution or complementarity. Kobayashi and Suehiro (2005) studied the team production problem in which every player has their own private information and every player can choose timings of moves at their own discretion.[1] This team production problem is precisely the one that this paper investigates. They found three possible types of leadership in teams: leadership by confidence, leadership by identity, and leadership by identity with confidence. Leadership by confidence is the outcome that we defined above. Leadership by identity means that a particular player moves first irrespective of his private information. Leadership by identity with confidence is a hybrid of the first two. They showed that if private information is independent across players: (i) leadership by identity always exists as a stable equilibrium; and (ii) no stable equilibria exist other than the ones by the three types of leadership.

An equilibrium with a particular leader in leadership by identity may emerge only depending on an outside context about the team beyond the description of the game. In contrast, leadership by confidence is independent

[1] This is an extension of Hermalin (1998) to the case of endogenous choice of timing. Hermalin (1998) studied a signaling game with specific rules: (i) one member exclusively holds information about team productivity; (ii) the member must commit a level of effort in the first period; and (iii) the other members choose their levels of efforts in the second period after observing the choice by the leader.

of an outside context. Therefore, unless there is a specific outside mechanism at work to induce a particular member to become a leader, leadership by confidence should naturally realize in the environment in which every player chooses his timings of moves. Hence, leadership by confidence is the most natural solution of the team game. Precisely the issue of under what condition leadership by confidence emerges as a stable outcome has remained unsolved. This paper explains the natural mode of leadership in teams as an equilibrium of endogenous timing games by identifying a sufficient condition for the emergence of leadership by confidence.

The rest of this paper is organized as follows. We describe our team production game in Section 2. In Section 3, we present the sufficient condition for leadership by confidence and prove our main result. Section 4 explains the role of each condition in realizing leadership by confidence. We conclude in Section 5.

2 The Model

We study the team production game of Kobayashi and Suehiro (2005). A team with two agents $i = 1, 2$ is engaged in production. Each agent i invests an effort $e_i \in R_+$ with cost $c_i(e_i)$. The output level x of the team is determined by $x = \theta \sum_i e_i$, where θ is a productivity parameter. Agent i receives a benefit $v_i(x)$ from the output x. When agent i invests e_i and the team output is x, his payoff is $u_i = v_i(x) - c_i(e_i)$.

We assume agent i is risk neutral in x, that is, $v_i(x) = s_i x$ for some $s_i > 0$. We assume the cost function c_i satisfies the following conditions.

A1 c_i is differentiable and $c_i'(e_i) > 0$ for any $e_i > 0$ with $\lim_{e_i \to 0} c_i'(e_i) = 0$ and $\lim_{e_i \to \infty} c_i'(e_i) = \infty$.

A2 c_i is twice-differentiable and $c_i''(e_i) > 0$ for any $e_i > 0$.

The cost is strictly increasing in e_i and is subject to increasing marginal cost.

The parameter θ is a realization of some random variable $\boldsymbol{\theta}$. Each agent i receives private information about θ before production. According to the information, he is in one of the two states of confidence about team productivity. He may be more confident or less confident. Agent i is called H-type in the former case and L-type in the latter. Let $p(t_i, t_j)$ denote a probability for an event that agent i is t_i-type and agent j is t_j-type. Let $E[\boldsymbol{\theta}|t_i, t_j]$ denote the conditional expectation of $\boldsymbol{\theta}$ given the event. The expectation by agent i of type t_i is expressed by:

$$E[\boldsymbol{\theta}|t_i] = \frac{p(t_i, H)}{\sum_{t_j} p(t_i, t_j)} E[\boldsymbol{\theta}|t_i, H] + \frac{p(t_i, L)}{\sum_{t_j} p(t_i, t_j)} E[\boldsymbol{\theta}|t_i, L].$$

The interpretation that H-type is more confident than L-type is modeled without loss of generality as:

$$E[\boldsymbol{\theta}|L, L] < E[\boldsymbol{\theta}|H, L], E[\boldsymbol{\theta}|L, H] < E[\boldsymbol{\theta}|H, H].$$

This implies:

$$E[\boldsymbol{\theta}|t_i = H] > E[\boldsymbol{\theta}|t_i = L].$$

Agents must choose levels of their efforts according to the following time sequence. There are two periods, 1 and 2. In period 1, each agent i may exert an effort level e_i or may choose to do nothing (denoted as $\emptyset$). If he implements his effort in period 1, then he cannot do anything in period 2. On the other hand, if he chooses to do nothing in period 1, then he must implement his effort in period 2.

In this sequence of moves, the two agents must move (taking some e_i or $\emptyset$) independently and simultaneously in period 1. Each agent i immediately observes the behavior that the other agent j has taken. Agent i can then utilize this information for his choice in period 2 if he has chosen to do nothing in period 1. If both agents have chosen to do nothing in period 1, both must invest some level of effort independently and simultaneously in period 2.

Agent i's strategy σ_i is a profile $(\sigma^1_{i,t_i}, \sigma^2_{i,t_i})_{t_i=H,L}$ of Bayesian strategies. The part σ^1_{i,t_i} prescribes his behavior in period 1 for t_i-type and it takes a value $\sigma^1_{i,t_i} = a^1_i$ in $R_+ \cup \{\emptyset\}$. The part σ^2_{i,t_i} prescribes his behavior in period 2 for t_i-type and it assigns a value $\sigma^2_{i,t_i}(a^1_j) = a^2_i$ in R_+ for each possible value a^1_j of agent j's choice from $R_+ \cup \{\emptyset\}$ in period 1.

A strategy profile (σ_1, σ_2) is called *leadership by confidence* when $\sigma^1_{i,H} \in R_+$ and $\sigma^1_{i,L} = \emptyset$ for $i = 1, 2$. In words, leadership by confidence means each agent moves first if and only if he is more confident about team productivity.

3 A Sufficient Condition for Leadership by Confidence to be a Stable Equilibrium

Let us develop a sufficient condition for leadership by confidence to be a stable equilibrium. Consider the following assumptions.

A3 (Type independence): Let $\rho_j \equiv \text{Prob}(t_j = H) = p(H, H) + p(L, H)$. Then, $\frac{p(t_i,H)}{\sum_{t_j} p(t_i,t_j)} = \rho_j$ for $t_i = L, H$.

A4 (Payoff symmetry): $s_i = s_j \equiv s$ and $c_i(e) = c_j(e) \equiv c(e)$ for all $e \in R_+$.

A5 (Informational symmetry): $E[\boldsymbol{\theta}|H, L] = E[\boldsymbol{\theta}|L, H]$.

A6 (Increasing convexity): c_i is differentiable three times and $c_i'''(e_i) \geq 0$ for any $e_i > 0$.

A7 (Decreasing returns to good news):

$$E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|H,L] \leq E[\boldsymbol{\theta}|L,H] - E[\boldsymbol{\theta}|L,L]$$
$$E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|L,H] \leq E[\boldsymbol{\theta}|H,L] - E[\boldsymbol{\theta}|L,L].$$

A3 means agent types are independently distributed. A4 means the payoff functions are symmetric. A5 means the conditional expectations of team productivity are symmetric with respect to the realization of types. In other words, good news and bad news have the same informational contents irrespective of who holds the news. A6 means the rate at which the marginal effort cost increases is higher for higher levels of effort. A7 means the increase in the conditional expectations of team productivity when an agent learns his partner is more confident is smaller when the agent himself is more confident than when he is less confident.

To state the stable outcome of leadership by confidence under these conditions, let us develop formulae for players' payoffs from a play in leadership by confidence. Suppose leadership by confidence (σ_1, σ_2) prevails. Let μ_{j,t_j} be a belief of player j of type t_j that agent i is H-type. Then, the expectation of his own contribution to his payoff by investing e_j is given by:

$$s\Big(\mu_{j,t_j} E[\boldsymbol{\theta}|H,t_j] + (1-\mu_{j,t_j})E[\boldsymbol{\theta}|L,t_j]\Big)e_j - c(e_j).$$

Let $e^*(t_j, \mu_{j,t_j})$ be the maximizer of this expectation.[2] The first order condition for the maximization problem is:

$$s\Big(\mu_{j,t_j} E[\boldsymbol{\theta}|H,t_j] + (1-\mu_{j,t_j})E[\boldsymbol{\theta}|L,t_j]\Big) = c'(e_j).$$

Assumptions A1 and A2 guarantee that the first order condition uniquely determines the interior solution for the maximization problem.

Suppose agent i of type t_i moves first with e_i. In the equilibrium, he expects that agent j of H-type moves first with $\sigma^1_{j,H}$ and agent j of L-type moves second. Suppose agent i believes the following: by observing e_i, agent j of L-type holds a belief $\mu_{j,L}$ and responds with the sequentially rational choice $e^*(L, \mu_{j,L})$. Then, under assumption A3, agent i's expected payoff from his first move is:

$$\begin{aligned} U_i(t_i, e_i, \mu_{j,L}) = sE[\boldsymbol{\theta}|t_i]e_i - c(e_i) \quad &+ \quad s\Big(\rho_j E[\boldsymbol{\theta}|t_i,H]\sigma^1_{j,H} \\ &+ \quad (1-\rho_j)E[\boldsymbol{\theta}|t_i,L]e^*(L,\mu_{j,L})\Big). \end{aligned}$$

[2]Note that we write e^* rather than e_j^* because the maximizer depends on j only through t_j and μ_{j,t_j} because of assumptions A4 and A5.

The payoff U_i also depends on player j's behavior $\sigma^1_{j,H}$ and a parameter ρ_j, but we drop them from its domain because our analysis is valid irrespective of their values.

Similarly, suppose agent i of type t_i moves second. In the equilibrium, he expects that agent j of H-type moves first with $\sigma^1_{j,H}$ and he invests the sequentially rational choice $e^*(t_i, 1)$ under the belief $\mu_{i,t_i} = 1$ after observing this move. He also expects that agent j of L-type moves second and he invests the sequentially rational choice $e^*(t_i, 0)$ under the belief $\mu_{i,t_i} = 0$ after observing this move while agent j responds with $e^*(L, 0)$ under the belief $\mu_{j,L} = 0$ after observing that agent i moves second. Then, under assumption A3, agent i's expected payoff from moving second is:

$$\begin{aligned} U_i(t_i, \emptyset, 0) &= \rho_j \Big(sE[\boldsymbol{\theta}|t_i, H](e^*(t_i, 1) + \sigma^1_{j,H}) - c(e^*(t_i, 1)) \Big) \\ &+ (1-\rho_j) \Big(sE[\boldsymbol{\theta}|t_i, L](e^*(t_i, 0) + e^*(L, 0)) - c(e^*(t_i, 0)) \Big). \end{aligned}$$

Here we do not explicitly write $\sigma^1_{j,H}$, ρ_j for the same reason as for $U_i(t_i, e_i, \mu_{j,L})$.

Our candidate for stable outcome of leadership by confidence is the quasi-Riley outcome defined as follows.

Definition. *A pair of $(\sigma^1_{1,H}, \sigma^1_{2,H})$ is the quasi-Riley outcome of leadership by confidence if for $i = 1, 2$:*

$$\begin{aligned} \sigma^1_{i,H} = &\arg\max_{e_i \in R_+} U_i(H, e_i, 1) \\ &\text{subject to } U_i(L, \emptyset, 0) \geq U_i(L, e_i, 1). \end{aligned} \tag{1}$$

Note that although U_i depends on $\sigma^1_{j,H}$, whether $\sigma^1_{i,H}$ satisfies (1) does not depend on $\sigma^1_{j,H}$. Therefore, a value of $\sigma^1_{i,H}$ that constitutes a quasi-Riley outcome is determined independent of $\sigma^1_{j,H}$.

First, we will show that there uniquely exists the quasi-Riley outcome in the described environment. Let:

$$M_i \equiv \{e_i \in R_+ | U_i(L, \emptyset, 0) \leq U_i(L, e_i, 1)\}.$$

Consider the equation of e_i:

$$U_i(L, \emptyset, 0) = U_i(L, e_i, 1).$$

If $M_i \neq \emptyset$, then there exists solutions e_i^-, e_i^+ ($e_i^- \leq e_i^+$) to this equation and $M_i = [e_i^-, e_i^+]$, because $U_i(L, e_i, 1)$ is strictly concave in e_i under assumption A2. With such a e_i^+, the quasi-Riley outcome is characterized as follows.

Proposition 1. *Under assumptions A1 through A5, there exists the unique quasi-Riley outcome of leadership by confidence, $(\hat{\sigma}^1_{i,H}, \hat{\sigma}^1_{j,H})$. Namely, if $e^*(H, \rho_j) \notin M_i$, then $\hat{\sigma}^1_{i,H} = e^*(H, \rho_j)$. If $e^*(H, \rho_j) \in M_i$, then $\hat{\sigma}^1_{i,H} = e_i^+$.*

Proof. Suppose $e^*(H, \rho_j) \notin M_i$. Then, $e^*(H, \rho_j)$ is the unique solution to:

$$\max_{e_i \in R_+ \setminus M_i} U_i(H, e_i, 1).$$

This means $e^*(H, \rho_j)$ constitutes the unique quasi-Riley outcome.

Suppose $e^*(H, \rho_j) \in M_i$. Then, either e_i^- or e_i^+ constitutes the quasi-Riley outcome, because $U_i(H, e_i, 1)$ is strictly concave in e_i under assumption A2. From the definition of e_i^- and e_i^+, we have:

$$U_i(L, e_i^+, 1) - U_i(L, e_i^-, 1) = 0.$$

This reduces to:

$$\Big(sE[\boldsymbol{\theta}|L]e_i^+ - c(e_i^+)\Big) - \Big(sE[\boldsymbol{\theta}|L]e_i^- - c(e_i^-)\Big) = 0.$$

Then, we have:

$$\Big(sE[\boldsymbol{\theta}|H]e_i^+ - c(e_i^+)\Big) - \Big(sE[\boldsymbol{\theta}|H]e_i^- - c(e_i^-)\Big) \geq 0,$$

because $E[\boldsymbol{\theta}|H] > E[\boldsymbol{\theta}|L]$. This inequality implies:

$$U_i(H, e_i^+, 1) \geq U_i(H, e_i^-, 1).$$

The equality holds for the case of $e_i^- = e_i^+$. Therefore, e_i^+ constitutes the unique quasi-Riley outcome. ■

The intuition is as follows. Let us consider the case $e^*(H, \rho_j) \in M_i$. In the definition of the quasi-Riley outcome, the maximization of agent i's payoff is equivalent to the maximization of agent i's own contribution, because his partner j is assumed to move first with $\sigma_{j,H}^1$ if he is H-type and to move second with response $e^*(L, 1)$ if he is L-type. Agent i is more willing to invest higher effort for his own contribution when he is H-type than when he is L-type. Therefore, when agent i of L-type is indifferent between e_i^- and e_i^+, agent i of H-type strictly prefers e_i^+ to e_i^-. This selects e_i^+ as the unique quasi-Riley outcome in this case.

Next, we will show that the quasi-Riley outcome is supported by a sequential equilibrium. We will prepare two basic lemmata that characterize the strength of agents' incentives to move first for the purpose of convincing partners that they are H-type. The first lemma shows that if an agent is H-type, the benefit of moving first with the interim optimal action and successfully signaling his type exceeds the value of information he will enjoy in moving second.

Lemma 1. *Assume A1 through A7. Then,* $U_i(H, e^*(H, \rho_j), 1) > U_i(H, \emptyset, 0)$.

Proof. See Appendix. ■

The intuition is as follows. The difference between the payoff from moving first and moving second is decomposed as follows:

$$
\begin{aligned}
&U_i(H, e^*(H,\rho_j), 1) - U_i(H, \emptyset, 0) \\
&= (1-\rho_j) sE[\boldsymbol{\theta}|H,L](e^*(L,1) - e^*(L,0)) \\
&-\Bigg[\Big\{\rho_j\Big(sE[\boldsymbol{\theta}|H,H]e^*(H,1) - c(e^*(H,1)\Big) + (1-\rho_j)\Big(sE[\boldsymbol{\theta}|H,L]e^*(H,0) - c(e^*(H,0))\Big)\Big\} \\
&-\Big\{s(\rho_j E[\boldsymbol{\theta}|H,H] + (1-\rho_j)E[\boldsymbol{\theta}|H,L])e^*(H,\rho_j) - c(e^*(H,\rho_j))\Big\}\Bigg].
\end{aligned}
$$

The first term represents the benefit of signaling by moving first, that is, the increased contribution by the partner. The second term is the value of information by moving second. The benefit of signaling is linear in ρ_j and vanishes at $\rho_j = 1$. The value of information is concave in ρ_j and vanishes at $\rho_j = 0$ and $\rho_j = 1$. Therefore, if it is the case that the benefit of signaling exceeds the value of information at $\rho_j = 1 - \epsilon$ for small $\epsilon > 0$, it is the case for any $\rho_j \in (0, 1)$.

Consider the case of $\rho_j = 1 - \epsilon$ for small $\epsilon > 0$. In this case, the benefit of signaling is:

$$\epsilon sE[\boldsymbol{\theta}|H,L](e^*(L,1) - e^*(L,0)). \tag{2}$$

On the other hand, the value of information is approximated by:

$$\epsilon\Big(sE[\boldsymbol{\theta}|H,L]e^*(H,0) - c(e^*(H,0))\Big) - \epsilon\Big(sE[\boldsymbol{\theta}|H,L]e^*(H,1) - c(e^*(H,1))\Big),$$

because $e^*(H,\rho_j) \approx e^*(H,1)$ for small $\epsilon > 0$. The approximated form of the value of information represents the improvement in payoff of choosing the correct action $e^*(H,0)$ instead of the wrong action $e^*(H,1)$ for $E[\boldsymbol{\theta}|H,L]$. Consider the parallel formula for the improvement in payoff of choosing the correct action $e^*(H,1)$ instead of the wrong action $e^*(H,0)$ for $E[\boldsymbol{\theta}|H,H]$. The following identity holds for the sum of the two improvements:

$$
\begin{aligned}
&\Big[\Big(sE[\boldsymbol{\theta}|H,L]e^*(H,0) - c(e^*(H,0))\Big) - \Big(sE[\boldsymbol{\theta}|H,L]e^*(H,1) - c(e^*(H,1))\Big)\Big] \\
&+\Big[\Big(sE[\boldsymbol{\theta}|H,H]e^*(H,1) - c(e^*(H,1))\Big) - \Big(sE[\boldsymbol{\theta}|H,H]e^*(H,0) - c(e^*(H,0))\Big)\Big] \\
&= s(E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|H,L])(e^*(H,1) - e^*(H,0)).
\end{aligned}
$$

Therefore, we know the value of information is bounded from above by:

$$\epsilon s(E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|H,L])(e^*(H,1) - e^*(H,0)), \tag{3}$$

that is, the increment of team productivity times the increase in own optimal effort corresponding to the good news that the partner is H-type rather than

L-type. This is smaller than the benefit of signaling for the following reasons. The partner's effort adjustment $e^*(L,1) - e^*(L,0)$ in formula (2) is larger than the agent's own adjustment $e^*(H,1) - e^*(H,0)$ in formula (3) because of the increased convexity of effort cost (assumption A6) and the decreasing returns to good news (assumption A7). The team productivity $E[\boldsymbol{\theta}|H,L]$ as the impact coefficient of the increase in partner's effort in formula (2) is larger than the increment in team productivity in formula (3) because of the decreasing returns to good news (assumption A7).

The second lemma shows that the relative benefit of signaling by moving first over the value of information by moving second is greater for H-type than for L-type. In other words, the single crossing property holds for the choice between moving first and moving second.

Lemma 2. *Assume A1 through A7. Then,* $U_i(H,e,1) - U_i(H,\emptyset,0) \geq U_i(L,e,1) - U_i(L,\emptyset,0)$ *for any* $e \geq e^*(H,\rho_j)$.

Proof. See Appendix. ■

The intuition is as follows. Consider the payoff difference $U_i(t_i,e,1) - U_i(t_i,\emptyset,0)$. The difference can be decomposed into three parts: (i) the loss of moving first with e instead of $e^*(t_i,\rho_j)$; (ii) the loss of moving first with $e^*(t_i,\rho_j)$ irrespective of the partner's type, that is, the loss from giving up the value of information from moving second; and (iii) the benefit from the change in partner's effort choice from $e^*(L,0)$ to $e^*(L,1)$. The benefit of (iii) is larger for H-type than L-type, because the expectation of θ as the multiplier of partner's effort is higher for H-type. The loss of (i) is smaller for H-type than L-type because the interim optimal level $e^*(t_i,\rho_j)$ is higher for H-type.

The loss of (ii) is rather complicated. Suppose hypothetically that player i did not know his type and believed with probability δ he is H-type. Consider the case in which assumption 7 holds with equality. In this case, when δ increases slightly, the increase in expected team productivity given $t_j = L$ is the same as the increase in expected team productivity given $t_j = H$. Call them the increase in expected team productivity simply. Then, using the envelope theorem, the change in the value of information from moving second is approximated by:

$$\begin{aligned} s \;\times\; & \text{the increase in expected team productivity} \\ \times\; & [\text{ the expected value of player } i\text{'s effort in moving second} \\ & -\text{the interim optimal effort level in moving first }]. \end{aligned}$$

Player i's effort choice $e^*(t_i,\rho_j)$ is concave in ρ_j under assumption 6. This implies the term in brackets is negative. Therefore, the value of information is decreasing in δ, and the value of information is smaller for H-type than L-type. In the case in which assumption 7 holds with strict inequality, H-type requires a smaller effort adjustment to partner's type in moving second.

Therefore, the value of information is even smaller for H-type in this case and the loss of (ii) is smaller for H-type than L-type.

Now we will establish our main result based on the prepared lemmata.

Proposition 2. *Under assumptions A1 through A7, the unique quasi-Riley outcome of leadership by confidence is supported by a sequential equilibrium.*

Proof. By Proposition 1, the quasi-Riley outcome uniquely exists. Denote it as $(\hat{\sigma}^1_{i,H}, \hat{\sigma}^1_{j,H})$. Consider the following strategy profile. For $i = 1, 2$:

$$\sigma^1_{i,H} = \hat{\sigma}^1_{i,H}$$
$$\sigma^1_{i,L} = \emptyset$$
$$\sigma^2_{i,t_i}(a^1_j) = \begin{cases} e^*(t_i, 1) & \text{if } a^1_j = \hat{\sigma}^1_{j,H} \\ e^*(t_i, 0) & \text{otherwise} \end{cases} \quad \text{for } t_i = H, L.$$

Consider the belief system:

$$\mu_i(a^1_j) \equiv \mu_{i,t_i}(a^1_j) = \begin{cases} 1 & \text{if } a^1_j = e_j = \hat{\sigma}^1_{j,H} \\ 0 & \text{if } a^1_j = \emptyset \text{ or } a^1_j = e_j \neq \hat{\sigma}^1_{j,H} \end{cases} \quad \text{for } t_i = H, L.$$

The constructed belief system is consistent with the strategy profile, that is, the beliefs $\mu_i(\hat{\sigma}^1_{j,H}) = 1$, $\mu_i(\emptyset) = 0$ along the equilibrium paths satisfy Bayes rule.

Let us verify sequential rationality of the strategies. Consider the incentive for L-type to mimic the behavior of H-type and to deviate to moving first with $\hat{\sigma}^1_{i,H}$. If $\hat{\sigma}^1_{i,H} \notin M_i$, obviously L-type has no incentive to deviate. If $\hat{\sigma}^1_{i,H} \in M_i$, then $\hat{\sigma}^1_{i,H} = e^+_i$ by Proposition 1 and L-type is indifferent between moving first with e^+_i and moving second. Consider the incentive for L-type to deviate to moving first with $e_i \neq \hat{\sigma}^1_{i,H}$. By the construction of the belief system, L-type must expect his partner of L-type to hold the same belief $\mu_j(e_i) = 0$ as the one his partner will hold when he moves second. Therefore, the expectation of his partner's contribution to his payoff remains the same when he deviates to the first move. On the other hand, the expectation of his own contribution to his payoff is strictly greater when he moves second than when he deviates to the first move, because he learns his partner's type and makes a better decision about his effort choice.

Consider the incentive for H-type to mimic the behavior of L-type and to deviate to moving second. In the case of $\hat{\sigma}^1_{i,H} = e^*(H, \rho_j)$, H-type does not deviate to moving second by Lemma 1. In the case of $\hat{\sigma}^1_{i,H} = e^+_i$, H-type does not deviate to moving second either, because Lemma 2 and the definition of e^+_i mean:

$$U_i(H, \hat{\sigma}^1_{i,H}, 1) - U_i(H, \emptyset, 0) \geq U_i(L, \hat{\sigma}^1_{i,H}, 1) - U_i(L, \emptyset, 0) = 0.$$

Consider the incentive for H-type to deviate to moving first with $e_i \neq \hat{\sigma}^1_{i,H}$. For the same argument made above for L-type, H-type prefers moving second to moving first with $e_i \neq \hat{\sigma}^1_{i,H}$. Because H-type was proved to prefer moving first with $\hat{\sigma}^1_{i,H}$ to moving second, he has no incentive to deviate to moving first with $e_i \neq \hat{\sigma}^1_{i,H}$. ■

The quasi-Riley outcome of leadership by confidence is not only an equilibrium outcome, but a stable outcome.

Proposition 3. *Under assumptions A1 through A7, the unique quasi-Riley outcome of leadership by confidence is stable.*

Proof. Consider the sequential equilibrium constructed in the proof of Proposition 2. In order to see that the belief system μ_i of player i satisfies the Cho–Kreps criterion, let us examine an incentive for player j to deviate from the equilibrium. Consider the case of $e^*(H, \rho_i) \notin M_j$. Because $\hat{\sigma}^1_{j,H} = e^*(H, \rho_i)$ by Proposition 1, H-type never has an incentive to deviate to $e_j \neq \hat{\sigma}^1_{j,H}$. If $M_j = \emptyset$, then L-type does not have an incentive to deviate to moving first with $e_j \neq \hat{\sigma}^1_{j,H}$ either. Hence, the Cho–Kreps criterion puts no restriction on $\mu_i(e_j)$ for any $e_j \neq \hat{\sigma}^1_{j,H}$. If $M_j \neq \emptyset$, then L-type is willing to deviate to moving first with $e_j \in M_j$ for some $\mu_i \in [0, 1]$. Therefore, the Cho–Kreps criterion requires $\mu_i(e_j) = 0$ for $e_j \in M_j$. On the other hand, the Cho–Kreps criterion puts no restriction on $\mu_i(e_j)$ for any $e_j \notin M_j$. Therefore, the constructed belief system satisfies the Cho–Kreps criterion.

Consider the case of $e^*(H, \rho_i) \in M_j$. Define $\tilde{e}^-_j$ as a solution to the equation of e_j :

$$U_j(H, e^+_j, 1) = U_j(H, e_j, 1).$$

The inequality:

$$U_j(H, e^-_j, 1) \leq U_j(H, e^+_j, 1)$$

in the proof of Proposition 1 implies $e^-_j \leq \tilde{e}^-_j$. For $e_j \in [e^-_j, \tilde{e}^-_j)$, L-type is willing to deviate to moving first with the e_j for some $\mu_i \in [0, 1]$ while H-type never has an incentive to deviate to the e_j. Therefore, the Cho–Kreps criterion requires $\mu_i(e_j) = 0$ for such e_j. For $e_j \in [\tilde{e}^-_j, e^+_j)$, both L-type and H-type are willing to deviate to moving first with the e_j for some $\mu_i \in [0, 1]$. Therefore, the Cho–Kreps criterion puts no restriction on $\mu_i(e_j)$ for such e_j. For $e_j \notin [e^-_j, e^+_j]$, neither L-type nor H-type has an incentive to deviate to the e_j. Therefore, the Cho–Kreps criterion puts no restriction on $\mu_i(e_j)$ for such e_j, and the constructed belief system satisfies the Cho–Kreps criterion.
■

4 Are the Assumptions Indispensable?

We showed that leadership by confidence is a stable equilibrium under the set of assumptions A3 to A7. Are these assumptions indispensable for emergence of leadership by confidence? The answer is no. However, leadership by confidence fails to emerge if any of the assumptions are violated severely. We will show where leadership by confidence fails to emerge for each of the assumptions when violated severely, and will explain the causes of the failure. The role of assumption A3 is explored in Kobayashi and Suehiro (2005), and it is shown that the assumption is important for the emergence of leadership in various patterns including leadership by confidence. Therefore, we will examine the remaining assumptions from A4 to A7 below. We applied these assumptions in various steps for establishment of the emergence of leadership by confidence. The most basic step is to prove Lemma 1, for which we employed all assumptions. We will show that Lemma 1 ceases to hold without the assumptions.

Lemma 1 asserts that H-type has an incentive to move first if he is believed to be H-type when he invests the interim optimal level of effort $e^*(H,\rho_j)$. After Lemma 1, we showed a decomposition of the difference between the payoff from moving first and the payoff from moving second. If we reproduce it for agent i of H-type without applying the assumptions of symmetry, the decomposition can be written as follows:

$$U_i(H, e_i^*(H,\rho_j), 1) - U_i(H, \emptyset, 0) = S_i - I_i,$$

where $e_i^*(t_i, \mu_{i,t_i})$ satisfies the first order condition:

$$s_i\Big(\mu_{i,t_i} E[\boldsymbol{\theta}|t_i, H] + (1-\mu_{i,t_i}) E[\boldsymbol{\theta}|t_i, L]\Big) = c_i'(e_i), \tag{4}$$

and:

$$\begin{aligned}
S_i &= (1-\rho_j) s_i E[\boldsymbol{\theta}|H,L](e_j^*(L,1) - e_j^*(L,0)),\\
I_i &= \Big\{\rho_j\Big(s_i E[\boldsymbol{\theta}|H,H] e_i^*(H,1) - c_i(e_i^*(H,1))\Big)\\
&\qquad +(1-\rho_j)\Big(s_i E[\boldsymbol{\theta}|H,L] e_i^*(H,0) - c_i(e_i^*(H,0))\Big)\Big\}\\
&\quad -\Big\{s_i(\rho_j E[\boldsymbol{\theta}|H,H] + (1-\rho_j)E[\boldsymbol{\theta}|H,L]) e_i^*(H,\rho_j) - c_i(e_i^*(H,\rho_j))\Big\}.
\end{aligned}$$

Now we write e_i^* rather than e^* to show explicitly that it depends not only on (t_i, μ_{i,t_i}), but also on s_i, $c_i(\cdot)$. The term S_i represents the benefit of signaling by moving first and the term I_i is the value of information by moving second. Agent i of H-type has an incentive to move first when S_i dominates I_i, that is, $S_i \geq I_i$.

Now we examine assumption A4 with the rest of the assumptions in place. Starting from the case of payoff symmetry, imagine we change s_j or $c_j(\cdot)$ for agent j. This affects $e_j^*(L,1) - e_j^*(L,0)$ in the benefit of signaling only. The first order conditions that determine the effort levels $e_j^*(L,1)$, $e_j^*(L,0)$ give:

$$s_j\Big(E[\boldsymbol{\theta}|H,L] - E[\boldsymbol{\theta}|L,L]\Big) = c_j'(e_j^*(L,1)) - c_j'(e_j^*(L,0)). \tag{5}$$

This shows that the increase in partner's effort $e_j^*(L,1) - e_j^*(L,0)$ will vanish if s_j falls to zero or if the slope of $c_j(\cdot)$ becomes steeper, that is, the slope of $c_j^{-1}(\cdot)$ becomes flat. Therefore, S_i approaches zero while I_i remains unchanged. Therefore, the condition for the first move incentive is no longer satisfied.

When agent i of H-type signals his type by moving first, the partner is reluctant to increase his level of effort if the reward rate s_j from the team production is negligibly small or if the cost of increasing effort is extremely high. If the partner will increase his level of effort negligibly, it is not worthwhile for agent i to give up the value of information and signal his type. The same argument applies when we change the role of agent i and j and we change s_i or $c_i(\cdot)$. Therefore, the equilibrium in leadership by confidence requires that payoff symmetry is not violated severely.

Let us show the tightness of the payoff symmetry requirement by an example. Consider the case of $\rho_i = \rho_j = \frac{1}{2}$, $E[\boldsymbol{\theta}|L,L] = 2$, $E[\boldsymbol{\theta}|H,L] = E[\boldsymbol{\theta}|L,H] = 3$, $E[\boldsymbol{\theta}|H,H] = 4$, and $c_i(e) = c_j(e) = e^2$ in which all assumptions except assumption A4 with respect to s_i and s_j are satisfied. Then, the first move incentive $S_i \geq I_i$ for agent i of H-type reduces to $s_j \geq \frac{1}{12}s_i$. The first move incentive $S_j \geq I_j$ for agent j of H-type is $s_j \leq 12s_i$. The parameters s_i and s_j must be in the symmetric fan along the line $s_i = s_j$.

Next, we examine assumption A5 with the rest of the assumptions in place. Starting from the case of informational symmetry, imagine we change $E[\boldsymbol{\theta}|H,L]$. Consider how the first move incentive for agent i of H-type will be affected. The benefit of signaling S_i will vanish as $E[\boldsymbol{\theta}|H,L]$ falls to $E[\boldsymbol{\theta}|L,L]$ for the same reason as in the analysis of assumption A4. That is, equation (5) indicates that when $E[\boldsymbol{\theta}|H,L] - E[\boldsymbol{\theta}|L,L]$ falls to zero, $e_j^*(L,1) - e_j^*(L,0)$ also falls to zero. On the other hand, the value of information I_i increases as $E[\boldsymbol{\theta}|H,L]$ decreases because agent i of H-type needs to adjust his level of effort downward more drastically when he learns his partner is L-type. Formally, we have by the envelope theorem:

$$\frac{\partial I_i}{\partial E[\boldsymbol{\theta}|H,L]} = s_i(1-\rho_j)\Big(e_i^*(H,0) - e_i^*(H,\rho_j)\Big) < 0.$$

Therefore, the first move incentive is no longer satisfied when $E[\boldsymbol{\theta}|H,L]$ is close to $E[\boldsymbol{\theta}|L,L]$ rather than equal to $E[\boldsymbol{\theta}|L,H]$. The same argument applies when we change the role of agents i and j and we change $E[\boldsymbol{\theta}|L,H]$. Therefore, the equilibrium in leadership by confidence requires that informational symmetry is not violated severely.

Let us examine the tightness of the informational symmetry requirement by an example. Consider the case of $\rho_i = \rho_j = \frac{1}{2}$, $s_i = s_j = 1$, and $c_i(e) = c_j(e) = e^2$. Suppose $E[\boldsymbol{\theta}|L,L] = 2$, $E[\boldsymbol{\theta}|H,L] = 3-\epsilon$, $E[\boldsymbol{\theta}|L,H] = 3+\epsilon$, and $E[\boldsymbol{\theta}|H,H] = 4$ for a parameter $\epsilon \in (-1,1)$. Then, all assumptions except assumption A5 are satisfied. The case of $\epsilon = 0$ corresponds to informational symmetry. The first move incentive $S_i \geq I_i$ for agent i of H-type reduces to $-1 < \epsilon \leq \frac{2}{3}$. The first move incentive $S_j \geq I_j$ for agent j of H-type is $-\frac{2}{3} \leq \epsilon < 1$. The parameter ϵ must be in the interval $[-\frac{2}{3}, \frac{2}{3}]$, which is symmetric around zero, so that informational symmetry is required.

Next, we examine assumption A6 with the rest of the assumptions in place. We claim there exists an effort cost function $c(\cdot)$ for which:

$$e^*(H,0) < e^*(H,\rho_j) < e^*(H,1), \tag{6}$$

$$e^*(L,0) \approx e^*(L,\rho_j) \approx e^*(L,1), \tag{7}$$

if we do not require the function $c(\cdot)$ to satisfy assumption 6. The inequality (6) means an agent of H-type adjusts his level of effort according to his expectation about his partner's type. When the inequality (6) holds, we have a positive value of information $I_i > 0$. The near-equality (7) means an agent of L-type chooses almost the same level of effort irrespective of his expectation about his partner's type. When the near-equality (7) holds, the benefit of signaling is almost zero, that is, $S_i \approx 0$. Therefore, the first move incentive for agent i of H-type is no longer satisfied.

The near-equality (7) occurs for a cost function $c(\cdot)$, which is almost linear below $e^*(L,1)$ and is very convex at $e^*(L,1)$. Without assumption A6, this property may be compatible with the regular convexity property of $c(\cdot)$, which guarantees inequality (6). An example of such a function is:

$$c(e) = \begin{cases} 0 & \text{if } 0 \leq e < 1 \\ e^2 - 1 & \text{if } e \geq 1. \end{cases} \tag{8}$$

Consider the case of $\rho_i = \rho_j = \frac{1}{2}$, $E[\boldsymbol{\theta}|L,L] = 1$, $E[\boldsymbol{\theta}|H,L] = E[\boldsymbol{\theta}|L,H] = 2$, and $E[\boldsymbol{\theta}|H,H] = 3$. Then:

$$e^*(H,0) = 1, e^*(H,\rho_j) = \frac{5}{4}, e^*(H,1) = \frac{3}{2},$$
$$e^*(L,0) = e^*(L,\rho_j) = e^*(L,1) = 1.$$

Note that this function $c(\cdot)$ does not satisfy assumptions A1 and A2. One can smooth the function so as to satisfy these assumptions and continue to have the inequality (6) and near-equality (7).[3]

Finally, we examine assumption A7 with the rest of the assumptions in place. Assume $E[\boldsymbol{\theta}|H,L] = E[\boldsymbol{\theta}|L,H]$ holds and these values fall to

[3]For example, consider an arbitrarily given $a > 1$ and a function $c(e) = \alpha e^{\beta}$ for $0 \leq e < a$. Then, connect this function to the function $c(e) = e^2 - 1$ at a "smoothly." Namely, find a set of values for parameters α and β such that the two functions cross at a, that is, $\alpha a^{\beta} = a^2 - 1$, and the two functions have the same slope at a, that is, $\alpha\beta a^{\beta-1} = 2a$. These values are $\alpha = \frac{a^2-1}{a^{\frac{2a^2}{a^2-1}}}$ and $\beta = \frac{2a^2}{a^2-1}$. The second derivative of the connected function is discontinuous and decreasing when a approaches 1. Therefore, assumption A6 is violated. This connected function approaches the cost function (8) when a approaches 1.

$E[\boldsymbol{\theta}|L, L]$. When $E[\boldsymbol{\theta}|H, L]$ approaches $E[\boldsymbol{\theta}|L, L]$, the analysis of assumption A5 holds with respect to the first move incentive for agent i of H-type. That is, the first move incentive is no longer satisfied. Therefore, the decreasing returns to good news should not be violated severely for agent i. The same is true for agent j.

5 Concluding Remarks

We studied a team production game in which each member holds his personal judgment about the team productivity and chooses a level of effort at a time he prefers. We examined the emergence of leadership by confidence, and derived a set of sufficient conditions for this leadership.

The mechanism of emergence of leadership by confidence is endogenous signaling. A sender-receiver relation is not determined by the rule of the game, but is rather an outcome of equilibrium play. The expectation of play of this sort induces a player to choose to be a leader or a follower depending only on the cost-benefits from becoming a sender or a receiver in teams with multi-sided private information and simple payoff externality.

The endogenous signaling explains endogenous choice of timings of moves in an environment without time discounting and without strategic substitution or complementarity. There are many possible applications for endogenous choice of timings of moves.

On the other hand, the sufficient condition for leadership by confidence to be a stable equilibrium also guarantees that leadership by identity is stable. We argued that leadership by confidence should naturally emerge unless there is a specific outside mechanism at work to induce a particular member to become a leader. It is a worthy challenge to identify theoretically and empirically under what kinds of play environment leadership by confidence prevails and under what other kinds of play environment leadership by identity emerges.

[2008.7.7 880]

Appendix

In this Appendix, we prove Lemma 1 and Lemma 2. To prove Lemma 1, let us prepare the following lemma.

Lemma A. *Assume A1, A2, A6, and A7. Let* $e_i^*(t_i, \mu_i)$ *satisfy the first order condition* (4). *Then,* $e_i^*(H,1) - e_i^*(H,0) \leq e_i^*(L,1) - e_i^*(L,0)$.

Proof. By differentiating the first order conditions (4), we have:

$$\begin{aligned}\frac{\partial}{\partial \mu_i} e_i^*(H,\mu_i) &= \frac{s_i(E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|H,L])}{c_i''(e_i^*(H,\mu_i))} \\ \frac{\partial}{\partial \mu_i} e_i^*(L,\mu_i) &= \frac{s_i(E[\boldsymbol{\theta}|L,H] - E[\boldsymbol{\theta}|L,L])}{c_i''(e_i^*(L,\mu_i))}.\end{aligned}$$

Assumption A7 implies:

$$s_i(E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|H,L]) \leq s_i(E[\boldsymbol{\theta}|L,H] - E[\boldsymbol{\theta}|L,L]).$$

On the other hand, assumption A2 implies $e_i^*(H,\mu_i) > e_i^*(L,\mu_i)$. Together with assumption A6, this in turn implies $c_i''(e_i^*(H,\mu_i)) \geq c_i''(e_i^*(L,\mu_i))$. Therefore, we have $\frac{\partial}{\partial \mu_i} e_i^*(H,\mu_i) \leq \frac{\partial}{\partial \mu_i} e_i^*(L,\mu_i)$ and we have:

$$\begin{aligned}e_i^*(H,1) - e_i^*(H,0) &= \int_0^1 \frac{\partial}{\partial \mu_i} e_i^*(H,\mu_i) d\mu_i \\ &\leq \int_0^1 \frac{\partial}{\partial \mu_i} e_i^*(L,\mu_i) d\mu_i \\ &= e_i^*(L,1) - e_i^*(L,0). \blacksquare\end{aligned}$$

Proof of Lemma 1. Define:

$$\begin{aligned}\Delta(\rho_j) \equiv\ & s\Big(\rho_j E[\boldsymbol{\theta}|H,H] + (1-\rho_j)E[\boldsymbol{\theta}|H,L]\Big)e^*(H,\rho_j) - c(e^*(H,\rho_j)) \\ & +(1-\rho_j)sE[\boldsymbol{\theta}|H,L](e^*(H,1) - e^*(H,0)) \\ & -\Big[\rho_j\{sE[\boldsymbol{\theta}|H,H]e^*(H,1) - c(e^*(H,1))\} \\ & \quad +(1-\rho_j)\{sE[\boldsymbol{\theta}|H,L]e^*(H,0) - c(e^*(H,0))\}\Big].\end{aligned}$$

Then, because of Lemma A, $U_i(H, e^*(H,\rho_j), 1) - U_i(H, \emptyset, 0) \geq \Delta(\rho_j)$ is obtained for any $\rho_j \in (0,1)$ by replacing $e^*(H,1) - e^*(H,0)$ in the definition of $\Delta(\rho_j)$ with $e^*(L,1) - e^*(L,0)$. Therefore, our proof is complete if we show $\Delta(\rho_j) > 0$. Observe $\Delta(1) = 0$ and $\Delta''(\rho_j) = s(E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|H,L])\frac{\partial}{\partial \rho_j} e^*(H,\rho_j) > 0$. Examine:

$$\begin{aligned}\Delta'(1) =\ & s\Big(E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|H,L] - E[\boldsymbol{\theta}|H,L]\Big)(e^*(H,1) - e^*(H,0)) \\ & -\Big[\{sE[\boldsymbol{\theta}|H,H]e^*(H,1) - c(e^*(H,1))\} - \{sE[\boldsymbol{\theta}|H,H]e^*(H,0) - c(e^*(H,0))\}\Big].\end{aligned}$$

Assumptions A5 and A7 imply:

$$\begin{aligned}&E[\boldsymbol{\theta}|H,H]-E[\boldsymbol{\theta}|H,L]-E[\boldsymbol{\theta}|H,L]\\&\leq E[\boldsymbol{\theta}|L,H]-E[\boldsymbol{\theta}|L,L]-E[\boldsymbol{\theta}|H,L]\\&=-E[\boldsymbol{\theta}|L,L]<0.\end{aligned}$$

Furthermore, $sE[\boldsymbol{\theta}|H,H]e^*(H,1)-c(e^*(H,1)) > sE[\boldsymbol{\theta}|H,H]e^*(H,0)-c(e^*(H,0))$ follows from the optimality of $e^*(H,1)$. Therefore, $\Delta'(1) < 0$. From the three facts combined, we conclude $\Delta(\rho_j) > 0$ for any $\rho_j \in (0,1)$. ■

Proof of Lemma 2. Define:

$$\begin{aligned}A_i &\equiv \Big[\Big(sE[\boldsymbol{\theta}|t_i=H]e-c(e)\Big)-\Big(sE[\boldsymbol{\theta}|t_i=H]e^*(H,\rho_j)-c(e^*(H,\rho_j))\Big)\Big]\\&\quad-\Big[\Big(sE[\boldsymbol{\theta}|t_i=L]e-c(e)\Big)-\Big(sE[\boldsymbol{\theta}|t_i=L]e^*(L,\rho_j)-c(e^*(L,\rho_j))\Big)\Big]\\B_i &\equiv \Big(sE[\boldsymbol{\theta}|t_i=H]e^*(H,\rho_j)-c(e^*(H,\rho_j))\Big)-\Big\{\rho_j(sE[\boldsymbol{\theta}|H,H]e^*(H,1)-c(e^*(H,1)))\\&\quad+(1-\rho_j)(sE[\boldsymbol{\theta}|H,L]e^*(H,0)-c(e^*(H,0)))\Big\}\\C_i &\equiv \Big(sE[\boldsymbol{\theta}|t_i=L]e^*(L,\rho_j)-c(e^*(L,\rho_j))\Big)-\Big\{\rho_j(sE[\boldsymbol{\theta}|L,H]e^*(L,1)-c(e^*(L,1)))\\&\quad+(1-\rho_j)(sE[\boldsymbol{\theta}|L,L]e^*(L,0)-c(e^*(L,0)))\Big\}.\end{aligned}$$

Then:

$$\begin{aligned}&\Big[U_i(H,e,1)-U_i(H,\emptyset,0)\Big]-\Big[U_i(L,e,1)-U_i(L,\emptyset,0)\Big]\\&=A_i+(B_i-C_i)+(1-\rho_j)s(E[\boldsymbol{\theta}|H,L]-E[\boldsymbol{\theta}|L,L])(e^*(L,1)-e^*(L,0)).\end{aligned}$$

The last term is obviously positive. The first term can be rewritten as:

$$\begin{aligned}A_i &= s(E[\boldsymbol{\theta}|t_i=H]-E[\boldsymbol{\theta}|t_i=L])(e-e^*(H,\rho_j))\\&\quad+\Big[\big(sE[\boldsymbol{\theta}|t_i=L]e^*(L,\rho_j)-c(e^*(L,\rho_j))\big)-\big(sE[\boldsymbol{\theta}|t_i=L]e^*(H,\rho_j)-c(e^*(H,\rho_j))\big)\Big].\end{aligned}$$

The first term is nonnegative for any $e \geq e^*(H,\rho_j)$. The second term is positive by the optimality of $e^*(L,\rho_j)$. Therefore, $A_i > 0$ for any $e \geq e^*(H,\rho_j)$, and the proof is complete if we show $(B_i - C_i)$ is positive.

For $\delta \in [0,1]$, define:

$$\begin{aligned}D_i(\delta) &= \rho_j\Big(sE[\boldsymbol{\theta}|\delta,H]e^*(\delta,1)-c(e^*(\delta,1))\Big)+(1-\rho_j)\Big(sE[\boldsymbol{\theta}|\delta,L]e^*(\delta,0)-c(e^*(\delta,0))\Big)\\&\quad-\Big[s(\rho_jE[\boldsymbol{\theta}|\delta,H]+(1-\rho_j)E[\boldsymbol{\theta}|\delta,L])e^*(\delta,\rho_j)-c(e^*(\delta,\rho_j))\Big],\end{aligned}$$

where we denote:

$$\begin{aligned}E[\boldsymbol{\theta}|\delta,H]&=\delta E[\boldsymbol{\theta}|H,H]+(1-\delta)E[\boldsymbol{\theta}|L,H],\\E[\boldsymbol{\theta}|\delta,L]&=\delta E[\boldsymbol{\theta}|H,L]+(1-\delta)E[\boldsymbol{\theta}|L,L],\end{aligned}$$

and we define $e^*(\delta,\mu)$ in a parallel way to the definition of $e^*(t,\mu)$ by using $E[\boldsymbol{\theta}|\delta,H]$ and $E[\boldsymbol{\theta}|\delta,L]$. Then, $B_i - C_i = D_i(0) - D_i(1)$. Examine:

$$\begin{aligned} D_i'(\delta) &= s\rho_j\Big(E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|L,H]\Big)e^*(\delta,1) + s(1-\rho_j)\Big(E[\boldsymbol{\theta}|H,L] - E[\boldsymbol{\theta}|L,L]\Big)e^*(\delta,0) \\ & -s\Big[\rho_j\Big(E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|L,H]\Big) + (1-\rho_j)\Big(E[\boldsymbol{\theta}|H,L] - E[\boldsymbol{\theta}|L,L]\Big)\Big]e^*(\delta,\rho_j). \end{aligned}$$

Assumption A7 and $e^*(\delta,0) < e^*(\delta,\rho_j)$ imply:

$$\begin{aligned} &(1-\rho_j)\Big(E[\boldsymbol{\theta}|H,L] - E[\boldsymbol{\theta}|L,L]\Big)e^*(\delta,0) - (1-\rho_j)\Big(E[\boldsymbol{\theta}|H,L] - E[\boldsymbol{\theta}|L,L]\Big)e^*(\delta,\rho_j) \\ &\leq (1-\rho_j)\Big(E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|L,H]\Big)e^*(\delta,0) - (1-\rho_j)\Big(E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|L,H]\Big)e^*(\delta,\rho_j). \end{aligned}$$

Therefore:

$$D_i'(\delta) \leq s\Big[\rho_j e^*(\delta,1) + (1-\rho_j)e^*(\delta,0) - e^*(\delta,\rho_j)\Big](E[\boldsymbol{\theta}|H,H] - E[\boldsymbol{\theta}|L,H]).$$

The first order conditions for $e^*(\delta,1)$, $e^*(\delta,0)$, and $e^*(\delta,\rho_j)$ are:

$$\begin{aligned} sE[\boldsymbol{\theta}|\delta,H] &= c'(e^*(\delta,1)), \\ sE[\boldsymbol{\theta}|\delta,L] &= c'(e^*(\delta,0)), \\ s\Big(\rho_j E[\boldsymbol{\theta}|\delta,H] + (1-\rho_j)E[\boldsymbol{\theta}|\delta,L]\Big) &= c'(e^*(\delta,\rho_j)). \end{aligned}$$

From this system of equations follows:

$$\rho_j c'(e^*(\delta,1)) + (1-\rho_j)c'(e^*(\delta,0)) = c'(e^*(\delta,\rho_j)). \tag{9}$$

On the other hand, assumption A6 implies:

$$c'(\rho_j e^*(\delta,1) + (1-\rho_j)e^*(\delta,0)) \leq \rho_j c'(e^*(\delta,1)) + (1-\rho_j)c'(e^*(\delta,0)). \tag{10}$$

Given (9) and (10), assumption A2 guarantees:

$$\rho_j e^*(\delta,1) + (1-\rho_j)e^*(\delta,0) \leq e^*(\delta,\rho_j).$$

This establishes $D_i'(\delta) \leq 0$. Therefore, $B_i - C_i = D_i(0) - D_i(1) \geq 0$. This completes the proof. ■

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