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# PRODUCT INNOVATION AND THE RATE OF PROFIT

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Marx stated that the rate of profit tends to fall because of technical change. Okishio pointed out that Marx ignored the profitability criterion for the introduction of new technologies. He proved that the introduction of new technology, which yields a higher profit rate measured in terms of old equilibrium prices, raises the rate of profit under new equilibrium prices when the real wage basket is kept constant (the Okishio Theorem).

In his argument, however, technical innovation was restricted to that of new production methods - i.e. process innovation - while the introduction of new products, namely product innovation, was not analyzed. For product innovation to be relevant in the history of technology, it is very important to know the eventual effects of this type of innovation on the rate of profit. In this paper, first we present the Okishio Theorem in its simplest form and summarize the controversies concerning this theorem. Next, we consider if the theorem holds when taking product innovation into account.

# I. A Simple Illustration of the Okishio Theorem

The Okishio Theorem states that a cost-reducing technological change necessarily raises the rate of profit, under constant real wages, when innovation occurs in a basic sector. Assume the economy consists of two sectors a production goods sector (the first sector) and a consumption goods sector (the second sector). Notations for input coefficients are listed in Table 1.

	TABLE 1			
	production goods	labor		
production goods	$a_1$	$\tau_{1}$		
consumption goods	$a_2$	$\tau_2$		

A laborer receives b units of consumption goods per unit of labor. Then the profit rates in the two sectors,  $r_1$  and  $r_2$ , are determined by the following equations.

$$p_{1} = (1 + r_{1})(a_{1}p_{1} + \tau_{1}bp_{2})$$

$$p_{2} = (1 + r_{2})(a_{2}p_{1} + \tau_{2}bp_{2})$$
(1)
(2)

We should note that  $a_1$  must be less than unity. If  $a_1 \ge 1$ , production goods cannot be reproduced. As is shown in Figure 1, the equilibrium profit rate r and the equilibrium relative price  $p(=p_1/p_2)$  are given by the intersection

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of curves  $L_1 L_1$  and  $L_2L_2$ , depicted by point A. Now, suppose that a new process innovation, described by the technology  $(a_1', \tau_1')$ , is introduced into the first sector. The condition for this new technology to be cost-reducing, measured by the equilibrium price p, is given by

$$a_1p + \tau_1b > a_1'p + \tau_1'b$$
 (3)

This condition can be shown in Figure 1 by drawing the profit rate curve  $L_3L_3$  of new technology as shifted upward from the old line  $L_1L_1$ .



It is apparent from this that the new equilibrium profit rate, which is given by the intersection of  $L_3L_3$  and  $L_2L_2$ , increases and the relative price decreases. This conclusion also holds if we consider that the innovation is introduced into the second sector.

## **II.** Issues in Dispute

The problems and questions expressed concerning this theorem can be listed as the following.

(1) The theorem is trivial because the cost-reducing technical change raises the profit rate.

(2) How does this theorem relate to the Marxian theory of a falling rate of profit?

(3) If the existence of durable capital equipment or more general joint production is considered, does the theorem still hold?

(4) In the real world, the evaluation of new technology is made based on disequilibrium prices. Does the theorem still hold in such a situation?

(5) Capitalists make decisions under dynamic circumstances where their expectations about prices or the economic life of durable equipment may not be fulfilled. In this uncertain and dynamic world, the theorem may not hold.

(6) Technological changes usually accompany completely new products. In this case, the theorem may not hold.

The first two problems relate to the meaning of the theorem itself; what it proves and what it does not prove. The next four relate to the robustness of the theorem, asking whether the theorem is valid if one or more assumptions made in the original proof are relaxed.

These points will be addressed in turn.

(1) The first critique, claiming that the theorem is trivial, ignores the difference between the temporal profit rate and the equilibrium profit rate. Of course, it is clear that the temporal profit rate, which is measured by the old prices, rises through the introduction of new technology. However, this is not obvious when we consider the equilibrium rate of profit established by the new equilibrium prices. In fact, the introduction of new technology into a non-basic sector does not raise the equilibrium rate of profit at all. This means that the theorem is not trivial.

(2) The Okishio Theorem relates to the logic of the Marxian theory of the falling rate of profit. Marx claimed that technological changes in a capitalist economy tend to suppress the profit rate via increases in the organic composition of capital. However, if we take the criterion for technical choice into consideration, such a technique would not be introduced by profit maximizing capitalists under a constant real wage rate. The Okishio theorem, however, does not preclude the possible validity of the Marxian claim. If the rate of profit falls in the long run, this must be because of a rise in the real wage rate. This is the implication of the Okishio Theorem.

(3) This theorem still holds when we consider durable equipment (Nakatani (1978) and Roemer (1979)). However, in the case of more general joint production, Salvadori (1981) presented one counter example. In his example where two goods and two production methods are available, the rates of profit of these two techniques are both increasing functions of the relative price as in Figure 2.



 $L_1$ : the rate of profit of the first technology  $L_2$ : the rate of profit of the second technology  $L_3$ : the rate of profit of the third technology

#### Figure 2

(4) In the Okishio Theorem, capitalists' evaluations of new techniques are based on old equilibrium prices. However, in the real economy this is not so. If they choose new techniques based on prevailing disequilibrium prices, the new equilibrium profit rate is not necessarily greater than the old equilibrium profit rate. For example, a technique which decreases unit cost if measured by disequilibrium prices but raises it if measured by equilibrium prices fits this case. However, if the profit rate differences between sectors are kept constant before and after the technical change, namely

 $r_i = \lambda_i r_i \ (i = 2, 3, \dots, n), \ \lambda_i s \text{ are constant},$  (4)

a cost-reducing technical change necessarily raises all profit rates in the economy if it is introduced into a basic sector. The Okishio Theorem is the special case of

 $\lambda_1 = 1$ .

(5) Shaikh (1978) and Alberro and Persky (1979) have argued that the theorem does not necessarily hold in competitive dynamic situations. Shaikh thinks that the source of falling profit rates is the introduction of new techniques under a continuous disequilibrium process where prices decrease as a result of the competitive battle among capitalists. On the other hand, Alberro and Persky (1979) stated that the discrepancy

between the expectation and reality of the economic life of durable capital equipment could be the source. The difficult points of these arguments lie in the assumption that the discrepancy between expectation and reality lasts in the long run<sup>1)</sup>.

(6) Product innovation has been poorly investigated, at least from a theoretical perspective, despite its importance in the history of technology. On this point, R. Boyer (1979) expressed the possibility that the theorem might not hold in the case of product innovation.

# **III.** New Production Goods

Here, we discuss the effect of product innovation on the rate of profit. First, we consider the case in which the new product is a production good. The case of consumption goods is discussed in the next section. For explanatory simplicity we discuss the simplest case and leave the general case for the Appendix. We assume the two-sector economy described by equations (1) and (2) in the first section. The rates of profit in the first and second sector are assumed to be equal as a result of competition. By dividing prices by the nominal wage rate  $(q_i = p_i/w)$  and denoting a reciprocal number of one plus the rate of profit by  $\beta(=1/(1+r))$ , we obtain

$\beta q_1 = a_1 q_1 + \tau_1$		(	(5)	

$$\beta q_2 = a_2 q_1 + \tau_2 \tag{6}$$

$$1=bq_2$$
.

We refer to  $(r, p_1, p_2, w)$  or  $(\beta, q_1, q_2)$  as the old price system. Now the new production goods (the third sector) appear. What conditions must be fulfilled for a new product to be introduced? The first condition is that it must be more profitable to produce the new product than the old. Without this condition, no one would dare to take the risk of introducing a new commodity. Even after the invention of new production goods, the new product is not necessarily accepted in the market. New products must find their market. Without fulfilling this condition, new production goods can not be sold at all and the profit in the third sector cannot be realized. In other words, there must exist two types of innovators. The first type of innovator introduces a new product into the market and the second uses it before others. We call the former the producer-innovator and the latter

(7)

<sup>1)</sup> This point is made by Roemer (1979).

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the user-innovator. Corresponding to these innovators, two conditions must be fulfilled. We assume that a unit production of new goods needs the old production goods by the amount  $a_3$  and labor by  $\tau_3$ . Furthermore, on the demand side of the new product, we assume that a unit of consumption goods is produced by  $a_2'$  new units of production goods and by  $\tau_2'$  units of labor. Then we have

$$\beta q_3 > a_3 q_1 + \tau_3 \tag{8}$$

$$\beta q_2 > a_2' q_3 + \tau_2' . \tag{9}$$

The former implies that the new product guarantees the extra profit rate  $r_3(>r)$  to its producer-innovator. The latter implies that the user-innovator can lower his unit cost under the old price system. Hereafter, we call (8) the supply condition and (9) the demand condition of the new product.

When both conditions (8) and (9) are satisfied, the new goods are accepted by the market. Here the old production goods are used only for the production of new production goods. The new equilibrium profit rate r' or  $\beta'(=1/(1+r'))$  and new prices  $q_i'$  are determined by the following equations.

$$\beta' q_1' = a_1 q_1' + \tau_1 \tag{10}$$

$$\beta' q_2' = a_2' q_3' + \tau_2' \tag{11}$$

$$\beta' q_{3}' = a_{3} q_{1}' + \tau_{3}$$
(12)

As the real wage b is kept constant, we have

 $1 = bq_{2}'$ .

We can easily show that r' > r or  $\beta > \beta'$  as follows.

Let us assume the converse,  $\beta \leq \beta'$ . Then from (9), (11) and  $q_2 = q_2'$  we have  $q_3' > q_3$  so that we know that  $q_1' > q_1$  from (8) and (12). Then from (10) and

$$\beta q_1 = a_1 q_1 + \tau_1 \tag{13}$$

we have

$$(\beta' - a_1)(q_1' - q_1) = (\beta - \beta')q_1 .$$
(14)

This contradicts  $\beta \leq \beta'$  and  $q_1' > q_1$  since we have  $\beta' - a_1 > 0$  from (10). Therefore we know that  $\beta > \beta'$ .

(3-1) The proof is essentially the same as the original one in Okishio (1961), because the introduction of new production goods can be reinterpreted as the process innovation of consumption goods if we eliminate  $q_3$ ' in (11) using equation (12). Namely (11), coupled with (12), means that the consumption goods are produced by means of old production goods and labor. Consider the case where the producer-innovator and the user-innovator are the same person or the same firm and every adopter of the innovation produces with the new production method. In this case, the new production goods may not be known by the public so that the situation may appear as a new process innovation.

(3-2) In the case of process innovation, the amount of labor commanded - the unit price divided by nominal wage rate  $p_i/w$  - by the commodity concerned necessarily decreases in the new equilibrium compared with that in the old equilibrium. In contrast, in the case of product innovation, this pattern is not necessarily the case. The amount of labor commanded by a new product may increase. This is because there exists special profit in both the supply sector and the demand sector of the new product. When the new product appears at a relatively low price, the demand sector takes much of the special profits so that in the new equilibrium the new commodity price, measured by labor, must increase.

(3-3) The labor commanded by a new commodity may increase in the new equilibrium. Then, is there any possibility of reswitching to the old product after prices are adjusted to the new equilibrium? We can show that this never happens. For the reswitching to occur, we need

$$\beta' q_2' > a_2 q_1' + \tau_2$$
 (15)

On the other hand, in the old equilibrium, we have

$$\beta q_2 = a_2 q_1 + \tau_2 \,. \tag{16}$$

From (15)(16), it follows

$$\beta' q_2' - \beta q_2 > a_2(q_1' - q_1)$$
 (17)

Noting  $q_1' > q_1$  and  $q_2' = q_2$ , we easily find that (17) is a contradiction of  $\beta > \beta'$ . Therefore (15) does not hold.

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# **IV.** New Consumption Goods

Next, suppose the new product is a consumption good. The simplest case here is the two consumption goods economy in which the first good is an old consumption good and the second is a new one. One unit of each good is produced with the input of  $\tau_1$  and  $\tau_2$  units of labor alone, without using production goods. New consumption goods can enter as either wage goods or luxury goods. As is well-known, in the latter case the equilibrium rate of profit does not change at all. Let the new product be a wage good. Then we have to reconsider the assumption originally made in Okishio (1961), namely that the laborer's wage basket is kept unchanged.

How should we consider this point? Suppose that the wage basket received per unit of labor is changed from b units of old goods to the basket  $(b_1 \text{ units of old goods and } b_2 \text{ units of new goods})$ . If the new wage basket  $(b_1, b_2)$  can be purchased at the old equilibrium prices, the new basket is revealed to be preferred by laborers to the old one because they have made a choice to buy new consumption goods instead of the old goods. From a utility viewpoint, we can say that the laborer's utility never decreases with this change of wage baskets.

The proposition to be presented in the case of new consumption goods is a modified Okishio Theorem, replacing the phrase "under a constant real wage rate" with "under the condition that the laborers' utility never decreases." The proof is as follows.

In the old equilibrium we have

$$\beta q_1 = \tau_1 . \tag{18}$$

The laborers' budget constraint is

 $1 = bq_1 . \tag{19}$ 

The supply condition for the producer-innovator is written as

$$\beta q_2 > \tau_2 . \tag{20}$$

Next, the demand condition for the user-innovator is that laborers buy  $(b_1, b_2)$  at current prices, namely

$$1 = b_1 q_1 + b_2 q_2 \quad (b_2 > 0). \tag{21}$$

If the change in the wage basket from the old one to the new one occurs once, the new equilibrium prices  $(q_1', q_2')$  and the new equilibrium rate of profit  $\beta'(=1/(1+r'))$  are determined by the following equations.

$$\beta' q_1' = \tau_1 \tag{22}$$
$$\beta' q_2' = \tau_2 \tag{23}$$

$$l = b_1 q_1 + b_2 q_2 \quad . \tag{24}$$

It is easy to prove r' > r. Substituting both sides of (23) from (20), we have

$$\beta(q_2-q_2') > (\beta'-\beta)q_2' . \tag{25}$$

Similarly from (18) and (22) we have

$$\beta(q_1 - q_1') = (\beta' - \beta)q_1'$$
(26)

and from (21) and (24)

$$b_1(q_1-q_1')+b_2(q_2-q_2')=0.$$
 (27)

Now suppose that  $\beta' - \beta \ge 0$ . Then from (25) and (26) we know that  $q_2 - q_2' > 0$  and  $q_1 - q_1' \ge 0$ , which is a contradiction of (27) in view of  $b_2 > 0$ . Therefore it must be that r' > r.

(4-1) The amount of labor commanded by new consumption goods necessarily decreases from the old equilibrium to the new, except in the special case where the only new goods are the wage goods. This is clear because we have  $q_1-q_1' < 0$  and  $q_2-q_2' \ge 0$  from (26) and (27). If the only new goods are the wage goods  $(b_1=0)$ , the labor commanded by new product  $q_2'$  remains constant.

The extra profit rate at the current prices is  $r_2^*$  or  $\beta_2^*$  determined by

$$\beta_2^* q_2 = \tau_2 \tag{28}$$

so that from (23) we have

 $\beta > \beta' \ge \beta_2^*$  or  $r < r' \le r_2^*$ .

The new equilibrium profit rate is lower than the extra profit rate at current prices. It is still higher than the old equilibrium rate of profit.

(4-2) As the price of new product decreases at the new equilibrium, the old consumption goods tend to be abandoned in the diffusion process of the new product. Even if the new product is introduced at first as a luxury good, it could happen that it becomes a basic good later through changes in the relative price in favor of the new product, which causes a shift in laborers' preference from the old good to the new product.

# V. Conclusion

We have shown that the Okishio Theorem still holds when taking the existence of product innovation into consideration. Then, in the case of consumption goods, we modified the original proposition by replacing

"under a constant real wage rate" into "under a non-decreasing utility level of laborers."

It is important to note that the introduction of new goods can change the composition of basic sectors in the economy. This happens via two phases. First, the new product replaces the old basic production goods or wage goods. Then, in the diffusion process, the non-basic new goods come to substitute for the old products and become basic. The second phase usually follows the first. In this sense we can say that product innovation induces process innovation.

## **APPENDIX: the General Case**

In this appendix, we discuss the *n*-commodity case. Clearly, the rate of profit will not be influenced if the new product remains non-basic throughout the argument. Therefore, we assume that the new product is basic in the following discussion. Product innovation sometimes occurs with a simultaneous introduction of a basket of new products. For ease of explanation, however, we assume that only one new product (labeled as the n+1st good) is added to the old *n*-commodity economy. By denoting the amount of production goods and labor necessary to produce a unit of the *i*-th commodity by  $a_{i1}$ ,  $a_{i2}$ , ...,  $a_{in}$ ,  $\tau_i$  and the real wage basket by  $b_1$ ,  $b_2$ , ...,  $b_n$  and labeling the *i*-th price measured by the nominal wage rate by  $q_i(=p_i/w)$  and the equilibrium rate of profit by r or  $\beta(=1/(1+r))$ , we have the following equations as the old equilibrium system.

$$\beta q_{i} = \sum_{j=1}^{n} a_{ij}q_{j} + \tau_{i}$$
(A-1)
$$1 = \sum_{j=1}^{n} b_{j}q_{j} .$$
(A-2)

Let the production coefficients of the new product be  $(a_{n+11}', a_{n+12}', ..., a_{n+1n+1}', \tau_{n+1}')$ . The supply condition of the new product can be written as

$$\beta q_{n+1} > \sum_{j=1}^{n+1} a_{n+1j}' q_j + \tau_{n+1}' . \qquad (A-3)$$

(1) New Production Goods

The new product is assumed to be used in the first sector and let the new technology be  $(a_{11}', a_{12}', ..., a_{1n+1}', \tau_1')$ . The demand condition of the new product is written as

$$\beta q_1 > \sum_{j=1}^{n+1} a_{1j}' q_j + \tau_1'$$
 (A-4)

Here the prices satisfy the following conditions from (A-1).

$$\beta q_i = \sum_{j=1}^n a_{ij} q_j + \tau_i \quad (i = 2, 3, ..., n)$$
(A-5)

The new equilibrium prices  $q_i'$  and profit rate r' or  $\beta'(=1/(1+r'))$  are determined by

$$\beta' q_1' > \sum_{i=1}^{n+1} a_{1i}' q_i' + \tau_1'$$
 (A-6)

$$\beta' q_i = \sum_{j=1}^n a_{ij} q_j' + \tau_i \quad (i=2,3,...,n)$$
(A-7)

$$\beta' q_{n+1}' = \sum_{j=1}^{n+1} a_{ij}' q_j' + \tau_{n+1}'$$
(A-8)

$$1 = \sum_{j=1}^{n} b_{j} q_{j}' .$$
 (A-9)

From (A-4) and (A-6)

$$\beta(q_1-q_1') > \sum_{j=1}^{n+1} a_{1j}'(q_j-q_j') + (\beta'-\beta)q_1' .$$
 (A-10)

From (A-5) and (A-7)

$$\beta(q_i - q_i') = \sum_{j=1}^n a_{ij}(q_j - q_j') + (\beta' - \beta) \quad (i = 2, 3, ..., n) .$$
 (A-11)

From (A-3) and (A-8)

$$\beta(q_{n+1}-q_{n+1}') > \sum_{j=1}^{n+1} a_{n+1j}'(q_j-q_j') + (\beta'-\beta)q_{n+1}'.$$
 (A-12)

From (A-2) and (A-9)

$$\sum_{j=1}^{n} (q_j - q_j') = 0 .$$
 (A-13)

From (A-10) to (A-12) we know that, if  $\beta' - \beta \ge 0$  is assumed,  $q_i - q_i' > 0$  (i=1,2,...,n+1) holds, which contradicts (A-13). Therefore we have  $\beta' - \beta < 0$ .

# (2)New Consumption Goods

The demand condition in this case is for laborers to purchase new goods at old equilibrium prices.

$$1 = \sum_{j=1}^{n+1} b_j q_j .$$
 (A-14)

The new equilibrium is determined by the following equations.

$$\beta' q_i' = \sum_{j=1}^n a_{ij} q_j' + \tau_i$$
 (A-15)

$$\beta' q_{n+1}' = \sum_{j=1}^{n} a_{n+1j}' q_j' + \tau_{n+1}'$$
(A-16)

$$1 = \sum_{j=1}^{n+1} b_j q_j' \tag{A-17}$$

From (A-1) and (A-15)

$$\beta(q_i - q'_i) = \sum_{j=1}^n a_{ij}(q_j - q'_j) + (\beta' - \beta)q'_i \quad (i = 1, 2, ..., n) .$$
 (A-18)

From (A-3) and (A-16)

$$\beta(q_{n+1}-q_{n+1}') > \sum_{j=1}^{n+1} a_{n+1j}'(q_j-q_j') + (\beta'-\beta)q_{n+1}'.$$
 (A-19)

From (A-14) and (A-17)

$$\sum_{j=1}^{n+1} b_j(q_j - q_j') = 0 .$$
 (A-20)

By the same reasoning we easily find that the assumption  $\beta' - \beta \ge 0$  is a contradiction. Therefore it follows that  $\beta' - \beta < 0$  or r' - r > 0.

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