RISK PERFORMANCES OF THE BIAS CORRECTED FEASIBLE MINIMUM MEAN SQUARED ERROR ESTIMATORS UNDER BALANCED LOSS

Ohtani, Kazuhiro

(Citation)
Kobe University Economic Review,47:1-11

(Issue Date)
2001

(Resource Type)
departmental bulletin paper

(Version)
Version of Record

(JaLCDOI)
https://doi.org/10.24546/81000929

(URL)
https://hdl.handle.net/20.500.14094/81000929

RISK PERFORMANCES OF THE BIAS CORRECTED FEASIBLE MINIMUM MEAN SQUARED ERROR ESTIMATORS UNDER BALANCED LOSS

By KAZUHIRO OHTANI

In this paper, we examine the risk performances of the bias corrected variants of the feasible minimum mean squared error (FMMSE) estimator and the adjusted FMMSE estimator under balanced loss. It is shown by numerical evaluations that although the bias correction can be effective under balanced loss for some cases, the bias correction is not effective for other cases.

1. Introduction

Consider a linear regression model,

$$y = X\beta + \epsilon,\ \epsilon \sim N(0,\ \sigma^2 I_n), \tag{1}$$

where y is an $n \times 1$ vector of observations on a dependent variable, X is an $n \times k$ matrix of full column rank of observations on non-stochastic independent variables, β is a $k \times 1$ vector of coefficients, and ϵ is an $n \times 1$ vector of normal error terms with $E[\epsilon] = 0$ and $E[\epsilon\epsilon'] = \sigma^2 I_n$.

The ordinary least squares (OLS) estimator of β is

$$b = S^{-1}X'y, \tag{2}$$

where $S = X'X$. If we use the quadratic loss function defined as $L(\tilde{\beta}; \beta) = (\tilde{\beta} - \beta)'S(\tilde{\beta} - \beta)$, where $\tilde{\beta}$ is any estimator of β, then $E[L(\tilde{\beta}; \beta)]$ is called the predictive mean squared error (PMSE). In terms of PMSE, the OLS estimator is dominated by a family of the Stein-rule estimators. [See, for example, Stein (1956), James and Stein (1961), Baranchik (1971), Judge and Bock (1976), and Ohtani (2000).]

As one of the improved estimators of β, Theil (1971) considered the minimum mean squared error (MMSE) estimator. However, since Theil's (1971) MMSE estimator includes unknown parameters, Farebrother (1975) proposed a feasible MMSE (FMMSE) estimator defined as

$$b_{FMM} = \left(\frac{b'Sb}{b'Sb + e'e/v}\right)b, \tag{3}$$

where $v = n - k$. There are many studies on large and small sample properties of the FMMSE estimator and its heterogeneous variants. Some examples are Vinod (1976), Dwivedi and Srivastava (1978), Stahlecker and Trenkler (1985), Liski et al. (1993), and Tracy and Srivastava (1994).

Since the FMMSE estimator satisfies Baranchik's (1971, example 2) condition, the FMMSE estimator dominates the OLS estimator in terms of PMSE when $k \geq 3$. Although the family of the Stein-rule estimators cannot be defined when $k \leq 2$, the FMMSE estimator is still valid even when $k \leq 2$. Since the OLS estimator is admissible in terms of PMSE when $k \leq 2$, the FMMSE estimator does not dominate the OLS estimator. However, Ohtani (1996a) showed by numerical evaluations that when $k = 2$, the gain in PMSE when using the FMMSE estimator instead of the OLS estimator is larger than the loss.

Since the degrees of freedom of $b'Sb$, which is a component of the FMMSE estimator, is k, Ohtani (1996b) considered the following adjusted FMMSE (AFMMSE) estimator:

$$b_{AFM} = \left(\frac{b'Sb/k}{b'Sb/k + e'e/v} \right) b. \tag{4}$$

Since the AFMMSE estimator does not meet Baranchik's (1971) condition, there is no theoretical guarantee for the AFMMSE estimator to dominate the OLS estimator. To examine the PMSE performance of the AFMMSE estimator, he derived the exact formula for the PMSE of b_{AFM}, and showed by numerical evaluations that the AFMMSE estimator has the smaller PMSE than the family of the Stein-rule estimators in a wide region of the noncentrality parameter. His numerical results also show that although the AFMMSE estimator does not dominate the OLS estimator when $k \leq 5$, the AFMMSE estimator dominates the OLS estimator when $k \geq 6$.

Kadiyala (1984) proposed a class of bias corrected shrinkage estimators, and showed that this class of estimators is not only bias corrected but also more efficient than the OLS estimator in terms of PMSE. However, since Kadiyala's (1984) bias corrected estimators include unknown parameters, several authors have examined the sampling properties of operational variants of the bias corrected shrinkage estimators. For example, Ohtani (1986), Singh et al. (1986) and Nomura (1988) examined the sampling properties of the operational variants of the bias corrected estimators based on the ridge regression estimators proposed by Hoerl and Kennard (1970). Also, Akdeniz and Kaçiranlar (1995) examined the sampling properties of the operational variant of the bias corrected estimator based on the Liu estimator proposed by Kejian (1993). Ohtani (2001) examined the sampling properties of the bias corrected estimators based on the FMMSE and AFMMSE estimators, and showed by numerical evaluations that although the bias can be corrected significantly when the bias correction term is incorporated, the PMSE increases conversely. Since the PMSE is a risk function when the loss function is quadratic, Ohtani's (2001) results show that the risk performance worsens under quadratic loss when the bias correction term is incorporated in the FMMSE and AFMMSE estimators.

When the bias correction term is incorporated in estimators, a researcher may consider that a reduction of bias is important as well as precision of estimation. The balanced loss function proposed by Zellner (1994) allow for both goodness of fit (*i.e.*, bias) and precision of estimation. Thus, when the bias correction term is incorporated, the balanced loss function may be more appropriate in risk comparison than the quadratic loss function, since the quadratic loss function allows for precision of estimation only. In this paper, using the balanced loss function, we examine the risk performances of the bias corrected FMMSE and AFMMSE estimators. In section 2 we derive the explicit formula for the risk functions of the bias corrected FMMSE and AFMMSE estimators, and in section 3 we compare the risk performances of the bias corrected FMMSE and AFMMSE estimators with those of the original MMSE and AFMMSE estimators by numerical evaluations.

2. Balanced Loss Function

First, we define the following formally general estimator:

$$\hat{\beta}_\alpha = \left(\frac{b'Sb}{b'Sb+\alpha e'e}\right)b. \tag{5}$$

Then, as is shown in Ohtani (2001), the bias corrected estimator based on $\hat{\beta}_\alpha$ is

$$\begin{aligned}\hat{\beta}_\alpha^* &= \hat{\beta}_\alpha + \left(\frac{\alpha e'e}{b'Sb+\alpha e'e}\right)\hat{\beta}_\alpha \\ &= \frac{[b'Sb+2\alpha e'e](b'Sb)}{(b'Sb+\alpha e'e)^2}b. \end{aligned}\tag{6}$$

It is easy to see that $\hat{\beta}_\alpha$ reduces to the bias corrected FMMSE estimator when $\alpha = 1/v$, and to the bias corrected AFMMSE estimator when $\alpha = k/v$. Although α in $\hat{\beta}_\alpha$ can take an arbitrary real value, our concern is just in the cases of $\alpha = 1/v$ and $\alpha = k/v$.

To allow for both goodness of fit (*i.e.,* unbiasedness) and precision of estimation, Zellner (1994) proposed the following balanced loss function:

$$L_B(\tilde{\beta}, \beta) = w(y-X\tilde{\beta})'(y-X\tilde{\beta})+(1-w)(\tilde{\beta}-\beta)'S(\tilde{\beta}-\beta), \tag{7}$$

where w $(0 \leq w \leq 1)$ is a nonstochastic weight of goodness of fit. The risk function of $\hat{\beta}_\alpha^*$ under balanced loss is

$$\begin{aligned}R(\hat{\beta}_\alpha^*) &= E[L_B(\hat{\beta}_\alpha^*, \beta)] \\ &= wE[(y-X\hat{\beta}_\alpha^*)'(y-X\hat{\beta}_\alpha^*)] \\ &\quad +(1-w)E[(\hat{\beta}_\alpha^*-\beta)'S(\hat{\beta}_\alpha^*-\beta)]. \end{aligned}\tag{8}$$

Since the second expectation in (8) is the PMSE itself, its formula has been obtained in Ohtani (2001). Thus, we evaluate the first expectation in (8).

Noting that $X'e = 0$, we obtain

$$E[(y-X\hat{\beta}_{\alpha}^{*})'(y-X\hat{\beta}_{\alpha}^{*})]$$
$$= E\left[b'Sb-2\left[\frac{(b'Sb)^{3}+2\alpha(b'Sb)^{2}(e'e)}{(b'Sb+\alpha e'e)^{2}}\right]+e'e\right.$$
$$\left.+\frac{(b'Sb)^{5}+4\alpha(b'Sb)^{4}(e'e)+4\alpha^{2}(b'Sb)^{3}(e'e)^{2}}{(b'Sb+\alpha e'e)^{4}}\right]. \tag{9}$$

Following Ohtani (2001), we define the functions $H(p, q, r; \alpha)$ and $J(p, q, r; \alpha)$ as

$$H(p, q, r; \alpha) = E\left[\frac{(b'Sb)^{p}(e'e)^{q}}{(b'Sb+\alpha e'e)^{r}}\right], \tag{10}$$

$$J(p, q, r; \alpha) = E\left[\frac{(b'Sb)^{p}(e'e)^{q}(\beta'Sb)}{(b'Sb+\alpha e'e)^{r}}\right]. \tag{11}$$

As is shown in Ohtani (2001), the explicit formulas of $H(p, q, r; \alpha)$ and $J(p, q, r; \alpha)$ are

$$H(p, q, r; \alpha) = (2\sigma^{2})^{p+q-r}\sum_{i=0}^{\infty} w_{i}(\lambda)G_{i}(p, q, r; \alpha), \tag{12}$$

$$J(p, q, r; \alpha) = (\beta'S\beta)(2\sigma^{2})^{p+q-r}\sum_{i=0}^{\infty} w_{i}(\lambda)G_{i+1}(p, q, r; \alpha), \tag{13}$$

where $w_{i}(\lambda) = exp(-\lambda/2)(\lambda/2)^{i}/i!$, $\lambda = \beta'S\beta/\sigma^{2}$ and

$$G_{i}(p, q, r; \alpha) = \frac{\Gamma((k+v)/2+i+p+q-r)}{\Gamma(k/2+i)\Gamma(v/2)}$$
$$\times\int_{0}^{1}\frac{t^{k/2+i+p-1}(1-t)^{v/2+q-1}}{[\alpha+(1-\alpha)t]^{r}}dt. \tag{14}$$

Noting that $E[b'Sb/\sigma^{2}] = k+\lambda$ and $E[e'e/\sigma^{2}] = v$, and using $J(p, q, r; \alpha)$ and $H(p, q, r; \alpha)$, we obtain

$$E[(y-X\hat{\beta}_{\alpha}^{*})'(y-X\hat{\beta}_{\alpha}^{*})]$$
$$= \sigma^{2}(k+\lambda-2[H(3, 0, 2; \alpha)+2\alpha H(2, 1, 2; \alpha)]+v$$
$$+[H(5, 0, 4; \alpha)+4\alpha H(4, 1, 4; \alpha)+4\alpha^{2}H(3, 2, 4; \alpha)]). \tag{15}$$

Substituting (18) given in Ohtani (2001) and (15) in (8), we obtain the risk function of $\hat{\beta}_{\alpha}^{*}$ under the balanced loss function. Since the theoretical analysis of the risk function of $\hat{\beta}_{\alpha}^{*}$ is difficult, we examine the risk performances by numerical evaluations in the next section.

3. Numerical Analysis

The parameter values used in the numerical evaluations were $k=3, 5, 8$, $v=10, 20, 30$, $w=0.1, 0.3, 0.5, 0.7, 0.9$, and various values for λ. The numerical evaluations were executed on a personal computer, using the FORTRAN code. Since the formula for $G_{i}(p, q, r; \alpha)$ given in (14) is expressed by an integral, we used Simpson's 3/8 rule with 200 equal subdivisions. Also, the infinite series in $H(p, q, r; \alpha)$ and $J(p, q, r; \alpha)$ were judged to converge when the increment of the series became less than 10^{-12}. The typical numerical results are shown in Tables 1 and 2. Since the entries in the Tables are the values of the relative risk of $\hat{\beta}_{\alpha}^{*}$ to the OLS estimator, $\hat{\beta}_{\alpha}^{*}$ has the smaller risk than the OLS estimator when the entry in the Tables is less than unity.

We see from Table 1 that when $k = 3$ and $w = 0.1$ (*i.e.*, the precision of estimation is much more important), the FMMSE estimator dominates the OLS estimator though the other estimators do not dominate the OLS estimator. Comparing the FMMSE estimator and the bias corrected FMMSE (AFMMSE) estimator, the former has smaller risk than the latter. This indicates that the bias correction has a negative effect on the risk performance of the FMMSE estimator when $k = 3$ and $w = 0.1$. Although the risk of the AFMMSE estimator is smaller than that of the bias corrected AFMMSE estimator for $\lambda \leq 10.0$, the risk performance is reversed for $\lambda \geq 15.0$.

When $k = 3$ and $w = 0.5$, all the estimators considered here do not dominate the OLS estimator. The FMMSE has the better risk performance than the bias corrected FMMSE estimator as a whole. However, although the risk of the AFMMSE estimator is smaller than that of the bias corrected AFMMSE estimator for $\lambda \leq 2.0$, the risk performance is reversed for $\lambda \geq 5.0$.

When $k = 3$ and $w = 0.7$, all the estimators considered here do not dominate the OLS estimator. Although the risk of the FMMSE estimator is smaller than that of the bias corrected FMMSE estimator for $\lambda \leq 5.0$, the risk performance is reversed for $\lambda \geq 10.0$. Since the bias corrected AFMMSE estimator dominates the AFMMSE estimator, the bias correction has a positive effect on the risk performance of the AFMMSE estimator.

When $k = 3$ and $w = 0.9$ (*i.e.*, the goodness of fit is much more important), the FMMSE and AFMMSE estimators have larger risk than the OLS estimator as a whole. Since the bias corrected FMMSE and AFMMSE estimators have slightly smaller risk than the FMMSE and AFMMSE estimators, the bias correction seems to be slightly effective.

We see from Table 2 that when $k = 8$ and $w = 0.1$ (*i.e.*, the precision of estimation is much more important), all the four estimators dominate the OLS estimator. In particular, the AFMMSE estimator seems to have the best risk performance among the estimators considered here. Also, when $w = 0.1$, the FMMSE and AFMMSE estimators have smaller risk than the bias corrected counterparts. This indicates that the bias correction is not effective even if the balanced loss function is used. When $w = 0.3$, the results are similar to the case of $w = 0.1$.

When $k = 8$ and $w = 0.5$, the FMMSE estimator and the bias corrected FMMSE estimator dominate the OLS estimator. However, since the risk of the bias corrected FMMSE estimator is larger than that of the FMMSE estimator, the bias correction has a negative effect on the risk performance of the FMMSE estimator. Although the

TABLE 1. Risks under balanced loss for $v = 20$ and $k = 3$

				bias corrected	
w	λ	FMMSE	AFMMSE	FMMSE	AFMMSE
0.1	.0	.7936	.6476	.9347	.8073
	.1	.7990	.6561	.9374	.8137
	.5	.8190	.6883	.9473	.8373
	1.0	.8407	.7241	.9576	.8627
	2.0	.8755	.7835	.9727	.9025
	5.0	.9354	.8950	.9937	.9669
	10.0	.9718	.9716	1.0007	.9981
	15.0	.9840	.9981	1.0011	1.0036
	20.0	.9892	1.0080	1.0008	1.0039
	25.0	.9920	1.0118	1.0006	1.0033
	30.0	.9936	1.0131	1.0004	1.0027
	35.0	.9947	1.0134	1.0003	1.0022
	40.0	.9955	1.0131	1.0003	1.0018
	50.0	.9965	1.0122	1.0002	1.0012
	75.0	.9978	1.0096	1.0001	1.0006
	100.0	.9984	1.0078	1.0000	1.0003
0.5	.0	.9604	.9438	.9865	.9641
	.1	.9616	.9458	.9871	.9654
	.5	.9659	.9533	.9892	.9704
	1.0	.9706	.9616	.9913	.9756
	2.0	.9779	.9752	.9945	.9837
	5.0	.9902	.9991	.9989	.9962
	10.0	.9969	1.0129	1.0003	1.0013
	15.0	.9988	1.0156	1.0003	1.0017
	20.0	.9995	1.0154	1.0002	1.0014
	25.0	.9998	1.0144	1.0001	1.0010
	30.0	.9999	1.0132	1.0001	1.0008
	35.0	1.0000	1.0121	1.0001	1.0006
	40.0	1.0000	1.0111	1.0001	1.0005
	50.0	1.0000	1.0095	1.0000	1.0003
	75.0	1.0001	1.0069	1.0000	1.0001
	100.0	1.0001	1.0054	1.0000	1.0001

TABLE 1. (continued)

				bias corrected	
w	λ	FMMSE	AFMMSE	FMMSE	AFMMSE
0.7	.0	.9868	.9905	.9946	.9888
	.1	.9873	.9915	.9949	.9894
	.5	.9891	.9951	.9958	.9914
	1.0	.9911	.9991	.9967	.9934
	2.0	.9941	1.0054	.9980	.9965
	5.0	.9988	1.0155	.9997	1.0008
	10.0	1.0009	1.0194	1.0002	1.0018
	15.0	1.0012	1.0184	1.0002	1.0014
	20.0	1.0011	1.0166	1.0001	1.0009
	25.0	1.0010	1.0148	1.0001	1.0007
	30.0	1.0009	1.0132	1.0000	1.0005
	35.0	1.0008	1.0119	1.0000	1.0004
	40.0	1.0007	1.0108	1.0000	1.0003
	50.0	1.0006	1.0091	1.0000	1.0002
	75.0	1.0004	1.0064	1.0000	1.0001
	100.0	1.0003	1.0050	1.0000	1.0000
0.9	.0	1.0033	1.0199	.9998	1.0044
	.1	1.0034	1.0202	.9998	1.0044
	.5	1.0037	1.0214	.9999	1.0045
	1.0	1.0039	1.0226	1.0000	1.0046
	2.0	1.0042	1.0244	1.0002	1.0046
	5.0	1.0042	1.0258	1.0002	1.0037
	10.0	1.0034	1.0235	1.0001	1.0021
	15.0	1.0026	1.0201	1.0001	1.0012
	20.0	1.0021	1.0173	1.0000	1.0007
	25.0	1.0018	1.0150	1.0000	1.0004
	30.0	1.0015	1.0132	1.0000	1.0003
	35.0	1.0013	1.0117	1.0000	1.0002
	40.0	1.0012	1.0106	1.0000	1.0001
	50.0	1.0009	1.0088	1.0000	1.0001
	75.0	1.0006	1.0062	1.0000	1.0000
	100.0	1.0005	1.0047	1.0000	1.0000

TABLE 2. Risks under balanced loss for $v = 20$ and $k = 8$

				bias corrected	
w	λ	FMMSE	AFMMSE	FMMSE	AFMMSE
0.1	.0	.8456	.4793	.9774	.7015
	.1	.8473	.4840	.9779	.7052
	.5	.8538	.5020	.9796	.7193
	1.0	.8612	.5233	.9815	.7358
	2.0	.8744	.5621	.9848	.7651
	5.0	.9035	.6543	.9910	.8306
	10.0	.9318	.7544	.9956	.8940
	15.0	.9478	.8160	.9975	.9284
	20.0	.9579	.8562	.9984	.9487
	25.0	.9648	.8839	.9989	.9615
	30.0	.9698	.9038	.9992	.9701
	35.0	.9735	.9185	.9994	.9760
	40.0	.9765	.9298	.9995	.9804
	50.0	.9807	.9457	.9997	.9862
	75.0	.9868	.9664	.9999	.9929
	100.0	.9899	.9762	.9999	.9957
0.5	.0	.9474	.8680	.9919	.9085
	.1	.9480	.8698	.9921	.9097
	.5	.9502	.8765	.9927	.9143
	1.0	.9528	.8844	.9934	.9197
	2.0	.9573	.8986	.9945	.9291
	5.0	.9674	.9313	.9968	.9498
	10.0	.9771	.9644	.9984	.9691
	15.0	.9825	.9827	.9991	.9792
	20.0	.9859	.9934	.9994	.9850
	25.0	.9882	.9999	.9996	.9887
	30.0	.9899	1.0039	.9997	.9911
	35.0	.9912	1.0064	.9998	.9928
	40.0	.9922	1.0080	.9998	.9940
	50.0	.9936	1.0096	.9999	.9957
	75.0	.9956	1.0101	.9999	.9977
	100.0	.9966	1.0092	1.0000	.9986

TABLE 2. (continued)

				bias corrected	
w	λ	FMMSE	AFMMSE	FMMSE	AFMMSE
0.7	.0	.9759	.9771	.9960	.9665
	.1	.9762	.9780	.9961	.9670
	.5	.9773	.9816	.9964	.9690
	1.0	.9785	.9857	.9967	.9712
	2.0	.9806	.9930	.9973	.9751
	5.0	.9853	1.0090	.9984	.9832
	10.0	.9898	1.0233	.9992	.9902
	15.0	.9922	1.0295	.9995	.9935
	20.0	.9938	1.0319	.9997	.9952
	25.0	.9948	1.0324	.9998	.9963
	30.0	.9956	1.0320	.9999	.9970
	35.0	.9961	1.0310	.9999	.9975
	40.0	.9966	1.0299	.9999	.9978
	50.0	.9972	1.0275	.9999	.9984
	75.0	.9981	1.0223	1.0000	.9991
	100.0	.9985	1.0185	1.0000	.9994
0.9	.0	.9972	1.0583	.9990	1.0098
	.1	.9972	1.0586	.9990	1.0098
	.5	.9974	1.0598	.9991	1.0097
	1.0	.9976	1.0611	.9992	1.0096
	2.0	.9979	1.0633	.9993	1.0094
	5.0	.9986	1.0669	.9996	1.0081
	10.0	.9992	1.0672	.9998	1.0059
	15.0	.9995	1.0644	.9999	1.0041
	20.0	.9996	1.0606	.9999	1.0028
	25.0	.9997	1.0566	.9999	1.0019
	30.0	.9998	1.0529	1.0000	1.0014
	35.0	.9998	1.0494	1.0000	1.0010
	40.0	.9998	1.0463	1.0000	1.0007
	50.0	.9999	1.0409	1.0000	1.0004
	75.0	.9999	1.0314	1.0000	1.0001
	100.0	.9999	1.0254	1.0000	1.0000

AFMMSE estimator does not dominate the OLS estimator, the bias corrected AFMMSE estimator dominates the OLS estimator. However, since the risk of the AFMMSE estimator for $\lambda \leq 10.0$ is smaller than that of the bias corrected AFMMSE estimator, the effect of the bias correction on the risk performance of the AFMMSE estimator is ambiguous.

When $k = 8$ and $w = 0.7$, the risk performances of the FMMSE estimator and the bias corrected FMMSE estimator are similar to the case of $w = 0.5$. However, when w increases from 0.5 to 0.7, the risk of the bias corrected AFMMSE estimator is smaller than that of the AFMMSE estimator. This indicates that the bias correction has a positive effect on the risk performance of the AFMMSE estimator.

When $k = 8$ and $w = 0.9$ (*i.e.*, the goodness of fit is much more important), the FMMSE estimator and the bias corrected FMMSE estimator dominate the OLS estimator. However, both the bias corrected AFMMSE estimator and the AFMMSE estimator are dominated by the OLS estimator. Comparing the AFMMSE estimator and the bias corrected AFMMSE estimator, the bias correction is slightly effective. However, since the risk of the bias corrected FMMSE estimator is larger than that of the FMMSE estimator, the bias correction has a negative effect on the risk performance of the FMMSE estimator. When $w = 0.9$, the risks are very close to unity. This indicates that using the FMMSE estimator may be meaningless when the goodness of fit is much more important than the precision of estimation.

REFERENCES

Akdeniz, F. and S. Kaçiranlar (1995), "On the almost unbiased generalized Liu estimator and unbiased estimation of the bias and MSE," *Communications in Statistics-Theory and Methods*, 24, pp. 1789-1797.

Baranchik, A. J. (1970), "A family of minimax estimators of the mean of a multivariate normal distribution," *Annals of Mathematical Statistics*, 41, pp. 642-645.

Dwivedi, T. D. and V. K. Srivastava (1978), "On the minimum mean squared error estimators in a regression model," *Communications in Statistics-Theory and Methods*, A7, pp. 487-494.

Farebrother, R. W. (1975), "The minimum mean square error linear estimator and ridge regression," *Technometrics*, 17, pp. 127-128.

Hoerl, A. E. and R. W. Kennard (1970), "Ridge regression: biased estimation for nonorthogonal problems," *Technometrics*, 12, pp. 55-67.

James, W. and C. Stein (1961), "Estimation with quadratic loss," *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability,* 1, Berkeley: University of California Press, pp. 361-379.

Judge, G. G. and M. E. Bock (1978), *The Statistical Implications of Pre-Test and Stein-Rule Estimators in Econometrics*, Amsterdam: North-Holland.

Kadiyala, K. (1984), "A class of almost unbiased and efficient estimators of regression coefficients," *Economics Letters*, 16, pp. 293-296.

Kejian, L. (1993), "A new class of biased estimate in linear regression," *Communications in Statistics-Theory and Methods*, 22, pp. 393-402.

Liski, E.P., H. Toutenburg and G. Trenkler (1993), "Minimum mean square error estimation in linear regression," *Journal of Statistical Planning and Inference*, 37, pp. 203-214.

Nomura, M. (1988), "On the almost unbiased ridge regression estimator," *Communications in Statistics-Simulation and Computation*, 17, pp. 729-743.

Ohtani, K. (1986), "On small sample properties of the almost unbiased generalized ridge estimator," *Communications in Statistics-Theory and Methods*, 15, pp. 1571-1578.

Ohtani, K. (1996a), "Exact small sample properties of an operational variant of the minimum mean squared error estimator," *Communications in Statistics-Theory and Methods*, 25, pp. 1223-1231.

Ohtani, K. (1996b), "On an adjustment of degrees of freedom in the minimum mean squared error estimator," *Communications in Statistics-Theory and Methods*, 25, pp. 3049-3058.

Ohtani, K. (2000), *Shrinkage Estimation of a Linear Regression Model in Econometrics*, New York: Nova Science Publishers.

Ohtani, K. (2001), "On the small sample properties of the almost unbiased minimum mean squared error estimators in regression," *Kokumin-Keizai Zasshi (Journal of Economics and Business Administration)*, 184, pp. 1-10 (In Japanese).

Singh, B., Y. P. Chaubey and T. D. Dwivedi (1986), "An almost unbiased ridge estimator," *The Indian Journal of Statistics*, B48, pp. 342-346.

Stahlecker, P. and G. Trenkler (1983), "On heterogeneous versions of the best linear and the ridge estimator," *Proc. First Tampere Sem. Linear Models*, pp. 301-322.

Stein, C. (1956), "Inadmissibility of the usual estimator for the mean of a multivariate normal distribution," *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability*, 1, Berkeley: University of California Press, pp. 197-206.

Theil, H. (1971), *Principles of Econometrics*, New York: John Wiley.

Tracy, D. S. and A. K. Srivastava (1994), "Comparison of operational variants of best homogenous and heterogeneous estimators in linear regression," *Communications in Statistics-Theory and Methods*, 23, pp. 2313-2322.

Vinod, H. D. (1976), "Simulation and extension of a minimum mean squared error estimator in comparison with Stein's," *Technometrics*, 18, pp. 491-496.

Zellner, A. (1994), "Bayesian and non-Bayesian estimation using balanced loss functions, in Gupta S. S. and J. O. Berger (eds.)," *Statistical Decision Theory and Related Topics*, V, New York: Springer-Verlag, pp. 377-390.