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Abstract

With the aim to propose a suitable countermeasure for the elevated structure noise, the sound field radiated by a steel plate girder, which is the main source of the elevated structure noise, have been theoretically analysed to gain a physical insight into the elevated structure noise problem. In actual condition of an elevated motorway, the reflection and diffraction due to the adjacent steel plate girders placed in parallel should be considered for analysis of the sound field. In the present study, the sound field radiated by some steel plate girders are analysed. Steel plate girders are modelled as infinitely long elastic plate strips placed in parallel. In the analysis of the radiated sound field, the equivalent source method is employed. The character of the sound field is discussed thorough numerical examples. The effects of an adjacent steel plate girder on the radiated sound field from a steel plate girder are discussed thoroughly: the adjacent steel plate girder affect significantly the sound field radiated by the plate girder. Especially, the increase of the sound pressure caused by the reflection due to the adjacent steel plate girder is shown. The variations of the effects due to varying number of the girders are also demonstrated.

Introduction

Recently, the elevated structure noise has become the one of main sources of road traffic noise around elevated motorways. Some studies in attempt to establish a prediction model of the elevated structure noise have been performed, and *in situ* measurement data have been published [1-4], however, few studies to discuss a countermeasure have been reported. To establish a reasonable countermeasure, a physical insight into the phenomenon nature of the problem is needed. To gain the physical insight, a theoretical study is helpful.

In the previous studies, the authors have analysed the sound field radiated by an infinitely long elastic plate strip, which is a simple model of a web plate of a steel plate girder [5, 6]. In those studies, it is confirmed that the calculated frequency characteristics of the radiated sound pressure from an infinitely long elastic plate show similar tendency to the main feature of the measured elevated structure noise, and concluded that the steel plate girder can be the main source of the elevated structure noise around the motorway which has a concrete slab. Furthermore, main features of the sound radiation from steel plate girder are discussed in those studies. The results are useful to understand the elevated structure noise so far as sound radiation is concerned.

In actual condition of an elevated motorway, the radiated sound field by a steel plate girder changes in a complex manner due to the presence of the various elements of the elevated structure, e.g., concrete slab, adjacent steel plate girders and braces. The elevated structure noise is affected by a reflection and diffraction due to the presence of these elements. This means that it is possible to reduce the elevated structure noise by controlling not only the sound radiation from the plate girders, but also the reflection and diffraction due to the other elements. Therefore, to establish an appropriate countermeasure, considering the some elements of the elevated structure as one sound source system is more advantageous than dealing with each element individually. Thus, it is necessary to clarify the effect of motorway elements on the sound field radiated by each steel plate girder. Considering the size and geometry among the motorway elements, a concrete slab and adjacent steel plate girders are likely to affect the most the sound field radiated by each steel plate girder due to those location and dimensions.

For the reason above, the effect of a concrete slab on the sound field radiated by a steel plate girder has been analysed in the previous study [7]. In the same study, a steel plate girder in contact with a concrete slab is modelled as an infinitely long elastic plate strip in contact at its edge with an infinite reflecting surface. The results obtained indicate that the effects of a slab appear mainly at lower frequency than the frequency range where the main component of the elevated structure noise is observed. This results imply the sound absorption on the under surface of a slab cannot be effective for reduction of the elevated structure noise. On the other hand, the effects of the other element, i.e., adjacent steel plate girders, on the sound field have not yet been discussed.

The aim of this study is to reveal the effect of adjacent steel plate girders on the sound field radiated by each steel plate girder. Steel plate girders are modelled as infinitely long elastic plate strips placed in parallel. The excitation conditions of the girders until now have not been clarified. Girders are usually connected to a slab, and the vibration of the slab is supposed to excite the girders. Therefore, in this study, each steel plate girder is assumed to be excited by a moment at its edge. The vibration pattern of a strip generated by the moment excitation is calculated from an analytical expression in which the acoustic loading to the strip is assumed to be negligible. This assumption is reasonable as long as the strip is much massive enough compared to the surrounding medium [8].

In the analysis of the radiated sound field, the equivalent source method is employed. The character of the sound field is discussed thorough numerical examples. For thorough investigation, the sound field radiated by a steel plate girder with an adjacent plate girder are analysed, and the effects of the adjacent steel plate girder on the sound field are discussed. The variation of the effects by the varying height of the girders and distance between the two plate girders are also demonstrated. Furthermore, the variations of the effects due to the varying number of the girders are demonstrated.

Analysis procedure

Figure 1 shows the two-dimensional calculation model of steel plate girders with a concrete slab. The number of the girders is varied between two and six. The steel plate girders are modelled by infinitely long elastic plate strips in contact at their edges with the concrete slab. The concrete slab is modelled as a rigid infinite reflecting surface, and its effect on the sound field is taken into account by introducing the image sources of the girders. The infinitely long plate strips of thickness h have the width L_x, and the distance between the girders is d as shown in Fig. 1. All boundary surfaces are assumed to have no sound absorptivity.

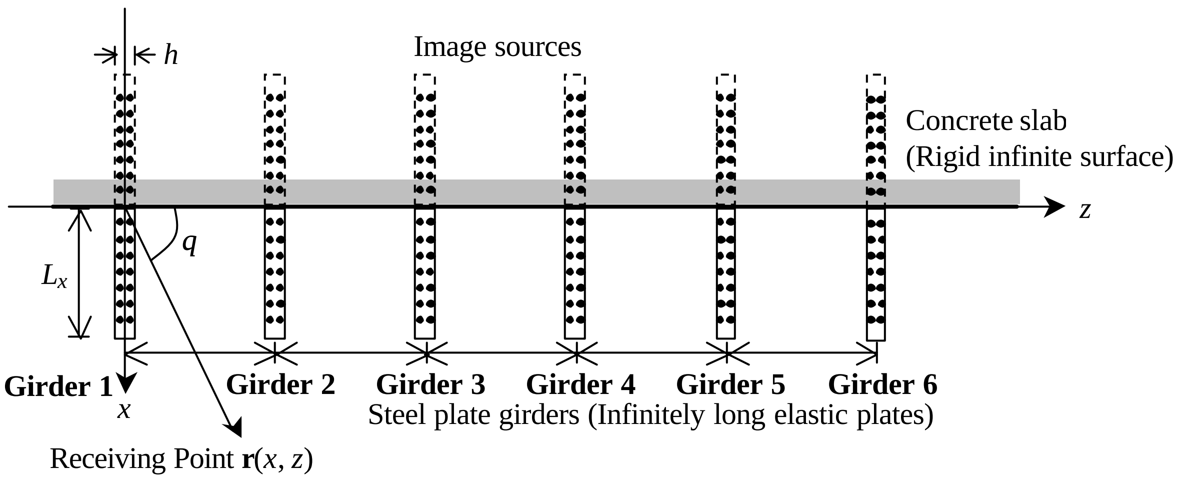

Figure 1 *The model of steel plate girders with a concrete slab for analysis by the equivalent source method: ● the equivalent sources (line sources). The calculated area shown in Figs. 2 - 4 is* $0\,\mathrm{m} \le x \le 15\,\mathrm{m}$ *and* $-30\,\mathrm{m} + d/2 \le z \le 30\,\mathrm{m} + d/2$.

Following the equivalent source method, the boundary conditions for both the real and image sources have to be fulfilled by equivalent sources. Line sources are used as the equivalent sources in the present study. The boundary condition for the sound field and the transfer function of the equivalent sources are needed for the analysis.

The particle velocity on the plate surfaces is identical to the vibration velocity of the plates, since the plate surfaces have no sound absorptivity. Therefore, the boundary condition for the sound field is expressed by the vibration velocity of the elastic plate. Each plate is assumed to be excited by a line moment at its edge, and the edges of each strip are assumed to be simply supported. When the plate strip is excited by a harmonic line force of amplitude F_0 at $x = l$ and vibrates with the displacement $w(x)$, the equation of motion of the plate strip is:

$$D\nabla^4 w(x) - r_p h w^2 w(x) = -F_0 d(x-l). \tag{1}$$

Here, $D = Eh^3(1-ih)/12(1-n^2)$ is the flexural rigidity of the plate, where E is Young's modulus, h is the thickness, h is the loss factor, and n is the Poisson's ratio of the plate. In the above equation, r_p is the plate's density, w is the angular frequency, and d denotes the Dirac delta function. The time factor $\exp(-iwt)$ is suppressed throughout. The x-dependent function $w(x)$ can be expanded into a series of the eigenfunctions satisfying the boundary condition for the plate vibration. The solution for a moment excitation can be derived in a similar procedure to Refs. [9, 10]. By adding the displacement due to a line force of equal magnitude with the opposite sign at $x = l - \Delta x$, and taking the limit of $\Delta x \to 0$ with $F_0 = M_0/\Delta x$, the displacement due to a line moment acting at $x = l$ is obtained. Substituting $l = 0$, the displacement by a moment excitation on the strip edge is obtained. Finally, the vibration velocity by a moment excitation on the strip edge is derived as:

$$v_{real}(x) = \sum_n \frac{2iwM_0 k_n}{DL_x(k_n^4 - m^4)} \sin k_n x; \quad k_n = np/L_x, \quad n = 1, 2, 3, \cdots, \quad 0 \le x \le L_x \tag{2}$$

where $m^4 = r_p h w^2 / D$. Equation (2) is used as the boundary condition for the sound field caused by the real source. The boundary condition for the image source is also obtained from Eq. (2) as:

$$v_{image}(x) = -\sum_n \frac{2iwM_0 k_n}{DL_x(k_n^4 - m^4)} \sin k_n x \quad , \qquad -L_x \le x \le 0 \tag{3}$$

Eqs. (2) and (3) have to be fulfilled by the equivalent sources.

The velocity potential $f_m(\mathbf{r}, \mathbf{r}_0)$ at an arbitrary point in a two-dimensional free field $\mathbf{r} = (x, z)$, produced by a monopole line source at $\mathbf{r}_0 = (x_0, z_0)$ with the volume velocity U per unit length, is given by [11]:

$$f_m(\mathbf{r}, \mathbf{r}_0) = U \frac{i}{4} H_0^{(1)}(k_0 |\mathbf{r} - \mathbf{r}_0|), \tag{4}$$

where $H_0^{(1)}$ is the first kind Hankel function of order zero, $k_0 = w/c_0$ is the acoustic wavenumber, where c_0 is the sound speed in the air. The z-directional particle velocity on the plate surface at $\mathbf{r} = (x, z)$, which is produced by the j-th line source located at $\mathbf{r}_0 = (x_0, z_0)$, is obtained from Eq. (4) as:

$$v_j(\mathbf{r}, \mathbf{r}_0) = -\frac{\partial f_m}{\partial z} = U_j \frac{ik_0(z - z_0)}{4|\mathbf{r} - \mathbf{r}_0|} H_1^{(1)}(k_0 |\mathbf{r} - \mathbf{r}_0|) \ , \tag{5}$$

where U_j is an unknown volume velocity to be determined later to fulfil the boundary conditions given by Eqs. (2) and (3). The particle velocity by equivalent sources is expressed as:

$$v = \sum_{j=1}^{n} v_j \,, \tag{6}$$

where n is the number of the line sources.

If U_j is determined, the sound pressure p is calculated as the sum of the sound pressure contributed by all the monopole line sources, i.e., the equivalent sources:

$$p = \sum_{j=1}^{n} p_j \,, \tag{7}$$

In Eq. (7), p_j denotes the sound pressure at a receiving point $\mathbf{r} = (x, z)$ which is produced by the j-th line source located at $\mathbf{r}_0$ obtained from Eq. (4) as:

$$p_j(\mathbf{r},\mathbf{r}_0) = r_0 \frac{\partial f_m}{\partial t} = U_j \frac{r_0 w}{4} H_0^{(1)}(k_0 \,|\mathbf{r} - \mathbf{r}_0|) \,, \tag{8}$$

where r_0 is the air density.

Numerical results and discussion

All numerical calculations have been performed assuming the following plate's properties: $E = 10^{11}$ $\mathrm{Nm^{-2}}$, $h = 10^{-3}$, $r_p = 8000$ $\mathrm{kgm^{-3}}$, $n = 0.3$, $h = 0.02$ m, $L_x = 2.0$ m, d = 3.5 m, unless otherwise noted. These are common values for actual steel plate girders of elevated motorways, except for L_x and d: in the actual motorways the values for L_x and d are determined according to structural considerations, i.e., they depend on the span and width of the elevated motorways. Thus, the variation of the sound field by changing these values is also demonstrated.

The effects of an adjacent plate strip

For thorough discussion on the effect of the plate girders on the radiated sound field, only the results for two girders are discussed in this section. Furthermore, to clarify the effect of reflection and diffraction due to the second girder, only the girder 1 is assumed to vibrate and radiate sound, and the girder 2 is assumed to be immobile. The sound pressure level difference (*LD*) between the cases with and without the girder 2 is calculated for 1/3-octave bands centred at 500 Hz and 1000 Hz. These are the frequencies where the main component of the elevated structure noise is observed. The *LD* is obtained by integrating the *LD*s for every 1/27-octave band within each 1/3-octave band. The calculation is performed at every 0.5 m grid points in the calculation area described in the caption to Fig. 1. Positive and negative *LD* indicate the increase and decrease of the sound pressure level due to the presence of the girder 2, respectively.

Figure 2 shows the calculated *LD*. The *LD* distribution is complicated because of the interference, however, there are similar tendencies found in both 1/3-octave bands. In the area $90° \le q \le 180°$, the positive *LD* is observed at many receiving points. These increases in sound pressure are caused by reflections due to the presence of the girder 2. In this area, the *LD* is small at $170° \le q \le 180°$ but especially large at $140° \le q \le 165°$. For the former, the reason is that the girder 1 works as a sound barrier on the reflected sound from the girder 2. For the latter, the reason will be discussed later. Turning now to the area $0° \le q \le 90°$, the negative *LD* is observed at $0° \le q \le 40°$ because the girder 2 works as a sound barrier on the radiated sound from the girder 1. However, the large positive *LD* observed at around $q = 20°$. This increase in sound pressure is caused by the reflected sound from the girder 1, i.e., multiple reflections. It is noted that these receiving points and the area $140° \le q \le 165°$ mentioned above locate symmetrically to each other. Therefore, the large increase in sound pressure observed at $140° \le q \le 165°$ can contain the effect of the multiple reflections.

It is supposed that the effects of the reflection due to the presence of the girder 2 change according to the geometry, i.e., the height of the girders L_x and the distance between the girders d. Figure 3 shows the *LD* in the case of d = 1 m. The grey scale becomes darker than the case of d = 3.5 m (Fig. 2).

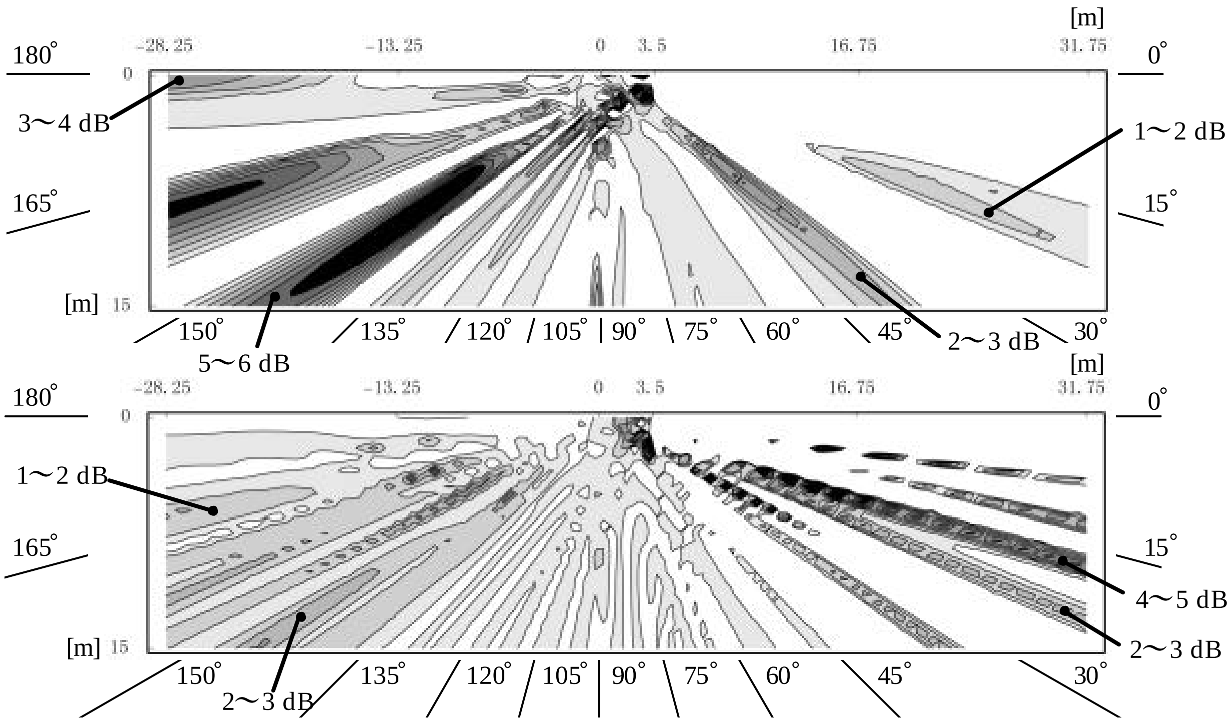

Figure 2 *Sound pressure level difference (LD) between the cases with and without the girder 2 on the sound pressure radiated by the girder 1 for 1/3 octave bands of centre frequencies at 500 Hz (top) and 1000 Hz (bottom) with 2.0 m girder height and 3.5 m distance between girders. Grey scale indicates 1 dB level difference. White zone indicates decrease of the sound pressure and black zone indicates more than 6 dB increases due to presence of the girder 2.*

Furthermore, in the area $90° \le \theta \le 180°$, the large positive *LD* is shown at about $120° \le \theta \le 140°$ for both 1/3-octave bands. Namely, the receiving points with the large positive *LD* shifts to downward relative to the case of in Fig. 2. The same tendency can be seen in the area $0° \le \theta \le 90°$. These results clearly indicate that the receiving points, which are significantly affected by the adjacent plate girder, change according to geometrical conditions. From the receiving points which are affected by multiple reflections, the half of the offside girder is invisible due to the presence of the nearside girder and the rest is visible. Figure 4 shows the *LD* in the case of L_x = 1 m. At most receiving points, the *LD* becomes small. It is confirmed that the effect due to the girders of 1 m height on the sound field is less than that due to the girders of 3.5 m height. According to the discussion above, it is expected that the receiving area with the large positive *LD* shift to upward than in the case of Fig. 2 because the area where the offside girder can be seen spread to upward. For the 1/3-octave band of centre frequency at 1000 Hz, this expected phenomenon can be seen, i.e., the effects of multiple reflections are shown at the receiving points that locate around $\theta = 10°$ and $170°$. However, for the case of 500 Hz, the phenomenon is not seen. It is supposed that the girders of 1 m height are not effective for the sound field in the case of the 1/3-octave band of centre frequency at 500 Hz. This consideration seems to be reasonable considering the wavelength relative to the girder height. These results suggest that the noise levels around motorways should be controlled by these parameters.

Variation of the effects by varying the number of plate strips

Considering the actual condition, every steel plate girder radiates the sound. Furthermore, the number of the girder is different from site to site. Therefore, change in the effects of the adjacent plate girders by varying the number of the plate girders when all the girders are excited and radiate the sound is demonstrated. In reality, girders are usually connected to a slab, and the vibration of the slab is mainly responsible for the excitation of the girders. Hence, each steel plate girder is assumed to be excited by a moment at its edge. The vibration of the slab is assumed to be in 'diffuse' condition at high frequency range. Therefore, the excitation force and the vibration of the girders are incoherent each other, and the sound pressures radiated from each steel plate girders are treated as incoherent in this study.

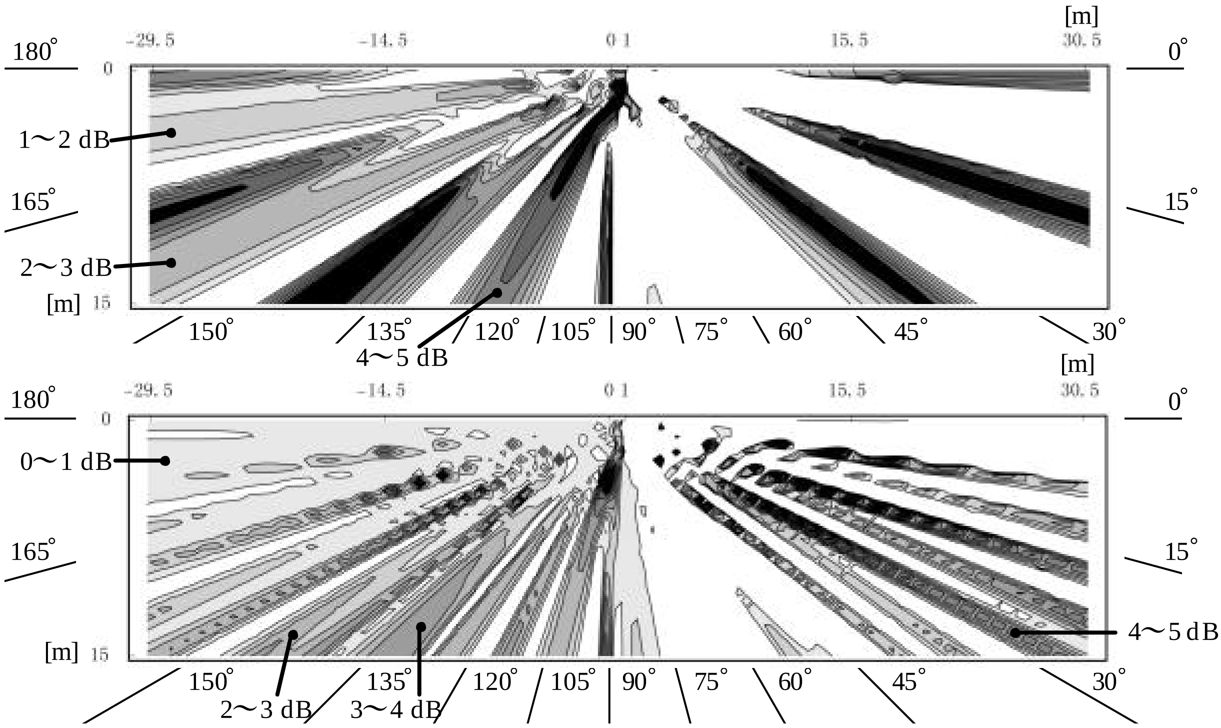

Figure 3 *The LD with 1.0 m distance between girders. Other parameters for the calculation and the definition of grey scale are the same as in Fig. 2.*

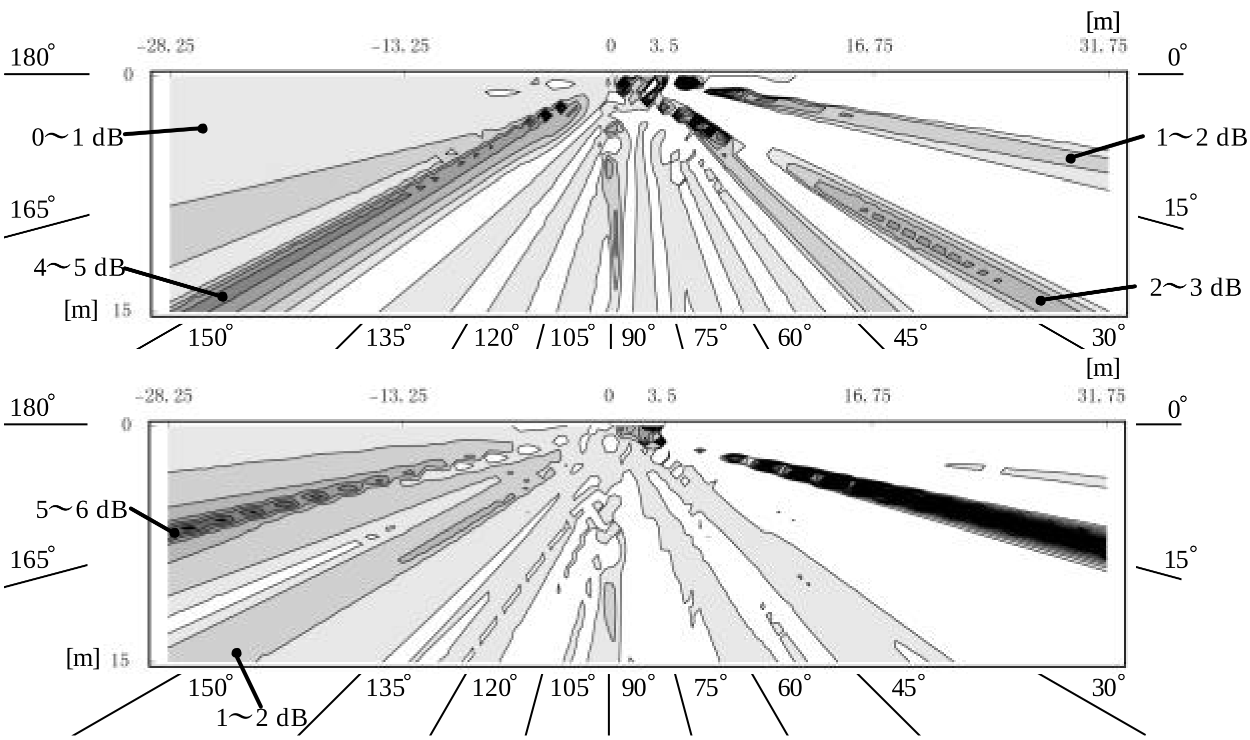

Figure 4 *The LD with 1.0 m girder height. Other parameters for the calculation and the definition of grey scale are the same as in Fig. 2.*

Figure 5 shows the *LD* calculated according to the following procedure. First, the sum of the sound pressure radiated by each plate girder in the absence of the other girders is calculated. Next, the sum of sound pressure radiated by each plate girder in the presence of the other girders that are assumed to be immobile is also calculated. Then, *LD* is defined as the difference between these two calculated results. The positive *LD* indicates that the sound pressure radiated from every plate girder, which is main interest of this study, is larger than the sum of the sound pressure from each girder without the other girders. The *LD* for 1/3-octave band centred at 1000 Hz is calculated. The *LD* is obtained by integrating the *LD*s for every 1/27-octave band within each 1/3-octave band. The calculation is performed at every 0.5 m grid points in the calculation area shown in Fig. 5.

The effect of the adjacent girders changes drastically as the number of the girder increases from 2 to 4. However, as the number of the girder increases from 4 to 6, the effect of the adjacent girders is hardly changed. The explanation of this results is twofold: first, the girders further added on are far apart from the sound radiating girder, and are also far from the receiving points, which makes their contribution to the sound field becomes smaller. Besides, the contribution from the girders further added on account for only small portion in the total power and is not reflected in decibel values. The large positive *LD* due to multiple reflections mentioned in Section 3.1 can be seen. These results suggest that the elevated structure noise level at residential area around motorways should be larger than the simple sum of the sound pressures radiated from each steel plate girders. Therefore, the noise level can be supposed to be reduced when the surfaces have absorptivity, because the reflections between the plate girders are suppressed.

Conclusions

The sound field radiated by infinitely long elastic plate strips in parallel has been numerically analysed. This is a simple model for the sound field radiated by steel plate girders in contact with a concrete slab.

Numerical examples indicate that the elevated structure noise measured around motorways is affected by adjacent steel plate girders. Especially, it is clarified that the increase of the noise level due to the presence of reflections between the girders occurs.

The variations in the effects by varying the height of and distance between girders were discussed through numerical examples. The close examination of numerical examples show that the area, where the sound field is significantly affected by the adjacent plate girder, changes according to the geometrical condition: it depends on whether the half of the girder is visible due to the presence of the nearside girder and the rest is visible from the receiving point. This affects the multiple reflections between the girders. Furthermore, it is shown that the effect becomes small if the height of the girder becomes small. Therefore, within the scope of noise control engineering, the geometrical condition of an elevated motorway should be carefully considered to minimize the effect of multiple reflections increasing noise level by the adjacent girders.

In this study, the further investigation to clarify the effect of multiple girders has been performed: the number of the girder is increased up to six, however, the results imply that the effect of the adjacent girders is hardly changed even if the number increases more than four.

According to the results above, the control or suppression of the reflections is suggested to be one of countermeasures for the elevated structure noise.

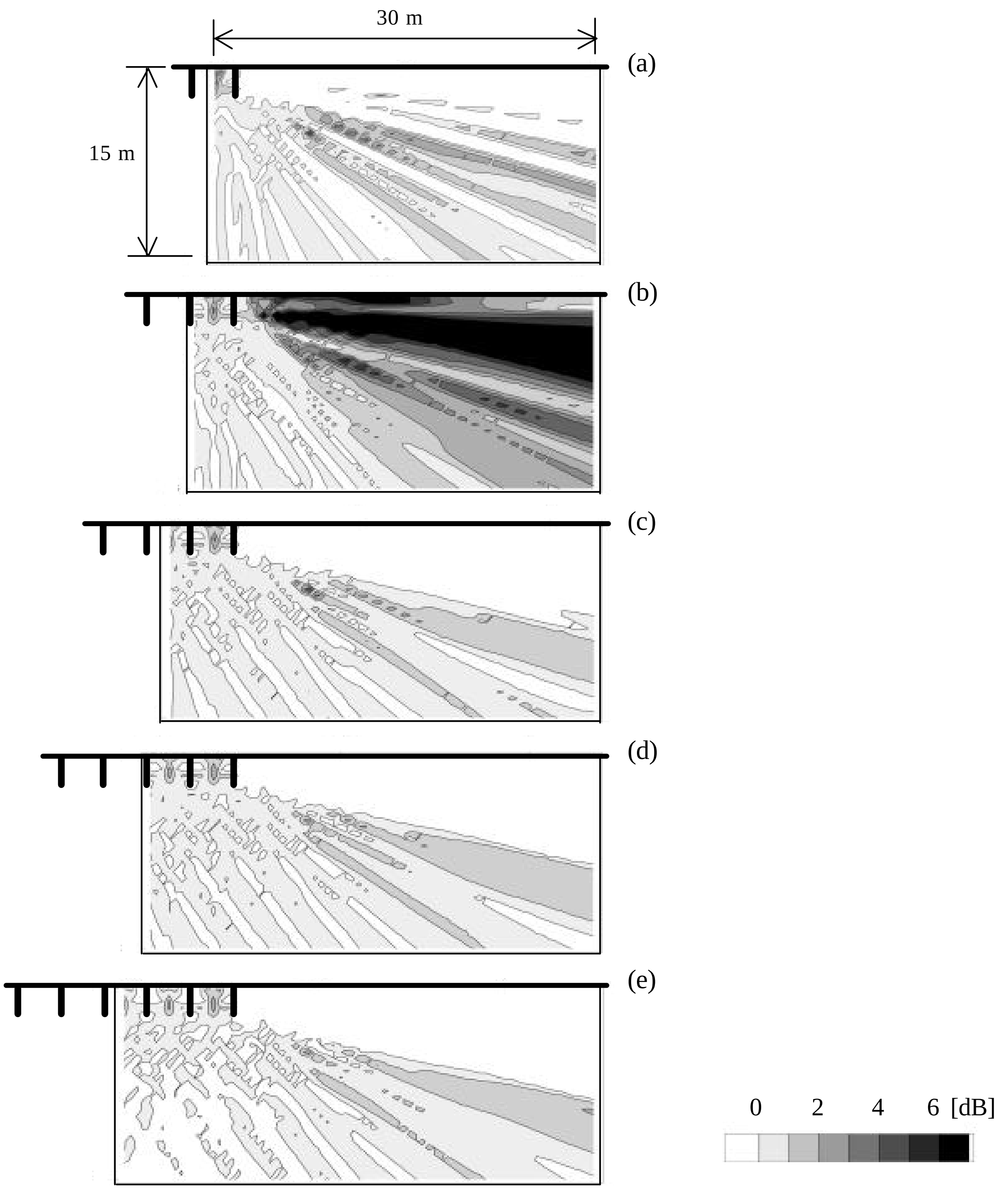

Figure 5 *The effect of the adjacent plate girders on the sound field radiated by all plate girders for 1/3 octave band of centre frequencies at 1000 Hz. The number of the girder is (a) 2, (b) 3, (c) 4, (d) 5 and (e) 6. White zone indicates decrease of the sound pressure and black zone indicates more than 6 dB increases due to presence of the adjacent plate girders.*

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