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R&D EFFICIENCY AND COLLABORATION NETWORKS*

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We investigate the relationships between collaboration networks and the efficiencies of R&D and collaboration. In our model, there are three firms, and firms' collaboration patterns are represented by networks. Since the number of firms is three, there are four possible shapes for collaboration networks (1) the complete network, (2) a star network, (3) an exclusive network, and (4) the empty network. Firms engage in Cournot competition, and we obtain four results on competition in the four respective networks. We show which network shapes are stable and optimal with respect to the efficiencies of R&D and collaboration.

Keywords: R&D; Collaboration; Network; Pairwise stability

1. Introduction

In a highly technological good market, firms face keen competition. Firms are going to obtain a large share of the market, because if a firm obtains a large share, then its product becomes the standard of the market, and the firm becomes large. To increase sales, firms make efforts to lower their production costs. Such efforts are considered as research and development (R&D). We consider that such an R&D activity is an important aspect of competition in the market. In such a market, we often observe that a few firms compete. Furthermore, since such a product requires a high level of technology, we also observe that firms collaborate with others to develop their production technology. We consider that a collaboration among a few firms is also an important aspect in competition.

To consider the two aspects noted in the above, we introduce two parameters that represent the efficiencies of technology and collaboration. Technological efficiency represents how much investment is needed to reduce the production cost. If technological efficiency is high, then cost reduction requires a small amount of investment. On the other hand, collaboration efficiency represents whether and by how much collaboration with other firms decreases a firm's investment cost. If collaboration efficiency is high, then collaboration among firms can significantly decrease investment costs. We study the relationships among market outcomes, collaborations, and the efficiencies of technology and collaboration.

Much of the recent industrial organization literature considers firms' collaboration patterns as networks. A collaboration between two firms is denoted by a link, and a set of all such links, *i.e.,* a network, represents a collaboration pattern among firms. For example, Goyal and Moraga (2001) and Goyal and Joshi (2002) study R&D collaboration among firms by using network formation games. In the empirical literature, Powell, White, Koput, and Owen-Smith (1996) investigate collaboration networks in the biotechnological sector. Their focuses are

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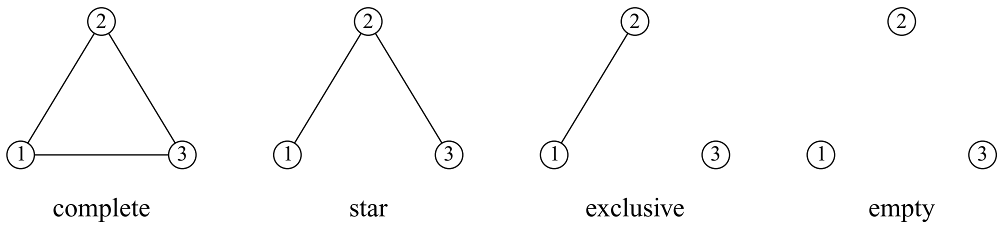

Figure 1: Four shapes of networks formed by three firms

the shapes of collaboration patterns, the resources that cause the shapes, and how market outcomes are affected by the resources and shapes. We also focus on these issues using a three stage game in which firms choose their collaboration partners and levels of R&D, and then engage in Cournot competition.

In our model, there are three firms, and accordingly there are four possible shapes for collaboration networks, as shown in Figure 1: (1) the complete network, in which each firm collaborates with the other two firms, (2) a star network, in which there is one hub firm that collaborates with the other two firms and there are two firms that collaborate only with the hub firm, (3) an exclusive network, in which there is one pair of firms that collaborate with each other and there is one firm that does not collaborate at all, and (4) the empty network, in which there is no collaboration. We show the conditions for which a network is pairwise stable and/or optimal, and that, for any network, there exist ranges of parameters for which the network is pairwise stable and optimal.

The present paper is closely related to Song and Vannetelbosch (2007). They show that the complete network is pairwise stable in similar settings to ours. In their model, if a firm collaborates with others, then the collaboration necessarily lowers the cost of R&D. In our model, if collaboration efficiency is high, then the collaboration necessarily lowers the cost of R&D. Besides the efficiency of collaboration, we also show that, if R&D is very costly, then the complete network, which is pairwise stable, maximizes social welfare.

By separating the efficiency of R&D into technological and collaboration efficiency, we obtain results that have implications for policy for R&D investment. According to our results, if R&D for highly technological good production is very costly, although a policy for promoting individual firms' R&D seems to increase social welfare, it possibly decreases social welfare, whereas a policy for promoting collaboration increases social welfare. Therefore, a policy for promoting R&D should be targeted towards collaboration among firms since the complete network or the star network is formed and is optimal when the policy sufficiently lowers the cost of collaboration.

This paper is organized as follows. Section 2 provides the model. Section 3 shows the outcomes of Cournot competition. Section 4 shows the pairwise stable networks. Section 5 shows the optimal networks. Section 6 shows conditions under which networks are stable and optimal. Section 7 concludes.

2. Model

Firms

Let $N = \{1, 2, 3\}$ be the set of firms. The firms face the (inverse) demand function defined by $p(Q) = a - Q$, where $a > 0$ is a constant and Q is the *total production* of firms in the market. Let $q_i \geq 0$ denote the production of firm i, and the profile of firms' productions be denoted by $q = (q_1, q_2, q_3)$. We assume that all firms have the same marginal cost, which is constant. Let c be the marginal cost of production, and we suppose that $a > c$. Each firm can reduce the marginal cost by doing research and development (R&D). Let x_i denote the *cost reduction of firm i*, and a *profile of firms' cost reductions* be denoted by $x = (x_1, x_2, x_3)$. If firm i's cost reduction is x_i, then firm i reduces the marginal cost c to $c - x_i$. When a firm's cost reduction is x_i, the firm incurs an investment cost, which depends on x_i. The investment cost function is specified in the following.

Networks and the R&D cost function

Each firm can collaborate with other firms to obtain the cost-reducing technology that other firms own. If a firm collaborates with another firm, then the firms make changes in their investment cost functions. To specify how firms that collaborate with each other change their investment cost functions, at first, we introduce collaboration networks. Let $ij = \{i, j\} \subset N$ denote that firms i and j collaborate with each other. We refer to ij as the *collaboration link ij* or, simply as the *link ij*. Let $g^N = \{ij : i, j \in N, i \neq j\}$ denote the set of all links. We call g^N the *complete network*, and a *network* is denoted by $g \subseteq g^N$. Let G be the set of networks. For example, $\{12, 23\}$ is a network, where firms 1 and 2 are linked, and firms 2 and 3 also are linked. We denote the *neighborhood* of firm i, which is a set of directly linked firms, by $N_i(g) = \{j \in N : ij \in g\}$. We refer to an element of a neighborhood as a *neighbor*. For example, $N_2(\{12, 23\}) = \{1, 3\}$, *i.e.*, firms 1 and 3 are neighbors of firm 2. Let $n_i(g)$ denote the number of firm i's neighbors in network g.

Next, we specify the collaboration by which each firm changes its investment cost. Let $\varphi(x_i) = \gamma x_i^2/2$ denote *firm i's investment cost function*, where $\gamma > 0$ is a constant. We call $1/\gamma$ the level of *technological efficiency*. The higher the level of technological efficiency, the lower the investment cost per cost reduction. We assume that all firms have the same investment cost function for cost reductions. However, the investment costs that firms face need not be same even if cost reductions are the same, because investment costs also depend on their positions in the network as follows. When firm i whose cost reduction is x_i has $n_i(g)$ neighbors, the firm incurs the investment cost, $\beta(n_i(g))\,\varphi(x_i)$, where

$$\beta(n_i(g)) = \begin{cases} \frac{\alpha}{n_i(g)+1} & \text{if } n_i(g) \geq 1, \\ 1 & \text{otherwise} \end{cases}$$

and $\alpha > 0$, which is a constant, represents the intensity of synergy and congestion effects in collaboration. Let $\psi(x_i, n_i(g)) = \beta(n_i(g))\,\varphi(x_i)$ denote the *R&D cost function*. When a firm collaborates with another firm and if α is sufficiently low, then $\beta(n_i(g)) < 1$. Then, a collaboration reduces the investment cost, indicating that the synergy effect is larger than the congestion effect. On the contrary, when α is sufficiently high, then $\beta(n_i\,(\mathrm{g})) > 1$. Then, a collaboration increases the investment cost, indicating that the congestion effect is larger than the synergy effect. We call $1/\alpha$ the level of *collaboration efficiency*.

Independent of whether α is high or low, if firms i and j collaborate, then each of them obtains its collaboration partner's technology, and enjoys a lower investment cost. The investment costs monotonically decreases in the number of neighbors. This monotonicity is simply represented by $1/(n_i(\mathrm{g})+1)$.

The profit function

We define the profit function as follows. Let π_i be *firm i's profit function*, which is defined by

$$\pi_i\,(q, x, g) = (p\,(Q) - (c_i - x_i))\; q_i - \psi\,(x_i, n_i(\mathrm{g}))$$

We also denote the profit simply as $\pi_i\,(\mathrm{g})$.

3. Cournot competition with cost reductions

This section analyzes an oligopolistic market in which three firms engage in Cournot competition with cost reductions. One of our main concerns is which collaboration pattern emerges in the market. We consider the networks that emerge to be stable networks. To analyze which network emerges, we need to derive firms' profits for each of the networks. In the following, therefore, we study the game of Cournot competition with cost reductions. The game is a two-stage perfect information game.

3.1 Game

Stage 1

The strategy of firm i is the cost reduction x_i. Since the coefficient of firm i's R&D cost function depends on the network, the cost reduction depends on the network. Let $x = (x_1, x_2, x_3)$ denote the *profile of all firms' cost reductions*. This strategy decides the R&D cost $\psi(x_i, n_i(\mathrm{g}))$.

Stage 2

All firms know the profile of cost reductions decided in Stage 1. Firms engage in Cournot competition under the cost reductions. Each firm chooses its production q_i to maximize its profit $\pi_i(q_i, q_{-i}, x, g)$ subject to the other firms' given levels of production q_{-i}. Firm i's production is a function $q_i(q_{-i}, x, g)$, or simply, $q_i(\mathrm{g})$. $q = (q_1, q_2, q_3)$ denote the *profile of all firms' production levels*.

3.2 Equilibria

This subsection shows the four results for the subgame perfect Nash equilibria of this two-stage game. Since the number of players is three, there are four possible network shapes: (1) the complete network, (2) stars including everyone, in which two links are formed and no other link exists, (3) exclusive networks, in which one link is formed and one player is isolated from the link, and (4) the empty network, in which no link exists. Hence, we obtain four results for Cournot competition.

When firm i collaborates with another firm, $\beta_i(n_i(\mathrm{g})) = \alpha\gamma/(n_i(\mathrm{g})+1)$. Therefore, many of the outcomes (quantities, cost reductions, and profits) include $\alpha\gamma$, as we can see in the following. We define $\delta = \alpha\gamma$, and we call $1/\delta$ the *R&D efficiency*. The higher the level of R&D efficiency, the lower the cost of R&D, *i.e.*, R&D is "efficient". On the contrary, R&D is "inefficient" if R&D efficiency is sufficiently low. To guarantee that production is positive in equilibrium, we assume that $\delta > 27/8$ and $\gamma > 9/8$. Let $\delta_0 = 27/8$ and $\gamma_0 = 9/8$.

The following subsections solve the game backwardly.

3.2.1 Stage 2

Firms decide their production levels. For all firms i, given the other firms' production levels q_{-i}, the cost reductions $x = (x_1, x_2, x_3)$, and the network g, the profit functions are

$$\pi_i(q_i, q_{-i}, x_i, \mathrm{g}) = (p(Q) - (c - x_i))q_i - \psi(x_i, n_i(\mathrm{g})).$$

In equilibrium, all firms i produce

$$q_i^*(x) = (d + 3x_i - x_j - x_k)/4,$$

where $d = a - c$. It is clear that the more firm i reduces the cost, the more its production increases, and the more other firms reduce their costs, the more firm i's production decreases.

3.2.2 Stage 1

Firms decide their cost reductions. Since the coefficient of the R&D cost depends on the number of collaboration links the firm has, we calculate Nash equilibria in the complete, a star, an exclusive, and the empty network, respectively.

The complete network

In the complete network, all firms have two neighbors. So, their profit functions are the same as

$$\pi_i(q^*(x), x_i, g^N) = (p(Q) - (c - x_i))\, q_i^*(x) - \delta x_i^2/6.$$

In equilibrium, for all firms i, cost reductions are

$$x_i^*(g^N) = \frac{9d}{8\delta - 9}.$$

Output and profits are

$$q_i^*(g^N) = \frac{2\delta d}{8\delta - 9} \text{ and } \pi_i(g^N) = \frac{\delta(8\delta - 27)d^2}{2(8\delta - 9)^2}$$

A star network

Suppose that firm i collaborates with firms j and k, and firms j and k do not collaborate. Let $g^s = \{ij, ik\}$ be the network, which is a star. Firm i is the hub of the star network, and we denote firm i as firm H. Firms j and k are in periphery of the star, and we denote firms j and k as firm P. The profit functions of the hub firm H and periphery firms P are

$$\pi_H(q^*(x), x, g^s) = (p(Q) - (c - x_H))q_H^*(x) - \delta x_H^2/6 \text{ and}$$
$$\pi_P(q^*(x), x, g^s) = (p(Q) - (c - x_P))q_P^*(x) - \delta x_P^2/4.$$

In equilibrium, the cost reductions of the hub firm and periphery firms are

$$x_H^*(g^s) = \frac{9(\delta - 3)d}{8\delta^2 - 39\delta + 27} \text{ and } x_P^*(g^s) = \frac{3(2\delta - 9)d}{8\delta^2 - 39\delta + 27}.$$

The outputs of the hub firm and periphery firms are

$$q_H^*(g^s) = \frac{2\delta(\delta - 3)d}{8\delta^2 - 39\delta + 27} \text{ and } q_P^*(g^s) = \frac{\delta(2\delta - 9)d}{8\delta^2 - 39\delta + 27}.$$

Thus, the profits of the hub firm and periphery firms are

$$\pi_H^*(g^s) = \frac{\delta(8\delta - 27)(\delta - 3)^2 d^2}{2(8\delta^2 - 39\delta + 27)^2} \text{ and } \pi_P^*(g^s) = \frac{\delta(4\delta - 9)(2\delta - 9)^2 d^2}{4(8\delta^2 - 39\delta + 27)^2}.$$

An exclusive network

Suppose that firms i and j collaborate with each other, and firm k does not collaborate with any others. Let $g^e = \{ij\}$ be the network, which is an exclusive network. We denote the linked firms i and j as firm L, and the isolated firm k as firm I. The profit functions of linked firms L and the isolated firm I are

$$\pi_L^*(q^*(x), x, g^e) = (p(Q) - (c - x_L))\, q_L^*(x) - \delta x_L^2/4 \text{ and}$$
$$\pi_I^*(q^*(x), x, g^e) = (p(Q) - (c - x_I))\, q_I^*(x) - \gamma x_I^2/2.$$

In equilibrium, the cost reductions of the linked firms and the isolated firm are

$$x_L^*(g^e) = \frac{3(2\gamma - 3)d}{8\delta\gamma - 9\delta - 12\gamma + 9} \text{ and } x_I^*(g^e) = \frac{3(2\delta - 3)d}{8\delta\gamma - 9\delta - 12\gamma + 9}.$$

The outputs of the linked firms and the isolated firm are

$$q_L^*(g^e) = \frac{3(2\gamma - 3)d}{8\delta\gamma - 9\delta - 12\gamma + 9} \text{ and } q_I^*(g^e) = \frac{2\gamma(\delta - 3)d}{8\delta\gamma - 9\delta - 12\gamma + 9}.$$

The profits of the linked firms and the isolated firm are

$$\pi_L(g^e) = \frac{\delta(4\delta - 9)(2\gamma - 3)^2 d^2}{4(8\delta\gamma - 9\delta - 12\gamma + 9)^2} \text{ and } \pi_I(g^e) = \frac{\gamma(8\gamma - 9)(\delta - 3)^2 d^2}{2(8\delta\gamma - 9\delta - 12\gamma + 9)^2}.$$

The empty network

Suppose that no firm collaborates. Let $g^{\emptyset}$ denote the network, which is the empty network. For all firms i, the profit functions are

$$\pi_i(q^*(x), x, g^{\emptyset}) = (p(Q) - (c - x_i))\, q_i^*(x) - \gamma x_i^2/2.$$

In equilibrium, the cost reductions are $x_i^*(g^{\emptyset}) = 3d/(8\gamma - 3)$. Then outputs and profits are

$$q_i^*(g^{\emptyset}) = \frac{2\gamma d}{8\gamma - 3} \text{ and } \pi_i(g^{\emptyset}) = \frac{\gamma(8\gamma - 9)d^2}{2(8\gamma - 3)^2}.$$

4. Pairwise stability

This section studies stable networks. “Stable” means that no player has an incentive to alter the network by forming or severing his available links. So we can consider that a stable network emerges in the market. There are several definitions of network stability in the strategic network formation literature.[1)] Among them, pairwise stability, introduced by Jackson and Wolinsky (1996), is widely used. A network is pairwise stable if no player has an incentive to sever an existing link, and no pair of players has an incentive to form a new link between them.

For example, the complete network is pairwise stable if no firm gains by severing one link, which alters the complete network to a star network. If a firm in the complete network severs one link, then the position of the firm becomes a periphery in a star network. Hence, if the profit in the complete network, $\pi_i(g^N)$, is greater than the profit of a periphery in a star, $\pi_P(g^s)$, *i.e.*, $\pi_i(g^N) > \pi_P(g^s)$, then firm i does not sever a link, and we can consider the complete network to have emerged in this market.

Formally, a network g is *pairwise stable* if for all firms i and j, $i \neq j$,

(i) $\pi_i(g) \geq \pi_i(g - ij)$ if $ij \in g$, and
(ii) $\pi_i(g+ij) > \pi_i(g) \Rightarrow \pi_j(g+ij) < \pi_j(g)$ if $ij \notin g$,

1) Goyal (2007) introduces many definitions of network stability.

where $g - ij = g \setminus \{i, j\}$ and $g + ij = g \cup \{i, j\}$.

The profits in networks, which are shown in the above section, are summarized as follows.

Fact 1. The profits in the complete network, a star network, an exclusive network, and the empty network respectively are

$$\pi_i(g^N) = \frac{\delta(8\delta - 27)}{2(8\delta - 9)^2},$$

$$\pi_H(g^s) = \frac{\delta(8\delta - 27)(\delta - 3)^2}{2(8\delta^2 - 39\delta + 27)^2}, \pi_P(g^s) = \frac{\delta(4\delta - 9)(2\delta - 9)^2}{4(8\delta^2 - 39\delta + 27)^2}$$

$$\pi_L(g^e) = \frac{\delta(4\delta - 9)(2\gamma - 3)^2}{4(8\delta\gamma - 9\delta - 12\gamma + 9)^2}, \pi_I(g^e) = \frac{\gamma(8\gamma - 9)(\delta - 3)^2}{2(8\delta\gamma - 9\delta - 12\gamma + 9)^2}, \text{ and}$$

$$\pi_i(g^\emptyset) = \frac{\gamma(8\gamma - 9)}{2(8\gamma - 3)^2},$$

where we omit d^2, the coefficient of all profits.

Thus, the conditions for pairwise stability for all networks are as follows.

Fact 2. (i) The complete network is pairwise stable if $\pi_i(g^N) > \pi_P(g^s)$.

(ii) A star network is pairwise stable if $\pi_H(g^s) > \pi_L(g^e)$, $\pi_P(g^s) > \pi_I(g^e)$, and $\pi_P(g^e) > \pi_i(g^N)$.

(iii) An exclusive network is pairwise stable if $\pi_L(g^e) > \pi_i(g^\emptyset)$ and $[\pi_L(g^e) > \pi_H(g^s)$ or $\pi_I(g^e) > \pi_P(g^s)]$.

(iv) The empty network is pairwise stable if $\pi_i(g^\emptyset) > \pi_L(g^e)$.

By using facts 1 and 2, we can find the ranges of R&D efficiency and technological efficiency over which networks are pairwise stable.

Proposition 1. (i) The complete network is pairwise stable if R&D efficiency is low $(\frac{1}{\delta} < \frac{1}{4.30507})$.

(ii) A star network is pairwise stable if R&D efficiency is not so low $(\frac{1}{\delta} \in (\frac{1}{3.94868}, \frac{1}{3.53865}))$ and technological efficiency is in a interval that depends on R&D efficiency.

(iii) Exclusive networks and the empty network are pairwise stable for all R&D efficiencies. If technological efficiency is sufficiently high or low, then an exclusive network is pairwise stable, and if technological efficiency is in a middle range, then the empty network is pairwise stable.

All proofs are collected in the Appendix.

5. Optimality

This section shows the optimal network. We measure network optimality using social welfare. We denote the *social welfare* by $S(g)$, which is defined by

$$S(g) = \frac{Q(g)(a - p\,(Q(g))}{2} + \sum_{i=1}^{3} \pi_i\,(g).$$

The first term of the right hand side is consumer surplus, and the second term is the sum of all firms' profits.

The social welfares for equilibria in networks are summarized as follows.

Fact 3. The social welfares in the complete network, a star network, an exclusive network, and the empty network are respectively

$$S(g^N) = \frac{60\delta^2 - 81\delta}{2(8\delta - 9)^2},$$
$$S(g^s) = \frac{60\delta^4 - 543\delta^3 + 1458\delta^2 - 972\delta}{2(8\delta^2 - 39\delta + 27)^2},$$
$$S(g^e) = \frac{3((20\delta^2 - 52\delta + 36)\gamma^2 + (-43\delta^2 + 78\delta - 27)\gamma + 24\delta^2 - 27\delta)}{2(8\delta\gamma - 12\gamma - 9\delta + 9)^2},$$
$$S(g^{\emptyset}) = \frac{60\gamma^2 - 27\gamma}{2(8\gamma - 3)^2},$$

where we omit d^2, the coefficient of all social welfares.

If a network provides the highest social welfare, then we call the network is optimal. Formally, a network g is *optimal* if $S(g) > S(g')$ for all $g' \in G \setminus \{g\}$.[2)]

We can find the ranges of technical and R&D efficiencies in which a network is optimal.

Proposition 2. (i) The complete network is optimal if R&D efficiency is low ($\frac{1}{\delta} < \frac{1}{5.50005}$), and collaboration efficiency is high ($\frac{1}{\alpha} > \frac{1}{3}$)

(ii) A star network is optimal if R&D efficiency is high ($\frac{1}{\delta} \in \frac{1}{5.50005}, \frac{1}{3.375}$)) and the technological efficiency is in an interval that depends on R&D efficiency.

(iii) An exclusive network is optimal if technological efficiency is low for all R&D efficiencies.

(iv) The empty network is optimal if R&D efficiency is low ($\frac{1}{\delta} < \frac{1}{6.3}$) and technological efficiency is high.

6. Compatibility

This section investigates compatibility between stability and optimality. We can consider that the market is well-worked if a network is pairwise stable and optimal. From propositions

2) In the present paper, the term "efficient" appears frequently. Hence we avoid referring to the condition as *strong efficiency*, which is used by the network literature.

1 and 2, we obtain the following results. All of them say that, for each network, there is a range of parameters in which the network is pairwise stable and optimal.

Corollary 1. The complete network is pairwise stable and optimal if the R&D efficiency is low ($\frac{1}{\delta} < \frac{1}{5.50005}$) and collaboration efficiency is high ($\frac{1}{\alpha} > \frac{1}{3}$).

Corollary 1 says that no matter how costly the cost decreasing technology is (γ is sufficiently large), if the synergy effect of collaboration is high, then all firms collaborate with all other firms and social welfare is maximized. Furthermore, in this range of δ, social welfare is increasing in δ. Therefore, if collaboration efficiency rises and R&D efficiency remains in the range, the complete network remains pairwise stable and optimal, and social welfare increases.

Corollary 2. A star network is pairwise stable and optimal if R&D efficiency is not so low ($\frac{1}{\delta} \in \frac{1}{3.94868}, \frac{1}{3.53865})$) and technological efficiency is not so high or low, which depends on R&D efficiency.

Note that, in the above range of $1/\delta$, the social welfare of a star network is increasing in δ, and therefore, social welfare sharply decreases if R&D efficiency increases.

The next two corollaries say that if the cost reducing technology is not costly, then collaborations are not necessary.

Corollary 3. An exclusive network is pairwise stable and optimal if technological efficiency is sufficiently high for all R&D efficiencies.

Corollary 4. The empty network is pairwise stable and optimal if R&D efficiency is low ($\frac{1}{\delta} < \frac{1}{6.3}$) and technological efficiency is high depending on R&D efficiency.

7. Concluding Remarks

We have studied strategic network formation in a market where firms engage in Cournot competition with cost reductions when the number of firms is three. We have shown that for each network, there exist ranges of parameters for collaboration efficiency, technological efficiency, and R&D efficiency, over which the network is pairwise stable and optimal. According to the results, we discuss implications for government policy that promotes R&D.

The complete network is pairwise stable and optimal if R&D efficiency is low, *i.e.*, R&D is inefficient, but collaboration efficiency is high, *i.e.*, collaboration is efficient, as shown by corollary 1. Moreover, in the complete network, social welfare is monotonically increasing in R&D efficiencies. Therefore, if the government adopts a policy that promotes collaboration among firms, *e.g.*, a subsidy for collaboration, social welfare will increase. Therefore, when R&D efficiency is sufficiently low, by promoting collaboration among firms, which raises collaboration efficiency, the complete network remains stable and optimal, and social welfare increases.

A star network is pairwise stable and optimal if R&D efficiency is high and technological efficiency is not so high or low, *i.e.*, the cost reducing technology is neither so efficient nor so inefficient, as shown by corollary 2. In this range of R&D efficiency, if the government adopts an R&D promoting policy, it sharply decreases social welfare. If the policy applies to cost reduction, which increases technological efficiency, then the stable network may alter to an exclusive network. In many of such cases, no matter whether an exclusive network is also optimal, social welfare is low, compared with that of the star network formed before adopting the policy since social welfare in a star network that is pairwise stable is very high. That is, when a star network is formed, it is difficult to use the promotion of R&D to increase social welfare.

An exclusive network is pairwise stable and optimal if the technological efficiency is sufficiently high, *i.e.*, the cost reducing technology is efficient, as shown by corollary 3. Hence, when we are concerned with a highly technological good production, which may imply that the technological efficiency is sufficiently low, then there is a room for a policy that promotes R&D to improve social welfare since an exclusive network is not optimal. For example, when $\gamma = 4$, the exclusive network is never optimal. Furthermore, there also exist ranges of parameters such that the empty network is pairwise stable and optimal. However, if a government adopts an R&D promoting policy that sufficiently raises collaboration efficiency, then the complete or a star network is pairwise stable and optimal, and social welfare after adopting the policy is greater than before adopting the policy.

We have studied the case of three firms. The general n firms case remains as a matter to be studied further but how far this approach can be extended is unclear because of the great complexity of network formation.

Appendix A: Proof of Proposition 1

A.1 The case that the complete network is pairwise stable

We find the range of R&D efficiency, δ, in which the complete network is pairwise stable. In the complete network, since all firms have all of the possible links, firms cannot form a new link. Hence, the complete network is pairwise stable if no firm has an incentive to sever a link. If a firm severs a link, then the complete network becomes a star network, and the firm's position is a periphery in the star. Thus, if $\pi_i\,(g^N) > \pi_i\,(g^s)$ for all firms i, then g^N is pairwise stable. $\pi_i\,(g^N) > \pi_P\,(g^s)$ if $\delta \in (\delta_{12}, \cdot)$, $\delta_{12} \approx 4.30507$.

A.2 The case that a star network is pairwise stable

We find ranges of R&D efficiency, δ, and technological efficiency, γ, in which a star network is pairwise stable. In a star network, if firm i is the hub, then firm i can only sever one link, and then he becomes a linked firm in an exclusive network. If firm i is a periphery, then firm i can sever the link or form a link between another periphery firm. Thus, if $\pi_H\,(g^s) > \pi_L\,(g^e)$, $\pi_P\,(g^s) > \pi_I\,(g^e)$, and $\pi_P\,(g^s) > \pi_i\,(g^N)$, then a star network is pairwise stable.

A.2.1 Conditions

- $\pi_P\,(g^s) > \pi_i\,(g^N)$

First, we find the range in which periphery firms do not have an incentive to form a link between them. If periphery firms form a link between them, then the star becomes the complete network. Hence if $\pi_P(g^s) > \pi_i(g^N)$, then the firms do not form a link. By the proof of (i) in the above, $\pi_P(g^s) > \pi_i(g^N)$ if and only if $\delta \in (\delta_0, \delta_{12})$

• $\pi_H(g^s) > \pi_L(g^e)$

Next, we find the range in which the hub firm has no incentive to sever a link. If the hub firm severs one link, then the star becomes an isolated network, and the firm's profit becomes $\pi_L(g^I)$. Hence, if $\pi_H(g^s) > \pi_L(g^e)$, then the hub firm does not sever any link. $\pi_H(g^s) > \pi_L(g^e)$ if

$$\begin{cases} \delta \in (\delta_0, \delta_4) & \text{and } \gamma \in (A_1(\delta), A_2(\delta)), \\ \delta \in (\delta_4, \delta_6) & \text{and } \gamma \in (\gamma_0, A_1(\delta)), \\ \delta \in (\delta_4, \delta_9) & \text{and } \gamma \in (A_2(\delta), A_1(\delta)), \\ \delta \in (\delta_9, \delta_{12}) & \text{and } \gamma \in (A_1(\delta), A_2(\delta)), \end{cases}$$

where $\delta_4 = 3.75$, $\delta_6 \approx 3.82006$, $\delta_9 \approx 4.03950$,

$$A_1(\delta) = \frac{\begin{pmatrix} 64\delta^5 + \sqrt{64\delta^2 - 360\delta + 486}\,(27\delta^4 - 87\delta^3 + 333\delta^2 - 513\delta + 243) \\ -792\delta^4 + 3024\delta^3 - 3429\delta^2 - 1134\delta + 2187 \end{pmatrix}}{64\delta^4 - 1224\delta^3 + 6426\delta^2 - 12204\delta + 7290},$$

and

$$A_2(\delta) = \frac{\begin{pmatrix} 64\delta^5 - \sqrt{64\delta^2 - 360\delta + 486}\,(27\delta^4 - 87\delta^3 + 333\delta^2 - 513\delta + 243) \\ -792\delta^4 + 3024\delta^3 - 3429\delta^2 - 1134\delta + 2187 \end{pmatrix}}{64\delta^4 - 1224\delta^3 + 6426\delta^2 - 12204\delta + 7290},$$

• $\pi_P(g^s) > \pi_I(g^e)$

Third, we find the range in which a periphery firm has no incentive to sever the link. If a periphery firm severs the link, then the star becomes the exclusive network, and the firm's profit becomes $\pi_I(g^e)$. Hence, if $\pi_P(g^S) > \pi_I(g^e)$, then a periphery firm does not sever the link. $\pi_P(g^S) > \pi_I(g^e)$ if

$$\begin{cases} \delta \in (\delta_0, \delta_9) & \text{and } \gamma \in (\gamma_0, A_3(\delta)) \cup (A_4(\delta), \cdot), \\ \delta \in (\delta_9, \delta_{12}) & \text{and } \gamma \in (\gamma_0, A_4(\delta)) \cup (A_3(\delta), \cdot), \end{cases}$$

where

$$A_3(\delta) = \frac{\begin{pmatrix} (24\delta^3 - 189\delta^2 + 432\delta - 243)\sqrt{64\delta^6 - 880\delta^5 + 4705\delta^4 - 12816\delta^3 + 19926\delta^2 - 17496\delta + 6561} \\ +192\delta^6 - 2256\delta^5 + 8109\delta^4 - 4536\delta^3 + 25758\delta^2 + 43740\delta - 19683 \end{pmatrix}}{512\delta^5 - 7344\delta^4 + 38016\delta^3 - 87264\delta^2 + 89424\delta - 34992},$$

and

$$A_4(\delta) = \frac{\begin{pmatrix} -(24\delta^3-189\delta^2+432\,\delta-243)\sqrt{64\delta^6-880\delta^5+4705\delta^4-12816\delta^3+19926\delta^2-17496\delta+6561} \\ +192\delta^6-2256\delta^5+8109\delta^4-4536\delta^3+25758\delta^2+43740\delta-19683 \end{pmatrix}}{512\delta^5-7344\delta^4+38016\delta^3-87264\delta^2+89424\delta-34992}.$$

A.2.2 Ranges

We find the ranges of δ and γ such that $\pi_P(g^s) > \pi_i(g^N)$, $\pi_H(g^s) > \pi_L(g^e)$, and $\pi_P(g^s) > \pi_I(g^e)$.

• $\delta \in (\delta_0, \delta_4)$.

Then, γ needs $\gamma \in (A_1(\delta), A_2(\delta))$ and $\gamma \in (\cdot, A_3(\delta)) \cup (A_4(\delta),\cdot)$. In this range of δ, $A_3(\delta) < A_1(\delta) < A_4(\delta)$. If $\delta \in (\delta_0, \delta_1)$, where $\delta_1 \approx 3.53865$, then $A_2(\delta) < A_4(\delta)$. Hence, no γ satisfies the inequalities. Thus, if $\delta \in (\delta_1, \delta_4)$ and $\gamma \in (A_4(\delta), A_2(\delta))$, a star is pairwise stable.

• $\delta \in (\delta_4, \delta_6)$.

Then, γ needs $\gamma \in (\gamma_0, A_1(\delta))$ and $\gamma \in (\cdot, A_3(\delta)) \cup (A_4(\delta),\cdot)$. In this range of δ, $\gamma_0 < A_3(\delta) < A_1(\delta) < A_4(\delta)$. Thus, if $\delta \in (\delta_4, \delta_6)$ and $\gamma \in (\gamma_0, A_3(\delta))$, then a star network is pairwise stable.

• $\delta \in (\delta_6, \delta_0)$.

Then, γ needs $\gamma \in (A_2(\delta), A_1(\delta))$ and $\gamma \in (\cdot, A_3(\delta)) \cup (A_4(\delta),\cdot)$. In this range of δ, $A_3(\delta) < A_1(\delta) < A_4(\delta)$. Thus, we ignore the conditio $\gamma \in (A_4(\delta),\cdot)$. If $\delta \in (\delta_4, \delta_8)$, where $\delta_8 \approx 3.94868$, then $A_2(\delta) < A_3(\delta)$. If $\delta \in (\delta_8, \delta_9)$, then $A_3(\delta) < A_2(\delta)$. Hence, if $\delta \in (\delta_8, \delta_9)$, then no γ satisfies the conditions. Thus, if $\delta \in (\delta_6, \delta_8)$ and $\gamma \in (A_2(\delta), A_3(\delta))$, then a star is pairwise stable.

• $\delta \in (\delta_9, \delta_{12})$.

Then, γ needs $\gamma \in (A_1(\delta), A_2(\delta))$ and $\gamma \in (\cdot, A_4(\delta)) \cup (A_3(\delta),\cdot)$. In this range of δ, $A_4(\delta) < A_1(\delta) < A_2(\delta) < A_3(\delta)$. Thus, no γ satisfies the above conditions.

A.2.3 Summary for pairwise stability of star network

A star network is pairwise stable if

$$\begin{cases} \delta \in (\delta_1, \delta_4) \text{ and } \gamma \in (A_4(\delta), A_2(\delta)), \\ \delta \in (\delta_4, \delta_6) \text{ and } \gamma \in (\gamma_0, A_3(\delta)), \\ \delta \in (\delta_6, \delta_8) \text{ and } \gamma \in (A_2(\delta), A_3(\delta)). \end{cases}$$

A.3 The case that an exclusive network is pairwise stable

We find ranges of R&D efficiency, δ, and technological efficiency, γ, in which an exclusive network is pairwise stable. An exclusive network is pairwise stable if a linked firm has no incentive to sever the link, and a pair of linked and isolated firms has no incentive to form a link. Thus, if $\pi_L(g^e) > \pi_i(g^\varnothing)$ and $[\pi_L(g^e) > \pi_I(g^s)$ or $\pi_I(g^e) > \pi_P(g^s)]$, then an exclusive network is pairwise stable.

A.3.1 Conditions

• $\pi_L(g^e) > \pi_i(g^\varnothing)$.

We find the range of δ and γ in which a linked firm has no incentive to sever the link. If a linked firm severs the link, then the exclusive network becomes the empty network, and the profit of a linked firm $\pi_L(g^e)$ becomes $\pi_i(g^{\emptyset})$. $\pi_L(g^e) > \pi_i(g^{\emptyset})$ if $\gamma \in (\gamma_0, A_6(\delta),\cdot)$ for all $\delta > \delta_0$, where

$$A_5(\delta) = Z(x) - \frac{3x^2 - 9x - 9}{64(x-3)^2 Z(x)} + \frac{6x - 21}{8x - 24},$$

where

$$Z(x) = \left(\frac{3^{\frac{3}{2}}\sqrt{976x^4 - 5520x^3 + 7767x^2 - 648x}}{1024(x-3)^2} + \frac{162x^3 - 945x^2 + 1377x - 54}{1024(x-3)^3} \right)^{\frac{1}{3}}$$

and $A_6(\delta) = \delta/2$.

• $\pi_L(g^e) > \pi_H(g^s)$.

If a linked and the isolated firms form a link between them, then the exclusive network becomes a star network. Firms profits, $\pi_L(g^e)$ and $\pi_I(g^e)$, become $\pi_H(g^s)$ and $\pi_P(g^s)$, respectively. Thus, if $\pi_L(g^e) > \pi_H(g^s)$ or $\pi_I(g^e) > \pi_P(g^s)$, then they do not have an incentive to form a link.

$\pi_L(g^e) > \pi_H(g^s)$ if

$$\begin{cases} \delta \in (\delta_0, \delta_4) & \text{and } \gamma \in (\gamma_0, A_1(\delta)) \cup (A_2(\delta),\cdot) \\ \delta \in (\delta_4, \delta_6) & \text{and } \gamma \in (A_1(\delta),\cdot), \\ \delta \in (\delta_6, \delta_9) & \text{and } \gamma \in (\gamma_0, A_2(\delta)) \cup (A_1(\delta),\cdot), \\ \delta \in (\delta_9, \delta_{14}) & \text{and } \gamma \in (\gamma_0, A_1(\delta)) \cup (A_2(\delta),\cdot), \\ \delta \in (\delta_{14},\cdot) & \text{and } \gamma \in (A_2(\delta),\cdot), \end{cases}$$

where $\delta_{14} \approx 4.46394$.

• $\pi_I(g^e) > \pi_P(g^s)$

$\pi_I(g^e) > \pi_P(g^s)$ if

$$\begin{cases} \delta \in (\delta_0, \delta_9) & \text{and } \gamma \in (A_3(\delta), A_4(\delta)) \\ \delta \in (\delta_9, \delta_{12}) & \text{and } \gamma \in (A_4(\delta), A_3(\delta)), \\ \delta \in (\delta_{12}, \delta_{15}) & \text{and } \gamma \in (\gamma_0, A_4(\delta)), \\ \delta \in (\delta_{15}, \delta_{19}) & \text{and } \gamma \in (\gamma_0, A_4(\delta)), \\ \delta \in (\delta_{19},\cdot) & \text{and } \gamma \in (A_4(\delta), A_3(\delta)), \end{cases}$$

where $\delta_{15} = 4.5$, and $\delta_{19} \approx 5.53763$.

A.3.2 Ranges of $\pi_L(g^e) > \pi_i(g^{\emptyset})$ and $\pi_L(g^e) > \pi_H(g^s)$

First, we find the range of δ and γ such that $\pi_L(g^e) > \pi_i(g^{\emptyset})$ and $\pi_L(g^e) > \pi_H(g^s)$.

• $\delta \in (\delta_0, \delta_4)$

Then, γ needs $\gamma \in (\gamma_0, A_5(\delta)) \cup (A_6(\delta),\cdot)$ and $\gamma \in (\gamma_0, A_1(\delta)) \cup (A_2(\delta),\cdot)$. If $\delta \in (\delta_0, \delta_2)$, where $\delta_2 \approx 3.66375$, then $A_5(\delta) < A_1(\delta) < A_2(\delta) < A_6(\delta)$. Hence, if $\delta \in (\delta_0, \delta_2)$, then we ignore the condition $\gamma \in (\gamma_0, A_1(\delta)) \cup (A_2(\delta),\cdot)$. If $\delta \in (\delta_2, \delta_3)$, where $\delta_3 \approx 3.69905$, then $A_1(\delta) < A_5(\delta) < A_2(\delta) < A_6(\delta)$. If $\delta \in (\delta_3, \delta_4)$, then $A_1(\delta) < A_5(\delta) < A_6(\delta) < A_2(\delta)$.

Thus, if $\delta \in (\delta_0, \delta_2)$ and $\gamma \in (\gamma_0, A_5(\delta)) \cup (A_6(\delta),\cdot)$, or $\delta \in (\delta_2, \delta_3)$ and $\gamma \in (\gamma_0, A_1(\delta)) \cup (A_6(\delta),\cdot)$, or $\delta \in (\delta_3, \delta_4)$ and $\gamma \in (\gamma_0, A_1(\delta)) \cup (A_2(\delta),\cdot)$, then an exclusive network is pairwise stable.

• $\delta \in (\delta_4, \delta_6)$

Then, $\gamma \in (\gamma_0, A_5(\delta)) \cup (A_6(\delta),\cdot)$ and $\gamma \in (A_1(\delta),\cdot)$. In this range of δ, $\gamma_0 < A_1(\delta) < A_5(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_4, \delta_6)$ and $\gamma \in (A_1(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot)$, then an exclusive network is pairwise stable.

• $\delta \in (\delta_6, \delta_9)$

Then, $\gamma \in (\gamma_0, A_5(\delta)) \cup (A_6(\delta),\cdot)$ and $\gamma \in (\gamma_0, A_2(\delta)) \cup (A_1(\delta),\cdot)$. In this range of δ, $A_2(\delta) < A_1(\delta) < A_5(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_4, \delta_9)$ and $\gamma \in (\gamma_0, A_2(\delta)) \cup (A_6(\delta),\cdot)$, then an exclusive network is pairwise stable.

• $\delta \in (\delta_9, \delta_{14})$

Then, $\gamma \in (\gamma_0, A_5(\delta)) \cup (A_6(\delta),\cdot)$ and $\gamma \in (\gamma_0, A_1(\delta)) \cup (A_2(\delta),\cdot)$. In this range of δ, $\gamma_0 < A_1(\delta) < A_2(\delta) < A_5(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_9, \delta_{14})$ and $\gamma \in (A_2(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot)$, then an exclusive network is pairwise stable.

• $\delta \in (\delta_{14},\cdot)$

Then, $\gamma \in (\gamma_0, A_5(\delta)) \cup (A_6(\delta),\cdot)$ and $\gamma \in (A_2(\delta),\cdot)$. In this range of δ, $\gamma_0 < A_2(\delta) < A_5(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_{14},\cdot)$ and $\gamma \in (A_2(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot)$, then an exclusive network is pairwise stable.

A.3.3 Ranges of $\pi_L(g^e) > \pi_i(g^{\emptyset})$ and $\pi_I(g^e) > \pi_P(g^s)$

• $\delta \in (\delta_0, \delta_9)$

Then, $\gamma \in (\gamma_0, A_5(\delta)) \cup (A_6(\delta),\cdot)$ and $\gamma \in (A_3(\delta), A_4(\delta))$. If $\delta \in (\delta_0, \delta_7)$, where $\delta_7 \approx 3.85911$, then $A_3(\delta) < A_5(\delta) < A_4(\delta) < A_6(\delta)$. If $\delta \in (\delta_7, \delta_9)$, then $A_3(\delta) < A_4(\delta) < A_5(\delta) < A_6(\delta)$.

Thus, if $\delta \in (\delta_0, \delta_7)$ and $\gamma \in (A_3(\delta), A_5(\delta))$, or $\delta \in (\delta_7, \delta_9)$ and $\gamma \in (A_3(\delta), A_4(\delta))$, then an exclusive network is pairwise stable.

• $\delta \in (\delta_9, \delta_{12})$

Then, γ needs $\gamma \in (\cdot, A_5(\delta)) \cup (A_6(\delta),\cdot)$ and $\gamma \in (A_4(\delta), A_3(\delta))$. If $\delta \in (\delta_9, \delta_{10})$, where $\delta_{10} \approx 4.14460$, then $A_4(\delta) < A_3(\delta) < A_5(\delta) < A_6(\delta)$. If $\delta \in (\delta_{10}, \delta_{11})$, where $\delta_{11} \approx 4.27258$, then $A_4(\delta) < A_5(\delta) < A_3(\delta) < A_6(\delta)$. If $\delta \in (\delta_{11}, \delta_{12})$, then $A_4(\delta) < A_5(\delta) < A_6(\delta) < A_3(\delta)$.

Thus, if $\delta \in (\delta_9, \delta_{10})$ and $\gamma \in (A_4(\delta), A_3(\delta))$, or if $\delta \in (\delta_{10}, \delta_{11})$ and $\gamma \in (A_4(\delta), A_5(\delta))$, or if $\gamma \in (\delta_{11}, \delta_{12})$ and $\gamma \in (A_4(\delta), A_5(\delta)) \cup (A_6(\delta), A_3(\delta))$, then an exclusive network is pairwise stable.

• $\delta \in (\delta_9, \delta_{12})$

Then, $\gamma \in (\cdot, A_5(\delta)) \cup (A_6(\delta),\cdot)$ and $\gamma \in (\gamma_0, A_4(\delta))$. In this range of δ, $A_4(\delta) < A_5(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_{12}, \delta_{15})$ and $\gamma \in (\gamma_0, A_4(\delta))$, then an exclusive network is pairwise stable.

• $\delta \in (\delta_{15}, \delta_{19})$

Then, $\gamma \in (\cdot, A_5(\delta)) \cup (A_6(\delta),\cdot)$ and $\gamma \in (\gamma_0, A_4(\delta))$. In this range of δ, $A_4(\delta) < A_5(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_{15}, \delta_{19})$ and $\gamma \in (\gamma_0, A_4(\delta))$, then and exclusive network is pairwise stable.

• $\delta \in (\delta_{19,}\cdot)$

Then, $\gamma \in (\cdot, A_5(\delta)) \cup (A_6(\delta),\cdot)$ and $\gamma \in (A_4(\delta), A_3(\delta))$. In this case, $A_4(\delta) < A_5(\delta) < A_6(\delta) < A_3(\delta)$. Thus, if $\delta \in (\delta_{19},\cdot)$ and $\gamma \in (A_4(\delta), A_5(\delta)) \cup (A_6(\delta), A_3(\delta))$, then an exclusive network is pairwise stable.

A.3.4 Summary for pairwise stability of exclusive network

An exclusive network is pairwise stable if

$$\begin{cases} \delta \in (\delta_0, \delta_3) & \text{and} \quad \gamma \in (\gamma_0, A_5(\delta)) \cup (A_6(\delta),\cdot), \\ \delta \in (\delta_3, \delta_4) & \text{and} \quad \gamma \in (\gamma_0, A_5(\delta)) \cup (A_2(\delta),\cdot), \\ \delta \in (\delta_4, \delta_6) & \text{and} \quad \gamma \in (A_3(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot), \\ \delta \in (\delta_6, \delta_7) & \text{and} \quad \gamma \in (\gamma_0, A_2(\delta)) \cup (A_3(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot), \\ \delta \in (\delta_7, \delta_8) & \text{and} \quad \gamma \in (\gamma_0, A_2(\delta)) \cup (A_3(\delta), A_4(\delta)) \cup (A_6(\delta),\cdot), \\ \delta \in (\delta_8, \delta_9) & \text{and} \quad \gamma \in (\gamma_0, A_4(\delta)) \cup (A_6(\delta),\cdot) \\ \delta \in (\delta_9, \delta_{12}) & \text{and} \quad \gamma \in (A_4(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot) \\ \delta \in (\delta_{12}, \delta_{19}) & \text{and} \quad \gamma \in (\gamma_0, A_4(\delta)) \cup (A_2(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot) \\ \delta \in (\delta_{19},\cdot) & \text{and} \quad \gamma \in (A_4(\delta), A_5(\delta)) \cup (A_6(\delta), A_3(\delta)). \end{cases}$$

A.4 The case that the empty network is pairwise stable

We find ranges of R&D efficiency, δ, and technological efficiency, γ, in which the empty network is pairwise stable. In the empty network, if all pairs do not form a link, then the network is pairwise stable. If a pair forms a link, then the network becomes an exclusive network, and their profits become $\pi_L(g^e)$. Thus, if $\pi_i(g^{\emptyset}) > \pi_L(g^e)$, then all firms have no incentive to form a link. $\pi_i(g^{\emptyset}) > \pi_L(g^e)$ if $\gamma \in (A_5(\delta), A_6(\delta))$. Thus, if $\delta \in (\delta_0,\cdot)$ and $\gamma \in (A_5(\delta), A_6(\delta))$, then the empty network is pairwise stable. □

Appendix B: Proof of Proposition 2

B.1 Social Welfare

In this subsection, we calculate social welfares. The consumer surplus in network g is

$$\frac{Q(g)\,(a - p\,(Q(g))}{2} = \frac{Q(g)\,(a - a + (Q(g))}{2} = \frac{Q(g)^2}{2}$$

Hence, the social welfare in network g is

$$S(g) = \frac{Q(g)^2}{2} + \sum_{i=1}^{3} \pi_i(g).$$

B.1.1 The complete network

In the complete network, total production and the consumer surplus are respectively

$$Q(g^N) = \frac{6\delta d}{8\delta - 9} \text{ and } \frac{Q(g^N)^2}{2} = \frac{36\delta^2 d^2}{2(8\delta - 9)^2}.$$

The sum of firms profits is

$$3\pi_i(g^N) = \frac{3\delta(8\delta - 27)d^2}{2(8\delta - 9)^2}.$$

Thus, the social surplus in the complete network is

$$S(g^N) = \left(\frac{36\delta^2}{2(8\delta - 9)^2} + \frac{3\delta(8\delta - 27)}{2(8\delta - 9)^2}\right) d^2 = \frac{60\delta^2 - 81\delta}{2(8\delta - 9)^2}\, d^2.$$

B.1.2 A star network

In a star network, total production is

$$Q(g^S) = \left(\frac{2\delta(\delta - 3)}{8\delta^2 - 39\delta + 27} + 2\frac{\delta(2\delta - 9)}{8\delta^2 - 39\delta + 27}\right) d.$$
$$= \frac{6\delta(\delta - 4)d}{8\delta^2 - 39\delta + 27}$$

Hence, the consumer surplus is

$$\frac{Q(g^s)^2}{2} = \frac{36\delta^2(\delta - 4)^2 d^2}{2(8\delta^2 - 39\delta + 27)^2}.$$

The sum of firms' profits is

$$\pi_H(g^s) + 2\pi_P(g^s) = \left(\frac{(8\delta-27)(\delta-3)^2}{2(8\delta^2-39\delta+27)^2} + \frac{2\delta(4\delta-9)(2\delta-9)^2}{4(8\delta^2-39\delta+27)^2}\right)d^2$$
$$= \frac{24\delta^4-255\delta^3+882\delta^2-972\delta}{2(8\delta^2-39\delta+27)^2}d^2$$

Thus, the social surplus is

$$S(g^s) = \left(\frac{36\delta^2(\delta-4)^2}{2(8\delta^2-39\delta+27)^2} + \frac{24\delta^4-255\delta^3+882\delta^2-972\delta}{2(8\delta^2-39\delta+27)^2}\right)d^2$$
$$= \frac{60\delta^4-543\delta^3+1458\delta^2-972\delta}{2(8\delta^2-39\delta+27)^2}d^2.$$

B.1.3 An exclusive network

In an exclusive network, total production is

$$Q(g^e) = \left(\frac{2\delta(2\gamma-3)}{8\delta\gamma-9\delta-12\gamma+9} + \frac{2\gamma(\delta-3)}{8\delta\gamma-9\delta-12\gamma+9}\right)d$$
$$= \frac{6(\delta\gamma-\delta-\gamma)d}{8\delta\gamma-9\delta-12\gamma+9}$$

Hence, the consumer surplus is

$$\frac{Q(g^e)^2}{2} = \frac{36(\delta\gamma-\delta-\gamma)^2d^2}{2(8\delta\gamma-9\delta-12\gamma+9)^2}.$$

The sum of firms' profits is

$$2\pi_L(g^e) + \pi_I(g^e)$$
$$= \left(\frac{2\delta(4\delta-9)(2\gamma-3)^2}{4(8\delta\gamma-9\delta-12\gamma+9)^2} + \frac{\gamma(8\gamma-9)(\delta-3)^2}{2(8\delta\gamma-9\delta-12\gamma+9)^2}\right)d^2$$
$$= \frac{(24\delta^2-84\delta+72)\gamma^2-(57\delta^2-162\delta+81)\gamma+36\delta^2-81\delta}{2(8\delta\gamma-9\delta-12\gamma+9)^2}d^2$$

Thus, the social surplus is

$$S(g^e) = \left(\frac{36(\delta\gamma-\delta-\gamma)^2}{2(8\delta\gamma-9\delta-12\gamma+9)^2}\right.$$
$$\left.+\frac{24(\delta^2-84\delta+72)\gamma^2-(57\delta^2-162\delta+81)+36\delta^2-81\delta}{2(8\delta\gamma-9\delta-12\gamma+9)^2}\right)d^2$$

$$= \frac{3((20\delta^2-52\delta+36)\gamma^2+(-43\delta^2+78\delta-27)\gamma+24\delta^2-27\delta)}{2(8\delta\gamma-9\delta-12\gamma+9)^2}\ d^2$$

B.1.4 The empty network

In the empty network, total production and the consumer surplus respectively are

$$Q(g^{\emptyset}) = \frac{6\gamma d}{8\gamma-3} \text{ and } \frac{Q(g^{\emptyset})^2}{2} = \frac{18\gamma^2 d^2}{(8\gamma^2-3)^2}$$

The sum of firms' profits is

$$3\pi_i(g^{\emptyset}) = \frac{3\gamma(8\gamma-9)d^2}{2(8\gamma-3)^2}.$$

Thus, the social surplus is

$$S(g^{\emptyset}) = \left(\frac{18\gamma^2}{(8\gamma-3)^2} + \frac{3\gamma(8\gamma-9)}{2(8\gamma-3)^2}\right) d^2 = \frac{60\gamma^2-27\gamma}{2(8\gamma-3)^2}\ d^2.$$

B.2 The case that the complete network is optimal

We find ranges of δ and γ such that the complete network is optimal.

$S(g^N) > S(g^s)$ for all $\delta > \delta_{o1}$, where $\delta_{18} \approx 5.50005$.

$S(g^N) > S(g^e)$ if $\gamma > B_1(\delta)$ for all $\delta > \delta_0$, where

$$B_1(\delta) = \frac{(8\delta^2-33\delta+27)\sqrt{256\delta^4-864\delta^3+4617\delta^2-10206\delta+6561} + 128\delta^4+96\delta^3-2403\delta^2+4374\delta-2187}{1280\delta^3-6696\delta^2+11016\delta-5832}.$$

$S(g^N) > S(g^{\emptyset})$ if $\gamma > B_2(\delta)$ for all $\delta > \delta_0$, where $B_2(\delta) = \delta/3$.

Furthermore, if $\delta < \delta_{18}$, then $B_2(\delta) < B_1(\delta)$ and if $\delta > \delta_{18}$, then $B_1(\delta) < B_2(\delta)$.

Thus, if $\delta \in (\delta_{18}, \cdot)$ and $\gamma > B_2(\delta)$, then the complete network is optimal. Furthermore, since $\gamma > B_2(\delta) = \delta/3 = \alpha\gamma/3$, we have $\alpha < 3$. Thus, if $\delta > \delta_{18}$ and $\alpha < 3$, then the complete network is optimal.

B.3 The case that a star network is optimal

We find ranges of δ and γ such that a star network is optimal.

$S(g^s) > S(g^N)$ for all $\delta \in (\delta_0, \delta_{18})$.

$S(g^s) > S(g^s)$ if

$$\begin{cases} \delta \in (\delta_0, \delta_9) & \text{and} \quad \gamma \in (\gamma_0, B_2(\delta)) \cup (B_3(\delta), \cdot), \\ \delta \in (\delta_9, \cdot) & \text{and} \quad \gamma \in (B_2(\delta), \cdot), \end{cases}$$

where

$$B_3(\delta) = \frac{84\delta^4 - 945\delta^3 + 3753\delta^2 - 5103\delta + 2187}{128\delta^4 - 1308\delta^3 + 4752\delta^2 - 6480\delta + 2916}.$$

$S(g^s) > S(g^{\emptyset})$ if

$$\begin{cases} \delta \in (\delta_0, \delta_{16}) & \text{and} \quad \gamma \in (\gamma_0, \cdot), \\ \delta \in (\delta_{16}, \cdot) & \text{and} \quad \gamma \in (B_4(\delta), \cdot), \end{cases}$$

where $\delta_{16} \approx 4.75920$, and

$$B_4(\delta) = \frac{\left(\begin{array}{r} (24\delta^2 - 117\delta + 81)\sqrt{256\delta^4 - 2720\delta^3 + 9801\delta^2 - 13770\delta + 6561} \\ +384\delta^4 - 3072\delta^3 + 5751\delta^2 + 3402\delta - 6561 \end{array} \right)}{1792\delta^3 - 15912\delta^2 + 42768\delta - 29160}.$$

Furthermore, if $\delta \in (\delta_{16}, \delta_{18})$, then $B_4(\delta) < B_2(\delta)$.

Thus, if

$$\begin{cases} \delta \in (\delta_0, \delta_9) & \text{and} \quad \gamma \in (\gamma_0, B_2(\delta)) \cup (B_3(\delta), \cdot), \\ \delta \in (\delta_9, \delta_{18}) & \text{and} \quad \gamma \in (B_2(\delta), \cdot), \end{cases}$$

then a star network is optimal.

B.4 The case that an exclusive network is optimal

We find ranges of δ and γ such that an exclusive network is optimal.

$S(g^e) > S(g^N)$ if $\gamma \in (\gamma_0, B_1(\delta))$ for all $\delta > \delta_0$.

$S(g^e) > S(g^s)$ if

$$\begin{cases} \gamma \in (B_2(\delta), B_3(\delta)) & \text{if } \delta \in (\delta_0, \delta_9), \\ \gamma \in (B_3(\delta), B_2(\delta)) & \text{if } \delta \in (\delta_9, \delta_{13}), \\ \gamma \in (\gamma_0, B_2(\delta)) & \text{if } \delta \in (\delta_{13}, \cdot), \end{cases}$$

where $\delta_{13} \approx 4.42847$.

$S(g^e) > S(g^{\emptyset})$ if $\gamma \in (\gamma_0, \gamma_1^o) \cup (B_5(\delta), \cdot)$, where $\gamma_1^o = 1.62$ and $B_5(\delta) = \delta/2$.[3)]

We ignore the condition of $B_5(\delta)$ since $B_5(\delta) > B_1(\delta)$ for all $\delta > \delta_0$. Furthermore, if $\delta \in (\delta_0, \delta_9)$, then $B_2(\delta) < B_3(\delta) < B_1(\delta) < \gamma_1^o$, and if $\delta \in (\delta_9, \delta_{13})$, then $B_3(\delta) < B_2(\delta) < \min\{\gamma_1^o, B_1(\delta)\}$, and if $\delta \in (\delta_{13}, \delta_{17})$, where $\delta_{17} = 4.86$, then $\gamma_1^o < B_2(\delta) < \gamma_1^o < B_1(\delta)$, and if $\delta \in (\delta_{17}, \cdot)$, then γ_0

3) We cannot solve analytically the case of $S(g^e) > S(g^{\emptyset})$. Hence, the value, 1.62, is a sufficiently small condition for $S(g^e) > S(g^{\emptyset})$.

$< \gamma_1^o < B_2(\delta) < B_1(\delta)$.

Thus, if

$$\begin{cases} \delta \in (\delta_0, \delta_9) & \text{and} \quad \gamma \in (B_2(\delta), B_3(\delta)), \\ \delta \in (\delta_9, \delta_{13}) & \text{and} \quad \gamma \in (B_3(\delta), B_2(\delta)), \\ \delta \in (\delta_{13}, \delta_{17}) & \text{and} \quad \gamma \in (\gamma_0, B_2(\delta)), \\ \delta \in (\delta_{17}, \cdot) & \text{and} \quad \gamma \in (\gamma_0, \gamma_1^o), \end{cases}$$

then an exclusive network is optimal.

B.5 The case that the empty network is optimal

We find ranges of δ and γ such that the empty network is optimal.

$S(g^{\emptyset}) > S(g^N)$ if $\gamma \in (\gamma_0, B_2(\delta))$ for all $\delta > \delta_0$. $S(g^{\emptyset}) > S(g^s)$ if $\delta > \delta_{16}$ and $\gamma \in (\gamma_0, B_4(\delta))$. $S(g^{\emptyset}) > S(g^e)$ if $\gamma > \gamma_2^o$, where $\gamma_2^o = 2.1$.[4)]

Thus, if $\delta \in (\delta_{20}, \cdot)$, and $\gamma \in (\gamma_2^o, B_2(\delta))$, where $\delta_{20} = 6.3$, then the empty network is optimal.

□

Appendix C: Proofs of Corollaries

C.1 Proof of Corollary 1

By proposition 1, the complete network is pairwise stable if $\delta > \delta_{12}$. By proposition 2, the complete network is optimal if $\delta > \delta_{18}$ and $\gamma > B_2(\delta)$. Since $\delta_{18} > \delta_{12}$, we have $\delta > \delta_{18}$.

Thus, $\delta > \delta_{18}$ and $\alpha < 3$, then the complete network is pairwise stable and optimal. □

C.2 Proof of Corollary 2

By proposition 1, a star network is pairwise stable if

$$\begin{cases} \delta \in (\delta_1, \delta_4) & \text{and } \gamma \in (A_4(\delta), A_2(\delta)), \\ \delta \in (\delta_4, \delta_6) & \text{and } \gamma \in (\gamma_0, A_3(\delta)), \\ \delta \in (\delta_6, \delta_8) & \text{and } \gamma \in (A_2(\delta), A_3(\delta)). \end{cases}$$

By proposition 2, a star network is optimal if

$$\begin{cases} \delta \in (\delta_0, \delta_9) & \text{and } \gamma \in (\gamma_0, B_2(\delta)) \cup (B_3(\delta), \cdot), \text{ and} \\ \delta \in (\delta_9, \cdot) & \text{and } \gamma \in (B_2(\delta), \cdot). \end{cases}$$

Since $\delta_8 < \delta_9$, we ignore the condition, $\delta \in (\delta_9, \cdot)$, and $\gamma \in (B_2(\delta), \cdot)$. If $\delta \in (\delta_1, \delta_4)$, then $\gamma_0 < B_2(\delta) < B_3(\delta) < A_4(\delta) < A_2(\delta)$, if $\delta \in (\delta_4, \delta_6)$, then $B_2(\delta) < B_3(\delta) < A_3(\delta)$, and if $\delta \in (\delta_6, \delta_8)$, then $A_2(\delta) < B_2(\delta) < A_3(\delta) < B_3(\delta)$.

Thus, if

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4) We cannot solve analytically the case of $S(g^{\emptyset}) > S(g^e)$. Hence, the value, 2.1, is a sufficiently large value that guarantees $S(g^{\emptyset}) > S(g^e)$.

$$\begin{cases} \delta \in (\delta_1, \delta_4) & \text{and } \gamma \in (A_4(\delta), A_2(\delta)), \\ \delta \in (\delta_4, \delta_6) & \text{and } \gamma \in (\gamma_0, B_2(\delta)) \cup (B_3(\delta), A_3(\delta)), \\ \delta \in (\delta_6, \delta_8) & \text{and } \gamma \in (A_2(\delta), B_2(\delta)), \end{cases}$$

then a star network is pairwise stable and optimal. □

C.3 Proof of Corollary 3

We find the ranges of δ and γ such that an exclusive network is pairwise stable and optimal.

An exclusive network is pairwise stable if

$$\begin{cases} \delta \in (\delta_0, \delta_3) & \text{and } \gamma \in (\gamma_0, A_5(\delta)) \cup (A_6(\delta), \cdot), \\ \delta \in (\delta_3, \delta_4) & \text{and } \gamma \in (\gamma_0, A_5(\delta)) \cup (A_2(\delta), \cdot), \\ \delta \in (\delta_4, \delta_6) & \text{and } \gamma \in (A_3(\delta), A_5(\delta)) \cup (A_6(\delta), \cdot), \\ \delta \in (\delta_6, \delta_7) & \text{and } \gamma \in (\gamma_0, A_2(\delta)) \cup (A_3(\delta), A_5(\delta)) \cup (A_6(\delta), \cdot), \\ \delta \in (\delta_7, \delta_8) & \text{and } \gamma \in (\gamma_0, A_2(\delta)) \cup (A_3(\delta), A_4(\delta)) \cup (A_6(\delta), \cdot), \\ \delta \in (\delta_8, \delta_9) & \text{and } \gamma \in (\gamma_0, A_4(\delta)) \cup (A_6(\delta), \cdot), \\ \delta \in (\delta_9, \delta_{12}) & \text{and } \gamma \in (A_4(\delta), A_5(\delta)) \cup (A_6(\delta), \cdot), \\ \delta \in (\delta_{12}, \delta_{19}) & \text{and } \gamma \in (\gamma_0, A_4(\delta)) \cup (A_2(\delta), A_5(\delta)) \cup (A_6(\delta), \cdot), \\ \delta \in (\delta_{19}, \cdot) & \text{and } \gamma \in (A_4(\delta), A_5(\delta)) \cup (A_6(\delta), A_3(\delta)). \end{cases}$$

An exclusive network is optimal if

$$\begin{cases} \delta \in (\delta_0, \delta_9) & \text{and } \gamma \in (B_2(\delta), B_3(\delta)), \\ \delta \in (\delta_9, \delta_{13}) & \text{and } \gamma \in (B_3(\delta), B_2(\delta)), \\ \delta \in (\delta_{13}, \delta_{17}) & \text{and } \gamma \in (\gamma_0, B_2(\delta)), \\ \delta \in (\delta_{17}, \cdot) & \text{and } \gamma \in (\gamma_0, \gamma_1^o). \end{cases}$$

• $\delta \in (\delta_0, \delta_3)$.

Then, $\gamma \in (\gamma_0, A_5(\delta)) \cup (A_6(\delta)\cdot)$, and $\gamma \in (B_2(\delta), B_3(\delta))$. In this range of δ, $B_2(\delta) < A_5(\delta) < B_3(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_0, \delta_3)$ and $\gamma \in (B_2(\delta), A_5(\delta))$, then an exclusive network is pairwise stable.

• $\delta \in (\delta_3, \delta_4)$.

Then, $\gamma \in (\gamma_0, A_5(\delta)) \cup (A_2(\delta), \cdot)$, and $\gamma \in (B_2(\delta), B_3(\delta))$. In this range of δ, $B_2(\delta) < B_3(\delta) < A_5(\delta) < A_2(\delta)$. Thus, if $\delta \in (\delta_3, \delta_4)$ and $\gamma \in (B_2(\delta), B_3(\delta))$, then an exclusive network is pairwise stable.

• $\delta \in (\delta_4, \delta_6)$.

Then, $\gamma \in (A_3(\delta), A_5(\delta)) \cup (A_6(\delta), \cdot)$, and $\gamma \in (B_2(\delta), B_3(\delta))$. If $\delta \in (\delta_4, \delta_5)$, where $\delta_5 \approx 3.79620$, then $B_2(\delta) < A_3(\delta) < A_5(\delta) < B_3(\delta) < A_6(\delta)$, and if $\delta \in (\delta_5, \delta_6)$, then $B_2(\delta) < A_3(\delta) < B_3(\delta) < A_5(\delta) < A_6(\delta)$.

Thus, if $\delta \in (\delta_4, \delta_5)$ and $\gamma \in (A_3(\delta), A_5(\delta))$, and if $\delta \in (\delta_5, \delta_6)$, and $\gamma \in (A_3(\delta), B_3(\delta))$, then an exclusive network is pairwise stable and optimal.

• $\delta \in (\delta_4, \delta_6)$.

Then, $\gamma \in (\gamma_0, A_2(\delta)) \cup (A_3(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot)$, and $\gamma \in (B_2(\delta), B_3(\delta))$. In this range of δ, $A_2(\delta) < B_2(\delta) < A_3(\delta) < B_3(\delta) < A_5(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_6, \delta_7)$ and $\gamma \in (A_3(\delta), B_3(\delta))$, then an exclusive network is pairwise stable and optimal.

• $\delta \in (\delta_7, \delta_8)$.

Then, $\gamma \in (\gamma_0, A_2(\delta)) \cup (A_3(\delta), A_4(\delta)) \cup (A_6(\delta),\cdot)$, and $\gamma \in (B_2(\delta), B_3(\delta))$. In this range of δ, $A_2(\delta) < B_2(\delta) < A_3(\delta) < B_3(\delta) < A_4(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_7, \delta_8)$ and $\gamma \in (A_3(\delta), B_3(\delta))$, then and exclusive network is pairwise stable and optimal.

• $\delta \in (\delta_8, \delta_9)$.

Then, $\gamma \in (\gamma_0, A_4(\delta)) \cup (A_6(\delta),\cdot)$, and $\gamma \in (B_2(\delta), B_3(\delta))$. In this range of δ, $B_2(\delta) < B_3(\delta) < A_4(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_8, \delta_9)$ and $\gamma \in (B_2(\delta), B_3(\delta))$, then an exclusive network is pairwise stable and optimal.

• $\delta \in (\delta_9, \delta_{12})$.

Then, $\gamma \in (A_4(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot)$, and $\gamma \in (B_3(\delta), B_2(\delta))$. In this range of δ, $A_4(\delta) < B_3(\delta) < B_2(\delta) < A_5(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_9, \delta_{12})$ and $\gamma \in (B_3(\delta), B_2(\delta))$, then an exclusive network is pairwise stable and optimal.

• $\delta \in (\delta_{12}, \delta_{13})$.

Then, $\gamma \in (\gamma_0, A_4(\delta)) \cup (A_2(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot)$, and $\gamma \in (B_3(\delta), B_2(\delta))$. In this range of δ, $A_4(\delta) < B_3(\delta) < A_2(\delta) < A_5(\delta) < B_2(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_{12}, \delta_{13})$, and $\gamma \in (A_2(\delta), A_5(\delta))$, then an exclusive network is pairwise stable and optimal.

• $\delta \in (\delta_{13}, \delta_{17})$.

Then, $\gamma \in (\gamma_0, A_4(\delta)) \cup (A_2(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot)$, and $\gamma \in (\gamma_0, B_2(\delta))$. In this range of δ, $A_4(\delta) < A_2(\delta) < A_5(\delta) < B_2(\delta) < A_6(\delta)$. Thus, if $\delta \in (\delta_{13}, \delta_{17})$ and $\gamma \in (A_2(\delta), A_5(\delta))$, then an exclusive network is pairwise stable and optimal.

• $\delta \in (\delta_{17}, \delta_{19})$.

Then, $\gamma \in (\gamma_0, A_4(\delta)) \cup (A_2(\delta), A_5(\delta)) \cup (A_6(\delta),\cdot)$, and $\gamma \in (\gamma_0, \gamma_1^o)$. In this range of δ, $A_4(\delta) < A_2(\delta) < A_5(\delta) < \gamma_1^o < A_6(\delta)$. Thus, if $\delta \in (\delta_{13}, \delta_{17})$ and $\gamma \in (\gamma_0, A_4(\delta)) \cup (A_2(\delta), A_5(\delta))$, then an exclusive network is pairwise stable and optimal.

• $\delta \in (\delta_{19},\cdot)$.

Then, $\gamma \in (A_4(\delta), A_5(\delta)) \cup (A_6(\delta), A_3(\delta))$, and $\gamma \in (\gamma_0, \gamma_1^o)$. In this range of δ, $A_4(\delta) < A_5(\delta) < \gamma_1^o < A_6(\delta) < A_3(\delta)$. Thus, if $\delta \in (\delta_{19},\cdot)$ and $\gamma \in (A_6(\delta), A_3(\delta))$, then an exclusive network is pairwise stable.

In summary, an exclusive network is pairwise stable if

$$\begin{cases} \delta \in (\delta_0, \delta_3) & \text{and } \gamma \in (B_2(\delta), A_5(\delta)), \\ \delta \in (\delta_3, \delta_4) & \text{and } \gamma \in (B_2(\delta), B_3(\delta)), \\ \delta \in (\delta_4, \delta_5) & \text{and } \gamma \in (A_3(\delta), A_5(\delta)), \\ \delta \in (\delta_5, \delta_8) & \text{and } \gamma \in (A_3(\delta), B_3(\delta)), \\ \delta \in (\delta_8, \delta_9) & \text{and } \gamma \in (B_2(\delta), B_3(\delta)), \\ \delta \in (\delta_9, \delta_{12}) & \text{and } \gamma \in (B_3(\delta), B_2(\delta)), \\ \delta \in (\delta_{17}, \delta_{19}) & \text{and } \gamma \in (\gamma_0, A_4(\delta)) \cup (A_2(\delta), A_5(\delta)), \\ \delta \in (\delta_{19}, \cdot) & \text{and } \gamma \in (A_6(\delta), A_3(\delta)). \end{cases}$$

□

C.4 Proof of corollary 4

The empty network is pairwise stable if $\delta \in (\delta_0, \cdot)$, and $\gamma \in (A_6(\delta), A_6(\delta))$. The empty network is optimal if $\delta \in (\delta_{20}, \cdot)$, and $\gamma \in (\gamma_2^o, B_2(\delta))$. If $\delta \in (\delta_{20}, \cdot)$, then $A_5(\delta) < \gamma_2^o < B_2(\delta) < A_6(\delta)$. Thus, the empty network is pairwise stable and optimal if $\delta \in (\delta_{20}, \cdot)$ and $\gamma \in (\gamma_2^o, B_2(\delta))$. □

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