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Patterns of Technology, Industry Concentration, and Productivity Growth Without Scale Effects

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Abstract

This paper investigates the relationship between geographic patterns of industrial activity and endogenous growth in a two region model of trade that exhibits no scale effect. The in-house process innovation of manufacturing firms drives productivity growth and is closely associated with firm-level scales of production and relative levels of accessible technical knowledge. Focusing on long-run industry shares and a cross-region productivity gap, we find that dispersed equilibria with positive industry shares for both regions always produce higher growth rates than core-periphery equilibria with all industry locating in one region. Moreover, the highest growth rate arises in a symmetric steady state that features no productivity gap and equal shares of industry leading to the conclusion that the geographic concentration of industry has a negative impact on overall growth. Convergence towards a dispersed equilibrium, however, is contingent on the levels of inter-regional transport costs and knowledge dispersion. Finally, we explore the implications of greater economic integration arising from reduced transport costs and greater knowledge dispersion for patterns of industry and productivity, and for regional welfare levels within a dispersed equilibrium.

Key Words: Industry Concentration, Industry Share, Productivity Gap, Productivity Growth, Scale Effect
JEL Classifications: F43; O30; O40; R12

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1 Introduction

Even the most casual observer of economic geography will recognize that the distribution of industrial activity is uneven at local, regional, and international levels. A distinctly more subtle issue, however, relates to unravelling the relationship between these patterns of industrial concentration and economic growth. Although a broadly historical perspective generally leads to the prediction that a higher concentration of industry supports a greater rate of economic growth (Baldwin et al. 2001), the results obtained by recent empirical studies are mixed. For example, Braunerhjelm and Borgman (2004) report a positive relationship between industry concentration and labor productivity growth in Sweden, while Brülhart and Sbergami (2009) investigate cross-country data that suggests the relationship between industry concentration and GDP growth depends on a country's level of economic development. Bosker (2007), on the other hand, finds that on average European regions with a denser spread of employment tend to experience slower rates of growth in GDP. In addition, Gardiner et al. (2010) report a negative relationship between a number of measures of industry concentration and GDP growth for several levels of agglomeration using European regional data. These mixed results are difficult to interpret as the existing theoretical models of the "new economic geography" literature predict a positive relationship between agglomeration and growth (Baldwin and Martin, 2004).

In this paper, we introduce a novel theoretical approach that supports a negative relationship between the geographic concentration of industry and aggregate economic growth. Building upon the in-house process innovation literature established by Smulders and van de Klundert (1995) and Peretto (1996), we develop a two region model of trade in which the distribution of manufacturing activity and productivity growth are determined endogenously by the innovation activities of monopolistically competitive manufacturing firms, and then explore the mechanisms through which growth effects the geography of industry and in turn the implications of this geography for growth.

Our model produces two types of long-run equilibrium: a core-periphery equilibrium in which all industry, and hence all productivity growth, occurs in one region, and a dispersed equilibrium in which industrial activity and productivity growth is spread across regions. Investigating the dynamics around a dispersed equilibrium, we find that the long-run pattern of industry is determined by the balance between a destabilizing *competition effect* and stabilizing *productivity effect*, the relative strengths of which depend on the level of transport costs and the degree of knowledge dispersion between regions. In particular, convergence in industry shares, the level of productivity, and the rate of productivity growth to a dispersed equilibrium is contingent on the levels of transport costs and knowledge dispersion between regions. Ascertaining which pattern of industrial activity the economy converges to in the long run is important as an equal dispersion of industry across regions is found to yield the highest rate of productivity growth but the lowest level of product variety. As such our framework lends support to empirical studies such as Bosker (2007) and Gardiner et al. (2010) that find a negative correspondence between the density of economic activity and the rate of economic growth.

Our paper is closely related with studies in the "new economic geography" literature that emphasize key elements of the variety-expansion model of innovation-based endogenous growth (Grossman and Helpman, 1991). These studies tend to find that agglomeration economies and growth are reinforcing processes, that is, a higher concentration of industry tends to promote economic growth (Baldwin and Martin, 2004). A key feature of the models adopted in this literature, however, is a scale effect in which the rate of growth is positively correlated with the labor endowment of the economy. Indeed, the scale effect appears to play a central role in the relationship between industry concentration and the endogenous pace of growth. The existence of the scale effect, however, has generally been rejected by empirical studies (Lainez and Peretto, 2006) indicating the need for a reassessment of this relationship in a framework that corrects for the scale effect. To this end, Minniti and Parello (2011) introduce population growth and decreasing returns in research and development (R&D) following Jones (1995) and investigate the relationship between trade integration and scale-invariant economic growth.[1] Interestingly, under this modification of the variety-expansion model, long-run growth is proportionate to population growth and determined independently of the level of industry concentration. In contrast, the in-house process innovation framework that we adopt in this paper shifts the focus from R&D activity at the aggregate level towards innovation at the level of individual product lines thereby removing the effects of population size on firm-level incentives for R&D and allowing for the endogenous determination of both the spatial distribution of industry and the long-run rate of productivity growth without the bias generated by scale effects.

A convenient feature of our framework is that the long-run equilibrium can be characterized completely in terms of the shares of manufacturing activity locating in each region and a productivity differential between regions. Examining stable dispersed equilibria, we find that the larger region always has a greater share of industry and a higher relative productivity, and thus that the model is consistent with the well established empirical result that firms are more productive in regions with a greater density of economic activity (Melo et al. 2009). We also investigate the impacts of greater economic integration between regions resulting from lower transport costs and greater knowledge dissemination. First, a decrease in the level of inter-regional transport costs increases both the share of industry and the relative productivity of the larger, more advanced region indicating that it attains a higher rate of productivity growth over the short-run transition. The increased concentration of industry in the larger region has a negative impact, however, on the overall long-run rate of productiv-

[1] See Futagami and Iwaisako (2007) for more detail on variety expansion models of scale-invariant endogenous growth. In addition, Segerstrom (1998) and Li (2003) develop rising product quality models of scale-invariant endogenous growth.

ity growth. On the other hand, an increase in the degree of inter-regional knowledge dissemination raises both the share of industry and the relative productivity of the smaller region providing credence to agglomeration theories based on localized knowledge spillovers. Although the relative pace of productivity growth rises for the less advanced region in the short run, the impact on overall long-run productivity growth is ambiguous owing to the fact that greater knowledge dissemination simultaneously raises the productivity of labor in innovation and lowers firm-level employment in innovation as an increase in the total number of incumbent firms crowds out R&D investment.

The remainder of this paper proceeds as follows. In Section 2 we present our basic model of endogenous productivity growth. Section 3 then examines the characteristics of core-periphery equilibria with the manufacturing industry concentrated in one region. The features of dispersed equilibria are then investigated in Section 4. We investigate the effects of several key variables on patterns of industry trade and location, and the rate of productivity growth in Section 5. Implications for regional welfare are then discussed in Section 6. Finally, we provide concluding remarks in Section 7. All calculations are provided in the Appendix.

2 The Model

Consider a two region general equilibrium model of trade and endogenous growth with two sectors: traditional production (Y) and manufacturing (X). The traditional sector is a standard constant returns to scale industry that is characterized by free trade and allows for a common wage rate in both regions. Firms in the manufacturing sector, on the other hand, compete according to monopolistic competition and incur iceberg transport costs on inter-regional transactions. Economic growth is driven by the in-house process innovation of manufacturing firms which leads to improvements in production technology. Labor (L), the sole factor of production, is supplied inelastically by households and, although there is no inter-regional migration, moves freely between the traditional and manufacturing sectors in each region. We refer to the two regions as the North and the South, and use an asterisk to indicate variables associated with the Southern region. In the following description of the model we focus on the North but analogous conditions can be obtained for the South.

2.1 Households

The demand side of the model is composed of dynastic households that optimize utility over an infinite time horizon by selecting optimal allocations of income flows across saving and expenditure. The aggregated preferences of households residing in the North are described by the following utility function:

$$U = \int_0^\infty e^{-\rho t} \left[\alpha \ln C_X(t) + (1-\alpha) \ln C_Y(t)\right] dt, \tag{1}$$

where $C_X(t)$ and $C_Y(t)$ are the respective consumptions at time t of a manufacturing composite and the traditional good, $\alpha \in (0,1)$ is the expenditure share for manufacturing goods, and ρ is the subjective discount rate. The manufacturing composite takes the following form:

$$C_X = \left[\int_0^n c_i^{\frac{\sigma-1}{\sigma}} di + \int_0^{n^*} c_j^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where c_i is the demand for variety i of the mass n varieties produced in the North, c_j is the demand for variety j of the mass n^* varieties produced in the South, and $\sigma > 1$ is the elasticity of substitution across varieties.

Households maximize lifetime utility subject to the following intertemporal budget constraint:

$$\int_0^\infty e^{-\int_0^t r(s)ds} E(t) dt \leq \int_0^\infty e^{-\int_0^t r(s)ds} w(t) L dt + B(0),$$

where $E(t)$ is aggregate household expenditure, $w(t)$ is the wage rate, L is the labor force, $r(t)$ is the nominal interest rate, and $B(0)$ is initial asset holdings in the Northern region.[2] Intertemporal utility maximization yields the standard Ramsey saving rule described by the following Euler condition:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho, \tag{3}$$

where a dot over a variable indicates differentiation with respect to time. We assume perfect capital mobility between regions and it follows that the North and the South share a common nominal interest rate and motions for household expenditure: $\dot{E}/E = \dot{E}^*/E^* = r - \rho$. Time notation is suppressed for the remainder of the paper unless required.

A constant share of household expenditure is allocated to the traditional good and the manufacturing composite each period:

$$P_X C_X = \alpha E, \qquad P_Y C_Y = (1-\alpha)E, \tag{4}$$

where P_X is the price index corresponding to the manufacturing composite and P_Y is the price of traditional goods in the Northern region. In particular, the price index is

$$P_X = \left[\int_0^n {p_i}^{1-\sigma} di + \int_0^{n^*} (\tau p_j^*)^{1-\sigma} dj \right]^{1/(1-\sigma)}, \tag{5}$$

where p_i is the price of variety i produced in the North, p_j^* is the price of variety j produced in the South, and $\tau > 1$ is an iceberg transport cost, under which τ units must be shipped for every unit sold in the export market (Samuelson, 1951).

Regarding the price index (5) as the household's unit expenditure function for manufacturing goods, the total Northern demands for representative varieties i and

[2] The value of asset holdings (B) will be driven to zero by free entry in the manufacturing sector. See Section 2.5 for more details.

j respectively produced in the North and the South can be obtained by using Sherphard's Lemma on (5) and aggregating across households:

$$c_i = \alpha p_i^{-\sigma} P_X^{\sigma-1} E, \qquad\qquad c_j = \alpha(\tau p_j^*)^{-\sigma} P_X^{\sigma-1} E. \tag{6}$$

2.2 Traditional Production

The traditional sector produces a homogeneous good for a perfectly competitive interregional market that is characterized by free trade. Firms in both regions produce goods using a similar constant returns to scale technology whereby one unit of output requires the employment of one unit labor. Under this production technology the price of the traditional good equals the wage rate and, with free trade and incomplete specialization of traditional production, the wage rate is common across regions.[3] We set the traditional good as the model numeraire and the traditional good price and the wage rate both equal one, $P_Y = P_Y^* = w = 1$, at all times.

2.3 Manufacturing

The manufacturing sector features firms that produce horizontally differentiated products and complete according to monopolistic competition (Dixit and Stiglitz, 1977). At any moment in time, there are n firms in the North and n^* firms in the South, each producing a unique variety and investing in process innovation with the objective of reducing future costs of production. Firms enter the market freely with negligible costs of product development and, with active process innovation, begin their operations with the same level of productivity as extant firms based in the same region. Once a firm has entered the market it incurs a per-period fixed labor cost (l_F) that corresponds with product marketing and the management of production and innovation activities.

[3] In particular, we assume that the traditional share of household expenditure $(1-\alpha)$ is sufficiently large to ensure that there is always an active traditional sector in each region.

Manufacturing firms employ a constant returns to scale production technology that is symmetric within a region but may differ across regions. Specifically, the production function for a representative firm based in the North is

$$x = \theta l_X, \tag{7}$$

where $\theta > 0$ is a firm-specific productivity coefficient, and l_X is firm-level employment in production. As indicated above, all firms based in the same region have the same level of productivity, but we allow for productivity differences across regions ($\theta \neq \theta^*$).

A firm faces demand for its product from both the domestic and the export markets. With iceberg transport costs, however, firms must produce additional units for sales to the export market in order to cover units lost in transit. Thus, referring to (6), a Northern firm produces $x = c_i + \tau c_i^*$ units in order to meet the combined demands of the Northern and Southern markets. As is well known, under monopolistic competition each firm maximizes operating profit on sales by setting price equal to a constant markup over unit cost. The price set by a Northern firm is therefore $p = \sigma/(\sigma - 1)\theta$. Combining this pricing rule with the demand functions (6) and the production function (7), optimal operating profit on sales can be obtained for a Northern firm as

$$\pi = \frac{\alpha p^{1-\sigma} \left[P_X^{\sigma-1} E + \varphi {P_X^*}^{\sigma-1} E^* \right]}{\sigma}, \tag{8}$$

where $\varphi \equiv \tau^{1-\sigma}$ is a measure of the freeness of trade with $\varphi = 0$ indicating prohibitively high transport costs and $\varphi = 1$ indicating perfectly free trade. With a large number of firms, the small market share of any single firm leads it to ignore the term in brackets in (8) when evaluating the impact of changes in price on operating profit on sales.

2.4 Process Innovation

Manufacturing firms invest in in-house process innovation with the objective of maximizing firm value. Improvements in technology reduce production costs and lead to lower prices allowing firms to increase their market shares and raise profits on sales.

Returning to the production function (7), we consider process innovations that generate improvements in productivity through increases in θ. In particular, the productivity of a representative Northern firm evolves according to

$$\dot{\theta} = K l_R, \tag{9}$$

where K is labor productivity in in-house process innovation, and l_R is firm-level R&D employment.

Following the in-house process innovation literature (Smulders and van de Klundert, 1995; Peretto, 1996), we model the productivity of labor in in-house innovation as a function of the weighted average productivity of technologies observable by the firm:

$$K = s\theta + \delta s^* \theta^*, \tag{10}$$

where $s \equiv n/N$ and $s^* \equiv n^*/N$ are respectively the shares of industry located in the North and the South, and $N \equiv n + n^*$ is the total number of firms. The exogenous parameter $\delta \in (0, 1)$ regulates the inter-regional scope of knowledge dissemination between firms.[4]

Implicit in the specification of (10) is the assumption that a firm's production technology is a culmination of the accumulation of both codifiable and tacit knowledge

[4] In order to focus on the implications of cross-region knowledge spillovers for industry location and growth within a simple framework, we have assumed that the number of firms is large enough to ensure that the influence of a single firm's technology on labor productivity in in-house innovation is negligible.

(Keller, 2004). While codifiable knowledge is easily conveyed across large distances using modern communication technology, the transfer of tacit knowledge tends to require face-to-face communication. Indeed, the imperfect nature of knowledge spillovers is well documented in empirical studies such as Jaffe et al. (1993), Mancusi (2008), and Coe et al. (2009). Accordingly, we interpret the parameter δ as describing the level of knowledge that can be transferred over large distances given the current state of communication technology. For example, when $\delta = 0$ knowledge spillovers are completely local in scope, and when $\delta = 1$ there is perfect inter-regional knowledge dispersion.[5]

A firm's intertemporal optimization problem entails choosing the level of investment in process innovation that maximizes the future flow of profits. The total per-period profit of a Northern firm equals operating profit on sales minus the cost of investment in process innovation and the per-period fixed labor cost l_F:

$$\Pi = \pi - l_R - l_F. \tag{11}$$

Firms based in the North, therefore, choose l_R to maximize $V = \int_0^\infty \Pi(t) e^{-\int_0^t r(s)ds} dt$ subject to the technological constraint (9). This optimization problem can be solved using the following current value Hamiltonian function: $H = \Pi + \mu K l_R$, where μ is the current shadow value of an improvement in the technology of a Northern firm.

The optimal investment path for process innovation satisfies conditions for both static and dynamic efficiency in addition to a standard transversality condition.[6] Static efficiency requires that a marginal increase in the value of technology equal the marginal cost of process innovation, that is, $\mu = 1/K$. Dynamic efficiency requires that the return to investment in process innovation equal the rate of return on a risk free asset, that is, $\mu r - \dot{\mu} = \partial\pi/\partial\theta$, where we assume that the market share of

[5] Baldwin and Forslid (2000) introduce a similar specification for the level of inter-regional knowledge spillovers in a variety expansion model of innovation-based endogenous growth.

[6] In the next subsection we show that $r = \rho$ at all moments in time. As a result, the transversality condition is $\lim_{t\to\infty} e^{-\rho t}\mu\theta = 0$.

each firm is small enough that it disregards the impact of its innovation efforts on both the composite price index (P_X) and knowledge spillovers to rival firms (K).[7] Combining these efficiency conditions yields the following no-arbitrage condition for in-house R&D investment:

$$r = \frac{(\sigma - 1)\pi K}{\theta} - \frac{\dot{K}}{K}. \tag{12}$$

This no-arbitrage condition must bind whenever there is active process innovation.

2.5 Free Entry

Manufacturing firms are free to enter and exit the market. As a result, the number of firms based in each region adjusts until per-period profits (11) are driven to zero. With firms earning zero profits, each region's expenditure equals its labor income and is therefore constant given our choice of the traditional good as the model numeraire, $E = L$ and $E^* = L^*$.[8] Consequently, from the Euler equation (3), the risk-free interest rate equals the discount rate and is common across regions at all moments in time, $r = \rho$.

The relative state of technology arising between firms based in the North and the South at any given moment in time is described using

$$\omega \equiv \frac{\theta}{\theta^*}, \tag{13}$$

which we refer to as the *productivity gap*. Given the pricing rules set by manufacturing firms, the productivity gap also describes the relative price of Southern produced varieties, that is $\omega = p^*/p$.

[7] The risk-free asset is simply used as a link between the internal rate of return to in-house innovation and the external rate of return to investment in the market for investment funds.

[8] More specifically, household wealth equals zero in equilibrium, $B = 0$, as firm value is driven to zero by free entry, $V = 0$.

The zero profit conditions can be conveniently expressed in terms of firm-level employment. First, using the demand functions (6) and the production function (7) with the pricing rules yields the optimal levels of employment in production for Northern and Southern firms respectively as

$$l_X = \frac{\alpha(\sigma-1)}{\sigma}\left[\frac{L}{n+\varphi\omega^{1-\sigma}n^*} + \frac{\varphi L^*}{\varphi n + \omega^{1-\sigma}n^*}\right], \tag{14}$$

$$l_X^* = \frac{\alpha(\sigma-1)}{\sigma}\left[\frac{\varphi\omega^{1-\sigma}L}{n+\varphi\omega^{1-\sigma}n^*} + \frac{\omega^{1-\sigma}L^*}{\varphi n + \omega^{1-\sigma}n^*}\right]. \tag{15}$$

Then, setting per-period profits (11) equal to zero and using (14) and (15) with (8), the free market entry conditions can be written as

$$l_X = (\sigma-1)\left(l_R + l_F\right), \qquad l_X^* = (\sigma-1)\left(l_R^* + l_F\right), \tag{16}$$

for the North and the South, respectively.

3 Core-periphery Long-run Equilibrium

In this section we investigate the characteristics of the core-periphery long-run equilibrium for which the manufacturing industry concentrates completely in one region. In particular, we are interested in state-state equilibria that feature a constant intersectoral allocation of labor in each region. To simplify the exposition we focus on the case where all manufacturing and innovation activity occurs in the North. In this case, although the North continues to produce traditional goods, the South specializes completely in traditional production and imports manufacturing goods from the North in return for the traditional good.

To begin with, consider a situation in which the no-arbitrage condition (12) binds in the North but not in the South. With no Southern employment in process innovation, the technology gap ω increases indefinitely with the North becoming increasingly advanced and the South becoming increasingly backward. At first Southern firms will earn positive per-period profits ($\Pi^* > 0$) as they no longer incur the fixed cost l_R^*. Without process innovation, however, there will be no new market entry in the South, and consequently, the number of Northern firms will rise to absorb these additional profits thereby driving Southern profits to zero, $l_X^* = (\sigma - 1)l_F$. It then follows that as Southern employment in production is a decreasing function of ω from (15), the rising technology gap steadily reduces the Southern share of manufacturing firms that can be supported by the zero profit condition. As a result, the Northern share of the manufacturing industry converges to one ($s = 1$).

With all manufacturing and innovation activity occurring in the North, the zero-profit condition (16) can be used with (9) and (10) to rewrite the binding no-arbitrage condition (12) for Northern firms as $\rho = (\sigma - 2)l_X/(\sigma - 1) + l_F$. In addition, setting $l_X^* = 0$, and using $n^* = 0$ with the pricing rules for manufacturing firms in (14) yields the product market clearing condition $nl_X = \alpha(\sigma - 1)(L + L^*)/\sigma$. Combining these two conditions, we find that the equilibrium number of firms is

$$N_C = \frac{\alpha(\sigma - 2)(L + L^*)}{\sigma(\rho - l_F)}, \tag{17}$$

where the subscript C denotes a concentrated equilibrium, and $N_C = n$ as all manufacturing firms are located in the North. Not surprisingly, an increase in the overall market size ($L + L^*$) increases the number of firms (N_C). On the other hand, an increase in l_F, or similarly, a decrease in ρ, lowers the level of employment in production (l_X) that satisfies the no-arbitrage condition and thus raises the number of firms through the product market clearing condition. These results parallel those obtained by Smulders and van de Klundert (1995) for a closed economy. Finally, a positive level of market entry requires that the return to process innovation (ρ) cover the per-period fixed costs (l_F). To ensure both positive market entry and productivity growth, we assume that $\rho > (\sigma - 1)l_F$ for the remainder of the paper.[9]

The long-run rate of productivity growth associated with the core-periphery equilibrium can now be obtained using (8), (9), and (10) with (12) and (17):

$$g_C = \frac{\alpha(\sigma - 1)(L + L^*)}{\sigma N_C} - \rho = \frac{\rho - (\sigma - 1)l_F}{\sigma - 2}. \tag{18}$$

From (18) we can see that there is no scale effect in the model as an increase in market size $(L + L^*)$ is neutralized by an increase in the number of manufacturing firms (N_C).[10]

Changes in the fixed cost and the discount rate have opposing effects on the rate of productivity growth. An increase in the fixed cost (l_F) has a negative effect on productivity growth as it raises the number of firms leading to a greater allocation of labor to per-period fixed cost that crowds out employment in process innovation. In contrast, although an increase in the discount rate (ρ) has both a direct negative effect and an indirect positive effect, through N_C, on the rate of productivity growth, the indirect effect dominates the direct effect and as a result g_C is positively related to ρ. These results are similar to those found by Smulders and van de Klundert (1995) and, in a different context, Dinopoulos and Thompson (1998), and highlight the tension between market concentration and productivity growth that arises in this type of endogenous growth model.

[9]Positive levels of market entry and innovation activity also require $\sigma > 2$. This requirement for the elasticity of substitution is supported by the average and median estimates obtained by Broda and Weinstein (2006) for various levels of industry disaggregation.

[10]The core-periphery rate of productivity growth is determined independently of the degree of inter-regional knowledge spillovers (δ) and the freeness of trade (φ). This is a common characteristic of models which assume that innovation and production are tied to the same location, as shown by a comparison of, for example, Martin and Ottaviano (1999) and Baldwin et al. (2001).

4 Dispersed Long-run Equilibrium

This section provides a characterization of the dispersed long-run equilibrium that occurs when both regions have active manufacturing sectors and positive productivity growth. Once again we are interested in steady states that feature constant labor allocations in both regions. For the dispersed long-run equilibrium, this requires a constant productivity gap and constant numbers of firms based in each region.

In order to describe the evolution of the productivity gap we take the time derivative of (13) and obtain the following differential equation:

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{\theta}}{\theta} - \frac{\dot{\theta}^*}{\theta^*} = \frac{Kl_R}{\theta} - \frac{K^* l_{R^*}}{\theta^*}, \tag{19}$$

where we have used (9). Clearly the evolution of the productivity gap depends on the relative levels of knowledge available to firms based in each region and their respective levels of investment in process innovation. Setting (19) equal to zero yields

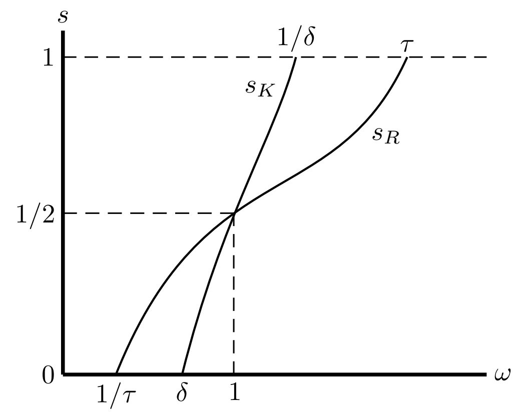
$$\frac{Kl_R}{\theta} = \frac{K^* l_R^*}{\theta^*}, \tag{20}$$

and a constant productivity gap therefore naturally implies equal rates of productivity growth in the North and the South.

With active process innovation in both regions the investment and entry decisions of manufacturing firms are determined by the no-arbitrage conditions of both regions (12), and the free entry conditions (16). In Appendix A we show that the dispersed equilibrium described by these investment and entry conditions can be reduced to a system of two equations that fully describe the long-run equilibrium in terms of the productivity gap (ω) and the North's share of industry ($s \equiv n/N$).

The first condition requires that the average level of accessible knowledge relative to firm level productivity be the same for both regions, that is, $K/\theta = K^*/\theta^*$. Sub-

Figure 1: Symmetric dispersed equilibrium



stituting (10) and (13) into this condition and rearranging, we solve for the North's share of industry as

$$s_K = \frac{1 - \delta\omega^{-1}}{2 - \delta\omega - \delta\omega^{-1}}. \tag{21}$$

As illustrated by the s_K curve in Figure 1, this condition is strictly increasing in ω as an increase in the industry share of Northern firms is necessary to offset the benefit that Southern firms derive from the increase in relative access to knowledge when the productivity gap rises. This tension between the industry share of the advanced region and the access of the backward region to relatively advanced knowledge is regulated by the degree of inter-regional knowledge dispersion (δ). Indeed, a close inspection of (21) indicates that $\omega \in (\delta, 1/\delta)$ is required for $s_K \in (0, 1)$ with equal access to knowledge in both regions.

The second condition requires that firm-level scales of production equalize across regions, $l_X = l_X^*$. Substituting (14) and (15) into this condition, we once again solve for the North's share of industry as

$$s_R = \left[\frac{L}{1 - \varphi\omega^{\sigma-1}} + \frac{\varphi L^*}{\varphi - \omega^{\sigma-1}}\right]\frac{1}{L + L^*}. \tag{22}$$

As shown by the s_R curve depicted in Figure 1 for the symmetric case where $L = L^*$, this condition is also strictly increasing in ω. The relationship between the productivity gap and industry share described by the s_R curve is determined by the link between firm-level scales of production and innovation that arises with free entry (16). An increase in ω raises the market shares, and therefore the profits, of Northern firms precipitating an increase in the North's share of industry. The size of this effect is determined by the freeness of trade with a rise in φ increasing the slope of the s_R curve.

Convergence to or divergence away from the dispersed long-run equilibrium illustrated in Figure 1 depends crucially on the level of transport costs and the degree of inter-regional knowledge spillovers. In Appendix A we approximate the local dynamics around the symmetric equilibrium using a Taylor expansion of (19) evaluated at $\omega = 1$ and $s = 1/2$, and obtain the following proposition:

Proposition 1 *(Convergence in the productivity gap): The symmetric dispersed equilibrium is saddle-path stable for $\varphi < \tilde{\varphi}$, where*

$$\tilde{\varphi} = (\kappa - 1) - \sqrt{(\kappa - 2)\kappa}, \qquad \kappa = \frac{(\sigma - 1)(1 + \delta)(2\rho - (1 + \delta)l_F)}{\delta(2\rho - (\sigma - 1)(1 + \delta)l_F)}.$$

Proof: See Appendix A. □

The threshold $\tilde{\varphi}$ determines the level of transport costs above which firm-level rates of productivity growth converge and is closely related to the degree of inter-regional knowledge spillovers as summarized in the following corollary:

Corollary 1 *The threshold $\tilde{\varphi}$ is increasing in the degree of inter-regional knowledge spillovers (δ) for $\delta < \tilde{\delta}$ and decreasing for $\delta > \tilde{\delta}$, where*

$$\tilde{\delta} = \frac{(\sigma - 1)(2\rho - l_F)}{(\sigma - 1)(2\rho - l_F) - 4\rho}.$$

Proof: See Appendix B. □

A change in the productivity gap affects firm-level productivity growth in the North and the South both directly and indirectly through adjustments in the number of firms based in each region. In the symmetric equilibrium, however, the indirect effects cancel out leaving two opposing direct effects that combine to determine the stability of long-run equilibrium. These direct effects can be identified by substituting (16) into (20), taking the partial derivative with respect to ω, and evaluating the result at the symmetric equilibrium:

$$\left.\frac{\partial \dot{\omega}}{\partial \omega}\right|_{\omega=1} = \left.\frac{\partial (K l_R/\theta - K^* l_R^*/\theta^*)}{\partial \omega}\right|_{\omega=1} = \frac{(1+\delta)2\varphi l_X}{(1+\varphi)^2} - \delta l_R. \tag{23}$$

A negative sign for this partial derivative implies a stable dispersed equilibrium.

Starting from the symmetric equilibrium, consider the effects of an exogenous perturbation that increases the productivity gap. The first term on the right-hand side of (23) describes the destabilizing *competition effect* associated with the ensuing increase in the production scale of Northern firms relative to Southern firms. As firm-level employment in innovation is determined proportionately with the scale of production through the free entry condition (16), the rate of productivity growth rises in the North and falls in the South causing subsequent increases in the productivity gap. The strength of this destabilizing force is increasing in both the freeness of trade (φ) and the degree of inter-regional knowledge spillovers (δ). In contrast, the second term on the right-hand side of (23) describes the stabilizing *productivity effect* corresponding with the rise of Southern labor productivity in innovation (K^*) relative to the North (K). The rate of productivity growth falls in the North and rises in the South causing the productivity gap to fall back to the symmetric equilibrium. The strength of this stabilizing force depends on the degree of inter-regional knowledge spillovers (δ). The stability of the symmetric dispersed equilibrium is determined by the balance of these opposing forces with convergence in technologies more likely for lower levels of φ and intermediates levels of δ.

To complete this section we investigate the relationship between the productivity gap and the total number of firms and the consequences of this relationship for long-run productivity growth in a more general setting. To facilitate this investigation we begin by deriving the relative level of accessible knowledge as a function of ω using (10) and (21):

$$\frac{K}{\theta} = \frac{K^*}{\theta^*} = \frac{1-\delta^2}{2-\delta\omega-\delta\omega^{-1}}. \tag{24}$$

The relative level of accessible knowledge is convex in the productivity gap with a minimum at $\omega = 1$, and therefore plays a key role in the determination of the total number of firms and the rate of productivity growth through its impact on the cost of innovation.

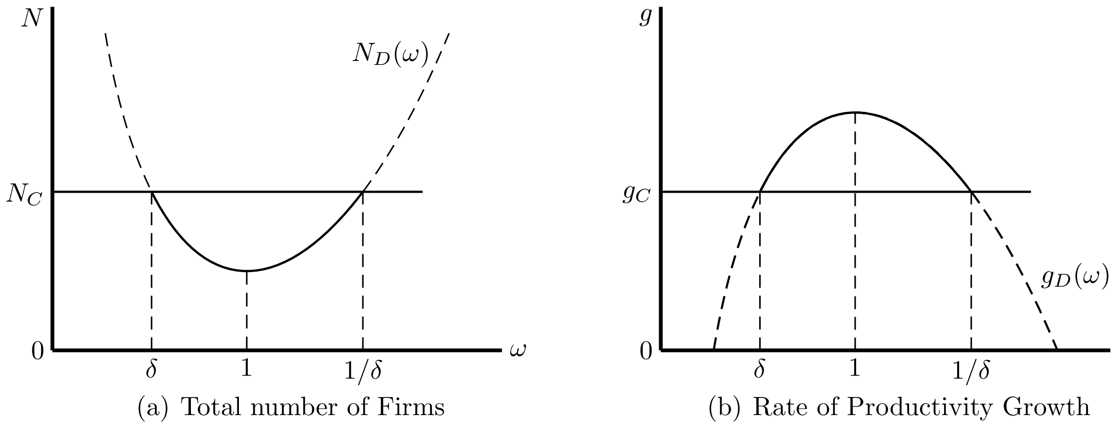
Next, combining (14) and (15), the total number of firms can be obtained as a function of the scale of production: $N_D = \alpha(\sigma-1)(L+L^*)/\sigma l_X$. Then, using (16), the steady-state no-arbitrage condition $\rho = [K/\theta][(\sigma-2)l_X/(\sigma-1)+l_F]$ shows that there is a negative relationship between the scale of production and the relative level of accessible knowledge. Using these conditions together with (24) yields the following expression for the long-run number of firms:

$$N_D = \frac{\alpha(\sigma-2)(1-\delta^2)(L+L^*)}{\sigma\left[\left(2-\delta\omega-\delta\omega^{-1}\right)\rho-(1-\delta^2)l_F\right]}, \tag{25}$$

where the subscript D indicates a dispersed equilibrium.[11] The relationship between the total number of firms and the productivity gap is also U-shaped with a minimum at $\omega = 1$, as illustrated by the $N_D(\omega)$ curve in Figure 2a. In particular, for $\omega < 1$, an

[11]In Proposition 2 we will show that $K/\theta < 1$ in a dispersed equilibrium. As such, our earlier assumption $\rho > (\sigma-1)l_F$ is sufficient to ensure positive market entry and productivity growth in both core-periphery and dispersed equilibria.

Figure 2: Productivity growth and the productivity gap



increase in ω lowers K/θ and raises the fixed cost of innovation leading to a decrease in the number of firms that the market can support under free entry. For $\omega > 1$, on the other hand, a rise in ω increases K/θ and the number of firms increases. Thus, the non-monotonic relationship between ω and N_D arises from the links between firm scale, the relative level of accessible knowledge, and the productivity gap. In addition, we can show that the total number of firms is always smaller in the dispersed equilibrium than in the core-periphery equilibrium, as seen in Figure 2a.

Finally, combining (9), (16), (24) and (25), the dispersed rate of productivity growth can be derived as a function of the productivity gap alone:

$$g_D = \frac{\alpha(\sigma-1)(1-\delta^2)(L+L^*)}{\sigma(2-\delta\omega-\delta\omega^{-1})N_D} - \rho = \frac{\rho}{\sigma-2} - \frac{(\sigma-1)(1-\delta^2)l_F}{(\sigma-2)(2-\delta\omega-\delta\omega^{-1})}. \tag{26}$$

As in the core-periphery equilibrium, the long-run rate of productivity growth is positively related to the discount rate (ρ) and negatively related to the per-period fixed cost (l_F). The first and second order derivatives of (26) with respect to ω provide the following lemma:

Lemma 1 *(Long-run productivity growth and the productivity gap): The long-run* *rate of productivity growth is concave in ω with a maximum at $\omega = 1$.*

Proof: Simple partial differentiation of (26). □

This result is a variant of the *rationalization* effect (Peretto, 2003) recast in terms of the productivity gap. The non-monotonic relationship between long-run productivity growth and the productivity gap stems from the U-shaped relationship between the relative level of accessible knowledge and the productivity gap shown in (24). For example, if $\omega < 1$, a rise in ω lowers K/θ. The consequent fall in the total number of firms (N_D) raises scales of employment in both production and innovation thereby increasing the productivity growth rate. In contrast, for $\omega > 1$, an increase in ω raises K/θ and N_D rises. The increase in the total number of firms lowers employment in innovation and the rate of productivity growth falls.

A comparison of the core-periphery rate of productivity growth (g_C) with that of the dispersed equilibrium (g_D) leads to the following proposition:

Proposition 2 *(Productivity growth comparison): Productivity growth is always higher in the dispersed equilibrium than in the core-periphery equilibrium.*

Proof: Subtracting (18) from (26) gives

$$g_D - g_C = \left[1 - \frac{1-\delta^2}{2 - \delta\omega - \delta\omega^{-1}}\right]\left[\frac{\sigma - 1}{\sigma - 2}\right] l_F.$$

This difference is positive for $K/\theta < 1$, which is always the case if the dispersed equilibrium exists as $\omega \in (\delta, 1/\delta)$ is required for $s_K \in (0, 1)$, as shown in Figure 1. □

With imperfect inter-regional knowledge dissemination ($0 < \delta < 1$), the relative level of accessible knowledge is always higher in the core-periphery equilibrium than it is in the dispersed equilibrium. Although we might expect this to lead to a greater rate of productivity growth for the core-periphery equilibrium, the rationalization effect lowers the firm-level scale of innovation activity below that for the dispersed equilibrium ($l_{RC} < l_{RD}$), and thus the dispersed rate of productivity growth dominates the core-periphery rate of productivity growth whenever the dispersed equilibrium exists, that is, for $\omega \in (\delta, 1/\delta)$.

5 Patterns of Industry, Productivity, and Growth

In this section we examine the implications of changes in relative market size, the level of transport costs, and the degree of inter-regional knowledge spillovers for patterns of industry location, the productivity gap, and the long-run rate of productivity growth that arise in a stable dispersed equilibrium. As we have seen, the relative slopes of the s_K and s_R curves are closely related to the freeness of trade and the degree of inter-regional knowledge spillovers. In Appendix C we compare these relative slopes with the necessary condition for a stable symmetric dispersed equilibrium given in Proposition 1 and obtain the following lemma:

Lemma 2 *(Relative slopes of the s_K and s_R curves): In the stable dispersed equilibrium the slope of the s_K curve is always greater than the slope of the s_R curve.*
Proof: See Appendix C. □

Therefore, in the following discussion we evaluate comparative statics using the slope ranking required by Lemma 2 and illustrated in Figure 1.

As the s_K curve is determined independently of labor endowments, comparative statics for the effect of a change in relative market size can be derived using the s_R curve alone. The results are summarized in the following proposition:

Proposition 3 *(Relative market size, industry share, and the productivity gap): In a dispersed equilibrium, an increase in the relative size of the Northern market (L/L^*) raises both North's share of industry (s) and the productivity gap (ω).*
Proof: From (22),

$$\frac{ds_R}{dL} = \frac{1+\varphi}{4(1-\varphi)L} > 0, \qquad \frac{ds_R}{dL^*} = -\frac{1+\varphi}{4(1-\varphi)L^*} < 0.$$

□

With an increase in the relative market size of the North, the s_R curve shifts upwards leading to a rise in the share of firms locating in the North and a higher productivity gap. This result stems from the home market effect of the trade literature whereby a larger share of firms locates in the larger market (Krugman, 1980). In our framework, however, the rise in the North's share of industry must be offset by an increase in the productivity gap to ensure equal rates of productivity growth in the North and the South.

The results of Proposition 3 provide a characterization of the trade patterns that arise in the dispersed equilibrium. Although both regions continue to produce both goods, the larger region becomes a net exporter of manufacturing goods and the smaller region becomes a net exporter of traditional goods. We henceforth adopt the assumption that the North is the larger region ($L > L^*$), and therefore that $\omega > 1$ and $s > 1/2$ when evaluating comparative statics.

The s_R curve can also be used to derive comparative statics for the effects of a change in the freeness of trade (φ), as summarized in the following proposition:

Proposition 4 *(Transport costs, industry share, and the productivity gap): In a dispersed equilibrium, for $L > L^*$ a decrease in transport costs (an increase in φ) increases both North's share of industry (s) and the productivity gap (ω).*

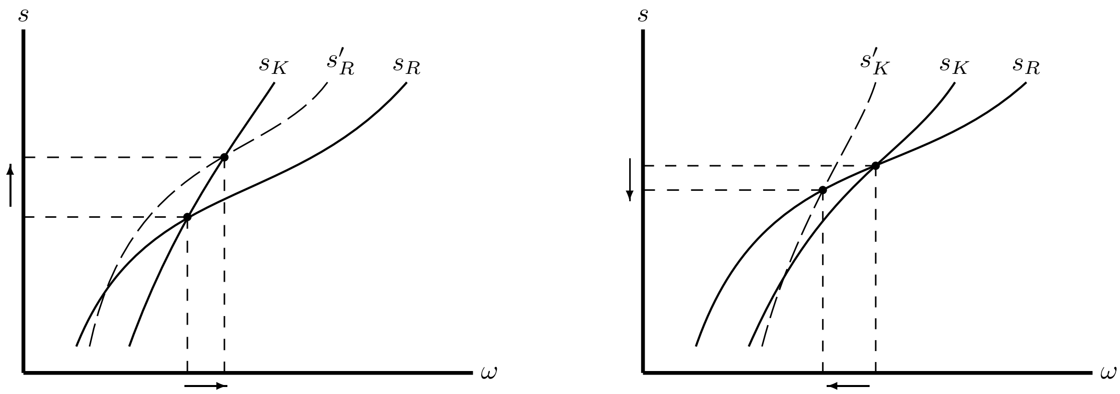
Proof: From (22),

$$\frac{ds_R}{d\varphi} = \left[\frac{L}{(1-\varphi\omega^{\sigma-1})^2} - \frac{L^*}{(\varphi-\omega^{\sigma-1})^2}\right]\frac{\omega^{\sigma-1}}{L+L^*} > 0,$$

where the term in parentheses is positive for $L > L^*$ as $\omega > 1$ from Proposition 3. □

Referring to Figure 3a, an improvement in the freeness of trade rotates the s_R curve counter-clockwise to s'_R and consequently for $L > L^*$ the share of industry based in the North and the productivity gap both rise. A decrease in transport costs

Figure 3: Steady-state comparative statics



(a) Effect of an increase in φ (b) Effect of an increase in δ

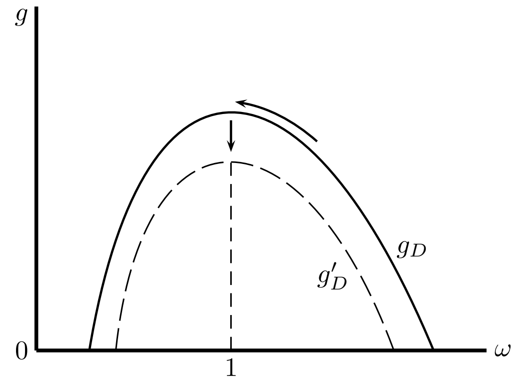
strengthens the home market effect since firms have a greater incentive to locate in the larger market as the cost of exporting to the smaller market has fallen. Once again, however, the increase in the larger region's share of industry must coincide with an increase in the productivity gap in order to maintain equal rates of productivity growth in the North and the South.

The results summarized in Propositions 3 and 4 have interesting consequences for economic growth. Referring back to Lemma 1 and Figure 2b, any movement in the productivity gap away from the symmetric equilibrium lowers the rate of productivity growth. Accordingly, increases in the relative market size of the North (L/L^*) and the freeness of trade (φ) both depress productivity growth.

Next, in order to examine the effect of a change in the degree of inter-regional knowledge spillovers, we now focus on the s_K curve as the s_R curve is determined independently of δ. The results are described in the following proposition:

Proposition 5 *(Knowledge spillovers, industry share, and the productivity gap): In a dispersed equilibrium, for $L > L^*$, an increase in the degree of inter-regional knowledge spillovers (δ) decreases both North's share of industry (s) and the productivity gap (ω).*

Figure 4: Effects of a rise in the level of inter-regional knowledge spillovers (δ)



Proof: From (21)

$$\frac{ds_K}{d\delta} = \frac{(\omega^2 - 1)\omega^{-1}}{(1 - \delta\omega^{-1} + 1 - \delta\omega)^2} > 0,$$

where $\omega > 1$ for $L > L^*$ from Proposition 3. □

This result is illustrated in Figure 3b. An increase in the degree of inter-regional knowledge dissemination (δ) rotates the s_K curve counter-clockwise to s'_K and the North's share of industry and the productivity gap decrease. In particular, the rise in δ raises Southern labor productivity in innovation relative to that of the North and industry shares adjust to equate the Northern and the Southern rates of productivity growth.[12]

The implications of Proposition 5 for long-run productivity growth are ambiguous. Referring to Figure 4, we can see that an increase in δ has both a direct and an indirect effect on g_D. The negative direct effect is shown by the downward shift in the g_D curve to g'_D, and results from an increase in the relative level of accessible knowledge that lowers innovation costs and raises the total number of firms. The positive indirect

[12]An increase in δ also reduces the concentration of industry in variety expansion models of endogenous growth, but through a different mechanism. Specifically, greater knowledge spillovers raise the rate of innovation thereby eroding monopolistic profits given the faster rate of entry by new firms. Lower profit reduces the level of industry concentration (Martin and Ottaviano, 2004).

effect $d\omega/d\delta < 0$ is described by a movement along the g_D curve towards $\omega = 1$ and entails a fall in the relative level of accessible knowledge and a subsequent decrease in the total number of firms. The balance of these direct and indirect effects determines whether an increase in the degree of inter-regional knowledge dissemination raises productivity growth or not (see Table 1 in the next section for numerical examples showing both cases). The ambiguous relationship between δ and g_D contrasts with the results of the variety expansion literature where an increase in the level of inter-regional knowledge spillovers always has pro-growth effects (Baldwin and Martin, 2004).

6 Welfare Analysis

In this section we briefly discuss the implications of greater market integration for region welfare levels in the dispersed long-run equilibrium. In particular, we are interested in ascertaining whether the North and the South benefit from or are hurt by increases in the freeness of trade and the degree of inter-regional knowledge dispersion. Continuing to assume a larger labor endowment for the North ($L > L^*$), we can use (1), (2), (5), (6), and (22) to obtain the indirect utilities for Northern and Southern households respectively as

$$U_0 = \frac{\ln[A\theta(0)^\alpha L]}{\rho} + \frac{\alpha}{(\sigma-1)\rho}\ln\left[\frac{L(1-\varphi^2)N_D}{(L+L^*)(1-\varphi\omega^{\sigma-1})}\right] + \frac{\alpha g_D}{\rho^2}, \tag{27}$$

$$U_0^* = \frac{\ln[A\theta^*(0)^\alpha L^*]}{\rho} + \frac{\alpha}{(\sigma-1)\rho}\ln\left[\frac{L^*(1-\varphi^2)N_D}{(L+L^*)(1-\varphi\omega^{1-\sigma})}\right] + \frac{\alpha g_D}{\rho^2}, \tag{28}$$

where $\theta(0)$ and $\theta(0)^*$ are the respective initial productivities of Northern and Southern firms, and $A = (\alpha(\sigma-1)/\sigma)^\alpha (1-\alpha)^{1-\alpha}$ is a constant.

Table 1: Welfare Effects

L	δ	φ	ω	s	$\frac{dg}{d\delta}$	$\frac{dg}{d\varphi}$	$\frac{dU_0}{d\delta}$	$\frac{dU_0}{d\varphi}$	$\frac{dU_0^*}{d\delta}$	$\frac{dU_0^*}{d\varphi}$
1	0.8	0.3	1	0.5	-0.015	0	1.3	2.6	1.3	2.6
1.2	0.8	0.3	1.08	0.66	-0.010	-0.005	-1.5	-0.1	2.4	2.2
1.2	0.85	0.3	1.04	0.62	-0.013	-0.002	-0.1	1.3	2.0	2.6
1.2	0.8	0.35	1.13	0.76	0.009	-0.021	-8.4	-8.5	3.6	0.5

Parameter values are $L^* = 1$, $\alpha = 0.5$, $\sigma = 2.5$, $l_F = 0.01$, and $\rho = 0.1$.

Beginning with the welfare effects of an increase in the freeness of trade, we obtain

$$\frac{\rho}{\alpha}\frac{dU_0}{d\varphi} = \frac{(\omega^{\sigma-1}-\varphi)-(1-\varphi\omega^{\sigma-1})\varphi}{(\sigma-1)(1-\varphi^2)(1-\varphi\omega^{\sigma-1})} + \frac{\varphi\omega^{\sigma-2}}{(1-\varphi\omega^{\sigma-1})}\frac{d\omega}{d\varphi} + \frac{1}{(\sigma-1)N_D}\frac{dN_D}{d\varphi} + \frac{1}{\rho}\frac{dg_D}{d\varphi},$$
$$\frac{\rho}{\alpha}\frac{dU_0^*}{d\varphi} = \frac{(1-\varphi\omega^{\sigma-1})-(\omega^{\sigma-1}-\varphi)\varphi}{(\sigma-1)(1-\varphi^2)(\omega^{\sigma-1}-\varphi)} - \frac{\varphi\omega^{\sigma-2}}{(\omega^{1-\sigma}-\varphi)}\frac{d\omega}{d\varphi} + \frac{1}{(\sigma-1)N_D}\frac{dN_D}{d\varphi} + \frac{1}{\rho}\frac{dg_D}{d\varphi}.$$

The first term on the right-hand side captures the impact of lower prices on traded goods and is positive for both regions. The second term describes the terms of trade effect associated with changes in the relative price of imported manufacturing varieties and is positive for Northern households and negative for Southern households given the results summarized in Proposition 4. The third and fourth terms capture the welfare effects associated with changes in the total number of firms (N_D) and the rate of productivity growth (g_D), respectively, and are the same for both Northern and Southern households. With the positive relationship between ω and φ, however, the total number of firms rises and the rate of productivity growth falls, and the overall welfare effect associated with an increase in the freeness of trade, therefore, depends on parameter values, as indicated by the numerical examples presented in Table 1.

Turning next to the degree of inter-regional knowledge spillovers, an increase in δ has several opposing welfare effects:

$$\frac{\rho}{\alpha}\frac{dU_0}{d\delta} = \frac{\varphi\omega^{\sigma-2}}{(1-\varphi\omega^{\sigma-1})}\frac{d\omega}{d\delta} + \frac{1}{(\sigma-1)N_D}\frac{dN_D}{d\delta} + \frac{1}{\rho}\frac{dg_D}{d\delta},$$
$$\frac{\rho}{\alpha}\frac{dU_0^*}{d\delta} = -\frac{\varphi\omega^{\sigma-2}}{(\omega^{\sigma-1}-\varphi)}\frac{d\omega}{d\delta} + \frac{1}{(\sigma-1)N_D}\frac{dN_D}{d\delta} + \frac{1}{\rho}\frac{dg_D}{d\delta}.$$

In this case, the first term describes the terms of trade effect and is negative for the North and positive for the South given the result that an increase in δ lowers the productivity gap from Proposition 5. As before, the remaining terms describe the changes in total variety (N_D) and productivity growth (g_D) and are common to both regions. The overall welfare effects of a change in the degree of inter-regional knowledge spillovers once again depend on parameter values as shown by the numerical examples presented in Table 1.

7 Concluding Remarks

In this paper we have investigated the relationship between productivity growth and spatial patterns of technology and industrial activity. In particular, we develop a two region model of trade and endogenous growth without scale effects in which manufacturing firms invest in in-house process innovation with the objective of reducing production costs. Using this framework we examine core-periphery equilibria for which all industrial activity occurs in one region and dispersed equilibria for which industrial activity is dispersed across regions, and find that a sufficiently high level of transport costs and an intermediate level of inter-regional knowledge dispersion is required for convergence in technologies to a dispersed long-run equilibrium. Determining which type of equilibrium the economy converges to is important as the highest rate of productivity growth arises in a symmetric equilibrium with no productivity gap and equal shares of industry across regions. As such, a key conclusion of our framework is that geographic concentration of industry has a negative impact on overall growth.

Considering the effects of greater economic integration, we find that within a dispersed long-run equilibrium although an increase in the freeness of trade leads to increases in both the larger region's share of industry and its relative level of technology, lower transport costs allow for a greater total number of manufacturing firms and as a result investment in innovation falls causing a decrease in the rate of productivity growth. On the other hand, an increase in the degree of inter-regional knowledge dispersion decreases the larger region's share of industry and its relative level of technology. In this case, the consequences for productivity are ambiguous as the number of manufacturing firms may rise or fall. Put together, these results have interesting implications for the impact of greater economic integration on regional welfare levels.

The framework presented in this paper is limited in two respects. Firstly, we have assumed that the production and innovation activities of a firm must occur in the same location. An interesting extension might include the separation of these activities across regions. Secondly, we have adopted an assumption of negligible entry costs for manufacturing firms. The inclusion of product development costs for manufacturing firms would, however, allow for a richer characterization of the dynamics of entry and exit associated with greater economic integration.

Appendix A: Proof of Proposition 1

The local dynamics around the symmetric steady state are determined by the differential equation for the productivity gap (19) and the no-arbitrage conditions for the North and the South:

$$\rho = \frac{Kl_X}{\theta} - sl_R - \frac{\delta s^* K^* l_R^*}{K}, \tag{A1}$$

$$\rho = \frac{K^* l_X^*}{\theta^*} - s^* l_R^* - \frac{\delta s K l_R}{K^*}. \tag{A2}$$

Respectively adding and subtracting Kl_R/θ and $K^* l_R^*/\theta^*$ from (A1) and (A2), these conditions can be rewritten as follows:

$$\triangle_1 + \triangle_2 = 0, \tag{A3}$$

$$\triangle_1^* + \triangle_2^* = 0, \tag{A4}$$

where

$$\triangle_1 = \frac{K(l_X - l_R)}{\theta} - \rho, \qquad \triangle_2 = \left[\frac{Kl_R}{\theta} - \frac{K^* l_R^*}{\theta^*}\right] \frac{\delta s^* \theta^*}{K},$$

$$\triangle_1^* = \frac{K^*(l_X^* - l_R^*)}{\theta^*} - \rho, \qquad \triangle_2^* = -\left[\frac{Kl_R}{\theta} - \frac{K^* l_R^*}{\theta^*}\right] \frac{\delta s \theta}{K^*}.$$

These conditions can be used to obtain s_K and s_R. First, in the steady-state equilibrium $\dot{\omega} = 0$ requires $Kl_R/\theta = K^* l_R^*/\theta^*$ and therefore $\triangle_1 = \triangle_1^* = \triangle_2 = \triangle_2^* = 0$. Next, using (16) to substitute l_X and l_X^* out of $\triangle_1 = \triangle_1^*$, we find that $K/\theta = K^*/\theta^*$. This implies that $l_R = l_R^*$ and thus we have $l_X = l_X^*$. Finally, the conditions $K/\theta = K^*/\theta^*$ and $l_X = l_X^*$ can be solved respectively for (21) and (22).

The no-arbitrage conditions (A3) and (A4) implicitly determine the numbers of firms based in each region for a given level of the productivity gap at each moment in time. Substituting in $l_R = l_X/(\sigma - 1) - l_F$ and $l_R^* = l_X^*/(\sigma - 1) - l_F$, from (16), taking the total derivatives of (A3) and (A4), and employing Cramer's rule yields

$$\frac{dn}{d\omega} = \frac{\frac{\partial(\triangle_1+\triangle_2)}{\partial\omega}\frac{\partial(\triangle_1^*+\triangle_2^*)}{\partial n^*} - \frac{\partial(\triangle_1+\triangle_2)}{\partial n^*}\frac{\partial(\triangle_1^*+\triangle_2^*)}{\partial\omega}}{\frac{\partial(\triangle_1+\triangle_2)}{\partial n}\frac{\partial(\triangle_1^*+\triangle_2^*)}{\partial n^*} - \frac{\partial(\triangle_1+\triangle_2)}{\partial n^*}\frac{\partial(\triangle_1^*+\triangle_2^*)}{\partial n}}, \tag{A5}$$

$$\frac{dn^*}{d\omega} = \frac{\frac{\partial(\triangle_1+\triangle_2)}{\partial n}\frac{\partial(\triangle_1^*+\triangle_2^*)}{\partial\omega} - \frac{\partial(\triangle_1+\triangle_2)}{\partial\omega}\frac{\partial(\triangle_1^*+\triangle_2^*)}{\partial n}}{\frac{\partial(\triangle_1+\triangle_2)}{\partial n}\frac{\partial(\triangle_1^*+\triangle_2^*)}{\partial n^*} - \frac{\partial(\triangle_1+\triangle_2)}{\partial n^*}\frac{\partial(\triangle_1^*+\triangle_2^*)}{\partial n}}, \tag{A6}$$

where

$$\begin{aligned}
\frac{\partial\triangle_1}{\partial n} &= \frac{\partial\triangle_1^*}{\partial n^*} = \frac{(1-\delta)(l_X-l_R)}{2N_D} - \frac{(\sigma-2)(1+\delta)(1+\varphi^2)l_X}{(\sigma-1)(1+\varphi)^2N_D}, \\
\frac{\partial\triangle_1}{\partial n^*} &= \frac{\partial\triangle_1^*}{\partial n} = -\frac{(1-\delta)(l_X-l_R)}{2N_D} - \frac{(\sigma-2)(1+\delta)\varphi 2l_X}{(\sigma-1)(1+\varphi)^2N_D}, \\
\frac{\partial\triangle_1}{\partial\omega} &= -\frac{\partial\triangle_1^*}{\partial\omega} = -\frac{\delta(l_X-l_R)}{2} + \frac{(\sigma-2)(1+\delta)\varphi l_X}{(1+\varphi)^2}, \\
\frac{\partial\triangle_2}{\partial n} &= -\frac{\partial\triangle_2^*}{\partial n^*} = -\frac{\partial\triangle_2}{\partial n^*} = \frac{\partial\triangle_2^*}{\partial n} = \frac{2(1-\delta)\delta l_R}{(1+\delta)N_D} - \frac{2\delta(1-\varphi)^2l_X}{(\sigma-1)(1+\varphi)^2N_D}, \\
\frac{\partial\triangle_2}{\partial\omega} &= -\frac{\partial\triangle_2^*}{\partial\omega} = \frac{2\delta\varphi l_X}{(1+\varphi)^2} - \frac{\delta^2 l_R}{1+\delta}.
\end{aligned}$$

Now, setting (A3) and (A4) equal, solving for $Kl_R/\theta - K^*l_R^*/\theta^*$, and substituting the result into (19), the motion for the productivity gap can be rewritten as

$$\frac{\dot\omega}{\omega} = \frac{(\triangle_1^*-\triangle_1)KK^*}{\delta(s\theta K + s^*\theta^*K^*)}. \tag{A7}$$

We approximate the local dynamics around the symmetric dispersed steady state ($\omega = 1$ and $n = n^*$) using a Taylor expansion of this differential equation:

$$\frac{\partial\dot\omega}{\partial\omega} = \frac{(1+\delta)}{2\delta}\left[\frac{\partial(\triangle_1^*-\triangle_1)}{\partial n}\frac{dn}{d\omega} + \frac{\partial(\triangle_1^*-\triangle_1)}{\partial n^*}\frac{dn^*}{d\omega} + \frac{\partial(\triangle_1^*-\triangle_1)}{\partial\omega}\right] = \frac{(1+\delta)}{\delta}\frac{\partial\triangle_2}{\partial\omega}.$$

Using (16) and (25) with $l_X = \frac{\alpha(\sigma-1)2L}{\sigma N_D}$, this Taylor expansion simplifies to

$$\frac{\partial\dot\omega}{\partial\omega} = \left[\frac{(\sigma-1)(1+\delta)2\varphi}{(1+\varphi)^2} - \delta\right]\left[\frac{2\rho-(1+\delta)l_F}{(\sigma-2)(1+\delta)}\right] + \delta l_F. \tag{A8}$$

As the productivity gap is a state variable, we require a negative sign for (A8) for a stable dispersed equilibrium. The sign of (A8) can be described as a function of the freeness of trade (φ) using

$$\varphi^2 + 2\varphi\left(1-\kappa\right) + 1 \geq 0, \tag{A9}$$

where $\kappa = \frac{(\sigma-1)(1+\delta)(2\rho-(1+\delta)l_F)}{\delta(2\rho-(\sigma-1)(1+\delta)l_F)} > 0$. This quadratic equation can be solved for two thresholds $\tilde{\varphi} = (\kappa-1) \pm \sqrt{(\kappa-2)\kappa}$. However, since $\kappa - 1 > 1$ is required for these thresholds to take real values, only one of them lies between zero and one: $\tilde{\varphi} = (\kappa-1) - \sqrt{(\kappa-2)\kappa}$. For $\varphi < \tilde{\varphi}$, the dispersed equilibrium is saddle path stable, and for $\varphi > \tilde{\varphi}$ it is unstable, as stated in Proposition 1.

Appendix B: Proof of Corollary 1

First, note that

$$\frac{d\tilde{\varphi}}{d\kappa} = 1 - \frac{\kappa-1}{\sqrt{(\kappa-2)K}} < 0,$$

and

$$\frac{d\kappa}{d\delta} = \frac{(\sigma-1)\left[(\sigma-1)(1+\delta^2)l_F(2\rho-l_F) - 2\rho(2\rho-(1-\delta^2)l_F)\right]}{\delta^2\left[2\rho-(\sigma-1)(1+\delta)l_F\right]^2}. \tag{B1}$$

The sign of $\frac{d\tilde{\varphi}}{d\kappa}\frac{d\kappa}{d\delta}$ depends on the sign of the numerator of (B1) which is determined by the sign of the following quadratic equation:

$$\delta^2 + 2\delta\left[1 + \frac{2\rho}{(\sigma-2)2\rho-(\sigma-1)l_F}\right] + 1 - \frac{4(1-l_F)\rho}{\left[(\sigma-2)2\rho-(\sigma-1)l_F\right]l_F}. \tag{B2}$$

Taking the partial derivative of (B2) with respect to δ and setting the result equal to zero, we can solve for a threshold value $\tilde{\delta}$, below which $\frac{d\tilde{\varphi}}{d\delta} > 0$ and above which $\frac{d\tilde{\varphi}}{d\delta} < 0$ as stated in Corollary 1.

Appendix C: Proof of Lemma 2

First, $\left(\frac{ds_K}{d\omega} - \frac{ds_R}{d\omega}\right)\Big|_{\omega=1} = \frac{\delta}{2(1-\delta)} - \frac{(\sigma-1)\varphi}{(1-\varphi)^2} > 0$ implies

$$\varphi^2 - 2\varphi\left[1 + \frac{(\sigma-1)(1-\delta)}{\delta}\right] + 1 > 0. \tag{C1}$$

Next, subtracting (A9) from (C1) we obtain

$$\frac{2\varphi(\sigma-2)\left[4\delta\rho + (\sigma-1)(1-\delta^2)l_F\right]}{\delta\left[2\rho - (\sigma-1)(1+\delta)l_F\right]} > 0,$$

for $\delta, \varphi \in (0,1)$ as we require $2\rho > (\sigma-1)(1+\delta)l_F$ for $g_D > 0$ in the symmetric steady state. We conclude that the slope of the s_K curve is always greater than that of the s_R curve for $\varphi < \tilde{\varphi}$ in the symmetric dispersed equilibrium. This completes the proof of Lemma 2.

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