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(Citation)
The Annals of the School of Business Administration, Kobe University,34:43-66

(Issue Date)
1990

(Resource Type)
departmental bulletin paper

(Version)
Version of Record

(JaLCDOI)
https://doi.org/10.24546/81003651

(URL)
https://hdl.handle.net/20.500.14094/81003651

VERTICAL MARKET STRUCTURE AND PRICE COMPETITION

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1. Introduction

This paper examines pricing behaviour of manufacturers and retailers in a differentiated product market with various types of consumers. Since Stigler (1961) draws our attention to the existence of price dispersion in a product class, a number of economic theorists, e.g., Rothschild (1973), Salop & Stiglitz (1977) and Varian (1980), have proposed models to explain this phenomenon. Salop & Stiglitz show a "two-price equilibrium" type of price dispersion; that is, only two prices can be sustained in the equilibrium. Varian yields a continuous equilibrium price distribution by assuming that firms use mixed strategies. These results come from the heterogeneity of individual consumers with respect to their search costs (or price-consciousness). Most of the models treat of the equilibrium price dispersion in a *homogeneous* product market. In my view, their analyses concern only the price dispersion of a particular brand of product at a retail level.

In a *differentiated* market, it is usually observed that there exist price differences among products. Price differences in the market generally come from the heterogeneity of consumers' brand-preferences. Although Bain (1968) has suggested some interesting points, the equilibrium configuration of prices in a differentiated market has not been fully explored through theoretical models.[1] In the present paper, this problem will be explored. With paying our attention both to the heterogeneity of consumer's price-consciousness and to the distribution of consumers' brand-preferences, we extend models of Salop & Stiglitz (1977) and Varian (1980).

On the one hand, each manufacturer captures some consumers' brand-loyalties through his sales-promotion, and has some monopoly power over

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1. See Bain (1968) (Chapter 7, pp. 223–250).

these consumers. But, recommendations from retailers to these consumers are not usually effective. If there are consumers who are both loyal to a particular brand and price-conscious about retail prices, then retailers are inclined to compete with each other in prices. That is, intra-brand price competition limits retailers' profits from sales to these consumers. On the other hand, when there are consumers who are not loyal to any brand, retailers will recommend the product which is the most profitable to them: i.e., the product with the lowest shipping price. Then there will arise inter-brand shipping price competition among manufacturers in order to gain their retailers' recommendation, which leads manufacturers' profits to be diminished. This suggests that the distribution of consumers' brand-preferences, or the degree of product differentiation of each manufacturer, governs the configuration of pricing behaviour of manufacturers and retailers.[2] It is the main purpose of this article to shed light on this issue.

The next section describes the concept of brand-preference and sets up the basic model. In section 3, by using the equilibrium concept of Nash-Stackelberg hybrid, the equilibrium pattern of transaction will be explored in several cases of consumers' brand-preferences. It will be shown that the market equilibrium takes only two forms; a single price equilibrium or a two-price equilibrium in terms of manufacturers' prices.

In some general case of consumers' brand-preferences, it will be shown that there exists a "two-price equilibrium". Some group of manufacturers set a monopolistic price and sell the products only to their brand-loyal consumers while another group of manufacturers set a competitive price and attract all non-brand-loyal consumers. As Varian (1980) pointed out, in the "two price equilibrium" of Salop & Stiglitz model, some stores are supposed to persistently sell their products at a lower price than other stores. If consumers can learn from experience, this persistence of price dispersion of homogeneous product seems implausible. The result of "two-price equilibrium" in this paper holds even when every consumer is informed, since this result will be derived from the heterogeneity of individual consumers with respect to brand-preferences. This is one of fundamental differences between my model and that of Salop & Stiglitz (1977). The

2. It is thus necessary to explicitly formulate a model including the distribution stage in a full analysis of manufacturers' pricing policies. On this point, the seminal work of Porter (1976) is suggestive, but the following analysis does not depend on his work.

price differences at the retail level reflect partly this aspect and partly another aspect of heterogeneity of consumer's price-consciousness.

It will also be made clear that there are some cases in which a Nash equilibrium does not exist. For these cases, by using the alternative "reactive equilibrium concept" due to Riley (1979), it will be shown that there exist "two-price equilibria" and "single-price equilibria".

Finally, following Varian (1980), the equilibrium distributions of prices will be examined when manufacturers use randomized pricing strategies. Then, some interesting results of comparative statics will be derived. Especially, price-conscious consumers themselves have external effects among consumers: the existence of price-conscious consumers, if they are non-brand-loyal, causes the external effect to *lower* prices as expected, but, if they are brand-loyal, causes the effect to *raise* prices. The latter point will be understood only when we can explicitly formulate a model of transactions between manufacturers and retailers. This result is due to my different formulation from Varian (1980). The final section will suggest the directions of further extensions and the area to which the results of this paper can be applied.

2. The Model

Suppose that there are several manufacturers $j \in J = \{ j \mid j = 1, 2, \ldots, n\}$ and retailers $i \in I = \{ i \mid i = 1, 2, \ldots, m\}$ who produce and distribute some kinds of consumer goods in a differentiated product class. Let H be the set of consumers, indexed by h. The total number of potential consumers is denoted by N. We assume that each consumer purchases none or just one unit of a product available in this product class. Hence, the total number of potential consumers is the same as the total level of potential demand. Depending on whether a consumer is loyal to a particular brand of this product or not, and whether he searches retail prices or not, the set of consumers H is divided into mutually exclusive four subsets:

$$
\begin{aligned}
H_{js} &= H_j \cap H_s, \\
H_{jn} &= H_j \cap H_n, \\
H_{os} &= H_o \cap H_s, \\
H_{on} &= H_o \cap H_n,
\end{aligned}
$$

where H_j : the set of consumers who are loyal to the j-th brand,
H_o : the set of consumers who are not loyal to any brand,
H_s : the set of consumers who searches retail prices, and
H_n : the set of consumers who do not search retail prices.

Denote by θ_j the ratio of all potential consumers loyal to the j-th brand to the total number of potential consumers N. Define $\theta = \Sigma_j \theta_j$. Then the number of potential consumers loyal to the j-th brand is given by $N\theta_j$ for all $j \in J$, and the number of potential consumers who are not loyal to any brand is given by $N(1 - \theta)$. Denote by δ_j the ratio of the number of consumers H_{js} to the number of consumers H_j. We also denote by δ_o the ratio of the number of consumers H_{os} to the number of consumers H_o. In what follows, we will treat the distribution of consumers' brand-preferences θ_j $(j = 1, 2, \ldots, n)$ and the ratio of price-conscious consumers δ_j $(j = 1, 2, \ldots, n)$ and δ_o as given parameters.

Ignoring income effects, the h-th consumer's valuation of the k-th brand is represented by his reservation price (i.e., the highest price the consumer is ready to pay for one unit), which is denoted by $p_r^h(k)$ and differs among consumers. For simplicity, we assume that for any consumer loyal to the j-th brand, $p_r^h(k) = 0$ if $k \neq j$; that is, he does not evaluate any other brand at all, and these consumers are unwilling to change their brands no matter what the differences between prices are there.[3] Among all consumers loyal to any j-th brand, each consumer's reservation price $p_r^h(j)$ differs, and which is assumed to be uniformly distributed in the closed interval $[0, \bar{p}_r]$ with equal density $(1/\bar{p}_r)$. We further assume that each non-brand-loyal consumer evaluates all brands equally. But, among non-brand-loyal consumers, $p_r^h(k)$ differs, and it is assumed to be uniformly distributed in $[0, \bar{p}_r]$ for any k. For notational convenience, we simply use the notation $f(p_r) = (1/\bar{p}_r)$ for the distribution of reservation prices among consumers. Given the parameter values $N, \theta_j, \theta_o, \delta_j$, and δ_o, the number of consumers whose reservation prices are higher than or equal to some price p is determined.[4] For example,

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3. Therefore, any consumers, who switch their brand in response to price differences among brands, are assumed to be classified into non-brand-loyal consumers. This specification will be a simplest possible way to represent consumers' brand-loyalties. I do not think this simplification loses an essential element of brand-loyalties.
4. We denote by $\#A$ the number of elements of the set A.

$$\#(H_{js} \cap \{ h \mid p_r^h(j) \geqq p \}) = N\theta_j \delta_j \int_p^{\bar{p}_r} f(p_r) dp_r$$
$$= N\theta_j \delta_j [\bar{p}_r - p] / \bar{p}_r$$

Now we will specify consumers' behaviour as follows: Each consumer $h \in H_{js}$ shops only among retailers who deal in the j-th brand, and buys from the lowest priced retailer. Each consumers $h \in H_{jn}$ chooses one of retailers at random who deal in the j-th brand, and buys it from the retailer. On the other hand, we have assumed that the consumers who are not loyal to any brands evaluate all brands equally. Hence, each consumer $h \in H_{os}$ searches all retail prices of all brands, and buys the product only at the retailer offering the lowest price. Furthermore, each consumer $h \in H_{on}$ visits only one retailer at random and buys the product recommended by this retailer. Here, it is assumed that each retailer can practice Pigouvian third-degree price discrimination in selling the product to consumers.[5]

Denote a shipping price and a quantity sold by the j-th manufacturer by p_j^m and q_j^m, respectively. Denote the average (= marginal) cost of production by c, which is simply assumed to be constant and the same for all manufacturers. Similarly, denote a retail price and a quantity of the j-th brand sold by the i-th retailer by p_{ij}^d and q_{ij}^d, respectively. It is also assumed that all retailers bear no cost except for the shipping prices charged by manufacturers.

We assume that each retailer sets his prices so as to maximize his profit, taking shipping prices of manufacturers and retail prices of others as given. Namely, each retailer is a Stackelberg follower with respect to manufacturers and a Nash-Bertrand player with respect to other retailers. Each manufacturer sets his shipping price so as to maximize his profit, taking shipping prices of others and the behavioural rule of each retailer as given. That is, each manufacturer is a Stackelberg leader with respect to retailers and a Nash-Bertrand player with respect to other manufacturers. When there is no incentive for any agent to change his strategy, given the above assump-

5. That is, it is assumed that each retailer can separate consumers of different types and set different prices for each groups of consumers. As a practical way to do this, we assume that each retailer employs a "dual pricing" with list price and discounted price for each good, and sells at a list price to the consumers H_n and at the discounted price to the consumers H_s. As later analyses will confirm, it must be noted that, under this retailers' dual pricing, our assumption on the behaviour of H_{on} is equivalent to assume that each consumer in H_{on} buys the product of the lowest list price in all products sold at a randomly chosen retailer.

tions, we call it a *market equilibrium*.[6] In what follows, this equilibrium concept will be employed. Here, it must be noted that, when focusing on the retail stage separately, the equilibrium configuration coincides with a usual Nash equilibrium.

3. The Analysis

In this section, we will explore the issue of transactions under the manufacturers' *open distribution policy*, i.e., manufacturers do not restrict the distribution channels of their products. Therefore, all retailers can deal in any brands at his discretion. The analysis will proceed from considering the four separate cases of consumers' brand-preferences listed below. Case (1), ($\theta_j = 0$ for all j): All consumers are non-brand-loyal. Case (2), ($\theta_j \neq 0$ for all j and $\Sigma_j \theta_j = 1$): Each manufacturer has his brand-loyal consumers and every consumer is brand-loyal to some manufacturers. Case (3), ($\theta_j \neq 0$ for all j and $\Sigma_j \theta_j < 1$): Each manufacturer has his brand-loyal consumers but there are some non-brand-loyal consumers. Case (4), ($\theta_j \neq 0$ for some j, $\theta_{j'} = 0$ for other j', and $\Sigma_j \theta_j < 1$): there is some manufacturer j who has his brand-loyal consumers, and other manufacturer j' who does not, and there are some consumers who are not loyal to any brands.

3.1. Case (1): $\theta_j = 0$ for all j

In this case, no consumer is loyal to any brand. Hence, each consumer $h \in H_s$ searches all retail prices of all brands and buys the lowest priced product. Each consumers $h \in H_n$ visits only one retailer at random and buys the lowest priced product of all products sold there. Therefore, each retailer deals with only the brand which has the lowest shipping price. Denote the lowest shipping price by $\underline{p}^m \equiv \min_{j \in J} p_j^m$. Then, due to the competition among retailers, the retail price to the consumers H_s goes down to $\underline{p}^m$. But, since each retailer can price-discriminate, the profit of any i-th retailer π_i^d from sales to the consumers H_n can be earned, and is formulated as

$$\pi_i^d = N[1 - \delta_o] \int_{p_i^d}^{\bar{p}_r} f(p_r)\, dp_r [p_i^d - \underline{p}^m]/m$$

6. The equilibrium concept of Nash-Stackelberg hybrid is employed in Salop & Stiglitz (1977).

$$= N[1-\delta_o]\,[\bar{p}_r - p_i^d]\,[p_i^d - \underline{p}^m]/(m\bar{p}_r), \qquad (1)$$

where it is assumed that each retailer has an equal number of visitors of H_n. The condition for profit maximization implies that the retail price to the consumers H_n is given by

$$p_i^d = [\bar{p}_r + \underline{p}^m]/2.$$

Any j-th manufacturer, taking retailers' behaviour as given, sets p_j^m so as to maximize his profit. Obviously, shipping price competition leads to the market equilibrium, $p_j^m = c$ for all j. The argument can be summarized as follows.

Proposition 1 If $\theta_j = 0$ for all j, then there prevail a *competitive* shipping price and two different retail prices in the market equilibrium:

$$\begin{array}{lll} p_j^m = c & & \text{for all } j, \\ p_i^d = c & \text{to the consumers } H_s & \text{for all } i, \\ p_i^d = [\bar{p}_r + c]/2 & \text{to the consumers } H_n & \text{for all } i. \end{array}$$

3.2. Case (2): $\theta_j \neq 0$ for all j and $\Sigma_j \theta_j = 1$.

In such a case, every consumer is loyal to his respective brand. Each consumer $h \in H_{js}$ searches all retail prices of the j-th brand and buys it at the lowest priced retailer. Hence, for the sales to the consumers H_{js}, owing to intra-brand price competition among retailers, it follows that $p_{ij}^d = p_j^m$ for all i and all j. Each consumer $h \in H_{jn}$ purchases the j-th brand from a randomly chosen outlet. Therefore, from the condition for profit maximization, each retailer sets his retail price $p_{ij}^d = [\bar{p}_r + p_j^m]/2$ for all i and j to the consumers H_{jn}. Taking these into consideration, any j-th manufacturer maximizes his profit π_j^m with respect to p_j^m,

$$\begin{aligned} \pi_j^m &= N\theta_j\delta_j \int_{p_j^m}^{\bar{p}_r} f(p_r)dp_r[p_j^m - c] \\ &\qquad + N\theta_j[1-\delta_j] \int_{p_{ij}^d}^{\bar{p}_r} f(p_r)dp_r\,[p_j^m - c] \\ &= N\theta_j[1+\delta_j]\,[\bar{p}_r - p_j^m]\,[p_j^m - c]/(2\bar{p}_r) \\ &= \hat{q}_j[p_j^m - c], \text{ where } \hat{q}_j = \hat{q}_j(p_j^m) = N\theta_j[1+\delta_j]\,[\bar{p}_r - p_j^m]/(2\bar{p}_r) \\ &\qquad \text{and } p_{ij}^d = [\bar{p}_r + p_j^m]/2. \end{aligned} \qquad (2)$$

From the condition for profit maximization, it follows that $p_j^m = [\bar{p}_r + c]/2$. The result can be summarized as follows.

Proposition 2 If $\theta_j \neq 0$ for all j and $\Sigma_j \theta_j = 1$, then there prevail a *monopolistic* shipping price and two different retail prices in the market equilibrium: for all i and all j,

$$p_j^m = [\bar{p}_r + c]/2$$

$$p_{ij}^d = [\bar{p}_r + c]/2 \quad \text{to the consumers } H_{js}$$

$$p_{ij}^d = [3\bar{p}_r + c]/4 \quad \text{to the consumers } H_{jn}.$$

In Case (1), where any manufacturer does not establish his brand name, owing to the shipping price competition among manufacturers in order to gain the retailers' recommendations of their brands, every manufacturer's profit eventually diminishes to zero. On the other hand, in Case (2), any manufacturer has a monopolistic power over his brand-loyal consumers. When there are price-conscious consumers, however, retail price competition arises within the brand and limits retailers' profits from sales to the brand-loyal consumers. In the similar terminology of Kaldor (1950), Case (1) corresponds to the *distributors' domination* and Case (2) to the *manufacturers' domination*. The above two cases are fundamental and quite easily tractable. We will now proceed to more sophisticated situations.

3.3. Case (3): $\theta_j \neq 0$ for all j and $\Sigma_j \theta_j < 1$.

According to our assumptions, each retailer can practice price discrimination. When selling the product to brand-loyal consumers, each retailer can set a price $[\bar{p}_r + p_j^m]/2$ to the consumers H_{jn}, but, each retailer has to set a price p_j^m to the consumers H_{js} due to the intra-brand price competition among retailers. When selling the product to non-brand-loyal consumers, each retailer sells the product of lowest shipping price $\underline{p}^m$ at a retail price $p_i^d = [\bar{p}_r + \underline{p}^m]/2$ to the consumers H_{on} and at $\underline{p}^m$ to the consumers H_{os}. When each manufacturer is able to adopt a discriminatory pricing policy in selling his product to retailers, it is obviously optimal for him to set

$$p_j^m = [\bar{p}_r + c]/2 \quad \text{for sales to his loyal consumers, and}$$

$$p_j^m = c \quad \text{for sales to non-loyal consumers.}$$

Because each retailer sells the lowest shipping priced product to the consumers H_o, shipping price competition among manufacturers leads to the price $\underline{p}^m = c$, but each manufacturer can set a monopolistic shipping price for the sales to his brand-loyal consumers. Hence, we have:

Proposition 3 If $\theta_j \neq 0$ for all j and $\Sigma_j \theta_j < 1$ and if each manufacturer can adopt a discriminatory pricing policy, then there prevail two different shipping prices in the market equilibrium: for all j,

$$p_j^m = [\bar{p}_r + c]/2 \quad \text{for sales to brand-loyal consumers,}$$

$$p_j^m = c \quad \text{for sales to non-loyal consumers,}$$

and there prevail different retail prices: for all i and all j,

$$p_{ij}^d = [\bar{p}_r + c]/2 \quad \text{to the consumers } H_{js},$$

$$p_{ij}^d = [3\bar{p}_r + c]/4 \quad \text{to the consumers } H_{jn},$$

$$p_{ij}^d = c \quad \text{to the consumers } H_{os},$$

$$p_{ij}^d = [\bar{p}_r + c]/2 \quad \text{to the consumers } H_{on}.$$

The casual empiricism suggests the way in which manufacturers practice price discrimination when selling their product to retailers. For example, the non-linear pricing in the form of *quantity discounts* may serve to this. That is, when any j-th manufacturer can perfectly monitor his demand, he can determine an optimal non-linear price schedule which sets a marginal price $\bar{p}^m \equiv [\bar{p}_r + c]/2$ to the total loyal consumers' demand $\hat{q}_j$ and the marginal price c to the larger demand than $\hat{q}_j$. See Figure 1. From equation (2),

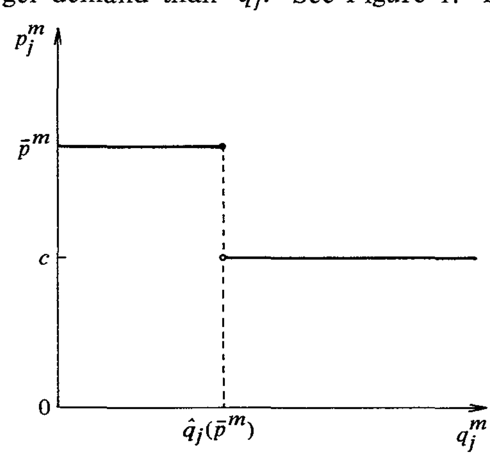

Figure 1: Quantity Discount

$\hat{q}_j$ is given by

$$\hat{q}_j = \hat{q}_j(\bar{p}^m) = N\theta_j[1+\delta_j][\bar{p}_r - c]/(4\bar{p}_r) \tag{3}$$

3.4. Case (4): $\theta_j \neq 0$ for some j, $\theta_j{}' = 0$ for some j', and $\Sigma_j \theta_j < 1$

When any j-th manufacturer who has his brand-loyal consumers can practice price discrimination, we can easily show:

Proposition 4 In Case (4), if any manufacturer with his established brand can practice price discrimination, then in the market equilibrium, his behaviour is the same as the one mentioned in Section 3.3. Whereas any j-th manufacturer without an established brand sets the price $p_j^m = c$, and sells his product only to non-brand-loyal consumers. At the retail level, there prevail the different prices as stated in Proposition 3.

Next, we will consider the case in which no manufacturer can price-discriminate. Though the full analysis will be undertaken in the next section, the following result can be easily shown:

Proposition 5 In Case (4), if no manufacturer can practice price discrimination and if $\#\{j \mid \theta_j = 0\} \geqq 2$, then, in the market equilibrium, any j-th manufacturer with his established brand sets $p_j^m = [\bar{p}_r + c]/2$ and sells his product only to his brand-loyal consumers. Whereas any manufacturer without an established brand sets $p_j^m = c$ and sells his product only to non-brand-loyal consumers. At the retail level, there prevail the different prices as stated in Proposition 3.

That is, when there are two or more manufacturers without established brands, the shipping price competition among them in order to sell their products to the non-brand-loyal consumers leads their shipping prices to $p_j^m = c$. When any j-th manufacturer such that $\theta_j \neq 0$ cannot price-discriminate, it would be more profitable for him to set a monopolistic shipping price $p_j^m = [\bar{p}_r + c]/2$ and to sell his product only to his brand-loyal consumers than to compete in prices with manufacturers without established brands. Where the market is not perfectly dominated by the existing manufacturers, there are rooms for new manufacturers to enter this market and sell their products to non-brand-loyal consumers with low-price policies. When there is no effective barrier preventing entry, the analysis of Case (4) shows that every existing manufacturer with his established brand holds high-price policy and abandons the part of market (H_o) to entrants.

3.5. Reactive Behaviour and Two-Price Equilibrium

The case in which no manufacturer can price-discriminate will be treated in more detail. Before examining the issue, it will be very useful to remark some points. First, from equation (2), we must note that any manufacturer, if setting the price $\bar{p}^m = [\bar{p}_r + c]/2$ and selling his product only to his brand-loyal consumers, can secure the profit

$$\bar{\pi}_j^m(\bar{p}^m) = N\theta_j[1+\delta_j][\bar{p}_r - c]^2/(8\bar{p}_r). \tag{4}$$

Second, when any j-th manufacturer with an established brand sets p_j^m which is the lowest and the number of lowest priced manufacturers is M, we denote his profit by $\pi_j^m(p_j^m(M))$. This profit is formulated as

$$\begin{aligned}\pi_j^m(p_j^m(M)) &= N\theta_j[1+\delta_j][\bar{p}_r - p_j^m][p_j^m - c]/(2\bar{p}_r)\\ &\quad + N[1-\theta][1+\delta_o][\bar{p}_r - p_j^m][p_j^m - c]/(2\bar{p}_r M).\end{aligned} \tag{5}$$

The p_j^m which satisfies conditions $\bar{\pi}_j^m(\bar{p}^m) = \pi_j^m(p_j^m(M))$ and $0 < p_j^m < \bar{p}^m$ is denoted by $\hat{p}_j(M)$. Then we can derive

$$\hat{p}_j(M) = [\bar{p}_r + c]/2 - \{[\bar{p}_r - c]\sqrt{1 - \theta_j[1+\delta_j]/A}\}/2 \tag{6}$$

where $A \equiv \theta_j[1+\delta_j] + [1-\theta][1+\delta_o]/M$.

This price will play an essential role in the following discussions. Here we must note the important properties of this price. From equation (6), it can be easily shown that as $\theta_j[1+\delta_j]$ is smaller and M is smaller, $\hat{p}_j(M)$ will be lower. We also define

$$\hat{p}(M) \equiv \min_j \hat{p}_j(M). \tag{7}$$

With these preliminaries, we can show:

Proposition 6 If no manufacturer can practice price discrimination, there exist no market equilibrium with pure strategies in Case (3) and in Case (4) where $\#\{j \mid \theta_j = 0\} = 1$.

[Proof] First, suppose that the lowest shipping price $\underline{p}^m \leq \hat{p}(1)$, where $\hat{p}(1)$ is given by equations (6) and (7) with $M = 1$. This supposition contradicts the assumption of equilibrium. The reason is that, in the equilibrium, only one manufacturer with the lowest $\theta_j[1+\delta_i]$ can profitably set the lowest price, but if this case arises, then there is an incentive for this manufacturer to raise his price since others set higher prices.

Next, suppose that $\underline{p}^m > \hat{p}(1)$. If there is only one manufacturer who sets $\underline{p}^m$, then he has an incentive to raise his price. If there are two or more manufacturers who set $\underline{p}^m$, then it must hold that $\underline{p}^m \geqq \hat{p}(2) > \hat{p}(1)$. Refer to Figure 2. But, in this situation, there will be an incentive for the manufacturer with the lowest $\theta_j[1+\delta_j]$ to lower his price slightly. Hence, the supposition $\underline{p}^m > \hat{p}(1)$ also contradicts the assumption of equilibrium.

Q.E.D.

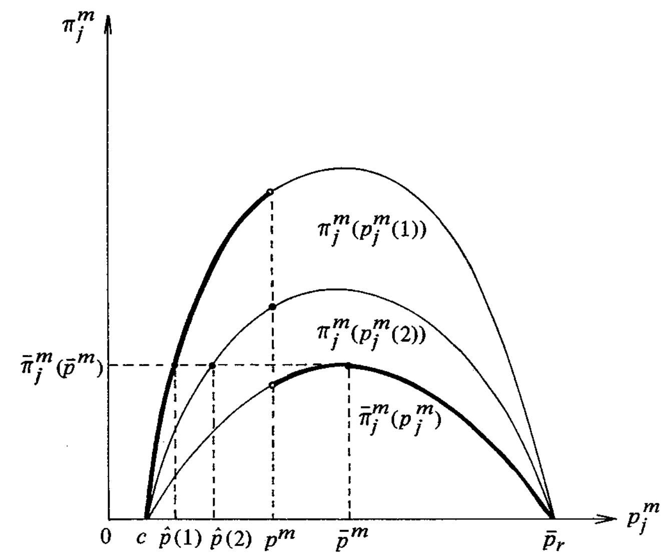

Figure 2

In the light of Proposition 6, we will now employ an alternative equilibrium concept and explore the issue further. For this purpose, we will define some preliminary concepts. Denote the present strategy of the j-th agent by s_j^* and the list of present strategies by $s^* = (s_j^*)_{j\in J}$. We will define,

$$S_j(s^*) \equiv \{s_j \mid s_j \text{ is feasible and } \pi_j(s_j, s^*_{)j(}) > \pi_j(s^*)\}$$

and we will call $S_j(s^*)$ the set of *better off strategies*, where $s^*_{)j(}$ is the list of present strategies except the j-th one. When any agent j changes his strategy s_j^* to $s_j \neq s_j^*$, we will define,

$$S_{j'}(s_j, s^*) \equiv \{s_{j'} \mid s_{j'} \text{ is feasible and } \pi_{j'}(s_j, s_{j'}, s^*_{)j,j'(}) > \pi_{j'}(s_j, s^*_{)j(})\}$$

and we will call $S_{j'}(s_j, s^*)$ the set of *credible counter strategies* of the j'-th agent to the change of the j-th agent's strategy. With these definitions we will introduce the following concepts.

Definition 1 (Reactive Equilibrium)
We call $s^* = (s_j^*)_{j \in J}$ a *reactive equilibrium*, if and only if

$$S_j(s^*) = \phi \text{ for any } j, \text{ or}$$

for any $s_j \in S_j(s^*)$, there exists some $s_{j'} \in S_{j'}(s_j, s^*)$ such that

$$\pi_j(s_j, s_{j'}, s^*_{)j,j'(}) < \pi_j(s^*).$$ [7]

Definition 2 (Reactive Behaviour)
Suppose that, given the list of strategies $(s_j^*)_{j \in J}$, any j-th agent changes his current strategy s_j^* to s_j if and only if there exists some $s_j \in S_j(s^*)$ such that $\pi_j(s_j, s_{j'}, s^*_{)j,j'(}) > \pi_j(s^*)$ for any $s_{j'} \in S_{j'}(s_j, s^*)$. Then we say that the j-th agent behaves in a *reactive* way.

Namely, we can say that the present state is a reactive equilibrium when there is no incentive for any agent to change his behaviour in the reactive way. From Definition 1, it can be easily seen that the Nash equilibrium list of strategies is always a reactive equilibrium, but the converse does not necessarily hold. The reactive equilibrium is a broader concept than the Nash equilibrium. Furthermore we will introduce:

Definition 3 (Reactive Market Equilibrium)
Suppose that any retailer, taking each manufacturer's shipping price as given, behaves in the reactive way to other retailers. Each manufacturer is supposed to behave in the Stackelberg leader way to retailers, and to behave in the reactive way to other manufacturers. When any agent has no incentive to change his behaviour, given the above assumptions, we call the state a *reactive market equilibrium.*

We will further explore the issues of transactions by using this equilibrium concept. Now we can show:

Proposition 7 The reactive market equilibria in Case (3) and in Case (4) would take only two forms; single-price equilibria or two-price equilibria in terms of manufacturers' prices.

[Proof] Suppose that, in the reactive market equilibrium, there prevails three different shipping prices p_1, p_2, p_3 such that $p_1 > p_2 > p_3$. Then,

7. This equilibrium concept slightly differs from the one proposed by Riley (1979).

any manufacturers who set p_1 or p_2 must sell their products only to their brand-loyal consumers. Note that the profits of these manufacturers in this case are maximized at the price $\bar{p}^m = [\bar{p}_r + c]/2$, and the fact $p_1 \neq p_2$ implies that there is some manufacturer who does not maximize his profit. This contradicts the assumption of equilibrium. Q.E.D.

Hence, we will examine the possibilities of two-price equilibria and single-price equilibria, separately. First, we will analyze two-price equilibria.

Proposition 8 If no manufacturer can price-discriminate in Case (3) and in Case (4) where $\#\{j \mid \theta_j = 0\} = 1$, then there exists a reactive market equilibrium such that:

(a) any j-th manufacturer, whose $\theta_j[1 + \delta_j]$ is not the lowest, sets $p_j^m = [\bar{p}_r + c]/2$ and sells his product only to his brand-loyal consumers,

(b) only one manufacturer with the lowest $\theta_j[1 + \delta_j]$ sets the lowest shipping price at $p_j^m = \underline{p}$ and sells his product to non-brand-loyal consumers (and also to his brand-loyal consumers in Case (3)), and other possible manufacturers with the lowest $\theta_j[1 + \delta_j]$ set $\bar{p}^m = [\bar{p}_r + c]/2$ and sell their products only to their brand-loyal consumers in Case (3), and

(c) there prevail retail prices such that: for all i and all j,

$$
\begin{aligned}
p_{ij}^d &= p_j^m && \text{to the consumers } H_{js}, \\
p_{ij}^d &= [\bar{p}_r + p_j^m]/2 && \text{to the consumers } H_{jn}, \\
p_i^d &= \underline{p} && \text{to the consumers } H_{os}, \\
p_i^d &= [\bar{p}_r + \underline{p}]/2 && \text{to the consumers } H_{on}.
\end{aligned}
$$

where $\underline{p}$ is defined as the second lowest value of $\hat{p}_j(1)$ given by equation (6) with $M = 1$, that is,

$$\underline{p} = \bar{p}^m - \{[\bar{p}_r - c]\sqrt{1 - \Omega}\}/2,$$

$$\Omega \equiv \frac{\theta_j[1 + \delta_j]}{\theta_j[1 + \delta_j] + [1 - \theta][1 + \delta_o]} \tag{8}$$

which is defined for the j-th manufacturer whose $\theta_j[1 + \delta_j]$ is the second lowest.

[Proof] In such a situation as stated above, there is a better off strategy for the lowest priced manufacturer to raise his price. However, for this better off strategy, there exists a credible counter strategy by the manufacturer whose $\theta_j[1+\delta_j]$ is the second lowest. Therefore, the payoff would be less than the present one as he takes this reaction into account. Next, for any other manufacturer's better off strategy of cutting price, there exists a credible counter strategy by the lowest priced manufacturer, then, taking this reaction into account, no one would cut his price. Moreover, it is also clear that there exists no better off strategy for any retailer. Hence, in this situation, there exists no incentive for any agent to change his behaviour in the reactive way. Q.E.D.

Remark 1: It must be noted that there are many other possibilities of two-price equilibria in Case (3) and in Case (4) where $\#\{j \mid \theta_j = 0\} = 1$. We do not explicitly formulate every case here, but we can readily seen that every possible two-price equilibrium has common properties: (1) the higher price is always a monopolistic price $\bar{p}^m$, and (2) the lower price always exceeds a competitive price c, even though it takes different values in each case.

Next, we will analyze single-price equilibria. As will be shown later, the possibilities of single-price equilibria depend on the number of manufacturers whose $\theta_j[1+\delta_j]$ is the lowest, which is denoted by $\#\{j \mid \theta_j[1+\delta_j]$ is the lowest$\}$. If $\#\{j \mid \theta_j[1+\delta_j]$ is the lowest$\} \geqq 2$, then we can show:

Proposition 9 If $\#\{j \mid \theta_j[1+\delta_j]$ is the lowest$\} \geqq 2$ in Case (3) and in Case (4), then there always exists a reactive market equilibrium in which every manufacturer sets a monopolistic price $\bar{p}^m$ and there prevail retail prices such that: for all i and all j

$$
\begin{aligned}
p_{ij}^d &= \bar{p}^m && \text{to the consumers } H_{js},\\
p_{ij}^d &= [\bar{p}_r + \bar{p}^m]/2 && \text{to the consumers } H_{jn},\\
p_i^d &= \bar{p}^m && \text{to the consumers } H_{os},\\
p_i^d &= [\bar{p}_r + \bar{p}^m]/2 && \text{to the consumers } H_{on}.
\end{aligned}
$$

[Proof] In such a situation as stated above, for any manufacturer's better off strategy of cutting his price, there exists a credible counter strategy by the manufacturer whose $\theta_j[1+\delta_j]$ is the lowest, and if he takes this reaction into account, his payoff would be less than the present one.

It is also clear that there exists no better off strategy for any retailer. Hence, in this situation, any agent has no incentive to change his behaviour in the reactive way. Q.E.D.

On the other hand, if $\#\{j \mid \theta_j[1+\delta_j]$ is the lowest$\} = 1$ both in (3) and in Case (4), then there does not necessarily exist a single-price equilibrium. In the following, we will derive the necessary and sufficient condition for the existence of single-price equilibria. Consider the single-price situation in which every manufacturer sets a same price p. In this situation, there exist better off strategies of cutting prices for any manufacturers. Then, we will denote by $\underline{p}(p)$ the lowest possible price among which all manufacturers can profitably set. That is, we will define

$$\underline{p}(p) \equiv \min_j \underline{p}_j(p) \tag{9}$$

where $\underline{p}_j(p)$ is the j-th manufacturer's lowest possible price, and $\underline{p}_j(p)$ is defined as the solution for p_j in the equation

$$\begin{aligned} &N\{\theta_j[1+\delta_j] + [1-\theta][1+\delta_o]/n\}[\bar{p}_r - p][p-c]/(2\bar{p}_r) \\ &= N\{\theta_j[1+\delta_j] + [1-\theta][1+\delta_o]\}[\bar{p}_r - p_j][p_j - c]/(2\bar{p}_r) \end{aligned} \tag{10}$$

and $0 < p_j < \bar{p}^m$. The left hand side of equation (10) is the j-th manufacturer's profit in the single price situation at p, and the right hand side is his profit at a price p_j when others set a price p $(p > p_j)$. We can derive

$$\underline{p}_j(p) = \bar{p}^m - \{[\bar{p}_r - c]\sqrt{1-\omega(p)}\}/2 \tag{11}$$

where

$$\omega = \omega(p) \equiv \left[\frac{\theta_j[1+\delta_j] + [1-\theta][1+\delta_o]/n}{\theta_j[1+\delta_j] + [1-\theta][1+\delta_o]}\right]\left[\frac{[\bar{p}_r - p][p-c]}{[\bar{p}_r - c]^2/4}\right]$$

From equations (9) and (11), it is easily seen that $\underline{p}(p) = \underline{p}_j(p)$ for the j-th manufacturer whose $\theta_j[1+\delta_j]$ is the lowest.

Next, for the cutting price strategy $\underline{p}(p)$ by the manufacturer with the lowest $\theta_j[1+\delta_j]$, the lowest possible counter price of any other manufacturers is just the $\underline{p}$ which is given in equation (8).

Then, we can state that there exists a single-price equilibrium, if and only if there exists some p such that $\underline{p}(p) > \underline{p}$ (or, equivalently, if and only if $\max_p \underline{p}(p) > \underline{p}$). From equations (9) and (11), we can easily show that the function $\underline{p}(p)$ is maximized with respect to p at $p = \bar{p}^m$, i.e., $\max_p \underline{p}(p) = \underline{p}(\bar{p}^m)$. Hence, we have:

Proposition 10 When $\#\{j \mid \theta_j[1+\delta_j]$ is the lowest$\} = 1$, there exists a single-price equilibrium in terms of manufacturers' prices both in Case (3) and in Case (4), if and only if

$$\max_p \underline{p}(p) > \underline{p}, \tag{12}$$

where

$$\max_p \underline{p}(p) = \underline{p}(\bar{p}^m) = \bar{p}^m - \{[\bar{p}_r - c]\sqrt{1-\omega(\bar{p}^m)}\}/2,$$

$$\omega(\bar{p}^m) \equiv \frac{\theta_j[1+\delta_j] + [1-\theta][1+\delta_o]/n}{\theta_j[1+\delta_j] + [1-\theta][1+\delta_o]} \tag{13}$$

which is defined for the j-th manufacturer whose $\theta_j[1+\delta_j]$ is the lowest,

$$\underline{p} = \bar{p}^m - \{[\bar{p}_r - c]\sqrt{1-\Omega}\}/2,$$

$$\Omega \equiv \frac{\theta_{j'}[1+\delta_{j'}]}{\theta_{j'}[1+\delta_{j'}] + [1-\theta][1+\delta_o]} \tag{14}$$

which is defined for the j'-th manufacturer whose $\theta_{j'}[1+\delta_{j'}]$ is the second lowest.

Remark 2: The single-price equilibrium, if it exists, would take several forms. But, Proposition 10 shows us that if there exist single-price equilibria, then the sate in which all of the manufacturers set a monopolistic price $\bar{p}^m$ is necessarily included in the single-price equilibria.

Here, we take note of the single-price equilibrium at a monopolistic price $\bar{p}^m$, where the joint profit of all manufacturers is maximized in this model. This situation is supported as an equilibrium not by manufacturers' collusive behaviour like a *price cartel* but by a non-cooperative mode of behaviour in a reactive way. This result suggests an interesting implication such that there would be a case in which the uniformity and stability of prices in a differentiated product class come from each manufacturer's non-myopic attitude toward his rivals.

3.6. Mixed Strategy and the Equilibrium Price Distribution

As a special case of Case (3), we will explore the symmetric case in which

$\theta_j = \hat{\theta}$ and $\delta_j = \hat{\delta}$ for all j, and $\Sigma_j \theta_j < 1$. In the light of Proposition 6, we will consider next the case where any manufacturer can use a mixed price strategy. We can show that there exists a symmetric market equilibrium in which each manufacturer uses the same mixed strategy. In order to show this, we must note the following. In view of equation (2), any j-th manufacturer, if setting the price $\bar{p}^m = [\bar{p}_r + c]/2$ and selling his product only to his brand-loyal consumers, can secure the profit

$$\bar{\pi}_j^m(\bar{p}^m) = N\hat{\theta}[1+\hat{\delta}][\bar{p}_r - c]^2/(8\bar{p}_r) \tag{15}$$

If any j-th manufacturer sets p_j^m which is the lowest and if the number of lowest priced manufacturers is only one, then by using equation (5) with $M = 1$, his profit can be expressed as

$$\begin{aligned}\pi_j^m(p_j^m) &= N\hat{\theta}[1+\hat{\delta}][\bar{p}_r - p_j^m][p_j^m - c]/(2\bar{p}_r) \\ &\quad + N[1-n\hat{\theta}][1+\delta_0][\bar{p}_r - p_j^m][p_j^m - c]/(2\bar{p}_r) \end{aligned} \tag{16}$$

Denote the p_j^m which satisfies conditions $\bar{\pi}_j^m(\bar{p}^m) = \pi_j^m(p_j^m)$ and $0 < p_j^m < \bar{p}^m$ by $\hat{p}(\hat{\theta})$. From equation (6) with $M = 1$, we can obtain

$$\hat{p}(\hat{\theta}) = [\bar{p}_r + c]/2 - \{[\bar{p}_r - c]\sqrt{1 - \hat{\theta}[1+\hat{\delta}]/\alpha}\}/2 \tag{17}$$

where $\alpha = \hat{\theta}[1+\hat{\delta}] + [1-n\hat{\theta}][1+\delta_o]$. With these preliminaries, we can show:

Proposition 11 In Case (3) where $\theta_j = \hat{\theta}$ and $\delta_j = \hat{\delta}$ for all j, there is a symmetric market equilibrium with mixed strategies such that any j-th manufacturer uses the following cumulative distribution function $F(p_j^m)$ for his mixed strategy:

$$F(p_j^m) = \begin{cases} 1 - \left[\dfrac{\bar{\pi}_j^m(\bar{p}^m) - \bar{\pi}_j^m(p_j^m)}{\pi_j^m(p_j^m) - \bar{\pi}_j^m(p_j^m)}\right]^{1/(n-1)} & \text{for } \hat{p}(\hat{\theta}) \leqq p_j^m \leqq \bar{p}^m \\ 0 & \text{for } p_j^m < \hat{p}(\hat{\theta}) \\ 1 & \text{for } p_j^m > \bar{p}^m \end{cases} \tag{18}$$

and there prevail different retail prices such that: for all i and all j,

$p_{ij}^d = p_j^m$ to the consumers H_{js},

$p_{ij}^d = [p_j^m + \bar{p}_r]/2$ to the consumers H_{jn},

$$p_i^d = \min_j p_j^m \equiv \underline{p}^m \quad \text{to the consumers } H_{os},$$

$$p_i^d = [\underline{p}^m + \bar{p}_r]/2 \quad \text{to the consumers } H_{on},$$

where

$$\bar{\pi}_j^m(\bar{p}^m) = N\hat{\theta}[1+\hat{\delta}][\bar{p}_r - c]^2/(8\bar{p}_r) \tag{19}$$

$$\bar{\pi}_j^m(p_j^m) = N\hat{\theta}[1+\hat{\delta}][\bar{p}_r - p_j^m][p_j^m - c]/(2\bar{p}_r) \tag{20}$$

$$\pi_j^m(p_j^m) = N\{\hat{\theta}[1+\hat{\delta}] + [1-n\hat{\theta}][1+\delta_o]\}[\bar{p}_r - p_j^m][p_j^m - c]/(2\bar{p}_r) \tag{21}$$

$$\bar{p}^m = [\bar{p}_r + c]/2 \tag{22}$$

$$\hat{p}(\hat{\theta}) = \bar{p}^m - \{[\bar{p}_r - c]\sqrt{1-\hat{\theta}[1+\hat{\delta}]/\alpha}\}/2 \tag{23}$$

[Proof] First, we can easily show that $F(\hat{p}(\hat{\theta})) = 0$ and $F(\bar{p}^m) = 1$, when substituting p_j^m by $\hat{p}(\hat{\theta})$ in equation (14) and using the equation $\pi_j^m(\hat{p}(\hat{\theta})) = \bar{\pi}_j^m(\bar{p}^m)$. Moreover, F is continuous and monotone non-decreasing. Hence, F is a cumulative distribution function. Next, suppose that some manufacturer uses pure strategies p_j^m which are higher than $\bar{p}^m$ while all others use F, then his expected profits $\bar{\pi}_j^m(p_j^m)$ are lower than $\bar{\pi}_j^m(\bar{p}^m)$. Similarly, pure strategies p_j^m which are lower than $\hat{p}(\hat{\theta})$ yield $\pi_j^m(p_j^m)$ lower than $\pi_j^m(\hat{p}(\hat{\theta})) = \bar{\pi}_j^m(\bar{p}^m)$ when all other manufacturers use F. Finally, suppose that some manufacturer j uses a pure strategy p_j^m such that $\hat{p}(\hat{\theta}) \leqq p_j^m \leqq \bar{p}^m$ when all other manufactures use F, then his expected profit is expressed as

$$[1 - F(p_j^m)]^{n-1}\pi_j^m(p_j^m) + \{1 - [1 - F(p_j^m)]^{n-1}\}\bar{\pi}_j^m(p_j^m) \tag{24}$$

Substituting $F(p_j^m)$ by the cumulative distribution function in Proposition 11, this expected profit coincides with $\bar{\pi}_j^m(\bar{p}^m)$. That is, there is no pure strategy which yields a higher expected profit than $\bar{\pi}_j^m(\bar{p}^m)$. Therefore, no mixed strategy which is a convex linear combination of pure strategies can yield a higher expected profit than $\bar{\pi}_j^m(\bar{p}^m)$. Any j-th manufacturer's expected profit is $\bar{\pi}_j^m(\bar{p}^m)$ when all manufacturers use F. F is the best reply when all other manufacturers use F. Consequently, $F(p_j^m)$ is a symmetric mixed equilibrium. Corresponding to these manufacturers' strategies, we can readily seen that there prevail different retail prices stated in Proposition 11. Q.E.D.

It is interesting to examine several issues of comparative statics concerning to parameter values $\hat{\theta}$, $\hat{\delta}$, and δ_o. First, we can show:

Proposition 12 It follows that $F(p_j^m \mid \hat{\theta}_1) \geqq F(p_j^m \mid \hat{\theta}_2)$ for any $\hat{\theta}_1$ and $\hat{\theta}_2$ such that $\hat{\theta}_1$, $\hat{\theta}_2 \in (0, 1/n)$ and $\hat{\theta}_1 < \hat{\theta}_2$.

[Proof] It can be seen that as $\hat{\theta}$ gets larger, $\hat{p}(\hat{\theta})$ increases, since

$$\partial \hat{p}(\hat{\theta})/\partial \hat{\theta} = [\bar{p}_r - c]\{1 - \hat{\theta}[1+\hat{\delta}]/\alpha\}^{-1/2}[1+\hat{\delta}][1+\delta_o]/(4\alpha^2) > 0. \quad (25)$$

As to the level of $F(p_j^m \mid \hat{\theta})$ we have

$$\partial F(p_j^m \mid \hat{\theta})/\partial \hat{\theta} = -B^{1/(n-1)}/\{[1 - n\hat{\theta}]\hat{\theta}[n-1]\} \leqq 0, \quad (26)$$

where

$$B \equiv \frac{\hat{\theta}[1+\hat{\delta}]\{[\bar{p}_r - c]^2/4 - \beta\}}{[1 - n\hat{\theta}][1+\delta_o]\beta}, \quad \beta \equiv [\bar{p}_r - p_j^m]\,[p_j^m - c].$$

Q.E.D.

In fact, the functional form of price distribution in the equilibrium can be explicitly solved as

$$F(p_j^m \mid \hat{\theta}) = 1 - B^{1/(n-1)} \quad (27)$$

and is illustrated by Figure 3.

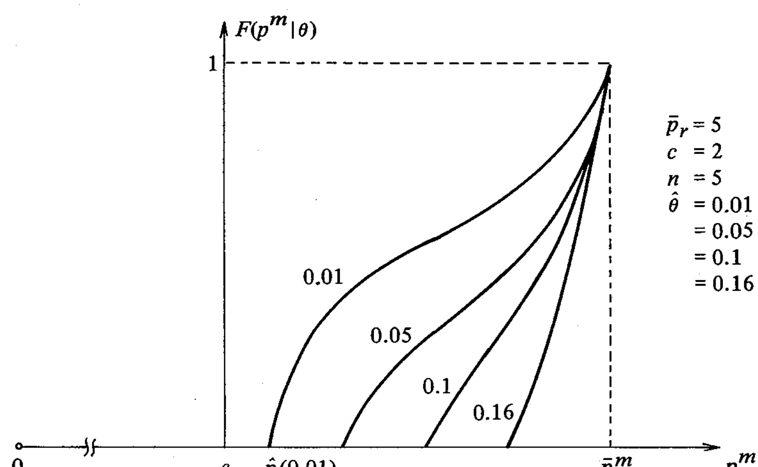

Figure 3: Cumulative Distribution Functions

This proposition shows that the larger is the ratio of brand-loyal consumers $\hat{\theta}$, the higher is the probability of setting a relatively high price, hence, the equilibrium shipping prices rise in the average sense.

Furthermore, the equilibrium density function of prices can be explicitly solved as

$$f(p_j^m \mid \hat{\theta}) = dF(p_j^m \mid \hat{\theta})/dp_j^m$$
$$= \{1/(n-1)\} B^{1/(n-1)-1} \frac{\hat{\theta}[1+\hat{\delta}][\bar{p}_r - c]^2 \beta'}{4[1+\delta_o][1-n\hat{\theta}]\beta^2} \qquad (28)$$

where $\beta' \equiv -2p_j^m + [\bar{p}_r + c] \geqq 0$. This is illustrated by Figure 4.

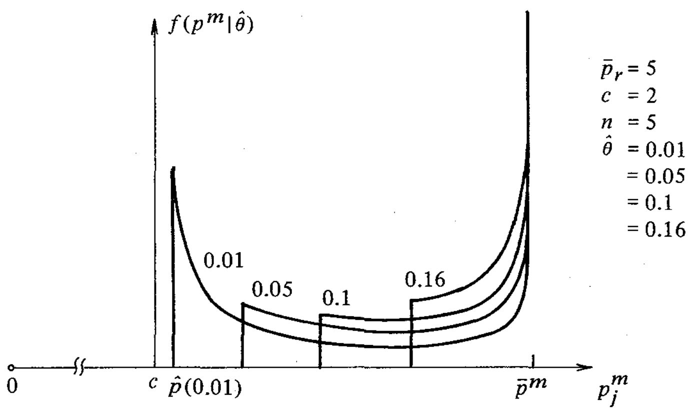

Figure 4: Probability Density Functions

In Figure 4, as $\hat{\theta}$ gets smaller (that is, as the ratio of non-brand-loyal consumers gets larger), we find that relatively large probability densities are attached both to the neighborhood of the competitive price c so as to capture non-brand-loyal consumers and to the neighborhood of the monopolistic price $\bar{p}^m$ so as to exploit the brand-loyal consumers, and relatively small probability densities are given in the middle range of prices. This results can be interpreted as a manufacturer's *probabilistic use of dual pricing.*

Next, we can show:

Proposition 13 It follows that

$F(p_j^m \mid \hat{\delta}_1) \geqq F(p_j^m \mid \hat{\delta}_2)$ for any $\hat{\delta}_1$ and $\hat{\delta}_2$ such that $\hat{\delta}_1, \hat{\delta}_2 \in (0, 1)$ and $\hat{\delta}_1 < \hat{\delta}_2$, and

$F(p_j^m \mid \delta_o) \leqq F(p_j^m \mid \delta_o')$ for any δ_o and δ_o' such that $\delta_o, \delta_o' \in (0, 1)$ and $\delta_o < \delta_o'$.

[Proof] It can be seen that as $\hat{\delta}$ gets larger, $\hat{p}(\hat{\theta})$ increases, since

$$\partial\hat{p}(\hat{\theta})/\partial\hat{\delta} = [\bar{p}_r - c]\{1 - \hat{\theta}[1+\hat{\delta}]/\alpha\}^{-1/2} \frac{\hat{\theta}[1-n\hat{\theta}][1+\delta_o]}{4\alpha^2} > 0. \quad (29)$$

As to the level of $F(p_j^m \mid \hat{\delta})$, we have

$$\partial F(p_j^m \mid \hat{\delta})/\partial\hat{\delta} = -B^{1/(n-1)}/\{[n-1][1+\hat{\delta}]\} \leqq 0. \quad (30)$$

And also, as δ_o gets larger, $\hat{p}(\hat{\theta})$ decreases, since

$$\partial\hat{p}(\hat{\theta})/\partial\delta_o = -[\bar{p}_r - c]\{1 - \hat{\theta}[1+\hat{\delta}]/\alpha\}^{-1/2} \frac{\hat{\theta}[1+\hat{\delta}][1-n\hat{\theta}]}{4\alpha^2} < 0. \quad (31)$$

As to the level of $F(p_j^m \mid \delta_o)$ we have

$$\partial F(p_j^m \mid \delta_o)/\partial\delta_o = B^{1/(n-1)}/\{[n-1][1+\delta_o]\} \geqq 0. \quad (32)$$

Q.E.D.

When $\hat{\delta}$ is getting larger, with other conditions as given, there are two opposite effects: the quantity sold to the consumers H_{js} increases but the quantity sold to the consumers H_{jn} decreases. However, overall the quantity demanded by the brand-loyal consumers increases. Hence, each manufacturer attaches a relatively higher probability density to the neighborhood of the monopolistic price so as to exploit the brand-loyal consumers. This leads to a higher shipping price on the average. On the other hand, when δ_o gets larger, with other conditions given, the demand for any manufacturer's product by non-brand-loyal consumers increases. Hence, each manufacturer attaches a relatively high probability density to the neighborhood of the competitive price so as to capture these non-brand-loyal consumers. Consequently, the average shipping price decreases. The analysis in this section has such an interesting implication that price-conscious consumers themselves have the opposite external effects on consumers as a whole. The existence of price-conscious consumers, if they are non-brand-loyal, causes the external effect which *lowers* the shipping prices as expected. If they are brand-loyal, however, it causes the external effect which *raises* the shipping prices. The main reason for the latter case can be explained by the existence of *successive monopoly* over the consumers H_{jn}; i.e., the retail prices set by retailers are higher than the level desired by manufactures. However, as $\hat{\delta}$ gets larger, the loss caused by the problem of successive monopoly is mitigated. That is, the demand for the product by brand-loyal consumers increases. Hence, for any manufacturer, the sales to his brand-loyal consumers is getting more valuable than the sales to non-brand-loyal consumers, which leads each manufacturer to attach higher probabilities to higher shipping prices.

4. Concluding Remarks

One of the major results of this paper concerns the existence of price dispersion for a differentiated product class, either in retail prices and/or in manufacturers' prices. The price dispersion reflects differences both of individual consumers' price-consciousness and brand-preferences.

The secondary result is the non-existence of a Nash equilibrium in some cases of consumers' brand-preferences in which no manufacturer can price-discriminate. Then, by using an alternative concept of equilibrium: the reactive equilibrium due to Riley (1979) which is a broader concept than a usual Nash equilibrium, we have characterized the equilibria. Furthermore, we investigate the equilibrium price distribution when manufacturers use randomized pricing strategies in Section 3.6. Then we have shown some interesting results: especially, price-conscious consumers themselves have two opposite external effects among consumers as a whole.

Thirdly, we have explored the aspect of *market* transaction between manufacturers and retailers. The issue of *closed* distribution policies of manufacturers which restrict their distribution channels and retailers' selling behaviours are not dealt here, but this paper seems to provide a basic model to set up such theoretical investigations. As the attempt of applying the result of this paper is given elsewhere (see Maruyama (1988)), we will show some points briefly. Let us note a general case in Section 3.4 when $\#\{j \mid \theta_j = 0\} \geqq 2$. Proposition 5 shows two important characteristics of "market" transaction between manufacturers and retailers: (1) any manufacturers with established brands cannot sell their products to the consumers who are indifferent to any brand, and (2) the problem of *successive monopoly* arises when selling them to the consumers who insists on a particular brand but do not search retail prices, that is, the retail price set to these consumers becomes higher than the desired level by manufacturers. However, if some manufacturer prohibits his agents from dealing in any other brands under *exclusive dealing arrangement*, then the manufacturer is able to sell his product to the consumers H_{on} who happen to come to his agents. Moreover, if the manufacturer sets an upper bound for retail prices and prohibits retailers from selling his product at a price higher than the upper bound, then this *standard price system* would solve the problem of successive monopoly.

Finally, we will state some limitation of this analysis and show the direction of further research. We have assumed, for simplicity, that each brand-loyal consumer does not switch his brand regardless of any price difference between brands, but there usually exist also brand-loyal consumers who evaluate other brands in some degree. Hence it may be interesting to explore further the problem of consumers' brand-switching in response to the price differences. In relation to this, the issue of product selection by manufacturers in a differentiated market may be worth-while to consider. These problems are left unsolved here.

Received August 7, 1989.

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