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R&D Subsidies, International Knowledge Dispersion, and Fully Endogenous Productivity Growth

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Abstract

This paper develops a two country model to investigate the effects of national R&D subsidies on aggregate product variety and endogenous productivity growth without scale effects. In particular, monopolistically competitive firms invest in process innovation with the aim of lowering production costs. With imperfect knowledge dispersion, the larger of the two countries has a larger share of firms and a greater level of productivity. The higher concentration of relatively productive firms increases the size of knowledge flows between firms, leading to an increase in firm-level employment in innovation. As a result, an economy with asymmetric countries produces a faster rate of growth than one with countries of similar size. The larger scale of firm-level innovation activity reduces market entry, however, and overall product variety falls. Using this framework, we find that a national R&D subsidy has a positive effect on the industry share, relative productivity, and wage rate of the implementing country. Moreover, if the smaller country introduces an R&D subsidy, overall product variety rises but the rate of productivity growth falls. Alternatively, if the larger country introduces an R&D subsidy, the rate of productivity growth rises, but overall product variety may rise or fall. Finally, we briefly consider the effects of a national R&D subsidy on national and world welfare levels.

Key Words: R&D Subsidy, Knowledge Dispersion, Productivity Growth, Scale Effect
JEL Classifications: F43; O30; O40

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1 Introduction

The discovery of novel ideas in the form of new products and production processes is broadly recognized to be an essential driver of economic growth. As such, policy makers are perpetually looking for opportunities to accelerate growth by stimulating research and development (R&D). In response, a diverse literature investigating the effects of economy policy designed to promote R&D activity has been developed with the objective of providing guidelines for policy makers. In this paper, we contribute to this literature with a study of the effects of national R&D subsidies on economic growth in the presence of imperfect knowledge dispersion.

A key theme of the innovation-based endogenous growth literature has been the role of technical knowledge dispersion amongst firms, and between regions and countries. Indeed, it is the non-rival nature of knowledge as an input in R&D that allows for perpetual economic growth when the knowledge created by current innovation activity generates an intertemporal knowledge spillover that raises the labor productivity of all future R&D efforts (Romer, 1990). Given then the central role of knowledge dispersion in growth theory, a large body of empirical work has set out to verify the existence and measure the extent of knowledge spillovers. While there is now convincing evidence that knowledge spillovers do occur, their strength appears to diminish significantly with distance (Jaffe et al., 1993; Mancusi, 2008; and Coe et al., 2009).[1] It is this localized aspect of knowledge dispersion that we focus on in our examination of the effects of national R&D subsidies.

Early comparative analysis of the policy implications of global versus local knowledge spillovers can be found in Rivera-Batiz and Romer (1991) and Grossman and Helpman (1991, chp. 8). In these studies, an improvement in the flow of ideas between countries, as a result of greater economic integration, raises labor productivity in R&D, thereby bolstering the rate of innovation. Within this context, we find that an exogenous reduction in

[1] See Keller (2004) for more detail on the various channels through which technical knowledge diffuses between firms and across regions.

innovation costs induces a reallocation of resources from production to innovation activity, leading us to conclude that R&D subsidies affect economic growth positively, with the strength of this effect increasing in the level of knowledge dispersion.

One potential concern with these results, however, is that they are inherently tied to a scale effect: a common feature of first-generation growth models whereby the growth rate is positively related to population size, with an increase in the labor force raising the number of researchers employed in innovation. In fact, there is a host of empirical evidence that generally rejects the existence of scale effects (Jones, 1995a; Dinopoulus and Thompson, 1999; Barro and Sala-i-Martin, 2004; Lainez and Peretto, 2006), indicating that an appraisal of national R&D policy within a framework that corrects for scale effects and allows for imperfect knowledge dispersion would be a valuable exercise. Along these lines, Minniti and Parello (2011) introduce diminishing technical opportunities into the intertemporal spillover associated with knowledge accumulation in a two country framework. This setting results in a second-generation growth model (Jones, 1995b; Kortum, 1997; Segerstrom, 1998), within which the rate of innovation depends solely on the rate of population growth and the rate of diminishing technical opportunities, thus leaving no role for either imperfect knowledge dispersion or R&D policy in the promotion of economic growth. In contrast, we develop a fully endogenous growth framework (Smulders and van de Klundert, 1991; Peretto, 1996; Aghion and Howitt, 1998; Dinopoulus and Thompson, 1998; Young, 1998) that accounts for scale effects while preserving a role for imperfect knowledge spillovers and R&D policy. As there is now strong evidence supporting the validity of the fully endogenous growth approach over the semi-endogenous growth approach (Zachariadis 2003, 2004; Lainez and Peretto, 2006; Ha and Howitt, 2007; Madsen 2008, 2010; Madsen et al. 2010a; Madsen et al. 2010b), this paper contributes to the literature with a reexamination of the implications for R&D policy when knowledge dispersion is incomplete, within an empirically robust framework.

In particular, we develop a two country model in which monopolistically competitive firms invest in process innovation with the objective of increasing firm value through improvements in their production technologies that lower unit costs. We suppose that labor productivity in process innovation reflects the average productivity of production technologies observable by the researchers employed at each firm. Then, with imperfect knowledge dispersion, firm-level employment in innovation determines the relative production scales and productivities of firms producing in each country. In order for innovation to occur in both countries, the costs of innovation must equalize, inferring that a rise in the share of firms producing in a given country must be paralleled by an increase in these firms' relative productivities. We thus find that the country with the larger market, as measured by national expenditure, attracts a larger share of firms, each with a greater level of productivity than the firms producing in the smaller country.

The positive relationship between relative productivity and the share of firms producing in a given country has important implications for overall product variety and the aggregate rate of productivity growth. Specifically, the greater concentration of relatively productive firms in the larger country generates a higher level of productivity in flows of technical knowledge between firms, thereby raising R&D productivity and inducing a greater level of employment in process innovation. With both labor productivity and labor employment rising at the firm level, we find that an economy with asymmetric market sizes will have a greater rate of productivity growth than an economy with symmetric market sizes. As the overall level of market entry is determined by optimal R&D employment, however, the lower cost of in-house process innovation associated with symmetric markets induces greater market entry such that the symmetric equilibrium allows for the largest number of firms, and hence the greatest level of product variety.

Our framework has interesting implications for the effects of national R&D subsidies on firm-level innovation activity. For example, consider the case where one country finances an R&D subsidy to the process innovation of domestic firms by collecting a lump-sum income tax from domestic households. The national R&D subsidy has a positive effect on the industry share, relative productivity, and wage rate of the implementing country, causing adjustments in its terms of trade. The effects of the R&D subsidy on overall product variety and aggregate productivity growth, however, depend on whether the implementing country initially has the smaller or larger market. If the smaller of the two countries introduces an R&D subsidy, the larger country's share of firms and relative productivity will fall, pushing the economy towards the symmetric equilibrium, causing a rise in the level of product variety, and inducing a fall in productivity growth. On the other hand, if the larger country implements the R&D subsidy, its share of firms and relative productivity will rise, thus pushing the economy away from the symmetric equilibrium and raising productivity growth. In this case, however, overall product variety may rise or fall.

While the large number of opposing effects associated with the R&D subsidy prevent a general analysis of welfare issues, we are able to extract several conclusions from our framework using simple numerical examples. First, when a national R&D subsidy is implemented by the larger country, its level will be too low from the perspective of the smaller country. Second, when countries are symmetric, there is no incentive for either country to promote domestic innovation activity, although both countries would prefer to have the other country provide an R&D subsidy. Third, if the smaller country implements an R&D subsidy, its own welfare level will fall, but the welfare of the larger country may rise or fall depending on parameter values. In general, these results suggest that it may be possible to raise average world welfare through policy coordination, particularly if the larger country agrees to provide an R&D subsidy that maximizes average world welfare. The results are, however, closely related to the level of knowledge spillovers with convergence in optimal subsidy rates when there is perfect knowledge dispersion.

The remainder of this paper proceeds as follows. In Section 2 we introduce our model of trade and endogenous productivity growth. Section 3 provides a characterization of long-run equilibrium, and Section 4 investigates the effects of R&D subsidies on the rate of growth and the level of product variety. In Section 5, we discuss the implications of R&D

subsidies for national and global welfare levels. Section 6 concludes.

2 Basic Model

This section develops a single sector model of trade in which firms produce horizontally differentiated product varieties and invest in in-house research with the aim of improving their production technologies. There are two countries, home and foreign, which are symmetric with the exception of population sizes, L and L^*, where an asterisk is used to denote variables associated with the foreign country. Investment capital is perfectly mobile between countries, but there is no international labor migration. Home-based firms receive an R&D subsidy, which is paid for with a lump-sum tax on home households. In what follows, we focus on the home setup, but similar conditions can be derived for foreign.

2.1 Households

Each country is populated by dynastic households that chooses an optimal expenditure-saving path in order to maximize lifetime utility over an infinite time horizon. The lifetime utility of a home household is

$$U = \int_0^\infty e^{-\rho t} \ln C(t) dt, \tag{1}$$

where $C(t)$ is household consumption at time t and $\rho > 0$ is the rate of time preference. Consumption takes the form of a constant elasticity of substitution quantity index:

$$C(t) = \left(\int_0^{N(t)} c_i(t)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

where N is the total number of product varieties, c_i is the consumption of a representative variety i, and $\sigma > 1$ is the elasticity of substitution between any two varieties.

At each moment in time, a home household allocates a given level of expenditure E

across all available product varieties: $N = n + n^*$, where n and n^* are respectively the numbers of product varieties produced in home and foreign. Thus, a home household's demand for a representative product variety m is

$$c_m(t) = \frac{p_m(t)^{-\sigma} E(t)}{\int_0^{n(t)} p_i(t)^{1-\sigma} di + \int_0^{n^*(t)} p_j^*(t)^{1-\sigma} dj}, \qquad m \in N, \tag{2}$$

where p_i and p_j^* are the respective prices of home and foreign produced varieties. It then follows that the price index associated with aggregate consumption can be defined as

$$P(t) = \left(\int_0^{n(t)} p_i(t)^{1-\sigma} di + \int_0^{n^*(t)} p_j^*(t)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \tag{3}$$

Thus, household expenditure is the product of the price and consumption indices: $E = PC$.

Households maximize lifetime utility (1) subject to the following intertemporal budget constraint:

$$\int_0^{\infty} e^{-\int_0^t r(s)ds} E(t) dt \leq \int_0^{\infty} e^{-\int_0^t r(s)ds} \left(w(t) - T(t) \right) dt + B(0),$$

where r is the interest rate, w is the wage rate, T is a lump-sum tax, and B is asset holdings in the home country.[2]

Substituting $C = E/P$ into lifetime utility (1), and solving the households optimization problem, we derive the following Euler equation:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho, \tag{4}$$

where a dot over a variable denotes time differentiation. Perfect capital mobility ensures a common evolution for home and foreign expenditures as the interest rate is equalized between countries: $\dot{E}/E = \dot{E}^*/E^* = r - \rho$.

[2]The value of asset holdings (B) will be driven to zero by free entry in the manufacturing sector. See Section 2.5 for more details.

Foreign households face a similar utility maximization problem and therefore have similar demand conditions. Time notation is suppressed where possible in order to economize on notation.

2.2 Production

There is a mass of firms locating in each country, with each firm producing a unique horizontally differentiated variety and competing according to Dixit and Stiglitz (1977) monopolistic competition. While no cost is incurred in market entry, in addition to production costs, each period incumbent firms face fixed costs in both the management (l_F) and the implementation (l_R) of innovation activity.

Home and foreign firms employ l_X and l_X^* units of labor with the following respective production technologies:

$$x = \theta l_X, \qquad\qquad x^* = \theta^* l_X^*, \tag{5}$$

where x and x^* are firm-level outputs, and θ and θ^* are firm-specific productivity coefficients. We suppose that firm-level productivity is symmetric within a country, but may differ across countries.

Given its current production technology, each firm produces to meet the combined demands of both countries. For example, the output of a home firm is $x = c_i L + c_i^* L^*$. The large number of firms operating in each market eliminates strategic interaction between firms as they set their optimal production levels. It thus follows that, under monopolistic competition, each firm maximizes operating profit on sales π ($\equiv px - wl_X$) by setting price equal to a constant markup over unit cost, that is,

$$p = \frac{\sigma w}{(\sigma - 1)\theta}, \qquad\qquad p^* = \frac{\sigma w^*}{(\sigma - 1)\theta^*}, \tag{6}$$

for home and foreign firms, respectively. Accordingly, optimal operating profit on sales can be obtained as $\pi = wl_X/(\sigma-1)$. Substituting the pricing rules (6), the demand conditions (2), and the production functions (5) with aggregate world expenditure $E^W \equiv EL + E^*L^*$ into $x = c_iL + c_i^*L^*$, and reorganizing the result, we obtain the following expressions for optimal firm levels of employment in production:

$$l_X = \frac{(\sigma-1)p^{1-\sigma}E^w}{\sigma w P^{1-\sigma}}, \qquad l_X^* = \frac{(\sigma-1)p^{*1-\sigma}E^w}{\sigma w^* P^{1-\sigma}}, \tag{7}$$

where $P = P^*$. These conditions determine the optimal production scales of home and foreign firms.

2.3 Process Innovation

Each firm invests in process innovation with the objective of raising profit through technology improvements that raise the productivity (θ) of production. In particular, the productivities of home and foreign firms evolve according to

$$\dot{\theta} = Kl_R^{\gamma}, \qquad \dot{\theta}^* = K^* l_R^{*\gamma}, \tag{8}$$

where l_R and l_R^* are firm-levels of employment in innovation, K and K^* are the labor productivities associated with innovation, and $\gamma \in (0,1)$ is a parameter that ensures diminishing marginal products to R&D investment.

A key aspect of the innovation process is the application of production processes employed by other incumbent firms. Following the in-house process innovation literature (Smulders and van de Klundert, 1995; Peretto, 1996), we assume that technical knowledge regarding production processes accumulates within the firm and can be proxied for using the productivity coefficients θ and θ^*. This allows us to model the labor productivities of home and foreign firms in in-house process innovation as a weighted average of the productivity of currently observable technical knowledge:

$$K = s\theta + \delta s^* \theta^*, \qquad\qquad K^* = s^*\theta^* + \delta s\theta, \tag{9}$$

where $s \equiv n/N$ and $s^* = 1 - s \equiv n^*/N$. The parameter $\delta \in (0,1)$ regulates the level of international knowledge spillovers. As δ approaches unity perfect knowledge spillovers arise between countries, and as δ approaches zero knowledge spillovers become completely national in scope.[3] Improvements in production technology raise the level of knowledge, thereby lowering the future cost of process innovation, and generating perpetual growth.

A firm's intertemporal optimization problem entails choosing the level of investment in process innovation that maximizes the future flow of profits. Total per-period profit equals operating profit on sales minus the cost of investment in process innovation l_R and the per-period fixed cost l_F:

$$\Pi = \pi - (1-\beta)w(l_R + l_F), \qquad\qquad \Pi^* = \pi^* - w^*(l_R^* + l_F), \tag{10}$$

where $\beta \in [0,1)$ captures the R&D subsidy rate provided to firms in the home country. We assume that per-period fixed costs are symmetric across countries ($l_F = l_F^*$).

Firms choose the optimal level of employment in innovation with the objective of maximizing firm value. For example, a home firm sets l_R to maximize $V = \int_0^\infty \Pi(\theta,t)e^{-\int_0^t r(s)ds}dt$ subject to the technology constraint (8). This intertemporal optimization problem can be solved using the following current-value Hamiltonian function: $H = \Pi + \mu K l_R^\gamma$, where μ is the current shadow value of an improvement in the productivity of production. Supposing that market shares are small enough that each firm takes the price index (3) and expenditure levels as constant when considering the impact of changes in its price on its profit flow, the first order conditions are $\partial H/\partial l_R = 0$ and $\partial H/\partial \theta = r\mu - \dot{\mu}$. These can be used to solved

[3] This method of modeling imperfect knowledge dispersion is adapted from Baldwin and Forslid (2000).

for the following no-arbitrage conditions:

$$r = \frac{\gamma l_X l_R^{\gamma-1} K}{(1-\beta)\theta} + \frac{(1-\gamma)\dot{l_R}}{l_R} + \frac{\dot{w}}{w} - \frac{\dot{K}}{K}, \quad r^* = \frac{\gamma l_X^* {l_R^*}^{\gamma-1} K^*}{\theta^*} + \frac{(1-\gamma)\dot{l}_R^*}{l_R^*} + \frac{\dot{w}^*}{w^*} - \frac{\dot{K}^*}{K^*}. \quad (11)$$

These conditions determine the optimal levels of R&D employment for home and foreign firms, respectively.

2.4 Government

As discussed above, the home government finances the R&D subsidy using a lump-sum tax on households. Aggregating the subsidies provided to home firms, a balanced government budget leads to a lump-sum tax of

$$TL = n\beta w(l_R + l_F), \quad (12)$$

where T is tax per household, and the lefthand side thus denotes aggregate tax revenue. The righthand side represents the total R&D subsidy payment. As we focus on the effects of R&D subsidies provided to firms in home, the R&D subsidy rate is set to zero in foreign, that is, $\beta^* = T^* = 0$.

2.5 Market Equilibrium

With zero costs incurred in product development and free market entry, per-period profits are driven to zero in both home and foreign. As a consequence, household expenditures are determined solely as a function of wages earned. To see this, first note that full labor employment requires

$$L = n(l_X + l_R + l_F), \qquad L^* = n^*(l_X^* + l_R^* + l_F). \quad (13)$$

Then, setting per-period profits (10) equal to zero, the free entry conditions for home and foreign based firms are

$$l_X = (\sigma - 1)(1 - \beta)(l_R + l_F), \qquad\qquad l_X^* = (\sigma - 1)(l_R^* + l_F). \tag{14}$$

Next, using (13) and (14), in the home country a balanced government budget leads to a lump-sum tax on each home household of $T = w\beta/(1 + (\sigma - 1)(1 - \beta))$. Once again, R&D subsidies are not provided in foreign. Thus, home and foreign expenditures are

$$E = (1 - \eta(\beta))w, \qquad\qquad E^* = w^*, \tag{15}$$

where $\eta(\beta) \equiv \beta/(1 + (1 - \beta)(\sigma - 1))$ is the effective tax rate; $\eta(0) = 0$ and $\eta'(\beta) > 0$. Thus, from (4), we have $\dot{w}/w = \dot{E}/E = \dot{w}^*/w^* = \dot{E}^*/E^* = r - \rho$ at all moments in time.

3 Steady-State Equilibrium

This section characterizes the long-run equilibrium of the economy in terms of the relative productivity of home firms and the share of firms based in home. We use this characterization of equilibrium to examine the effects of changes in relative labor endowments and the R&D subsidy rate on relative productivity and firm-level employment in process innovation.

We begin by defining the relative productivity of home firms as follows:

$$\tilde{\theta} \equiv \theta/\theta^*. \tag{16}$$

The evolution of relative productivity is determined by the difference between the home and foreign productivity growth rates, as seen from the time derivative of (16):

$$\frac{\dot{\tilde{\theta}}}{\tilde{\theta}} = \frac{\dot{\theta}}{\theta} - \frac{\dot{\theta}^*}{\theta^*} = \frac{K l_R^\gamma}{\theta} - \frac{K^* l_R^{*\gamma}}{\theta^*}, \tag{17}$$

where we have used the technology constraints (8).

As we are interested in a steady state with a constant allocation of labor across production and innovation activities, given a constant level of R&D employment, the rate of productivity growth is constant and common across countries in long-run equilibrium. Thus, setting the evolution of relative productivity (17) equal to zero yields

$$\frac{Kl_R^{\gamma}}{\theta} = \frac{K^* l_R^{*\gamma}}{\theta^*}. \tag{18}$$

With productivity growth equalized across countries, an examination of the time derivatives of K and K^* then reveals that $\dot{\theta}/\theta = \dot{\theta}^*/\theta^* = \dot{K}/K = \dot{K}^*/K^*$ in equilibrium.

Next, we consider the equilibrium investment conditions. Combining $\dot{\theta}/\theta = \dot{K}/K$ and $\dot{\theta}^*/\theta^* = \dot{K}^*/K^*$ with the technology constraints (8) and the free entry conditions (14) yields the steady-state no-arbitrage conditions for home and foreign as follows:

$$\rho = R(l_R) \equiv \left[\gamma(\sigma-1)\left(1+\frac{l_F}{l_R}\right)-1\right]\frac{l_R^{\gamma}K}{\theta}, \tag{19}$$

$$\rho = R^*(l_R^*) \equiv \left[\gamma(\sigma-1)\left(1+\frac{l_F}{l_R^*}\right)-1\right]\frac{l_R^{*\gamma}K^*}{\theta^*}, \tag{20}$$

where $R(l_R)$ and $R^*(l_R^*)$ represent the internal rates of return on investment in process innovation. Invoking (18), it is clear from these conditions that $l_R = l_R^*$. Therefore, from the free entry conditions (14), we also have $K/\theta = K^*/\theta^*$ and $l_X/(1-\beta) = l_X^*$ in long-run equilibrium.

We are now ready to solve for relative productivity. The equilibrium conditions enable the derivation of two expressions for the share of firms based in home (s). The first is obtained directly using the technology constraints (9) in $K/\theta = K^*/\theta^*$ to solve for $s = s(\tilde{\theta})$. The second is found using $l_R = l_R^*$ and the free-entry conditions (14) in the labor market conditions (13) to solve for $s = s(L/L^*, \beta)$. Setting these solutions equal yields the following condition for the steady-state value of $\tilde{\theta}$:

$$s = \frac{1-\delta\tilde{\theta}^{-1}}{1-\delta\tilde{\theta}+1-\delta\tilde{\theta}^{-1}} = \frac{L/L^*}{L/L^* + (1+(1-\beta)(\sigma-1))/\sigma}. \tag{21}$$

Thus, as illustrated in Figure 1, relative productivity is determined implicitly by the relative labor endowment and the R&D subsidy rate: $\tilde{\theta} = \tilde{\theta}(L/L^*, \beta)$. Proportionate changes in the labor endowments have no effect on relative productivity. Moreover, setting $L/L^* = 1$ and $\beta = 0$ leads to a symmetric equilibrium with $s = 1/2$ and $\tilde{\theta}(1,0) = 1$. A change in the R&D subsidy rate has a similar effect to a change in the relative labor endowment (L/L^*), and accordingly $s > L/(L+L^*)$ for $\beta > 0$. Note that with perfect knowledge dispersion ($\delta = 1$), the $s(\tilde{\theta})$ curve becomes a vertical line and productivity is symmetric across countries ($\tilde{\theta} = 1$). We summarize the determinants of relative productivity in the following lemma.

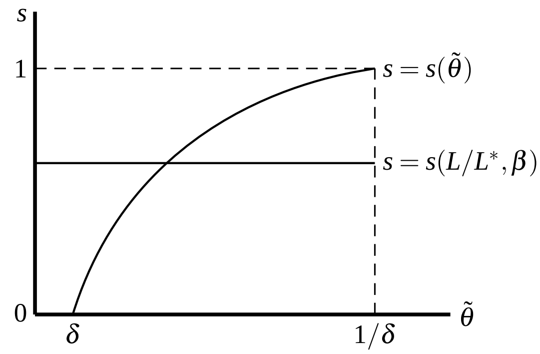
Lemma 1 *Increases in the relative labor endowment (L/L^*) and the R&D subsidy rate (β) raise the relative productivity of home firms.*

Proof: Total differentiation of (21) gives

$$\frac{d\tilde{\theta}}{d(L/L^*)} = \frac{(1+(1-\beta)(\sigma-1))(\delta-\tilde{\theta})^2}{\sigma\delta(1-\delta^2+(\tilde{\theta}-\delta)^2)(L/L^*)^2} > 0,$$
$$\frac{d\tilde{\theta}}{d\beta} = \frac{(\sigma-1)(\delta-\tilde{\theta})^2}{\sigma\delta(1-\delta^2+(\tilde{\theta}-\delta)^2)(L/L^*)} > 0.$$

Changes in L/L^* and β affect relative productivity through adjustments in the shares of firms locating in each country. For instance, returning to Figure 1, consider a rise in the R&D subsidy rate. The $s = s(L/L^*, \beta)$ line shifts upwards resulting in an increase in the relative productivity of home firms. Intuitively, the increase in the R&D subsidy lowers per-period fixed costs for home firms leading to an increase in profits and raising the share of firms locating in home. This rise in home's share of firms coincides with an increase in relative productivity that brings knowledge spillovers back into balance, $K/\theta = K^*/\theta^*$, as the economy returns to steady-state equilibrium.

Figure 1: Firm Shares and Relative Productivity



Next, we consider the relationship between relative productivity and knowledge spillovers, as it will provide an important link in the determination of firm-level R&D employment. Substituting $s = s(\tilde{\theta})$ from (21) into the expression for observable knowledge (9), and reorganizing the result, yields the equilibrium level of relative knowledge spillovers as follows:

$$\frac{K}{\theta} = \frac{K^*}{\theta^*} = \frac{1-\delta^2}{2-\delta\tilde{\theta}-\delta\tilde{\theta}^{-1}}. \tag{22}$$

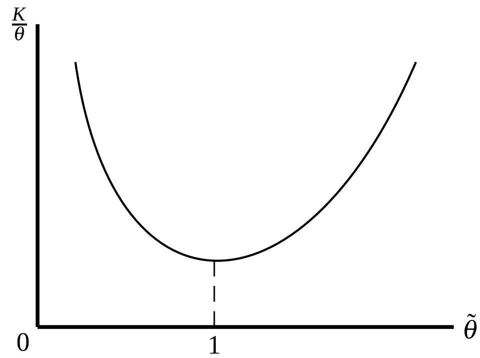
This equilibrium condition provides the following result:

Lemma 2 *The steady-state level of relative knowledge spillovers is convex in relative productivity with a minimum at* $\tilde{\theta} = 1$.

As shown in Figure 2, an increase in relative productivity lowers relative knowledge spillovers for $\tilde{\theta} < 1$ and raises relative knowledge spillovers for $\tilde{\theta} > 1$. From (21), we can see that $\tilde{\theta} = 1/\delta$ when all firms locate in home ($s = 1$), and that $\tilde{\theta} = \delta$ when all firms locate in foreign ($s = 0$). Thus, it is clear that an increase in the concentration of industry in one country raises labor productivity in R&D activity. This result is a common feature of new economic geography models that assume imperfect knowledge dispersion between countries (Baldwin and Martin, 2004).

Finally, we investigate the relationship between firm-level innovation employment and the internal rate of return. Figure 3 provides an illustration of the no-arbitrage condition for

Figure 2: Relative Knowledge Spillovers



home. At every moment in time, firms choose optimal levels of in-house innovation with the aim of maximizing firm value. Thus, when the internal rate of return $R(l_R)$ is greater than the risk free rate of return $\rho(=r)$, firms increase R&D employment, and when $R(l_R) < \rho$, they reduce R&D employment. This investment behavior implies that the internal rate of return should be decreasing in R&D employment, as depicted by the negative slope of $R(l_R)$ in Figure 3, in order to ensure a positive and finite level of employment in process innovation, thereby ensuring a balanced growth path. In the Appendix, we examine the local dynamics of the economy around a symmetric long-run equilibrium for $L/L^* = 1$, and $\beta = 0$. We find that if $2 > (\sigma-1)(1-\delta+2(1+\delta)\gamma)$, then

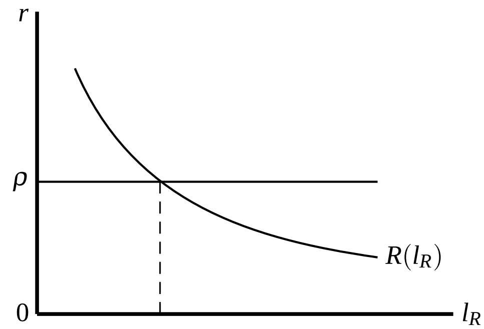
$$\frac{\partial R(l_R)}{\partial l_R} = -\left[1-\gamma(\sigma-1)+\frac{(1-\gamma)(\sigma-1)l_F}{l_R}\right]\frac{\gamma l_R^{\gamma-1}K}{\theta} < 0, \tag{23}$$

and the symmetric equilibrium is saddle-point stable.[4] Henceforth, we characterize the long-run equilibrium and investigate the determinants of productivity growth and product variety under the assumption that $\partial R(l_R)/\partial l_R < 0$.

With relative productivity determined as a function of the relative labor endowment and the R&D subsidy rate, and relative knowledge spillovers determined as a function of relative productivity, it is now possible to examine the effects of changes in L/L^* and β on

[4]See condition (A6) in the Appendix.

Figure 3: In-house R&D Employment



firm-level employment in process innovation.

Lemma 3 *Increases in the relative labor endowment (L/L^*) and the R&D subsidy rate (β) lower firm-level employment in innovation for $\tilde{\theta} < 1$ and raise it for $\tilde{\theta} > 1$.*

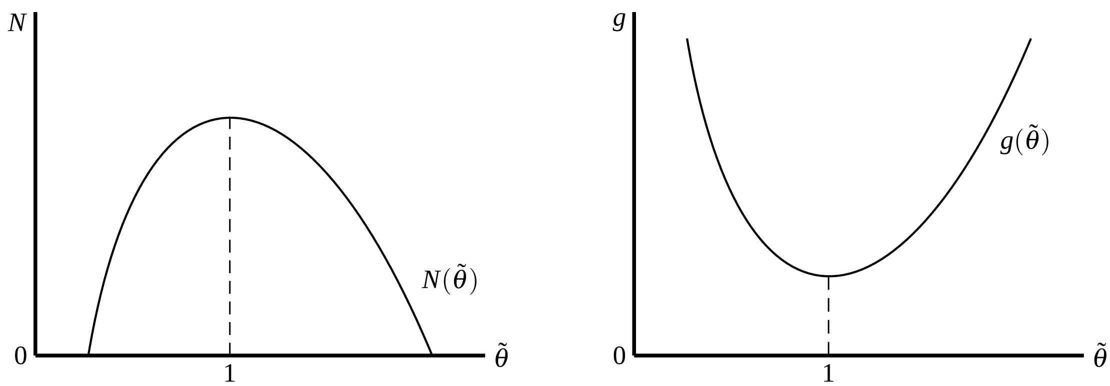
Proof: Substituting (22) into (19) and taking the derivative with respect to l_R and $\tilde{\theta}$ yields:

$$\frac{dl_R}{d\tilde{\theta}} = -\frac{1}{R'(l_R)} \frac{\rho\delta(1-\tilde{\theta}^{-2})(K/\theta)}{1-\delta^2}.$$

This derivative can be signed using (23) and Lemma 1.

Changes in L/L^* and β affect l_R indirectly through $\tilde{\theta}$ and subsequently K/θ. For example, consider the effect of an increase in the R&D subsidy rate. When home has a greater share of firms and $\tilde{\theta} > 1$, the rise in β increases $\tilde{\theta}$, thereby raising relative knowledge spillovers. Returning to Figure 3, the internal rate of return to process innovation $R(l_R)$ shifts upward and firms increase employment in process innovation until the internal rate of return falls back to the risk free level ρ. On the other hand, when home has a smaller share of firms and $\tilde{\theta} < 1$, the increase in β shifts the $R(l_R)$ curve downwards and firms reduce innovation employment.

Figure 4: Product Variety and Productivity Growth



(a) Total number of Firms (b) Rate of Productivity Growth

4 Product Variety and Productivity Growth

This section investigates the effects of changes in the home R&D subsidy on equilibrium product variety and the rate of productivity growth.

Combining the labor market conditions (13) with the free-entry conditions (14) and the steady-state condition $l_R = l_R^*$, we can solve for the equilibrium level of product variety as

$$N = \frac{1}{l_R + l_F}\left(\frac{L}{1+(1-\beta)(\sigma-1)} + \frac{L^*}{\sigma}\right). \tag{24}$$

As illustrated in Figure 4a, equilibrium product variety and relative productivity have a concave relationship with a maximum occurring at $\tilde{\theta} = 1$. The mechanics of this relationship can be understood by beginning from a symmetric equilibrium, in which productivity is equalized across countries, and then considering the effects of a change in relative productivity. Either an increase or a decrease in $\tilde{\theta}$ raises relative knowledge spillovers, and firm-level innovation employment rises, leading to an increase in per-period fixed costs. As a result, a portion of firms exit the market and equilibrium product variety falls.

Investigating the effects of the home R&D subsidy on equilibrium product variety, we obtain the following result.

Proposition 1 *For $\tilde{\theta} \leq 1$, an increase in the R&D subsidy rate (β) raises product variety. For $\tilde{\theta} > 1$, however, product variety may rise or fall.*

Proof: Taking the total derivative of (24) and applying Lemma 3, we have

$$\frac{dN}{d\beta} = \frac{(\sigma-1)L}{(l_R+l_F)(1+(1-\beta)(\sigma-1))^2} - \frac{N}{l_R+l_F}\frac{dl_R}{d\beta}.$$

The R&D subsidy has two effects on equilibrium product variety. The first is a positive direct effect that shifts the $N(\tilde{\theta})$ curve upward in Figure 4a. A rise in β directly lowers per-period fixed costs causing a decrease in firm scale, through the free entry conditions (14), and allowing the market to support a larger number of firms for all levels of relative productivity. The second, in contrast, is an indirect effect that moves the economy along the $N(\tilde{\theta})$ curve as firms adjust their optimal investment levels in response to the change in relative knowledge spillovers which results from the increase in the R&D subsidy rate. Invoking Lemma 3, through changes in l_R, an increase in β will raise N for $\tilde{\theta} < 1$ and reduce N for $\tilde{\theta} > 1$. Thus, we conclude that when home has the larger number of firms and $\tilde{\theta} < 1$, an R&D subsidy can be used to raise product variety, but for $\tilde{\theta} > 1$ the overall effect will be ambiguous.

Next, we calculate the steady-state rate of productivity growth. Substituting the no-arbitrage conditions (19) into the innovation function (8) and manipulating the result yields

$$g \equiv \frac{\dot{\theta}}{\theta} = \frac{l_R^{\gamma}K}{\theta} = \frac{\rho}{\gamma(\sigma-1)-1+\gamma(\sigma-1)l_F/l_R}. \tag{25}$$

An examination of (25) shows that the rate of productivity growth is determined independently of the overall size of the labor force, and is therefore not biased by a scale effect, as $\tilde{\theta} = \tilde{\theta}(L/L^*, \beta)$ is unaffected by proportionate changes in the labor endowments. Intuitively, an increase in the overall labor force is absorbed fully by a rise in the number of incumbent firms (24). Then, although the aggregate level of employment associated with innovation activity $(l_R + l_F)N = L/(1+(1-\beta)(\sigma-1)) + L^*/\sigma$ rises with proportionate increases in L and L^*, firm-level employment in process innovation remains unchanged. As such, our model does not include a scale effect and is consistent with empirical evidence.

Figure 4b depicts the convex relationship between productivity growth and relative productivity, with the rate of productivity growth minimized at $\tilde{\theta} = 1$. Starting once again from a symmetric equilibrium in which productivity is equalized across countries, either a rise or a fall in relative productivity increases relative knowledge spillovers, leading to increases in both labor employment and labor productivity in process innovation. As a consequence, the rate of productivity growth rises. Intuitively, as investment in process innovation rises, per-period fixed costs $(l_R + l_F)$ increase and firms are pushed out of the market. With the number of incumbent firms decreasing, the resources required to cover economy-wide per-period fixed costs Nl_F fall, and the amount of labor available for employment in innovation rises.

We examine the relationship between the home R&D subsidy and productivity growth and obtain the following proposition:

Proposition 2 *An increase in the R&D subsidy (β) lowers productivity growth for $\tilde{\theta} < 1$ and raises productivity growth for $\tilde{\theta} > 1$.*

The R&D subsidy only effects productivity growth indirectly through changes in relative productivity. Returning once again to Lemma 1, $d\tilde{\theta}/d\beta > 0$ and an increase in the R&D subsidy leads to a rightward movement along the $g(\tilde{\theta})$ curve in Figure 4b. Accordingly, the R&D subsidy depresses productivity growth for $\tilde{\theta} < 1$ and accelerates it for $\tilde{\theta} > 1$. Note that the R&D subsidy has no effect on productivity growth for a symmetric equilibrium evaluated at $\beta = 0$.

5 Welfare Analysis

The welfare issues associated with our model are rather complex, given the opposing effects of a national R&D subsidy on product variety, productivity growth, and the terms of trade. In this section, we provide a simple discussion of welfare and use a numerical example to show the optimal levels of the home R&D subsidy for the home, foreign, and world economies.

Although the relative wage drops out of the model in the equilibrium analysis of the preceding two sections, it assumes a role in the welfare effects associated with adjustments in the terms of trade. Substituting the pricing rules (6), the price index (3), and optimal firm-level employment in production (7) into $l_X/(1-\beta) = l_X^*$ and simplifying yields the relative wage rate as $w/w^* = (1-\beta)^{-\frac{1}{\sigma}}\tilde{\theta}(L/L^*,\beta)^{\sigma-1}$, where $w/w^* = 1$ in the symmetric equilibrium for $L/L^* = 1$ and $\beta = 0$. Hence, the terms of trade for the home country are

$$\frac{p}{p^*} = \left[(1-\beta)\tilde{\theta}(L/L^*,\beta)\right]^{\frac{1}{\sigma}}.$$

Thus, while the direct effect of an increase in the R&D subsidy rate is a deterioration of home's terms of trade, a rise in relative productivity will lead to an improvement in p/p^*.

Moving on to national welfare levels, the present values of utility flows to home and foreign households are found by substituting (3), (6), (15), (21), and the terms of trade into lifetime utility (1):

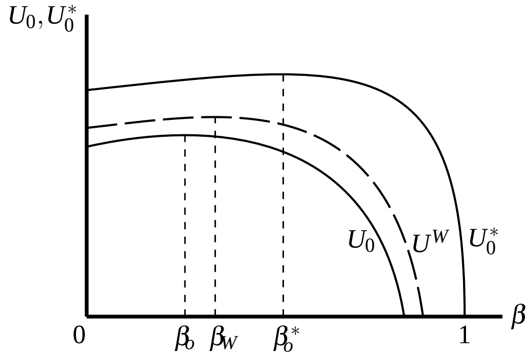
$$\rho U_0 = \ln(1-\eta(\beta))A + \frac{1}{(\sigma-1)}\ln\left[s(\beta) + (1-s(\beta))\left[(1-\beta)\tilde{\theta}\right]^{-\frac{\sigma-1}{\sigma}}\right]N + \frac{g}{\rho}, \quad (26)$$

$$\rho U_0^* = \ln A^* + \frac{1}{(\sigma-1)}\ln\left[s(\beta)\left[(1-\beta)\tilde{\theta}\right]^{\frac{\sigma-1}{\sigma}} + (1-s(\beta))\right]N + \frac{g}{\rho}, \quad (27)$$

where $A = (\sigma-1)\theta(0)/\sigma$ and $A^* = (\sigma-1)\theta^*(0)/\sigma$ are constants, and $s(\beta)$ is defined in the second expression of (21).

Focusing again on the home country, a change in the R&D subsidy affects welfare

Figure 5: Optimal Home R&D Subsidies



Parameter values are L/L^* = 2, σ = 1.75, δ = 0.5, γ = 0.5, l_F = 0.01, and ρ = 0.02.

through four separate channels. First, $\eta(\beta)$ captures the negative income effect of the lump-sum tax placed on home households to pay for the R&D subsidy. Next, the second term in parenthesis describes the terms of trade effect that results from changes in production shares (s), the relative wage rate (w/w^*), and relative productivity ($\tilde{\theta}$). The third term is the effect of the R&D subsidy on the level of product variety. Finally, the fourth term describes the effect of the R&D subsidy on the productivity growth rate. The opposing effects of the home R&D subsidy on each of these terms makes a general analytical analysis of welfare intractable. As an alternative, we use a simple numerical example to discuss the implications of changes in the R&D subsidy for specific cases.

Figure 5 plots home and foreign welfare against the home R&D subsidy rate for the case where $L/L^* = 2$, and the population of home is twice that of foreign. In this case, home hosts a greater share of firms, which are relatively more productive, that is, $s > 1/2$ and $\tilde{\theta} > 1$. The home R&D subsidy rates that maximize home and foreign welfare levels are respectively β_0 and β_0^*, and accordingly, we can see that home will set the subsidy rate at a lower level than is optimal for foreign. This suggests an opportunity for policy coordination between home and foreign with the aim of maximizing the welfare level of the average household. Denoting welfare for the average household in the world economy by $U^W \equiv (L/(L+L^*))U_0 + (L^*/(L+L^*))U_0^*$, the optimal home subsidy rate would then become β_W, as depicted in Figure 5. Thus, as $\beta_W > \beta_o$, average world welfare can be increased by raising the home R&D subsidy rate above the optimal level for home residents.

Generally, a positive level for β_o only arises when the home country has a relatively large population. For example, when country sizes are symmetric, that is $L/L^* = 1$, we can evaluate the welfare effects of change in the R&D subsidy as follows:

$$\left.\frac{dU_0}{d\beta}\right|_{\beta=0} = -\frac{(\sigma-1)(1-\delta)}{4\sigma^2\delta\rho} < 0, \qquad \left.\frac{dU_0^*}{d\beta}\right|_{\beta=0} = \frac{(\sigma-1)(1-\delta)}{4\sigma^2\delta\rho} > 0.$$

In this symmetric case, we find that the R&D subsidy lowers home welfare and raises foreign welfare. Hence, while the optimal home R&D subsidy rate would be positive for foreign ($\beta_o^* > 0$), home would prefer to set the R&D subsidy to zero ($\beta_o = 0$), or perhaps even to implement an R&D tax ($\beta_o < 0$). The optimal R&D subsidy rate for the average world household is zero in this case, as $(dU^W/d\beta)|_{\beta_W} = 0$. Numerical analysis shows that these results continue to hold for $L/L^* < 1$, although $\beta_o^* > 0$ falls to zero as L/L^* approaches zero. As a final point, we note that an increase in the level of knowledge dispersion leads to policy convergence with the optimal subsidies β_o and β_o^* equalized at $\delta = 1$, given that productivity is symmetric across countries, $\tilde{\theta} = 1$.

6 Conclusion

In this paper we have developed a two country model of fully endogenous growth in order to investigate the effects of national R&D subsidies on aggregate product variety and economic growth. Monopolistically competitive firms assume a central role in the model as they invest in process innovation with the aim of improving production technologies and lowering unit costs. In a world of perfect capital mobility, but imperfect knowledge dispersion, the country with the larger market maintains a larger share of manufacturing firms, a greater level of productivity, and a higher wage rate. The greater concentration of relatively productive firms in the larger country generates higher average productivity in flows of technical knowledge between firms, thereby raising R&D productivity and inducing a greater level of employment in in-house process innovation. As a result, we find that equilibria with asymmetric country sizes produce faster rates of growth than a symmetric equilibrium. As the overall level of market entry is determined by optimal R&D employment, however, the lower cost of process innovation associated with symmetric markets generates greater market entry so that the symmetric equilibrium produces the largest number of firms, and the greatest level of product variety.

We use this framework to investigate how national R&D subsidies affect firm-level innovation activity. In particular, we consider the case where one country finances an R&D subsidy to the process innovation of domestic firms through a lump-sum income tax collected from domestic households. The R&D subsidy has positive effects on industry share, relative productivity, and the wage rate in the implementing country. The effects of the R&D subsidy on overall product variety and aggregate productivity growth, however, depend on whether the implementing country initially has the smaller or larger market. If the smaller of the two countries introduces an R&D subsidy, the larger country's share of firms and relative productivity will fall, pushing the economy towards the symmetric equilibrium, causing a rise in product variety, and inducing a fall in productivity growth. In contrast, if the larger country implements the R&D subsidy, its share of firms and relative productivity will rise, thus pushing the economy away from the symmetric equilibrium and raising productivity growth. In this case, however, overall product variety may rise or fall.

Although the large number of opposing effects generated by national R&D subsidies makes a direct study of welfare issues intractable, simple numerical examples lead us to conclude that the optimal level for a national R&D subsidy that is implemented by the larger country will be too low from the perspective of the smaller country. As such, there may be opportunities for policy coordination between the two countries with the aim of raising average world welfare by increasing the R&D subsidy.

Given these preliminary results, an interesting extension of our framework might be an analysis of strategic R&D policy. For example, Kondo (2012) investigates the relationship between R&D subsidies and economic growth in a variety expansion model that allows for strategic policy interaction in the present of imperfect knowledge dispersion, but that is subject to a scale effect. An simplified version of the model presented in this paper might allow for an investigation of the relationship between strategic R&D policy and endogenous growth that is not biased by a scale effect. We leave this issue as a topic for future work.

Appendix: Stability of Symmetric Equilibrium

This appendix derives a sufficient condition for the stability of the symmetric equilibrium. In order to examine the stability of the model, we first require a condition for home's share of firms outside the steady-state equilibrium. As both countries have the same rate of time preference and there are no flows of investment income, trade flows balance at every moment in time. Therefore, using the product demand conditions (2), the pricing rules (6), and household expenditures (15) into the trade balance condition $npc_i^*L^* = n^*p^*c_jL$, we obtain the home share of firms on the dynamic path as follows:

$$\bar{s} = \frac{(1-\eta(\beta))L/L^*}{(1-\eta(\beta))L/L^* + (w/w^*)^{-\sigma}\tilde{\theta}^{\sigma-1}}. \tag{A1}$$

Next, conditions (11) and (17) can be combined with $K/\theta = \delta\tilde{\theta}^{-1} + \bar{s}(1-\delta\tilde{\theta}^{-1})$, and $K^*/\theta^* = 1 + \bar{s}(\delta\tilde{\theta} - 1)$ to obtain the following dynamic system:

$$\dot{\tilde{\theta}} = \tilde{\theta}\left[\frac{l_R^\gamma K}{\theta} - \frac{l_R^{*\gamma} K^*}{\theta^*}\right], \tag{A2}$$

$$\dot{l_R} = \frac{l_R}{1-\gamma}\left[\rho - \frac{\gamma(\sigma-1)l_R^\gamma K}{\theta}\left(1 + \frac{l_F}{l_R}\right) + \frac{\dot{K}}{K}\right], \tag{A3}$$

$$\dot{l_R^*} = \frac{l_R^*}{1-\gamma}\left[\rho - \frac{\gamma(\sigma-1)l_R^{*\gamma} K^*}{\theta^*}\left(1 + \frac{l_F}{l_R^*}\right) + \frac{\dot{K}^*}{K^*}\right], \tag{A4}$$

where

$$\frac{\dot{K}}{K} = \bar{s}l_R^\gamma + \frac{(1-\bar{s})\left(1+(\delta\tilde{\theta}-1)\bar{s}\right)\delta l_R^{*\gamma}}{\left(\delta + (\tilde{\theta}-\delta)\bar{s}\right)} + \frac{\bar{s}(1-\bar{s})(\sigma-1)(\delta-\tilde{\theta})}{\left(\delta+(\tilde{\theta}-\delta)\bar{s}\right)}\frac{\dot{\tilde{\theta}}}{\tilde{\theta}},$$

$$\frac{\dot{K}^*}{K^*} = (1-\bar{s})l_R^{*\gamma} + \frac{\bar{s}\left(\delta+(\tilde{\theta}-\delta)\bar{s}\right)\delta l_R{}^\gamma}{\left(1+(\delta\tilde{\theta}-1)\bar{s}\right)} + \frac{\bar{s}(1-\bar{s})(\sigma-1)(1-\delta\tilde{\theta})}{\left(1+(\delta\tilde{\theta}-1)\bar{s}\right)}\frac{\dot{\tilde{\theta}}}{\tilde{\theta}}.$$

In our study of this system, we set the relative wage equal to its steady-state level, $w/w^* = (1-\beta)^{-1/\sigma}\tilde{\theta}(L/L^*,\beta)^{\sigma-1}$, since $(\dot{w/w^*}) = (1-\eta(\beta))^{-1}(\dot{E/E^*}) = 0$. Then, (A1), (A2), (A3), and (A4) provide an autonomous system in $\tilde{\theta}$, l_R, and l_R^*. We investigate the local dynamics around the symmetric equilibrium that arises for $L/L^* = 1$ and $\beta = 0$:

$$\tilde{\theta} = 1, \qquad l_R = l_R^* = \overline{l}_R, \qquad \frac{w}{w^*} = 1, \qquad s = \frac{1}{2}, \qquad \frac{K}{\theta} = \frac{K^*}{\theta^*} = \frac{1+\delta}{2}. \tag{A5}$$

Taking a linear expansion of (A2), (A3), and (A4) and evaluating it around (A5) yields the following Jacobian matrix:

$$J = \begin{bmatrix} -\frac{(2\delta+(1-\delta)(\sigma-1))l_R^\gamma}{2} & \frac{\gamma(1+\delta)l_R^{\gamma-1}}{2} & \frac{\gamma(1+\delta)l_R^{\gamma-1}}{2} \\ \frac{\partial \dot{l_R}}{\partial \tilde{\theta}} & \frac{\partial \dot{l_R}}{\partial l_R} & \frac{(2\delta+(1-\delta)(\sigma-1))\gamma l_R^\gamma}{4(1-\gamma)} \\ \frac{\partial \dot{l_R^*}}{\partial \tilde{\theta}} & \frac{(2\delta+(1-\delta)(\sigma-1))\gamma l_R^\gamma}{4(1-\gamma)} & \frac{\partial \dot{l_R^*}}{\partial l_R^*} \end{bmatrix},$$

where

$$\frac{\partial \dot{l_R}}{\partial \tilde{\theta}} = -\frac{\partial \dot{l_R^*}}{\partial \tilde{\theta}} = \frac{[2\delta+(1-\delta)(\sigma-1)][(1-\delta)(\sigma-2)l_R+\gamma(\sigma-1)(1+\delta)(l_R+l_F)]\, l_R^\gamma}{4(1-\gamma)(1+\delta)},$$
$$\frac{\partial \dot{l_R}}{\partial l_R} = \frac{\partial \dot{l_R^*}}{\partial l_R^*} = \frac{[2(1-\gamma)(1+\delta)(\sigma-1)l_F-(2(\sigma-2)+(1-\delta)(\sigma-1))\, l_R]\,\gamma l_R^{\gamma-1}}{4(1-\gamma)}.$$

As the system consists of two jump variables (l_R and l_R^*) and one state variable ($\tilde{\theta}$), we require two eigenvalues with positive real parts and one eigenvalue with a negative real part for saddle-point stability. While we cannot solve for the eigenvalues directly, saddle-point stability is established for $tr(J) > 0 > |J|$, where $tr(J)$ and $|J|$ denote the trace and determinant of J, respectively. Calculating $tr(J)$ and $|J|$, we obtain

$$tr(J) = -\frac{(2\delta+(1-\delta)(\sigma-1))l_R^\gamma}{2(1-\gamma)} - \frac{2l_R}{(1-\gamma)}\frac{\partial R(l_R)}{\partial l_R},$$
$$|J| = \frac{\gamma(1+\delta)(\sigma-1)(2\delta+(1-\delta)(\sigma-1))l_R^{2\gamma}l_F}{4(1-\gamma)^2}\frac{\partial R(l_R)}{\partial l_R},$$

where $\partial R(l_R)/\partial l_R = -[1-\gamma(\sigma-1)+(1-\gamma)(\sigma-1)l_F/l_R]\,\gamma(1+\delta)l_R^{\gamma-1}/2$. Thus, saddle-point stability is achieved if $\partial R(l_R)/\partial l_R$ is sufficiently negative. Returning to the steady-state no-arbitrage condition (19), we can see that $l_F/l_R > (1-\gamma(\sigma-1))/(\gamma(\sigma-1))$ is required for active process innovation. Then, substituting $l_F/l_R = (1-\gamma(\sigma-1))/(\gamma(\sigma-1))$ with $\partial R(l_R)/\partial l_R$ into $tr(J)$, and manipulating the result, we obtain

$$(1+\delta)(1-\gamma(\sigma-1)) > \frac{2\delta+(1-\delta)(\sigma-1)}{2} > 0. \tag{A6}$$

This inequality can be reduced to $2 > (\sigma-1)(1-\delta+2(1+\delta)\gamma)$, which represents a sufficient condition for $\partial R(l_R)/\partial l_R < 0$, $tr(J) > 0$, $|J| < 0$, and the saddle-point stability of the dynamic path around the symmetric equilibrium.

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